Parameters: polynomials u, v
precondition: v != "0"
$\frac{u}{v} = q\ (quotient), r\ (remainder)$
postcondition: $u = q * v + r$

Assume for any polynomial p: $p.leadingTerm = p[p.degree] * x^{p.degree}$

Pseudocode:
$q = 0$
$r = u$
while r != "0" and $r.degree \geq v.degree$
$t = \frac{r_{old}.leadingTerm}{v.leadingTerm}$ (t = temporary, single-term poly)
$q_{new} = q_{old} + t$
$r_{new} = r_{old} - t * v$

invariant: $u = q * v + r$

Base case:
$u = q * v + r = 0 * v + u = 0 + u = u$ $[TRUE]$
Iteration k (assume true):
$$u = q_{old} * v + r_{old}$$
Iteration k + 1:
$$u = q_{new} * v + r_{new} \Longrightarrow \frac{u}{v} = q_{new} + \frac{r_{new}}{v} = (q_{old} + t) + \frac{(r_{old} - t * v)}{v} = q_{old} + \frac{r_{old}}{v}$$
$$\frac{u}{v} = q_{old} + \frac{r_{old}}{v} \Longrightarrow u = q_{old} * v + r_{old}\ [TRUE]$$

Termination:
$D = r.degree - v.degree + 1$
==> Loop exits when $r.degree < v.degree$ ($D_{minimum} = 0$, given degrees are integers)
==> Initially: $D > 0$, when $r.degree \geq v.degree$

For $r.degree$ to decrease after every iteration, r_{old}'s leading term must be removed at every iteration.

Given: $r_{new} = r_{old} - t * v$. To ensure r_{old}'s leading term is removed after every iteration, the following cases must hold true:

A) degrees must cancel out ==> $r_{old}.degree = t.degree + v.degree$

$$t = \frac{r_{old}.leadingTerm}{v.leadingTerm} \Longrightarrow t.degree = r_{old}.degree - v.degree$$

$$r_{old}.degree = (r_{old}.degree - v.degree) + v.degree = r_{old}.degree\ [TRUE]$$

B) coefficients must cancel out ==> $r_{old}[r_{old}.degree] = t[t.degree] * v[v.degree]$

$$t = \frac{r_{old}.leadingTerm}{v.leadingTerm} \Longrightarrow t[t.degree] = \frac{r_{old}[r_{old}.degree]}{v[v.degree]}$$

$$r_{old}[r_{old}.degree] = \left(\frac{r_{old}[r_{old}.degree]}{v[v.degree]}\right) * v[v.degree] = r_{old}[r_{old}.degree]\ [TRUE]$$

$==>$ r_{old}'s leading term is always removed $==>$ $r.degree$ decreases after every iteration, $v.degree$ does not change $==>$ D decreases after every iteration until $D \leq 0$ $==>$ LOOP EXITS $[TRUE]$