Q-10.13.3.10

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Question: Eight coins are tossed together. The probability of getting exactly 3 heads is

1) $\frac{1}{256}$
2) $\frac{7}{32}$
3) $\frac{5}{32}$
4) $\frac{3}{32}$

Solution: Defining variables:

Parameter	Value	Description
n	8	Number of coins tossed
p	0.5	probability of getting heads
$\mu = np$	4	mean of the distribution
$\sigma^2 = np(1-p)$	2	variance of the distribution
Y	0-8	denotes number of heads obtained

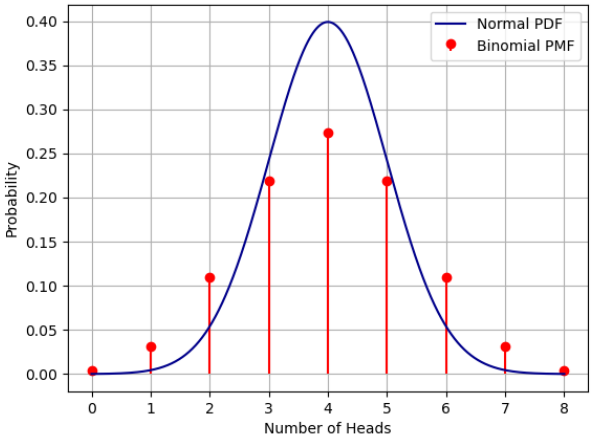

Fig. 1. Binomial vs Gaussian

Gaussian Distribution:
The gaussian distribution for Y is

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{1}$$

For getting 3 exactly heads

$$Y = 3 \tag{2}$$

Substituting in equation (1), probability for getting exactly 3 heads is

$$Y = 3 \tag{3}$$

$$p_Y(3) = \frac{1}{\sqrt{2\pi \times 2}} e^{\frac{-(3-4)^2}{2\times 2}} \tag{4}$$

$$= 0.35206 \tag{5}$$

And, by binomial distribution, the probability of getting exactly 3 heads is

$$= \binom{8}{3} \times 0.5^3 \times 0.5^5 \tag{6}$$

$$= 0.21875 \tag{7}$$

$\therefore$ option 2 is correct.

Comparing Binomial and Gaussian distribution: