Adding/Deleting Elements/Relations To/From a Poset

Let $P = (X, \leq)$ be a partially order set (or poset). Here X is a set of *elements* and $\leq$ is a reflexive, antisymmetric, transitive *relation* on X.

We wish to consider the following four operations: (1) add an element to a poset, (2) delete an element from a poset, (3) add a relation to a poset, and (4) delete a relation from a poset.

Adding/Deleting Elements

Operations on elements are relatively simple to describe. Let $P = (X, \leq)$ be a poset.

If $a \notin X$, then to add the element a to P results in a new poset that includes a in which a is related only to itself. Formally, let $P' = P + a$ be the poset $P' = (X', \leq')$ where we have the following:

(1) $X' = X \cup \{a\}$.
(2) $\leq' = \leq \cup \{(a, a)\}$. That is,
  - $\forall x, y \in X, x \leq' y \iff x \leq y$,
  - $\forall x \in X$, $x \not\leq' a$ and $a \not\leq' x$, and
  - $a \leq' a$.

Element deletion is also easy to describe. Deleting an element a from P deletes a from the set X and all remaining elements have the same relations they had before. Formally, for $a \in X$, let $P' = P - a$ be the poset $P' = (X', \leq')$ where we have the following:

(1) $X' = X - \{a\}$.
(2) $\forall x, y \in X', x \leq' y \iff x \leq y$.