

Solution to problem number 1.5.11

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Question:

Obtain p, q, r in terms of a, b, c, the sides of the triangle using a matrix equation. Obtain the numerical values.

Solution:

Given in the question:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

Now, the side lengths a, b and c can be calculated as:

$$a = \sqrt{(\mathbf{C} - \mathbf{B})^\top \cdot (\mathbf{C} - \mathbf{B})} \quad (1)$$

$$= \sqrt{\begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} 1 \\ -11 \end{pmatrix}} \quad (2)$$

$$= \sqrt{1 + 121} \quad (3)$$

$$= \sqrt{122} \quad (4)$$

$$b = \sqrt{(\mathbf{A} - \mathbf{C})^\top \cdot (\mathbf{A} - \mathbf{C})} \quad (6)$$

$$= \sqrt{\begin{pmatrix} 4 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix}} \quad (7)$$

$$= \sqrt{16 + 16} \quad (8)$$

$$= \sqrt{32} \quad (9)$$

$$c = \sqrt{(\mathbf{B} - \mathbf{A})^\top \cdot (\mathbf{B} - \mathbf{A})} \quad (11)$$

$$= \sqrt{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix}} \quad (12)$$

$$= \sqrt{25 + 49} \quad (13)$$

$$= \sqrt{74} \quad (14)$$

it can be said that

$$AB = AF_3 + BF_3 \quad (15)$$

$$BC = BD_3 + CD_3 \quad (16)$$

$$CA = AE_3 + BE_3 \quad (17)$$

$$\therefore c = m + n, \quad (18)$$

$$a = n + p, \quad (19)$$

$$b = m + p \quad (20)$$

$$\quad (21)$$

these 3 equations can be written as:

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad (22)$$

$$\Rightarrow \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad (23)$$

solving by Gaussian elimination method,

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_2 + R_3} \quad (24)$$

$$\Rightarrow \begin{pmatrix} 2 & 2 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a + b + c \\ b \\ c \end{pmatrix} \quad (25)$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \quad (26)$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{a+b+c}{2} \\ b \\ c \end{pmatrix} \quad (27)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \quad (28)$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{a+b+c}{2} \\ \frac{a-b+c}{2} \\ c \end{pmatrix} \quad (29)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_3} \quad (30)$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{a+b+c}{2} \\ \frac{a-b+c}{2} \\ \frac{a+b-c}{2} \end{pmatrix} \quad (31)$$

AB being a straight line with F_3 a point on it,

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$$\overleftrightarrow{R_1 \leftarrow R_1 - R_2 - R_3} \quad (32)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{-a+b+c}{2} \\ \frac{a-b+c}{2} \\ \frac{a+b-c}{2} \end{pmatrix} \quad (33)$$

$$\Rightarrow \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{-a+b+c}{2} \\ \frac{a-b+c}{2} \\ \frac{a+b-c}{2} \end{pmatrix} \quad (34)$$

$$\Rightarrow \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{74} + \sqrt{32} - \sqrt{122}}{2} \\ \frac{\sqrt{74} + \sqrt{122} - \sqrt{32}}{2} \\ \frac{\sqrt{122} + \sqrt{32} - \sqrt{74}}{2} \end{pmatrix} \quad (35)$$

$$\therefore m = \frac{\sqrt{74} + \sqrt{32} - \sqrt{122}}{2}, \quad (36)$$

$$n = \frac{\sqrt{74} + \sqrt{122} - \sqrt{32}}{2}, \quad (37)$$

$$p = \frac{\sqrt{122} + \sqrt{32} - \sqrt{74}}{2} \quad (38)$$