Solution to problem number 1.1.3

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Question: The points are defined to be collinear if

$$\text{rank}\begin{pmatrix}1 & 1 & 1\\ \mathbf{A} & \mathbf{B} & \mathbf{C}\end{pmatrix} = 2 \tag{1}$$

Are the points collinear?
Solution: Given in the question:

$$\mathbf{A} = \begin{pmatrix}1\\ -1\end{pmatrix} \tag{2}$$

$$\mathbf{B} = \begin{pmatrix}-4\\ 6\end{pmatrix} \tag{3}$$

$$\mathbf{C} = \begin{pmatrix}-3\\ -5\end{pmatrix} \tag{4}$$

$$\text{rank}\begin{pmatrix}1 & 1 & 1\\ \mathbf{A} & \mathbf{B} & \mathbf{C}\end{pmatrix} = \text{rank}\begin{pmatrix}1 & 1 & 1\\ -1 & -4 & -3\\ 1 & 6 & -5\end{pmatrix} \tag{5}$$

Solving by row-echelon method,

$$\begin{pmatrix}1 & 1 & 1\\ 1 & -4 & -3\\ 1 & 6 & -5\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix}1 & 1 & 1\\ 0 & -5 & -4\\ 1 & 6 & -5\end{pmatrix} \tag{6}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix}1 & 1 & 1\\ 0 & -5 & -4\\ 0 & 7 & -4\end{pmatrix} \tag{7}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 + \frac{7}{5}R_2} \begin{pmatrix}1 & 1 & 1\\ 0 & -5 & -4\\ 0 & 0 & \frac{-48}{5}\end{pmatrix} \tag{8}$$

$\therefore$ rank of matrix = number of non-zero rows = 3, Hence the points are not collinear

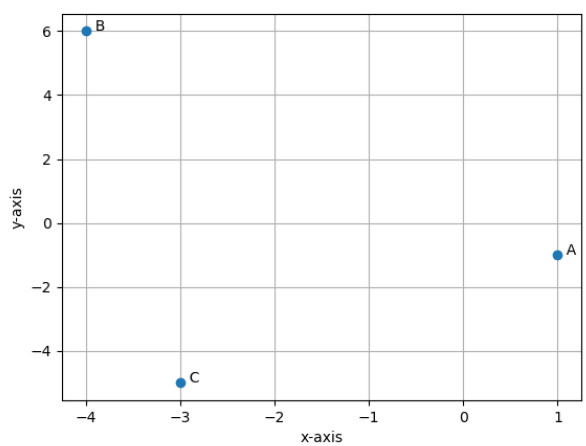

Fig. 0. Proof that A, B and C or non-collinear