Q-10.13.3.10

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Question: Eight coins are tossed together. The probability of getting exactly 3 heads is

1) $\frac{1}{256}$
2) $\frac{7}{32}$
3) $\frac{5}{32}$
4) $\frac{3}{32}$

Solution: Defining variables:

Parameter	Value	Description
n	8	Number of coins tossed
p	0.5	probability of getting heads
$\mu = np$	4	mean of the distribution
$\sigma^2 = np(1-p)$	2	variance of the distribution
Y	0-8	denotes number of heads obtained

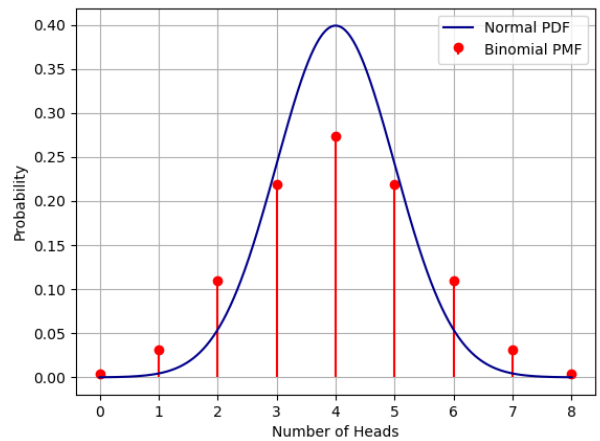

Fig. 1. Binomial distribution vs Gaussian distribution

Binomial distribution: the probability of getting exactly 3 heads is

$$= \binom{8}{3} \times 0.5^3 \times 0.5^5 \tag{1}$$

$$= 0.21875 \tag{2}$$

$\therefore$ option 2 is correct.

Gaussian Distribution:

The gaussian distribution for Y is

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{3}$$

For getting 3 exactly heads

$$Y = 3 \tag{4}$$

Substituting in equation (3), probability for getting exactly 3 heads is

$$Y = 3 \tag{5}$$

$$p_Y(3) = \frac{1}{\sqrt{2\pi \times 2}} e^{\frac{-(3-4)^2}{2\times 2}} \tag{6}$$

$$= 0.35206 \tag{7}$$

Using Q function:

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{8}$$

CDF of Y is given by

$$F_Y(y) = 1 - \Pr(Y > y) \tag{9}$$

$$= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{y-\mu}{\sigma}\right) \tag{10}$$

As

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{11}$$

$$\implies F_Y(y) = 1 - Q\left(\frac{y-\mu}{\sigma}\right) \tag{12}$$

including a correction upto 0.5,

$$p_Y(2.5 < Y < 3.5) = F_Y(3.5) - F_Y(2.5) \tag{13}$$

$$= Q\left(\frac{2.5-\mu}{\sigma}\right) - Q\left(\frac{3.5-\mu}{\sigma}\right) \tag{14}$$

$$= Q(-1.0608) - Q(-0.3536) \tag{15}$$

$$= 0.855610 - 0.638181 \tag{16}$$

$$= 0.1974 \tag{17}$$