Part II – Decision Systems for Engineering Design
Lecture 6: Multi-disciplinary design optimization

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1 Lecture overview

This lecture examines the topic of *multi-disciplinary design optimization* (MDO), which consists of a family of methods that can be used to perform decision support for the design of complex engineered products, such as vehicles or critical infrastructure.

In this lecture, we will cover:

- The motivation for using MDO within a decision system;
- A detailed look at a specific MDO architecture known as *collaborative optimization*;
- An engineering design example where collaborative optimization has been used successfully;
- A high-level overview of the MDO architectures available.

2 Motivation for multi-disciplinary design optimization

2.1 The organisation of engineering design activities

The design of a complex engineered product, such as the Boeing Dreamliner or the Jaguar I-PACE, is itself a complex endeavour, involving hundreds of designers distributed across teams both within the product manufacturing company itself and also across the company's supply chain. These design teams may be geographically distributed and only communicate with each other on an infrequent basis. Some teams may use proprietary design and simulation software that is not accessible to the other teams, and will typically be keen to retain their own independence as designers, rather than relinquish design responsibilities to a central authority.

In this setting it is not realistic for an optimization algorithm (such as those we have considered in previous lectures) to be set up as a 'design god' with control of all aspects of design and with access to all the simulations required to evaluate the objectives and constraints. Rather, it is necessary to embed optimization algorithms *within* each design team. Such embedding poses a coordination problem: how do we ensure that all the optimizers are working to a common purpose? MDO architectures have been developed to help answer this question.

Traditionally, owing to its roots in the aerospace industry, MDO has assumed that design teams are organised according to engineering 'disciplines'—e.g. aerodynamics, propulsion, structures, noise and vibration, control systems, weight and cost engineering. In this set up, the disciplines typically 'own' distinct performance criteria (e.g. drag, thrust, strength) but share responsibility for aspects of the design. Meanwhile, other industries (e.g. the automotive industry) tend to organise their teams according to the components that are integrated to form the product—e.g. powertrain, body, brake systems. In this case, the teams tend to have sole responsibility for an aspect of design, but share the ownership of the performance criteria.

MDO, as a concept, is agnostic to the particular type of organisation employed by the lead manufacturer and its supply chain. However particular MDO architectures may be better suited than others to the configuration of teams in a particular setting.

2.2 Aims of MDO

There are three main aims that MDO is trying to deliver, in order of priority from highest to lowest:

- Ensure that the overall design is internally consistent—i.e. that all design teams are using the same assumptions about the shared design variables and interfaces. This is equivalent to saying that the design is, in principle, physically realisable;
- Ensure that all hard constraints are satisfied;
- Achieve optimal performance against the objectives.

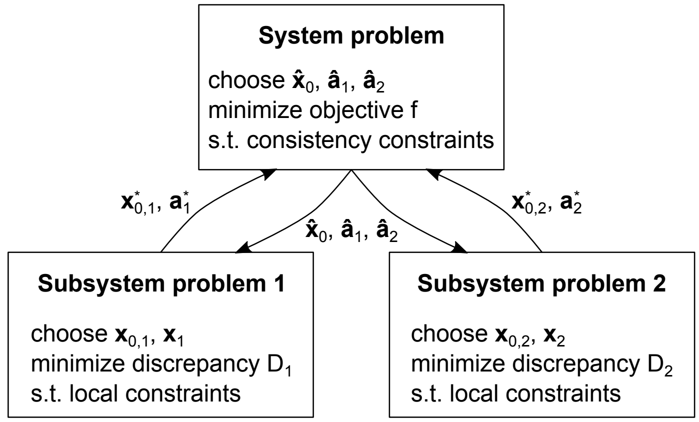

Figure 1: Collaborative optimization architecture

3 Collaborative Optimization

3.1 Design scenario

Consider a generic engineering design problem with two design teams, $i = 1$ and $i = 2$. We will assume that each design team is responsible for its own local design variables, $\mathbf{x}_1$ and $\mathbf{x}_2$ respectively, and that there are additional global design variables $\mathbf{x}_0$ that are *shared* by the teams (for which the teams need to agree on common choices).

