Theory

In order to solve

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}\left(t\right) \tag{1}$$

numerically, it must be returned to first order system. Let

$$\boldsymbol{v} = \dot{\boldsymbol{u}}, \tag{2}$$
$$\dot{\boldsymbol{v}} = \ddot{\boldsymbol{u}}, \tag{3}$$

so

$$\boldsymbol{M}\dot{\boldsymbol{v}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}\left(t\right) \tag{4}$$
$$\dot{\boldsymbol{u}} = \boldsymbol{v} \tag{5}$$

Rearranging terms:

$$\dot{\boldsymbol{u}} = \boldsymbol{v} \tag{6}$$
$$\dot{\boldsymbol{v}} = \boldsymbol{M}^{-1}\left(\boldsymbol{f}\left(t\right) - \boldsymbol{K}\boldsymbol{u}\right) \tag{7}$$

So

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{u}} \\ \dot{\boldsymbol{v}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{M}^{-1}\boldsymbol{K} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{v} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{f}\left(t\right) \end{bmatrix}. \tag{8}$$

Note: $\boldsymbol{v}$ is velocity.