Q-10.13.3.10

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Question: A die is tossed twice. A 'success' is getting an even number on a toss. Find the variance of the number of successes.

Solution:

Parameter	Value	Description
X_i	0,1	0-Not a success, 1-Success and it represents outcome of i^{th} throw
Y	0-2	Represents number of successes in 2 throws

pmf of X:

$$p_{X_i}(k) = \begin{cases} \frac{1}{2}, & k = 0 \\ \frac{1}{2}, & k = 1 \end{cases} \tag{1}$$

pmf of Y:

$$p_Y(k) = \binom{2}{k}(0.5)^{2-k} \times (0.5)^k \quad \forall \quad 0 \le k \le 2 \tag{2}$$

$$\therefore p_Y(k) = \begin{cases} \frac{1}{4}, & k = 0 \\ \frac{1}{2}, & k = 1 \\ \frac{1}{4}, & k = 2 \end{cases} \tag{3}$$

Variance of number of successes is given by

$$\sigma^2 = E(Y^2) - [E(Y)]^2 \tag{4}$$

$$= \sum_{k=0}^{2}(k^2 \times p_Y(k)) - \left[\sum_{k=0}^{2} k \times p_Y(k)\right]^2 \tag{5}$$

$$= \frac{3}{2} - 1 \tag{6}$$

$$= \frac{1}{2} \tag{7}$$