Q-10.13.3.10

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Question: All the jacks, queens and kings are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 similar value for other cards, find the probability that the card has a value

1) 7
2) greater than 7
3) less than 7

Solution: Number of cards left after removing all jacks, queens and kings(=N)

$$= 52 - 4 \times 3 \tag{1}$$
$$= 40 \tag{2}$$

Parameter	Value	Description
X	1-10	Represents the value of the card picked
Y	Spades(1), Diamond(2), Clubs(3), Hearts(4)	Represents suit of the card picked up

Proving X and Y represent independant events by showing:

$$\Pr(X.Y) = \Pr(X) \times \Pr(Y) \tag{3}$$
$$\tag{4}$$

Say, finding probabilities for card of value 'p' and suit 'q'

$$\Pr(X.Y) = \frac{\text{Number of cards with value 'p' and suit 'q'}}{N} \tag{5}$$
$$= \frac{1}{40} \tag{6}$$
$$\Pr(X = p) = \frac{1}{10} \tag{7}$$
$$\Pr(Y = q) = \frac{1}{4} \tag{8}$$
$$\therefore \Pr(X.Y) = \Pr(X) \times \Pr(Y) = \frac{1}{40} \tag{9}$$

Hence, both events are independant.

1) Probability that card has value equal to 7
   = (Probability of getting value equal to 7)×(Probability of picking any suit)

$$= \Pr(X = 7) \times \sum_{i=1}^{4} \Pr(Y = i) \tag{10}$$
$$= \frac{1}{10} \times \frac{1}{1} \tag{11}$$
$$= \frac{1}{10} \tag{12}$$

2) Probability that card has value greater than 7
= (Probability of getting value greater than 7)×(Probability of picking any suit)

$$= \sum_{j=8}^{10} \Pr(X = j) \times \sum_{i=1}^{4} \Pr(Y = i) \tag{13}$$

$$= \frac{3}{10} \times \frac{1}{1} \tag{14}$$

$$= \frac{3}{10} \tag{15}$$

3) Probability that card has value less than 7
= (Probability of getting value less than 7)×(Probability of picking any suit)

$$= \sum_{j=1}^{6} \Pr(X = j) \times \sum_{i=1}^{4} \Pr(Y = i) \tag{16}$$

$$= \frac{6}{10} \times \frac{1}{1} \tag{17}$$

$$= \frac{6}{10} \tag{18}$$