

Q-10.13.3.10

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Question: All the jacks, queens and kings are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 similar value for other cards, find the probability that the card has a value

- 1) 7
- 2) greater than 7
- 3) less than 7

Solution: Number of cards left after removing all jacks, queens and kings(=N)

$$= 52 - 4 \times 3 \quad (1)$$

$$= 40 \quad (2)$$

Parameter	Value	Description
X	1-10	Represents the value of the card picked

Finding pmf:

$$p_X(k) = \Pr(X = k) \quad \forall \quad 1 \leq k \leq 10 \quad (3)$$

$$= \frac{4 \times 1}{40} \quad (4)$$

$$= \frac{1}{10} \quad (5)$$

$$\therefore p_X(k) = \begin{cases} \frac{1}{10} & 1 \leq k \leq 10 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

CDF for the following pmf is:

$$F_X(k) = \sum_{m=0}^k p_X(m) \quad \forall \quad 1 \leq k \leq 10 \quad (7)$$

$$= k \times \frac{1}{10} \quad (8)$$

$$= \frac{k}{10} \quad (9)$$

$$\therefore F_X(k) = \begin{cases} 0 & k \leq 0 \\ \frac{k}{10} & 1 \leq k \leq 10 \\ 1 & k > 10 \end{cases} \quad (10)$$

1) Probability that card has value equal to 7:

$$= p_X(7) \quad (11)$$

$$= 1 \times \frac{1}{10} = \frac{1}{10} \quad (12)$$

2) Probability that card has value greater than 7

$$= F_X(10) - F_X(7) \quad (13)$$

$$= 1 - \frac{7}{10} = \frac{3}{10} \quad (14)$$

3) Probability that card has value less than 7

$$= F_X(6) \quad (15)$$

$$= \frac{6}{10} \quad (16)$$