Solution to problem number 1.5.11

Question:
Obtain p, q, r in terms of a, b, c, the sides of the triangle using a matrix equation. Obtain the numerical values.

Solution:
Given in the question:
A = $\begin{pmatrix}1\\-1\end{pmatrix}$, B = $\begin{pmatrix}-4\\6\end{pmatrix}$ and C = $\begin{pmatrix}-3\\-5\end{pmatrix}$

Now, the side lengths a, b and c can be calculated as:

$$AB = B - A \tag{1}$$
$$= \begin{pmatrix}-4-1\\6+1\end{pmatrix} = \begin{pmatrix}-5\\7\end{pmatrix} \tag{2}$$
$$BC = C - B \tag{3}$$
$$= \begin{pmatrix}-3+4\\-5-6\end{pmatrix} = \begin{pmatrix}1\\-11\end{pmatrix} \tag{4}$$
$$CA = A - C \tag{5}$$
$$= \begin{pmatrix}1+3\\-1+5\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix} \tag{6}$$

Now, the side lengths a, b and c can be calculated as:

$$a = \sqrt{BC^\top . BC} \tag{7}$$
$$= \sqrt{\begin{pmatrix}1 & -11\end{pmatrix}\begin{pmatrix}1\\-11\end{pmatrix}} \tag{8}$$
$$= \sqrt{1+121} \tag{9}$$
$$= \sqrt{122} \tag{10}$$

$$b = \sqrt{CA^\top . CA} \tag{11}$$
$$= \sqrt{\begin{pmatrix}4 & 4\end{pmatrix}\begin{pmatrix}4\\4\end{pmatrix}} \tag{12}$$
$$= \sqrt{16+16} \tag{13}$$
$$= \sqrt{32} \tag{14}$$

$$c = \sqrt{AB^\top . AB} \tag{15}$$
$$= \sqrt{\begin{pmatrix}-5 & 7\end{pmatrix}\begin{pmatrix}-5\\7\end{pmatrix}} \tag{16}$$
$$= \sqrt{25+49} \tag{17}$$
$$= \sqrt{74} \tag{18}$$

AB being a straight line with F_3 a point on it, it can be said that

$$AB = AF_3 + BF_3 \tag{19}$$
$$BC = BD_3 + CD_3 \tag{20}$$
$$CA = AE_3 + BE_3 \tag{21}$$

$$\therefore c = m + n, \tag{22}$$
$$a = n + p, \tag{23}$$
$$b = m + p \tag{24}$$

adding these 3 equations (1), (2) and (3) gives:

$$2(m+n+p) = a+b+c \tag{25}$$
$$\implies m+n+p = (a+b+c)/2 \tag{26}$$
$$= s \tag{27}$$
$$= \frac{\sqrt{74}+\sqrt{32}+\sqrt{122}}{2} \tag{28}$$

subtracting equations (1), (2) and (3) from the above gives us the values of p, m and n respectively

$$\therefore m = s - a \tag{29}$$
$$= \frac{\sqrt{74}+\sqrt{32}-\sqrt{122}}{2} \tag{30}$$
$$n = s - b \tag{31}$$
$$= \frac{\sqrt{74}+\sqrt{122}-\sqrt{32}}{2} \tag{32}$$
$$p = s - c \tag{33}$$
$$= \frac{\sqrt{122}+\sqrt{32}-\sqrt{74}}{2} \tag{34}$$