

Q-10.13.3.10

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Question: Eight coins are tossed together. The probability of getting exactly 3 heads is

- 1) $\frac{1}{256}$
- 2) $\frac{7}{32}$
- 3) $\frac{5}{32}$
- 4) $\frac{3}{32}$

Solution: Defining variables:

Parameter	Value	Description
n	8	Number of coins tossed
p	0.5	probability of getting heads
$\mu = np$	4	mean of the distribution
$\sigma^2 = np(1 - p)$	2	variance of the distribution
Y	0-8	denotes number of heads obtained

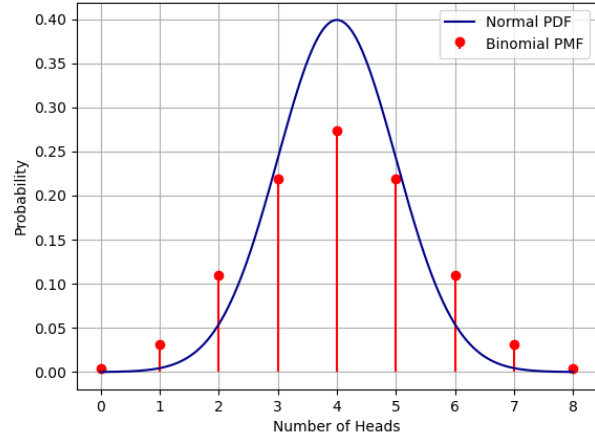


Fig. 1. Binomial distribution vs Gaussian distribution

- 1) **Binomial distribution:** the probability of getting exactly 3 heads is

$$= \binom{8}{3} \times 0.5^3 \times 0.5^5 \quad (1)$$

$$= 0.21875 \quad (2)$$

\therefore option 2 is correct.

- 2) **Gaussian Distribution:**

The gaussian distribution for Y is

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (3)$$

For getting 3 exactly heads

$$Y = 3 \quad (4)$$

Substituting in equation (3), probability for getting exactly 3 heads is

$$Y = 3 \quad (5)$$

$$p_Y(3) = \frac{1}{\sqrt{2\pi \times 2}} e^{-\frac{-(3-4)^2}{2 \times 2}} \quad (6)$$

$$= 0.35206 \quad (7)$$

- 3) **Using Q function:**

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (8)$$

CDF of Y is given by

$$F_Y(y) = 1 - \Pr(Y > y) \quad (9)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{y - \mu}{\sigma}\right) \quad (10)$$

As

$$\frac{Y - \mu}{\sigma} \sim \mathcal{N}(0, 1) \quad (11)$$

$$\Rightarrow F_Y(y) = 1 - Q\left(\frac{y - \mu}{\sigma}\right) \quad (12)$$

including a correction upto 0.5,

$$p_Y(2.5 < Y < 3.5) = F_Y(3.5) - F_Y(2.5) \quad (13)$$

$$= Q\left(\frac{2.5 - \mu}{\sigma}\right) - Q\left(\frac{3.5 - \mu}{\sigma}\right) \quad (14)$$

$$= Q(-1.0608) - Q(-0.3536) \quad (15)$$

$$= 0.855610 - 0.638181 \quad (16)$$

$$= 0.1974 \quad (17)$$

4) Comparing all three techniques:

Event	Binomial	Gaussian	Q function
Getting exactly 3 heads	0.21875	0.35206	0.1974