这是标题

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1 李代数的有用公式

以下公式对于左扰动和右扰动均成立。通过级数展开:

$$\exp(\lfloor\boldsymbol{\theta}\rfloor_\times) = \boldsymbol{I} + \lfloor\boldsymbol{\theta}\rfloor_\times + \frac{1}{2!}\lfloor\boldsymbol{\theta}\rfloor_\times^2 + \frac{1}{3!}\lfloor\boldsymbol{\theta}\rfloor_\times^3 + \dots$$

可以得到:

$$\exp(\lfloor\delta\boldsymbol{\theta}\rfloor_\times) = \boldsymbol{I} + \lfloor\delta\boldsymbol{\theta}\rfloor_\times \tag{1}$$

$$\exp(t\lfloor\boldsymbol{\theta}\rfloor_\times) = \exp(\lfloor t\boldsymbol{\theta}\rfloor_\times) = \exp(\lfloor\boldsymbol{\theta}\rfloor_\times)^t \tag{2}$$

$$\boldsymbol{R}\exp(\lfloor\delta\boldsymbol{\theta}\rfloor_\times)\boldsymbol{R}^{-1} = \exp(\boldsymbol{R}\lfloor\delta\boldsymbol{\theta}\rfloor_\times\boldsymbol{R}^{-1}) \tag{3}$$

$$\boldsymbol{R}^{-1}\exp(\lfloor\delta\boldsymbol{\theta}\rfloor_\times)\boldsymbol{R} = \exp(\boldsymbol{R}^{-1}\lfloor\delta\boldsymbol{\theta}\rfloor_\times\boldsymbol{R}) \tag{4}$$

$$\boldsymbol{R}^{-1} = \boldsymbol{R}^T \tag{5}$$

$$\mathbf{Ad}_{\boldsymbol{R}}\delta\boldsymbol{\theta} = (\boldsymbol{R}\lfloor\delta\boldsymbol{\theta}\rfloor_\times\boldsymbol{R}^{-1})^\vee = \boldsymbol{R}\delta\boldsymbol{\theta} \tag{6}$$

$$\mathbf{Ad}_{\boldsymbol{R}}{}^{-1}\delta\boldsymbol{\theta} = (\boldsymbol{R}^{-1}\lfloor\delta\boldsymbol{\theta}\rfloor_\times\boldsymbol{R})^\vee = \boldsymbol{R}^{-1}\delta\boldsymbol{\theta} \tag{7}$$

以下公式以右扰动模型为依据:

$$\delta^{\boldsymbol{R}}\boldsymbol{\theta} = \boldsymbol{Q}\ominus\boldsymbol{R} = \mathrm{Log}(\boldsymbol{R}^{-1}\boldsymbol{Q}) \tag{8}$$

以下公式以左扰动模型为依据:

$$\delta^{\boldsymbol{I}}\boldsymbol{\theta} = \boldsymbol{Q}\ominus\boldsymbol{R} = \mathrm{Log}(\boldsymbol{Q}\boldsymbol{R}^{-1}) \tag{9}$$

将右扰动 (局部扰动) 化为左扰动 (全局扰动) 的公:

$$\begin{cases} \boldsymbol{R}\exp(\lfloor\delta^{\boldsymbol{R}}\boldsymbol{\theta}\rfloor_\times) = \exp(\boldsymbol{R}\lfloor\delta^{\boldsymbol{R}}\boldsymbol{\theta}\rfloor_\times\boldsymbol{R}^{-1})\boldsymbol{R} \\ \lfloor\delta^{\boldsymbol{I}}\boldsymbol{\theta}\rfloor_\times = \boldsymbol{R}\lfloor\delta^{\boldsymbol{R}}\boldsymbol{\theta}\rfloor_\times\boldsymbol{R}^{-1} \end{cases} \tag{10}$$

将左扰动化为右扰动的公式:

$$\begin{cases} \exp(\lfloor\delta^{\boldsymbol{I}}\boldsymbol{\theta}\rfloor_\times)\boldsymbol{R} = \boldsymbol{R}\exp(\boldsymbol{R}^{-1}\lfloor\delta^{\boldsymbol{I}}\boldsymbol{\theta}\rfloor_\times\boldsymbol{R}) \\ \lfloor\delta^{\boldsymbol{R}}\boldsymbol{\theta}\rfloor_\times = \boldsymbol{R}^{-1}\lfloor\delta^{\boldsymbol{I}}\boldsymbol{\theta}\rfloor_\times\boldsymbol{R} \end{cases} \tag{11}$$

