Q-10.13.3.10

Yash Patil - EE22BTECH11058

Question: A lot of 100 watches is known to have 10 defective watches. If 8 watches are selected (one by one with replacement) at random, what is the probability that there will be at least one defective watch?
Solution:

Parameter	Value	Description
X_i	0,1	0-Defective watch, 1-Good watch and X_i denotes i^{th} selection
Y	$\sum_{i=1}^{8} X_i$	Represents number of defective watches selected in 8 selections

pmf of X_i:

$$p_{X_i}(k) = \begin{cases} \frac{1}{10}, & k = 0 \\ \frac{9}{10}, & k = 1 \end{cases} \quad \forall \quad 1 \le i \le 8 \tag{1}$$

pmf of Y is given by:

$$p_Y(k) = \binom{8}{k} \times (0.1)^k \times (0.9)^{8-k} \quad \forall \quad 0 \le k \le 8 \tag{2}$$

$$\tag{3}$$

CDF of Y:

$$F_Y(k) = \sum_{i=0}^{k} \binom{8}{i} \times (0.1)^i \times (0.9)^{8-i} \quad \forall \quad 0 \le k \le 8 \tag{4}$$

$$\tag{5}$$

$\therefore$ probability of choosing atleast one defective watch in 8 selections

$$= \sum_{k=1}^{8} p_Y(k) \tag{6}$$

$$= \sum_{k=0}^{8} p_Y(k) - p_Y(0) \tag{7}$$

$$= F_Y(8) - F_Y(0) \tag{8}$$

$$= 1 - \binom{8}{0} \times (0.1)^0 \times (0.9)^8 \tag{9}$$

$$= 1 - 0.430467 \tag{10}$$

$$= 0.569533 \tag{11}$$