

Solution to problem number 1.1.3

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Question: The points are defined to be collinear if

$$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix} = 2 \quad (1)$$

Are the points collinear?

Solution: Given in the question:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (3)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (4)$$

$$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix} = \text{rank} \begin{pmatrix} 1 & 1 & 1 \\ 1 & -4 & -3 \\ 1 & 6 & -5 \end{pmatrix} \quad (5)$$

Solving by row-echelon method,

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -4 & -3 \\ 1 & 6 & -5 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -5 & -4 \\ 1 & 6 & -5 \end{pmatrix} \quad (6)$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -5 & -4 \\ 0 & 5 & -6 \end{pmatrix} \quad (7)$$

$$\xrightarrow{R_3 \leftarrow R_3 + R_2} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -5 & -4 \\ 0 & 0 & -10 \end{pmatrix} \quad (8)$$

\therefore rank of matrix = number of non-zero rows = 3,
Hence the points are not collinear