Exam

Andrejs Komisarovs

May 2019

To compute L_q, we use

$$L_q = L - (1 - p_0) = L - \left(1 - \frac{1-\rho}{1-\rho^{K+1}}\right) = L - \frac{1-\rho^{K+1} - 1 + \rho}{1-\rho^{K+1}} = L - \frac{\rho(1-\rho^K)}{1-\rho^{K+1}}.$$

To find W, we use Little's law, ($W = L/\lambda$). However, what we need is the *effective* arrival rate, which we denote by λ'. This is the mean rate of customers actually entering the system and is given by

$$\lambda' = \lambda(1 - p_K)$$

since customers only enter when space is available. W and W_q may now be obtained from Little's law,

$$W = \frac{1}{\lambda'}L \quad \text{and} \quad W_q = \frac{1}{\lambda'}L_q.$$

Alternatively, we could also use

$$W_q = W - \frac{1}{\mu}.$$

Notice that in computing L_q we did not use the formula $L_q = L - \rho$ as we did in the *M/M/1* queue with $\rho = \lambda/\mu$, instead choosing to write this as $L_q = L - (1 - p_0)$. The reason should now be clear, as we require the effective arrival rate λ' and not λ. Thus, we could have used $L_q = L - \lambda'/\mu$.

Throughput and Utilization in the *M/M/1/K* Queue

In a queueing situation in which customers may be lost, the throughput cannot be defined as being equal to the customer arrival rate—not all arriving customers actually enter the queue. The probability that an arriving customer is lost is equal to the probability that there are already K customers in the system, i.e., p_K. The probability that the queue is *not* full, and hence the probability that an arriving customer is accepted into the queueing system is $1 - p_K$. Thus the throughput, X, is given by

$$X = \lambda(1 - p_K).$$

It is interesting to examine the throughput in terms of what actually leaves the *M/M/1/K* queue. So long as there are customers present, the server serves these customers at rate μ. The probability that no customers are present is p_0, so the probability that the server is working is given by $1 - p_0$. It follows that the throughput must be given by $X = \mu(1 - p_0)$. Thus

$$X = \lambda(1 - p_K) = \mu(1 - p_0).$$

As we have just seen,

$$p_0 = \frac{(1-\rho)}{(1-\rho^{K+1})}$$

and

$$p_n = \frac{(1-\rho)\rho^n}{1-\rho^{K+1}}.$$

Therefore

$$\frac{1-p_0}{1-p_K} = \frac{1 - [(1-\rho)/(1-\rho^{K+1})]}{1 - [(1-\rho)\rho^K/(1-\rho^{K+1})]} = \frac{(1-\rho^{K+1}) - (1-\rho)}{(1-\rho^{K+1}) - (1-\rho)\rho^K} = \frac{\rho - \rho^{K+1}}{1-\rho^K} = \rho$$

and hence

$$\lambda(1 - p_K) = \mu(1 - p_0).$$

Observe that $(1 - p_K)$ is the probability that the queue is not full: it is the probability that new customers can enter the system and $\lambda(1 - p_K)$ is the effective arrival rate, the rate at which customers enter the system. Similarly, $(1 - p_0)$ is the probability that the system is busy and so $\mu(1 - p_0)$ is

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In a queueing situation in which customers may he lost, the throughput cannot he defined as being equal to the customer arrival rate—not all arriving customers actually enter the queue. The probability that an arriving customer is lost is equal to the probability that there are already K customers in the system, i.e., pa. The probability that the queue is not full, and hence the probability that an arriving customer is accepted into the queueing system is 1 — pa. Thus the throughput, X, is given by

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It is interesting to examine the throughput in terms of what actually leaves the M/M/1/K queue. So long as there are customers present, the server serves these customers at rate u. The probability that no customers are present is pp, so the probability that the server is working is given by $1 - p_0$. It follows that the throughput must he given by $X = \mu(1 - p_0)$. Thus

$$X = \lambda(1 - p_K) = \mu(1 - p_0).$$

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$$\frac{1-p_0}{1-p_K} = \frac{1 - [(1-\rho)/(1-\rho^{K+1})]}{1 - [(1-\rho)\rho^K/(1-\rho^{K+1})]} = \frac{(1-\rho^{K+1}) - (1-\rho)}{(1-\rho^{K+1}) - (1-\rho)\rho^K} = \frac{\rho - \rho K + 1}{1-\rho^K} = \rho$$

and hence

$$\lambda(1 - \rho_K) = \mu(1 - p_0).$$

Observe that $(1 - p_K)$ is the probability that the queue is not full: it is the probability that new customers can enter the system and $\lambda(1-p_K)$ is the effective arrival rate, the rate at which customers enter the system. Similarly, $(1 - p_0)$ is the probability that the system is busy and so $\mu(1 - p_0)$ is

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\documentclass[10pt]{extarticle}
\usepackage[utf8x]{inputenc}
\usepackage{fancyhdr}
\usepackage{ragged2e}
\usepackage{amsmath,amssymb}
\usepackage{graphicx}
\usepackage{incgraph,tikz}
\usepackage{tikz,lipsum,lmodern}
\usepackage[most]{tcolorbox}
\usepackage[margin=2.9cm,paperwidth=210mm,paperheight=308mm]{geometry}
\usepackage[bottom]{footmisc}
\renewcommand{\thefootnote}{\fnsymbol{footnote}}
\renewcommand*\footnoterule{}

\pagestyle{fancy}
\fancyhf{}
\rhead{11.5 Finite-Capacity Systems|The M/M/1/K Queue \quad \textbf{429}}

\title{Exam}
\author{Andrejs Komisarovs}
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\begin{document}

\maketitle

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\hspace{-0.55cm}To compute $L_q$, we use
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Therefore
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