Tutorial - 4

Unzila
Sec - I

Ans 1 → $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a \geq 1, \ b > 1$

On Comparing -

$a = 3, \ b = 2, \ f(n) = n^2$

Now

$c = \log_b a = \log_2 3$

$\Rightarrow c = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$T(n) = \theta(n^2)$

Ans 2 → $T(n) = 4T(n/2) + n^2$

$a \geq 1, \ b \geq 1$

$a = 4, \ b = 2, \ f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n)$

$T(n) = \theta(n^2 \log n)$

Ans 3 → $T(n) = T(n/2) + 2^n$

$a = 1, \ b = 2, \ f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \theta(2^n)$

Ans 4 → $T(n) = 2^n T(n/2) + n^n$

Here Master's Theorem can't be applied as 'a' must be constant.

Ans 5 → $T(n) = 16T(n/4) + n$

$a = 16, \ b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 4^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$\Rightarrow T(n) = \theta(n^2)$

Ay.

Ans 6 → $T(n) = 2T(n/2) + n \cdot \log n$

$a = 2, \ b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\because n \log n > n$

$f(n) > n^c$

$\Rightarrow T(n) = \theta(n \log n)$

Ans 7 → $T(n) = 2T(n/2) + n/\log n$

$a = 2, \ b = 2$

$f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n' = n$

$\because n/\log n < n$

$\Rightarrow T(n) = \theta(n)$

Ans 8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2,\ b = 4,\ f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$\because n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\Rightarrow T(n) = \Theta(n^{0.51})$

Ans 9) $T(n) = 0.5\,T(n/2) + 1/n$

$a = 0.5,\ b = 2$

$\because a < 1$

$\Rightarrow$ Master Theorem can't be applied.

Ans 10) $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

$n! > n^2$

$\Rightarrow T(n) = \Theta(n!)$

Ans 11) $T(n) = 4T(n/2) + \log n$

$a = 4,\ b = 2,\ f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\log n < n^2$

$\Rightarrow T(n) = \Theta(n^2)$

Ans 12) $T(n) = sqrt(n)\,T(n/2) + \log n$

Here, Master Theorem cannot be applied as 'a' must be constant.

Ans 13) $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2,\ f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n < n^{1.5849}$

$f(n) < n^c$

$\Rightarrow T(n) = \Theta(n^{1.5489})$

Ans 14) $T(n) = 3T(n/3) + sqrt(n)$

$a = 3,\ b = 3,\ c = \log_3 3 = 1$

$n^c = n^1 = n$

$sqrt(n) < n$

$f(n) < n^c$

$\Rightarrow T(n) = \Theta(n)$

Ans 15) $T(n) = 4T(n/2) + cn$

$a = 4,\ b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\because cn < n^2$

$\Rightarrow T(n) = \Theta(n^2)$

Ans 16) $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$\because n^{0.792} < n\log n$

$\Rightarrow T(n) = \Theta(n\log n)$

Ans 17) $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\because n^c = n^1 = n$

as $n/2 < n$

$\Rightarrow T(n) = \Theta(n)$

Ans 18, $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \quad b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

as $n^{1.6309} < n^2 \log n$

$\Rightarrow T(n) = \theta(n^2 \log n)$

Ans 19, $T(n) = 4T(n/2) + n/\log n$

$a = 4, \ b = 2, \ f(n) = n/\log n$

$c = \log_2 4 = 2$

$n^c = n^2 > n/\log n$

$\Rightarrow T(n) = \theta(n^2)$

Ans 20, $T(n) = 64\,T(n/8) - n^2 \log n$

Here, Master Theorem can't be applied because $f(n)$ is negative.

Ans 21, $T(n) = 7\,T(n/3) + n^2$

$a = 7, \ b = 3, \ f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712} < n^2$

$\Rightarrow T(n) = \theta(n^2)$

Ans 22, $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \ b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$\because n(2 - \cos n) > n^c$

$\Rightarrow T(n) = \theta(n(2 - \cos n))$

Ay.