Solution to problem number 1.5.11

Question:
Obtain p, q, r in terms of a, b, c, the sides of the triangle using a matrix equation. Obtain the numerical values.

Solution:
Given in the question:
$\text{A} = \begin{pmatrix}1\\-1\end{pmatrix}$, $\text{B} = \begin{pmatrix}-4\\6\end{pmatrix}$ and $\text{C} = \begin{pmatrix}-3\\-5\end{pmatrix}$

Now, the side lengths a, b and c can be calculated as:

$$\begin{aligned} AB &= B - A \\ &= \begin{pmatrix}-4-1\\6+1\end{pmatrix} = \begin{pmatrix}-5\\7\end{pmatrix} \\ BC &= C - B \\ &= \begin{pmatrix}-3+4\\-5-6\end{pmatrix} = \begin{pmatrix}1\\-11\end{pmatrix} \\ CA &= A - C \\ &= \begin{pmatrix}1+3\\-1+5\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix} \end{aligned}$$

Now, the side lengths a, b and c can be calculated as: AB being a straight line with F_3 a point on it, it can be said that

$$AB = AF_3 + BF_3$$

and similary,

$$\begin{aligned} BC &= BD_3 + CD_3 \\ CA &= AE_3 + BE_3 \end{aligned}$$

$$\begin{aligned} \therefore c &= m + n, && (1)\\ a &= n + p, && (2)\\ b &= m + p && (3) \end{aligned}$$

adding these 3 equations (1), (2) and (3) gives:

$$\begin{aligned} 2(m+n+p) &= a+b+c \\ \implies m+n+p &= (a+b+c)/2 \\ &= s \\ &= \frac{\sqrt{74}+\sqrt{32}+\sqrt{122}}{2} \end{aligned}$$

subtracting equations (1), (2) and (3) from the above gives us the values of p, m and n respectively

$$\begin{aligned} \therefore m &= s - a \\ &= \frac{\sqrt{74}+\sqrt{32}-\sqrt{122}}{2} \\ n &= s - b \\ &= \frac{\sqrt{74}+\sqrt{122}-\sqrt{32}}{2} \\ p &= s - c \\ &= \frac{\sqrt{122}+\sqrt{32}-\sqrt{74}}{2} \end{aligned}$$