Q-10.13.3.10

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Question: A lot of 100 watches is known to have 10 defective watches. If 8 watches are selected (one by one with replacement) at random, what is the probability that there will be at least one defective watch?
Solution:

Parameter	Value	Description
X	0,1	0-Defective watch, 1-Good watch
Y	0-8	Represents number of defective watches selected in 8 selections

Finding pmf of X and Y:

$$p_X(k) = Pr(X = k) \quad k \in \{0, 1\} \tag{1}$$

$$= \begin{cases} \frac{1}{10}, & k = 0 \\ \frac{9}{10}, & k = 1 \end{cases} \tag{2}$$

$$p_Y(k) = Pr(Y = k) \ \forall \ 0 \le k \le 8 \tag{3}$$

$$= \binom{8}{k} \prod_{i=0}^{k} p_X(0) \prod_{j=0}^{8-k} p_X(1) \tag{4}$$

$$= \binom{8}{k} \times (0.1)^k \times (0.9)^{8-k} \tag{5}$$

CDF of Y:

$$F_Y(k) = \sum_{i=0}^{k} p_Y(i) \tag{6}$$

$$= \sum_{i=0}^{k} \binom{8}{i} \times (0.1)^i \times (0.9)^{8-i} \tag{7}$$

$\therefore$ probability of choosing atleast one defective watch in 8 selections

$$= F_Y(8) - F_Y(0) \tag{8}$$

$$= 1 - \binom{8}{0} \times (0.1)^0 \times (0.9)^8 \tag{9}$$

$$= 1 - 0.430467 \tag{10}$$

$$= 0.569533 \tag{11}$$