Q-10.13.3.10

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Question: A die is tossed twice. A 'success' is getting an even number on a toss. Find the variance of the number of successes.

Solution:

Parameter	Value	Description
X_i	0,1	0-Not a success, 1-Success and it represents outcome of i^{th} throw
X	0,1,2	$X = X_1 + X_2$, denoting number of outcomes in two throws

pmf of X_i is

$$p_{X_i}(k) = \begin{cases} \frac{1}{2}, & k = 0\\ \frac{1}{2}, & k = 1 \end{cases} \quad \forall \quad 1 \le i \le 2$$
 (1)

Mean value of X_i is

$$\mu_{X_i} = E[X_i], \quad i = 0, 1$$
 (2)

$$=\frac{1}{2}\tag{3}$$

Variance of X_i is

$$\sigma_{X_i}^2 = E[(X_i - \mu_{X_i})^2], \quad i = 0, 1$$
 (4)

$$=\frac{1}{4}\tag{5}$$

Variance of getting successes in two throws of a die is

$$\sigma_X^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2 \tag{6}$$

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 (6)
= $\frac{1}{2}$ (7)