Q-10.13.3.10

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Question: All the jacks, queens and kings are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 similar value for other cards, find the probability that the card has a value

1) 7
2) greater than 7
3) less than 7

Solution: Number of cards left after removing all jacks, queens and kings(=N)

$$= 52 - 4 \times 3 \tag{1}$$
$$= 40 \tag{2}$$

Parameter	Value	Description
X	1-10	Represents the value of the card picked
Y	1-4	Represents suit of the card: 1-Spades, 2-Diamond, 3-Clubs, 4-Hearts

Proving X and Y represent independant events by calculating marginal and joint probabilities
Finding pmf of marginal probability:

$$p_X(j) = \Pr(X = j) \tag{3}$$
$$= \frac{4 \times 1}{40} \tag{4}$$
$$= \frac{1}{10} \tag{5}$$
$$p_Y(k) = \Pr(Y = k) \tag{6}$$
$$= \frac{10}{40} \tag{7}$$
$$= \frac{1}{4} \tag{8}$$

Finding pmf of joint probability:

$$p_{X,Y}(j,k) = \Pr(X = j, Y = k) \tag{9}$$
$$= \frac{1}{40} \tag{10}$$

As

$$p_{X,Y}(j,k) = \frac{1}{40} \tag{11}$$
$$= p_X(j) \times p_Y(k) \tag{12}$$
$$\tag{13}$$

we can say that both the events are independant of each other.

∴ pmf of getting number 'j' and suit 'k'=

$$p_{X,Y}(j,k) = \frac{1}{40} \tag{14}$$

CDF for the following pmf is:

$$F_{X,Y}(j,k) = \sum_{k=k_0}^{k=k_1} \sum_{j=j_0}^{j=j_1} p_{X,Y}(j,k) \tag{15}$$

$$= \sum_{k=k_0}^{k=k_1} \sum_{j=j_0}^{j=j_1} \frac{1}{40} \tag{16}$$

$$= (k_1 - k_0 + 1) \times (j_1 - j_0 + 1) \times \frac{1}{40} \tag{17}$$

1) Probability that card has value equal to 7:

$$= F_{X,Y}(j,k);\;\; j = 7 \; and \; 1 \leq k \leq 4 \tag{18}$$

$$= 4 \times 1 \times \frac{1}{40} \tag{19}$$

$$= \frac{1}{10} \tag{20}$$

2) Probability that card has value greater than 7

$$= F_{X,Y}(j,k);\;\; 8 \leq j \leq 10 \; and \; 1 \leq k \leq 4 \tag{21}$$

$$= 4 \times 3 \times \frac{1}{40} \tag{22}$$

$$= \frac{3}{10} \tag{23}$$

3) Probability that card has value less than 7

$$= F_{X,Y}(j,k);\;\; 1 \leq j \leq 6 \; and \; 1 \leq k \leq 4 \tag{24}$$

$$= 4 \times 6 \times \frac{1}{40} \tag{25}$$

$$= \frac{6}{10} \tag{26}$$