Solution to problem number 1.4.2

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Question: Find the intersection **O** of the perpendicular bisectors of AB and AC.

Solution: Given in the question:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

Perpendicular bisector of:

1) Line AB:

$$\begin{pmatrix} 5 & -7 \end{pmatrix} \mathbf{x} = -25 \tag{4}$$

2) Line AC:

$$\begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{x} = -8 \tag{5}$$

Combining the two equations, we get

$$\begin{pmatrix} 5 & -7 \\ 4 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -25 \\ -16 \end{pmatrix} \tag{6}$$

Solving by row reduction method:

$$\begin{pmatrix} 5 & -7 & -25 \\ 4 & 4 & -16 \end{pmatrix} \tag{7}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{7}{4}R_2} \begin{pmatrix} 12 & 0 & -53 \\ 4 & 4 & -16 \end{pmatrix} \tag{8}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - \frac{1}{3}R_1} \begin{pmatrix} 12 & 0 & -53 \\ 0 & 4 & \frac{5}{3} \end{pmatrix} \tag{9}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{4}R_2} \begin{pmatrix} 12 & 0 & -53 \\ 0 & 1 & \frac{5}{12} \end{pmatrix} \tag{10}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{1}{12}R_1} \begin{pmatrix} 1 & 0 & \frac{-53}{12} \\ 0 & 1 & \frac{5}{12} \end{pmatrix} \tag{11}$$

$\therefore$ the point of intersection **O** is $\begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix}$