

Q-10.13.3.10

Yash Patil - EE22BTECH11058

Question: A die is tossed twice. A ‘success’ is getting an even number on a toss. Find the variance of the number of successes.

Solution:

Parameter	Value	Description
X_i	0,1	0-Not a success, 1-Success and it represents outcome of i^{th} throw

pmf of X_i :

$$p_{X_i}(k) = \begin{cases} \frac{1}{2}, & k = 0 \\ \frac{1}{2}, & k = 1 \end{cases} \quad \forall \quad 1 \leq i \leq 2 \quad (1)$$

Probability of 0 successes:

$$= \Pr(X_1 + X_2 = 0) \quad (2)$$

$$= \frac{1}{2} \times \frac{1}{2} \quad (3)$$

$$= \frac{1}{4} \quad (4)$$

Probability of 1 success:

$$= \Pr(X_1 + X_2 = 1) - \Pr(X_1 X_2 = 1) \quad (5)$$

$$= 3 \times \frac{1}{2} \times \frac{1}{2} - \frac{1}{4} \quad (6)$$

$$= \frac{1}{2} \quad (7)$$

Probability of 2 successes:

$$= \Pr(X_1 X_2 = 1) \quad (8)$$

$$= \frac{1}{2} \times \frac{1}{2} \quad (9)$$

$$= \frac{1}{4} \quad (10)$$

Mean of given distribution is

$$= 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4} \quad (11)$$

$$= 1 \quad (12)$$

Variance of number of successes is given by:

$$\sigma^2 = (0 - 1)^2 \times \frac{1}{4} + (1 - 1)^2 \times \frac{1}{2} + (2 - 1)^2 \times \frac{1}{4} \quad (13)$$

$$= \frac{1}{2} \quad (14)$$