# Q-10.13.3.10

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**Question:** Eight coins are tossed together. The probability of getting exactly 3 heads is

1) $\frac{1}{256}$
2) $\frac{7}{32}$
3) $\frac{5}{32}$
4) $\frac{3}{32}$

**Solution:** Let X be a random variable with parameters as

| Parameter | Value | Description |
|---|---|---|
| $n$ | 8 | Number of coins tossed |
| $p$ | 0.5 | probability of getting heads |
| $q$ | 0.5 | probability of getting tails |

Mean and Variance of $X$ are

$$\mu = n \times p \tag{1}$$
$$= 4 \tag{2}$$
$$\sigma^2 = n \times p \times q \tag{3}$$
$$= 2 \tag{4}$$

Defining another random variable:

| Parameter | Value | Description |
|---|---|---|
| $Y$ | 0-8 | denotes number of heads obtained |

**Gaussian Distribution:**
The gaussian distribution for $Y$ is

$$P_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{5}$$

Defining a random variable Z such that

$$Z = \frac{Y - \mu}{\sigma} \tag{6}$$

$\therefore$ Z $\sim \mathcal{N}(0, 1)$
Hence gaussian distribution function becomes

$$P_Z(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \tag{7}$$

$\therefore$ Finding probability of getting three heads

$$Z = \frac{3-4}{2} \tag{8}$$
$$= -0.5 \tag{9}$$
$$P_Z(-0.5) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(-0.5)^2}{2}} \tag{10}$$
$$= 0.35206 \tag{11}$$

**Binomial Distribution:**
The pmf of Y is given by

$$P_Y(k) = \binom{n}{k} \times p^k \times q^{n-k} \tag{12}$$
$$\tag{13}$$

CDF of Y is

$$F_Y(k) = \sum_{i=0}^{k} \binom{n}{i} \times p^i \times q^{n-i} \tag{14}$$
$$\tag{15}$$

$\therefore$ probability of getting exactly 3 heads is

$$P_Y(3) = \binom{8}{3} \times 0.5^3 \times 0.5^5 \tag{16}$$
$$= 0.21875 \tag{17}$$

$\therefore$ option 2 is correct.
**Comparing Binomial and Gaussian distribution:**

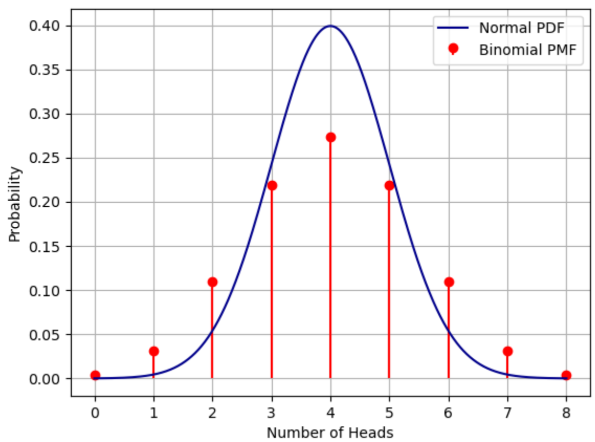


Fig. 1. Binomial vs Guassian