Solution to problem number 1.5.11

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Given in the question:
A = $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, B = $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$, C = $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$

Using the given vertices, sides AB, BC and CA can be found out as:

$$\begin{aligned} AB &= B - A \\ &= \begin{pmatrix} -4 - 1 \\ 6 + 1 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \\ BC &= C - B \\ &= \begin{pmatrix} -3 + 4 \\ -5 - 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -11 \end{pmatrix} \\ CA &= A - C \\ &= \begin{pmatrix} 1 + 3 \\ -1 + 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \end{aligned}$$

Now, the side lengths a, b and c can be calculated as:

$$\begin{aligned} a &= \sqrt{\mathbf{BC}^{\top} \cdot \mathbf{BC}} \\ &= \sqrt{\begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} 1 \\ -11 \end{pmatrix}} \\ &= \sqrt{1 + 121} \\ &= \sqrt{122} \end{aligned}$$

$$\begin{aligned} b &= \sqrt{\mathbf{CA}^{\top} \cdot \mathbf{CA}} \\ &= \sqrt{\begin{pmatrix} 4 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix}} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \\ c &= \sqrt{\mathbf{AB}^{\top} \cdot \mathbf{AB}} \\ &= \sqrt{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix}} \\ &= \sqrt{25 + 49} \\ &= \sqrt{74} \end{aligned}$$

Looking at the figure, it can be said that

$$\begin{aligned} AB &= AF_3 + BF_3 \\ BC &= BD_3 + CD_3 \\ CA &= AE_3 + BE_3 \end{aligned}$$

$$\therefore c = m + n, \tag{1}$$
$$a = n + p, \tag{2}$$
$$b = m + p \tag{3}$$

adding these 3 equations (1), (2) and (3) gives:

$$2(m + n + p) = a + b + c$$
$$\implies m + n + p = (a + b + c)/2 = s = \frac{\sqrt{74} + \sqrt{32} + \sqrt{122}}{2}$$

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subtracting equations (1), (2) and (3) from the eabove gives us the values of p, m and n respectively

$$\therefore m = s - a$$
$$= \frac{\sqrt{74} + \sqrt{32} - \sqrt{122}}{2}$$
$$n = s - b$$
$$= \frac{\sqrt{74} + \sqrt{122} - \sqrt{32}}{2}$$
$$p = s - c$$
$$= \frac{\sqrt{122} + \sqrt{32} - \sqrt{74}}{2}$$