Circle Problem

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1 Problem Outline

Suppose that you're given a list of n points in the Cartesian plane, each defined by coordinates (x, y) where $x, y \in \mathbb{R}$. Each point is also associated with a letter tag from the set $\{A, B, C, D, E, F\}$.

Formally, the data set is $S = \{ (x_i, y_i, l_i)\}_{i=1}^{n}$

Your task is to write a function that determines the largest valid circle centered at the origin $(0, 0)$, encompassing the maximum number of points such that no two points within or on the boundary of the circle can share the same tag.

1.1 Radius Set Definition

1. Define a "radius set" R, which is a set of values derived from the data points in S. Each element in R denoted as r_i is derived from a unique transformation of the properties of each point (x_i, y_i) in S and doesn't explicitly require each l_i since those aren't necessary for generating radi. Let this transformation be represented by a function f.

   Formally, the radius set R can be defined as:

$$R = \{r_i | r_i = f(x_i, y_i),\ \forall (x_i, y_i, l_i) \in S\}$$

   Where $f : \mathbb{R}^2 \to \mathbb{R}$ is a transformation function that you design. This function f should uniquely transform the properties of the data points in S into a set of potential radii. Note that uniquely transform here means one-to-one and onto (injective and surjective $\iff$ bijective)

1.2 Circle Definition

- The circle is centered at the origin $(0, 0)$.
- The radius of the circle must be selected from the "radius set" R.

1.3 Point Definition

- Points are represented by (x, y) coordinates.
- Each point is tagged with a letter (non-empty) string.

1.4 Objective

- Find the radius from R that allows the largest valid circle.
- If no valid solution exists within R, the function should indicate this.

1.5 Handling Edge Cases

- If two or more points with the same tag lie exactly on the boundary of a potential circle drawn using a radius from R, the circle is considered invalid.

2 Example

Using the following color-mapping:

Tag	Color
A	Red
B	Blue
C	Green
D	Orange
E	Purple
F	Black

Table 1: Color mapping for labels.

And the following data set:

$$S = \left\{ \begin{array}{l} (0,-5,D),(4,-1,E),(-5,3,C),(-2,-2,E), \\ (3,3,E),(-1,1,B),(-5,-3,C),(3,-2,C), \\ (3,-3,C),(-4,-3,E),(-4,4,D),(4,1,C), \\ (-3,1,F),(2,4,E),(-4,3,B),(-5,0,D), \\ (4,-2,F),(-5,-1,F) \end{array} \right\}$$

We can observe the following examples of a solution circle and an invalid circle

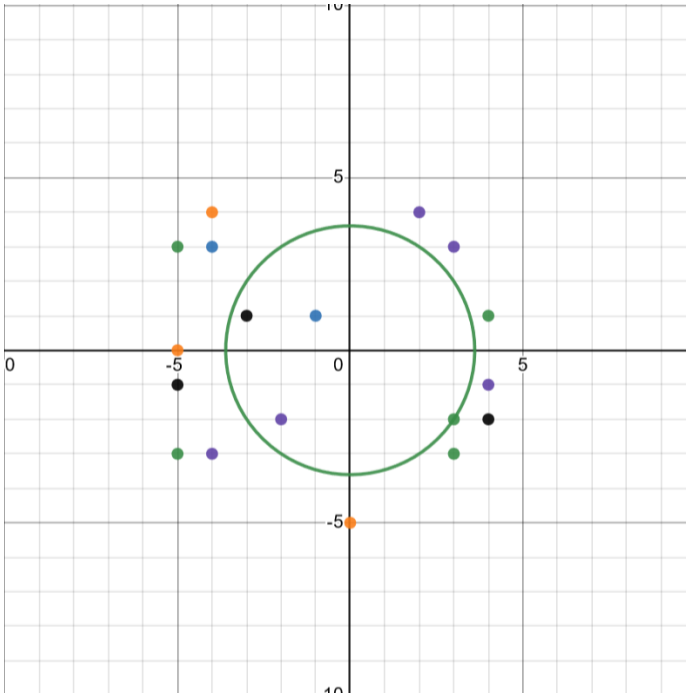

Figure 1: Correct Circle

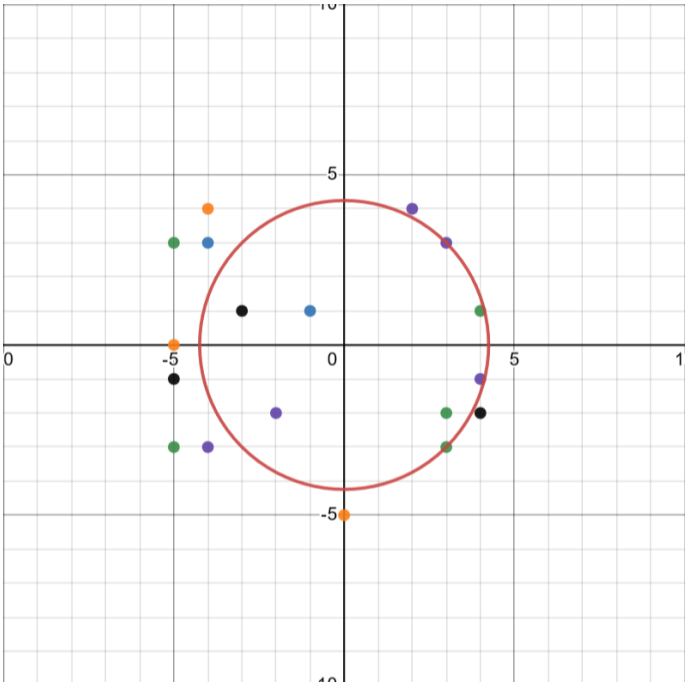

Figure 2: Invalid Circle

As shown in Figure 1 1, the circle does not contain any duplicate points and is also the largest circle we can create where its radius $r_i \in R$.

In Figure 2 2, we can see that this is not a valid circle because it contains duplicate points.

This example is not exhaustive. Another examples of an invalid circle could be one that doesn't contain duplicate points, but there exists a larger circle.