ETH Zurich
Institut für Theoretische Informatik
Prof. Dr. Angelika Steger
Prof. Dr. Emo Welzl
Prof. Dr. Peter Widmayer

Algorithms Lab

Exercise – *Goldfinger*

James Bond is tracking Auric Goldfinger, the leader of an international smuggling conglomerate. Using a tiny GPS sensor planted in a car, Bond locates Goldfinger and follows him all the way up to the Furka pass in the Swiss Alps. After a bit of scouting Bond discovers a factory that appears to serve as Goldfinger's headquarters. He decides to infiltrate the compound so as to gain information about Goldfinger's plans.

Unfortunately, the factory is heavily secured by motion sensors and henchmen. But climbing a nearby summit overlooking the area James knows (1) exactly where both sensors and henchmen are located and (2) that neither move around and therefore can be regarded as stationary.

Most dangerous are the magnetic motion *sensors*: They detect any kind of motion and once detected Bond would be caught almost certainly. Therefore all sensors must be deactivated. Luckily, the Q division has provided Bond with a latest prototype of a *magnetic chain supercharger*. It consists of a series of magnetic pulse emitters (*MPE*s) that—provided enough energy is used—can deactivate the sensors. Using a special gun Bond has shot all MPEs into the compound. This process comes with limited control only and so he has to live with wherever the MPEs landed.

For every sensor s Bond has a safe estimate of the magnetic energy E_s needed to deactivate s. Every MPE p comes with two individually adjustable parameters: an *intensity* $i_p \geq 0$ and a *range* $r_p \geq 0$. A point $q \in \mathbb{R}^2$ is *reached* by the pulse from an MPE p if $||p - q|| \leq r_p$. If q is reached by the pulse from p, it *receives* a pulse of energy $i_p||p - q||^{-2}$ from p. The *total pulse energy* that hits q is the sum of the energies of all pulses that q receives from MPEs. A sensor s is *deactivated* if it is hit by a total pulse energy of at least E_s.

Once the sensors are deactivated Bond can work around the henchmen easily. However, if one of the henchmen would notice the supercharger activation, (s)he would most certainly raise an alarm. Therefore, *none* of the pulses from an MPE must reach a henchman.

Also, working with a prototype comes with limitations:

(1) the MPEs are numbered $p_0, \ldots, p_{m-1}$ and they are chained in this particular order. Not all MPEs need to be *activated*. But the set of activated MPEs must form a *valid chain*, that is, a chain that starts with p_0 and ends with p_{k-1}, for some $k \in \{1, \ldots, m\}$.

(2) The supercharger can handle a limited amount of total intensity only. The intensities of the activated MPEs must be set so that their total sum does not exceed a given threshold $I_{\max}$.

The less MPEs are activated, the more likely it is that the supercharger works as expected. Therefore you must find an active chain of *minimum length* that suffices to deactivate all sensors.

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that contains four integers `n m h` $\mathtt{I}_{\max}$, separated by a space. They denote

- n, the number of sensors ($1 \leq n \leq 10^2$);
- m, the number of MPEs ($1 \leq m \leq 2 \cdot 10^4$);
- h, the number of henchmen ($0 \leq h \leq 10^5$);
- $I_{\max}$, the maximum total intensity the supercharger can handle ($0 \leq I_{\max} \leq 10^6$).

- The following n lines define the individual positions $s_0, \ldots, s_{n-1}$ of the sensors along with the deactivation energy. Each line contains three integers `x y E`, separated by a space and such that $|x|, |y| < 2^{24}$ and $1 \leq E < 2^{14}$.
- The following m lines denote the individual positions $p_0, \ldots, p_{m-1}$ of the MPEs. Each line contains two integers `x y`, separated by a space and such that $|x|, |y| < 2^{24}$.
- The following h lines denote the individual positions of the henchmen. Each line contains two integers `x y`, separated by a space and such that $|x|, |y| < 2^{24}$.

You may assume that any two entities—sensors, MPEs, and henchmen—are at pairwise distinct positions.

Output For each test case the corresponding output appears on a separate line. If it is impossible to deactivate all sensors with the available MPEs and total intensity, then output the string `impossible`. Otherwise, output the smallest integer k such that activating the chain $p_0, \ldots, p_{k-1}$ is enough to deactivate all sensors (when intensities and ranges are set appropriately).

Points There are five groups of test sets. Each is worth 20 points, which amounts to 100 points in total.

1. For the first group of test sets, you may assume that there are no henchmen and no more than 20 MPEs ($h = 0$ and $m \leq 20$).
2. For the second group of test sets, you may assume that there are no more than 20 MPEs and no more than 10^3 henchmen ($m \leq 20$ and $h \leq 10^3$).
3. For the third group of test sets, you may assume that a chain of no more than 20 MPEs is sufficient to deactivate all sensors ($k \leq 20$). In particular, there is always a solution.
4. For the fourth and fifth group of test sets, there are no additional assumptions.

Corresponding sample test sets are contained in `test`*i*`.in/out`, for $i \in \{1, 2, 3, 4\}$.

Sample Input

```
3
2 2 0 2
0 0 2
1 1 2
0 1
1 0
2 1 1 2
0 0 2
2 2 2
2 0
1 1
1 3 1 8
1 2 2
1 0
0 2
2 2
2 0
```

Sample Output

```
1
impossible
2
```