

# Q-10.13.3.10

Yash Patil - EE22BTECH11058

**Question:** Let  $X_1, X_2, \dots, X_{10}$  be a random sample of size 10 from a  $N_3(\mu, \Sigma)$  distribution, where  $\mu$  and a non-singular  $\Sigma$  are unknown parameters. If

$$\bar{X}_1 = \frac{1}{5} \sum_{i=1}^5 X_i \quad (1)$$

$$\bar{X}_2 = \frac{1}{5} \sum_{i=6}^{10} X_i \quad (2)$$

$$S_1 = \frac{1}{4} \sum_{i=1}^5 (X_i - \bar{X}_1)(X_i - \bar{X}_1)^r \quad (3)$$

$$S_2 = \frac{1}{4} \sum_{i=6}^{10} (X_i - \bar{X}_2)(X_i - \bar{X}_2)^r \quad (4)$$

Then which one of the following statements is not true?

- 1)  $\frac{5}{6}(\bar{X}_1 - \mu)^T S_1^{-1}(\bar{X}_1 - \mu)$  follows a  $F$ -distribution with 3 and 2 degrees of freedom
- 2)  $\frac{6}{5(\bar{X}_1 - \mu)^T S_1^{-1}(\bar{X}_1 - \mu)}$  follows a  $F$ -distribution with 3 and 2 degrees of freedom
- 3)  $4(S_1 + S_2)$  follows a Wishart distribution of order 3 with 8 degrees of freedom
- 4)  $5(S_1 + S_2)$  follows a Wishart distribution of order 3 with 10 degrees of freedom

**Solution:**