HPC for mathematicians Assignment 2

Yasmin Hengster

February 2021

Exercise 3

The purpose of the exercise to produce a parallel code for the $1D$ heat equation:

$$\frac{\partial}{\partial t}u = \frac{\partial^2}{\partial x^2}u$$

for $t \in (0, T)$ and $x \in (0, 1)$. The given initial condition and boundary conditions are:

$$u(x, t = 0) = \sin(2\pi x) + 2\sin(5\pi x) + 3\sin(20\pi x) u(x = 0, t) \quad = u(x = 1, t) = 0$$

The numerical scheme which is used is the forward Euler:

$$u_m^{n+1} = u_m^n + \frac{\Delta t}{(\Delta x)^2}\left(u_{m-1}^n - 2u_m^n + u_{m+1}^n\right)$$

where m represents the spatial discretisation and n the time discretisation with Δx and Δt as step size and time step.

In the first step, each processes calculated values for equally sized sub-intervals of length J with an overlap of two values. The processes calculated the initial condition in $[process\, J - 1 : process\, J - 1 + J]$ and the next time step for the sub-interval $[process\, J - 1 : process\, J - 1 + J]$. Secondly, each process sent the second and the second-last value of u to the $process - 1$ and $process + 1$ and set the new values as the new "initial condition". With MPI_Barrier it was ensured that all processes have finished their calculations for the first time step. With the same method I iterated over N time steps. In the last step all processes sent their final values to process 0 which printed the result. In Figure 1 shows the exact solution and the numerical solution for $T = 0.01$. The size of M and J has to be chosen in a way that the following equation is satisfied:

$$M = size(J - 2) + 1$$

To compare the computation time, I choose M to be approximately 100 for a different number of processes. If the number of processes is too large the time for sending and receiving is higher than savings due to parallel programming (Figure 2).

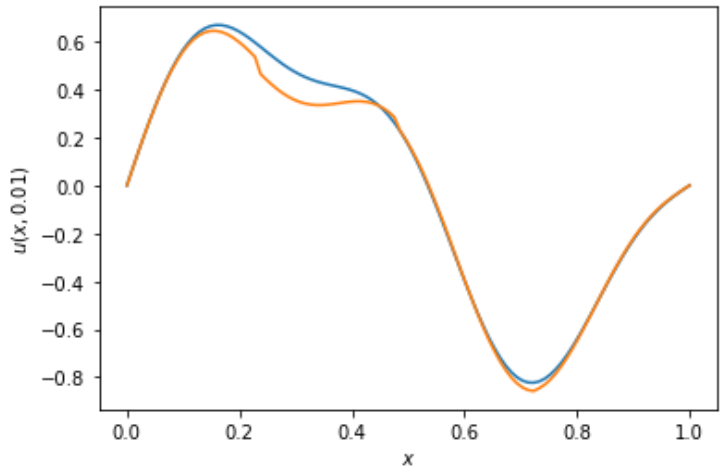

Figure 1: Solution of the heat equation at $T = 0.01$. The blue line is the exact solution, the orange line is the calculated solution.

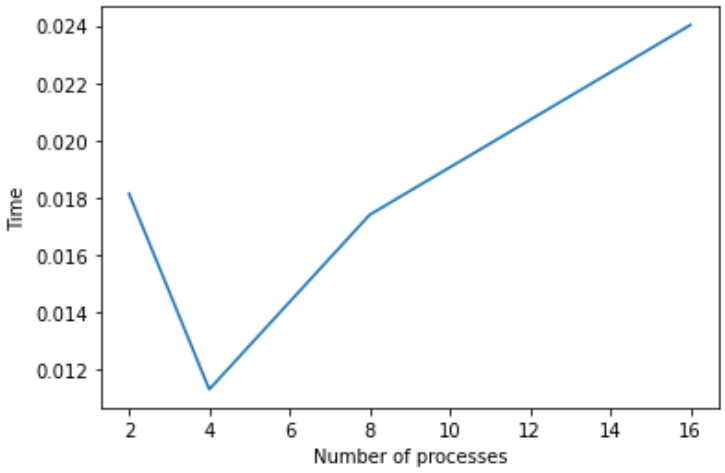

Figure 2: Time for the calculation as a function of the number of processes.