注意: 大写 LOG 直接将旋转矩阵变成三维向量, 小写 log 将旋转矩阵变成李代数, Exp 和 exp 有同样意韵, 但是恰好相反:

$$\begin{cases} \exp(\lfloor\boldsymbol{\theta}\rfloor_\times) = \mathrm{Exp}(\boldsymbol{\theta}) \\ \log(\boldsymbol{R})^\vee = \mathrm{Log}(\boldsymbol{R}) \end{cases}$$

2 线性化

记非线性函数为 $f(\boldsymbol{R})$, 则对于右扰动模型:

$$f(\boldsymbol{R}\oplus\delta^{\boldsymbol{R}}\boldsymbol{\theta}) = f(\boldsymbol{R})\oplus\frac{{}^{\boldsymbol{R}}\partial f(\boldsymbol{R})}{\partial\delta^{\boldsymbol{R}}\boldsymbol{\theta}}\delta^{\boldsymbol{R}}\boldsymbol{\theta}$$

对于左扰动模型:

$$f(\delta^{\boldsymbol{I}}\boldsymbol{\theta}\oplus\boldsymbol{R}) = \frac{{}^{\boldsymbol{I}}\partial f(\boldsymbol{R})}{\partial\delta^{\boldsymbol{I}}\boldsymbol{\theta}}\delta^{\boldsymbol{I}}\boldsymbol{\theta}\oplus f(\boldsymbol{R})$$

且二者导数之间应有如下的关系 (可以用来验证推导结果):

$$\begin{aligned} \mathbf{Ad}_{f(\boldsymbol{R})}\frac{{}^{\boldsymbol{R}}\partial f(\boldsymbol{R})}{\partial\delta^{\boldsymbol{R}}\boldsymbol{\theta}} &= \frac{{}^{\boldsymbol{I}}\partial f(\boldsymbol{R})}{\partial\delta^{\boldsymbol{I}}\boldsymbol{\theta}}\mathbf{Ad}_{\boldsymbol{R}} \\ \to \mathbf{Ad}_{\boldsymbol{R}} &= \boldsymbol{R} \end{aligned}$$

3 公式应用

注意, 以下推导过程中 $\boldsymbol{R}$、$\boldsymbol{Q}$ 为旋转矩阵, $\delta\boldsymbol{\theta}$ 为对旋转矩阵 $\boldsymbol{R}$ 施加的扰动量, $\boldsymbol{p}$ 为点向量。

3.1 左扰动

$$\begin{aligned} \frac{{}^{\boldsymbol{I}}\partial\boldsymbol{R}}{\partial\delta\boldsymbol{\theta}} &= \frac{\exp(\lfloor\delta\boldsymbol{\theta}\rfloor_\times)\boldsymbol{R}\ominus\boldsymbol{R}}{\delta\boldsymbol{\theta}} \\ &= \frac{\mathrm{Log}(\exp(\lfloor\delta\boldsymbol{\theta}\rfloor_\times)\boldsymbol{R}\boldsymbol{R}^{-1})}{\delta\boldsymbol{\theta}} \\ &= \frac{\mathrm{Log}(\exp(\lfloor\delta\boldsymbol{\theta}\rfloor_\times))}{\delta\boldsymbol{\theta}} \\ &= \frac{\mathrm{LOG}(\mathrm{Exp}(\delta\boldsymbol{\theta}))}{\delta\boldsymbol{\theta}} = \boldsymbol{I} \end{aligned}$$

