Solution to problem number 1.5.11

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Question:
Obtain p, q, r in terms of a, b, c, the sides of the triangle using a matrix equation. Obtain the numerical values.

Solution:
Given in the question:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$
$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$
$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

Now, the side lengths a, b and c can be calculated as:

$$a = \sqrt{(\mathbf{C}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B})} \tag{4}$$
$$= \sqrt{\begin{pmatrix} 1 & -11 \end{pmatrix}\begin{pmatrix} 1 \\ -11 \end{pmatrix}} \tag{5}$$
$$= \sqrt{1+121} \tag{6}$$
$$= \sqrt{122} \tag{7}$$

$$b = \sqrt{(\mathbf{A}-\mathbf{C})^\top(\mathbf{A}-\mathbf{C})} \tag{8}$$
$$= \sqrt{\begin{pmatrix} 4 & 4 \end{pmatrix}\begin{pmatrix} 4 \\ 4 \end{pmatrix}} \tag{9}$$
$$= \sqrt{16+16} \tag{10}$$
$$= \sqrt{32} \tag{11}$$

$$c = \sqrt{(\mathbf{B}-\mathbf{A})^\top(\mathbf{B}-\mathbf{A})} \tag{12}$$
$$= \sqrt{\begin{pmatrix} -5 & 7 \end{pmatrix}\begin{pmatrix} -5 \\ 7 \end{pmatrix}} \tag{13}$$
$$= \sqrt{25+49} \tag{14}$$
$$= \sqrt{74} \tag{15}$$

AB being a straight line with F_3 a point on it, it can be said that

$$AB = AF_3 + BF_3 \tag{16}$$
$$BC = BD_3 + CD_3 \tag{17}$$
$$CA = AE_3 + BE_3 \tag{18}$$

$$\therefore c = m + n, \tag{19}$$
$$a = n + p, \tag{20}$$
$$b = m + p \tag{21}$$

these 3 equations can be written as:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c \\ a \\ b \end{pmatrix} \tag{22}$$
$$\implies \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} c \\ a \\ b \end{pmatrix} \tag{23}$$

solving by Guassian elimination method,

$$\begin{pmatrix} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \end{pmatrix} \tag{24}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 + R_3 - R_2} \begin{pmatrix} 2 & 0 & 0 & c+b-a \\ 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \end{pmatrix} \tag{25}$$
$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & 0 & 0 & \frac{c+b-a}{2} \\ 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \end{pmatrix} \tag{26}$$
$$\xleftrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & 0 & 0 & \frac{c+b-a}{2} \\ 0 & 1 & 1 & a \\ 0 & 0 & 1 & \frac{a+b-c}{2} \end{pmatrix} \tag{27}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 - R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{c+b-a}{2} \\ 0 & 1 & 0 & \frac{a+c-b}{2} \\ 0 & 0 & 1 & \frac{a+b-c}{2} \end{pmatrix} \tag{28}$$

$$\therefore m = \frac{c + b - a}{2} \tag{29}$$

$$= \frac{\sqrt{74} + \sqrt{32} - \sqrt{122}}{2} \tag{30}$$

$$n = \frac{a + c - b}{2} \tag{31}$$

$$= \frac{\sqrt{74} + \sqrt{122} - \sqrt{32}}{2} \tag{32}$$

$$p = \frac{a + b - c}{2} \tag{33}$$

$$= \frac{\sqrt{122} + \sqrt{32} - \sqrt{74}}{2} \tag{34}$$