HPC4M - Assignment 2 Report

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Exercise 3 - Heat Equation

For this task we work to implement a numerical scheme to solve the heat equation:

$$u_t = u_{xx} \qquad \text{for } x \in (0,1), t \in (0,T)$$

subject to Dirichelet boundary conditions

$$u(x=0,t) = u(x=1,t) = 0$$

with an initial condition of

$$u(x,t=0) = \sin(2\pi x) + 2\sin(5\pi x) + 3\sin(20\pi x).$$

To do this we use a forward Euler scheme, which when rearranged gives the relation:

$$U_m^{n+1} = U_m^n + \mu\left[U_{m-1}^n - 2U_m^n + U_{m+1}^n\right]$$

where $\frac{\Delta t}{(\Delta x)^2} = \mu$, m and n represent spacing in space and time respectively and U_m^n is the solution at grid point m, n. Careful choice of Δt and Δx must be made to ensure stability, enforced by $\mu < 0.5$.

Solution

To parallelise this computation we make use of 'Halo-Swapping'. This technique involves dividing our space domain into overlapping segements (overlap of 2 points) allowing us to compute all points ina segment bar the edge points in each segment using the above scheme. We can then update the remaining edge points in each segment using the neighbouring segments which we overlap with and their newly computed values for these edge points. In this code we use MPI_Send and MPI_Recv to allow our processes to share information with each other.

The number of total grid points M, processes employed P and size of each segment J must be balanced witht the relation $M = P(J-2) + 2$. ensuring that all values are integers. Here I chose $M \approx 1000$ for a number of different processes (tailorng J to fit the stated relation). From this I found that while initially computation time is decreased for increasing processes, eventually the cost of communicating between the processes outweighs the gains made by the parallesisation.