$$\begin{aligned} \frac{{}^{\boldsymbol{I}}\partial\boldsymbol{R}^{-1}}{\partial\delta\boldsymbol{\theta}} &= \frac{(\exp(\lfloor\delta\boldsymbol{\theta}\rfloor_\times)\boldsymbol{R})^{-1}\ominus\boldsymbol{R}^{-1}}{\delta\boldsymbol{\theta}} \\ &= \frac{\mathrm{Log}((\exp(\lfloor\delta\boldsymbol{\theta}\rfloor_\times)\boldsymbol{R})^{-1}\boldsymbol{R})}{\delta\boldsymbol{\theta}} \\ &= \frac{\mathrm{Log}(\boldsymbol{R}^{-1}\exp(-\lfloor\delta\boldsymbol{\theta}\rfloor_\times)\boldsymbol{R})}{\delta\boldsymbol{\theta}} \\ &= \frac{\mathrm{Log}(\exp(-\boldsymbol{R}^{-1}\lfloor\delta\boldsymbol{\theta}\rfloor_\times\boldsymbol{R}))}{\delta\boldsymbol{\theta}} \\ &= \frac{\mathrm{Log}(\mathrm{Exp}((-\boldsymbol{R}^{-1}\lfloor\delta\boldsymbol{\theta}\rfloor_\times\boldsymbol{R})^\vee))}{\delta\boldsymbol{\theta}} \\ &= \frac{(-\boldsymbol{R}^{-1}\lfloor\delta\boldsymbol{\theta}\rfloor_\times\boldsymbol{R})^\vee}{\delta\boldsymbol{\theta}} \\ &= \frac{-\boldsymbol{R}^{-1}\delta\boldsymbol{\theta}}{\delta\boldsymbol{\theta}} = -\boldsymbol{R}^{-1} \end{aligned}$$

$$
\begin{aligned}
\frac{{}^{I}\partial \boldsymbol{R}\boldsymbol{p}}{\partial \delta\boldsymbol{\theta}} &= \frac{\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R}\boldsymbol{p} - \boldsymbol{R}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{(\boldsymbol{I} + \lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R}\boldsymbol{p} - \boldsymbol{R}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{\lfloor \delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{R}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{-\lfloor \boldsymbol{R}\boldsymbol{p} \rfloor_\times \delta\boldsymbol{\theta}}{\delta\boldsymbol{\theta}} = -\lfloor \boldsymbol{R}\boldsymbol{p} \rfloor_\times
\end{aligned}
$$

$$
\begin{aligned}
\frac{{}^{I}\partial \boldsymbol{R}^{-1}\boldsymbol{p}}{\partial \delta\boldsymbol{\theta}} &= \frac{(\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R})^{-1}\boldsymbol{p} - \boldsymbol{R}^{-1}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{\boldsymbol{R}^{-1}\exp(-\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{p} - \boldsymbol{R}^{-1}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{\boldsymbol{R}^{-1}(\boldsymbol{I} - \lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{p} - \boldsymbol{R}^{-1}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{-\boldsymbol{R}^{-1}\lfloor \delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{\boldsymbol{R}^{-1}\lfloor \boldsymbol{p} \rfloor_\times \delta\boldsymbol{\theta}}{\delta\boldsymbol{\theta}} = \boldsymbol{R}^{-1}\lfloor \boldsymbol{p} \rfloor_\times
\end{aligned}
$$

$$
\begin{aligned}
\frac{{}^{I}\partial \boldsymbol{R}\boldsymbol{Q}}{\partial \delta\boldsymbol{\theta}} &= \frac{\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R}\boldsymbol{Q} \ominus \boldsymbol{R}\boldsymbol{Q}}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R}\boldsymbol{Q}\boldsymbol{Q}^{-1}\boldsymbol{R}^{-1})}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times))}{\delta\boldsymbol{\theta}} = \boldsymbol{I}
\end{aligned}
$$

