

Q-10.13.3.10

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Question: A lot of 100 watches is known to have 10 defective watches. If 8 watches are selected (one by one with replacement) at random, what is the probability that there will be at least one defective watch?

Solution:

Parameter	Value	Description
X_i	0,1	0-Defective watch, 1-Good watch and X_i denotes i^{th} selection
Y	$\sum_{i=1}^8 X_i$	Represents number of defective watches selected in 8 selections

pmf of X_i :

$$p_{X_i}(k) = \begin{cases} \frac{1}{10}, & k = 0 \\ \frac{9}{10}, & k = 1 \end{cases} \quad \forall \quad 1 \leq i \leq 8 \quad (1)$$

pmf of Y is given by:

$$p_Y(k) = \Pr(Y = k) = \binom{8}{k} \times (0.1)^k \times (0.9)^{8-k} \quad \forall \quad 0 \leq k \leq 8 \quad (2)$$

(3)

CDF of Y :

$$F_Y(k) = \sum_{i=0}^k \binom{8}{i} \times (0.1)^i \times (0.9)^{8-i} \quad \forall \quad 0 \leq k \leq 8 \quad (4)$$

(5)

\therefore probability of choosing atleast one defective watch in 8 selections

$$= \sum_{k=1}^8 \Pr(Y = k) \quad (6)$$

$$= F_Y(8) - F_Y(0) \quad (7)$$

$$= 0.569533 \quad (8)$$