

Q-10.13.3.10

Yash Patil - EE22BTECH11058

Question: A die is tossed twice. A ‘success’ is getting an even number on a toss. Find the variance of the number of successes.

Solution:

Parameter	Value	Description
X_i	0,1	0-Not a success, 1-Success and it represents outcome of i^{th} throw

pmf of X_i is

$$p_{X_i}(k) = \begin{cases} \frac{1}{2}, & k = 0 \\ \frac{1}{2}, & k = 1 \end{cases} \quad \forall \quad 1 \leq i \leq 2 \quad (1)$$

Mean value of X_i is

$$\mu_X = E[X_i] \quad i = 0, 1 \quad (2)$$

$$= \frac{1}{2} \quad (3)$$

Variance of X_i is

$$\sigma_X = E[(X_i - \mu_X)^2] \quad i = 0, 1 \quad (4)$$

$$= \frac{1}{4} \quad (5)$$

Variance of getting successes in two throws of a die is

$$= E[((X_1 - \mu_X) + (X_2 - \mu_X))^2] \quad (6)$$

$$= E[(X_1 - \mu_X)^2] + E[(X_2 - \mu_X)^2] + E[(X_1 - \mu_X) \times (X_2 - \mu_X)] \quad (7)$$

$$= \frac{1}{4} + \frac{1}{4} + 0 \quad (8)$$

$$= \frac{1}{2} \quad (9)$$