$$
\begin{aligned}
\frac{{}^{I}\partial \boldsymbol{R}^{-1}\boldsymbol{Q}}{\partial \delta\boldsymbol{\theta}} &= \frac{(\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R})^{-1}\boldsymbol{Q} \ominus \boldsymbol{R}^{-1}\boldsymbol{Q}}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\boldsymbol{R}^{-1}\exp(-\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{Q}\boldsymbol{Q}^{-1}\boldsymbol{R})}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\exp(\boldsymbol{R}^{-1}\lfloor -\delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{R}))}{\delta\boldsymbol{\theta}} \\
&= \frac{(\boldsymbol{R}^{-1}\lfloor -\delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{R})^\vee}{\delta\boldsymbol{\theta}} = -\boldsymbol{R}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{{}^{I}\partial \boldsymbol{Q}\boldsymbol{R}}{\partial \delta\boldsymbol{\theta}} &= \frac{\boldsymbol{Q}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R} \ominus \boldsymbol{Q}\boldsymbol{R}}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\boldsymbol{Q}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{Q}^{-1})}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\boldsymbol{Q}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{Q}^{-1})}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\exp(\boldsymbol{Q}\lfloor \delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{Q}^{-1}))}{\delta\boldsymbol{\theta}} \\
&= \frac{(\boldsymbol{Q}\lfloor \delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{Q}^{-1})^\vee}{\delta\boldsymbol{\theta}} = \boldsymbol{Q}
\end{aligned}
$$

3.2 右扰动

$$
\begin{aligned}
\frac{{}^{R}\partial \boldsymbol{R}}{\partial \delta\boldsymbol{\theta}} &= \frac{\boldsymbol{R}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times) \ominus \boldsymbol{R}}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\boldsymbol{R}^{-1}\boldsymbol{R}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times))}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times))}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{LOG}(\mathrm{Exp}(\delta\boldsymbol{\theta}))}{\delta\boldsymbol{\theta}} = \boldsymbol{I}
\end{aligned}
$$

$$
\begin{aligned}
\frac{{}^{R}\partial \boldsymbol{R}^{-1}}{\partial \delta\boldsymbol{\theta}} &= \frac{(\boldsymbol{R}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times))^{-1} \ominus \boldsymbol{R}^{-1}}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\boldsymbol{R}\exp(-\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R}^{-1})}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\exp(-\boldsymbol{R}\lfloor \delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{R}^{-1}))}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\mathrm{Exp}((-\boldsymbol{R}\lfloor \delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{R}^{-1})^\vee))}{\delta\boldsymbol{\theta}} \\
&= \frac{(-\boldsymbol{R}\lfloor \delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{R}^{-1})^\vee}{\delta\boldsymbol{\theta}} \\
&= \frac{-\boldsymbol{R}\delta\boldsymbol{\theta}}{\delta\boldsymbol{\theta}} = -\boldsymbol{R}
\end{aligned}
$$

$$
\begin{aligned}
\frac{{}^{R}\partial \boldsymbol{R}\boldsymbol{p}}{\partial \delta\boldsymbol{\theta}} &= \frac{\boldsymbol{R}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{p} - \boldsymbol{R}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{\boldsymbol{R}(\boldsymbol{I} + \lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{p} - \boldsymbol{R}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{\boldsymbol{R}\lfloor \delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{-\boldsymbol{R}\lfloor \boldsymbol{p} \rfloor_\times \delta\boldsymbol{\theta}}{\delta\boldsymbol{\theta}} = -\boldsymbol{R}\lfloor \boldsymbol{p} \rfloor_\times
\end{aligned}
$$

$$
\begin{aligned}
\frac{{}^{R}\partial \boldsymbol{R}^{-1}\boldsymbol{p}}{\partial \delta\boldsymbol{\theta}} &= \frac{(\boldsymbol{R}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_\times))^{-1}\boldsymbol{p} - \boldsymbol{R}^{-1}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{\exp(-\lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R}^{-1}\boldsymbol{p} - \boldsymbol{R}^{-1}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{(\boldsymbol{I} - \lfloor \delta\boldsymbol{\theta} \rfloor_\times)\boldsymbol{R}^{-1}\boldsymbol{p} - \boldsymbol{R}^{-1}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{-\lfloor \delta\boldsymbol{\theta} \rfloor_\times \boldsymbol{R}^{-1}\boldsymbol{p}}{\delta\boldsymbol{\theta}} \\
&= \frac{\lfloor \boldsymbol{R}^{-1}\boldsymbol{p} \rfloor_\times \delta\boldsymbol{\theta}}{\delta\boldsymbol{\theta}} = \lfloor \boldsymbol{R}^{-1}\boldsymbol{p} \rfloor_\times
\end{aligned}
$$

$$
\begin{aligned}
\frac{{}^{R}\partial \boldsymbol{R}\boldsymbol{Q}}{\partial \delta\boldsymbol{\theta}} &= \frac{\boldsymbol{R}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_{\times})\boldsymbol{Q} \ominus \boldsymbol{R}\boldsymbol{Q}}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\boldsymbol{Q}^{-1}\boldsymbol{R}^{-1}\boldsymbol{R}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_{\times})\boldsymbol{Q})}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\boldsymbol{Q}^{-1}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_{\times})\boldsymbol{Q})}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\exp(\boldsymbol{Q}^{-1}\lfloor \delta\boldsymbol{\theta} \rfloor_{\times}\boldsymbol{Q})}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\mathrm{Exp}((\boldsymbol{Q}^{-1}\lfloor \delta\boldsymbol{\theta} \rfloor_{\times}\boldsymbol{Q})^{\vee}))}{\delta\boldsymbol{\theta}} \\
&= \frac{(\boldsymbol{Q}^{-1}\lfloor \delta\boldsymbol{\theta} \rfloor_{\times}\boldsymbol{Q})^{\vee}}{\delta\boldsymbol{\theta}} = \boldsymbol{Q}^{-1} \\
\frac{{}^{I}\partial \boldsymbol{R}^{-1}\boldsymbol{Q}}{\partial \delta\boldsymbol{\theta}} &= \frac{(\boldsymbol{R}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_{\times}))^{-1}\boldsymbol{Q} \ominus \boldsymbol{R}^{-1}\boldsymbol{Q}}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\boldsymbol{Q}^{-1}\boldsymbol{R}\exp(-\lfloor \delta\boldsymbol{\theta} \rfloor_{\times})\boldsymbol{R}^{-1}\boldsymbol{Q})}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\exp(-\boldsymbol{Q}^{-1}\boldsymbol{R}\lfloor \delta\boldsymbol{\theta} \rfloor_{\times}\boldsymbol{R}^{-1}\boldsymbol{Q}))}{\delta\boldsymbol{\theta}} \\
&= \frac{(-\boldsymbol{Q}^{-1}\boldsymbol{R}\lfloor \delta\boldsymbol{\theta} \rfloor_{\times}\boldsymbol{R}^{-1}\boldsymbol{Q})^{\vee}}{\delta\boldsymbol{\theta}} \\
&= \frac{-\boldsymbol{Q}^{-1}\boldsymbol{R}\delta\boldsymbol{\theta}}{\delta\boldsymbol{\theta}} = -\boldsymbol{Q}^{-1}\boldsymbol{R} \\
\frac{{}^{R}\partial \boldsymbol{Q}\boldsymbol{R}}{\partial \delta\boldsymbol{\theta}} &= \frac{\boldsymbol{Q}\boldsymbol{R}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_{\times}) \ominus \boldsymbol{Q}\boldsymbol{R}}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\boldsymbol{R}^{-1}\boldsymbol{Q}^{-1}\boldsymbol{Q}\boldsymbol{R}\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_{\times}))}{\delta\boldsymbol{\theta}} \\
&= \frac{\mathrm{Log}(\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_{\times}))}{\delta\boldsymbol{\theta}} = \boldsymbol{I}
\end{aligned}
$$

4 符号说明

符号 $\lfloor \delta\boldsymbol{v} \rfloor_{\times}$ 表示将向量 $\delta\boldsymbol{v}$ 变换成反对称矩阵 (从向量空间到李代数空间)。比如对于三维维空间旋转而言:

$$
\begin{cases}
\delta\boldsymbol{\theta} = \begin{pmatrix} \delta\theta_x & \delta\theta_y & \delta\theta_z \end{pmatrix}^T \\
\lfloor \delta\boldsymbol{\theta} \rfloor_{\times} = \begin{pmatrix} 0 & -\delta\theta_z & \delta\theta_y \\ \delta\theta_z & 0 & -\delta\theta_x \\ -\delta\theta_y & \delta\theta_x & 0 \end{pmatrix}
\end{cases}
$$

事实上:

$$
\exp(\lfloor \delta\boldsymbol{\theta} \rfloor_{\times}) = \boldsymbol{I} + \lfloor \delta\boldsymbol{\theta} \rfloor_{\times} = \boldsymbol{R}(\delta\boldsymbol{\theta})
$$

正好对应了小角度旋转情况下的旋转矩阵。