We will further assume that each design team has a number of inequality constraints that its local designs need to satisfy, i.e. $\mathbf{g}_{1,2}(.) \leqslant 0$, and that there is an overall objective $f(.)$ that the integrated global design is aiming to minimize.

Finally, we assume that, in addition to the *coupling* between the teams introduced by the shared design variables $\mathbf{x}_0$, there is further coupling relating to parameters needed by the teams to evaluate their constraints and also the overall objective—i.e. that team 1 produces interdisciplinary information $\mathbf{a}_1$ needed by team 2, and that team 2 produces interdisciplinary information $\mathbf{a}_2$ needed by team 1.

Note that, for simplicity, we are considering here only two design teams; however, the following description is generalisable to problems with greater than two teams.

3.2 Architecture

Collaborative Optimization (CO) is a popular architecture that attempts to deliver the three aims of MDO described in Section 2.2. CO defines a subsystem problem for each design team and a further system problem that coordinates the subsystems. A schematic of the architecture is given in Figure 1.

3.2.1 System problem

The CO system problem attempts to minimize the objective $f(.)$ whilst maintaining a consistent overall design. To do this it defines *targets* or *master copies* of all the coupled variables in the design—i.e. the shared design variables $\mathbf{x}_0$ and the interdisciplinary coupling variables $\mathbf{a}_{1,2}$ between the subsystems. These targets are denoted using circumflex (ˆ) notation:

- $\hat{\mathbf{x}}_0$ – targets for shared design variables;
- $\hat{\mathbf{a}}_1$ – targets for interdisciplinary coupling variables produced by subsystem 1;
- $\hat{\mathbf{a}}_2$ – targets for interdisciplinary coupling variables produced by subsystem 2.

We will also now introduce star notation to indicate the choices presently made by each subsystem and the corresponding results of subsystem analyses:

- $\mathbf{x}^\star_{0,1}$ – choice made by subsystem 1 for the global design variables;
- $\mathbf{x}^\star_{0,2}$ – choice made by subsystem 2 for the global design variables;
- $\mathbf{x}^\star_1$ – choice made by subsystem 1 for its local design variables;
- $\mathbf{x}^\star_2$ – choice made by subsystem 2 for its local design variables;
- $\mathbf{a}^\star_1$ – results of disciplinary analysis of subsystem 1 using $\mathbf{x}^\star_{0,1}$, $\mathbf{x}^\star_1$ and $\hat{\mathbf{a}}_2$;
- $\mathbf{a}^\star_2$ – results of disciplinary analysis of subsystem 2 using $\mathbf{x}^\star_{0,2}$, $\mathbf{x}^\star_2$ and $\hat{\mathbf{a}}_1$.

To evaluate the consistency of the subsystems with respect to the global problem, CO defines a set of equality constraints, known as *consistency constraints* based on *discrepancy functions* that calculate the squared Euclidean distance between the subsystem choices/results, and the system-level targets. The overall system problem formulation is therefore given by:

$$\begin{aligned} \underset{\hat{\mathbf{x}}_0,\hat{\mathbf{a}}_1,\hat{\mathbf{a}}_2}{\text{minimize}} \quad & f(\hat{\mathbf{x}}_0, \hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2) \\ \text{subject to} \quad & c_1(\hat{\mathbf{x}}_0, \hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2) = 0 \\ & c_2(\hat{\mathbf{x}}_0, \hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2) = 0 \end{aligned} \tag{1}$$

where:

$$\begin{aligned} c_1(\hat{\mathbf{x}}_0, \hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2) &= \frac{1}{2}\left(||\mathbf{x}^\star_{0,1} - \hat{\mathbf{x}}_0||^2 + ||\mathbf{a}^\star_1 - \hat{\mathbf{a}}_1||^2\right) \\ c_2(\hat{\mathbf{x}}_0, \hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2) &= \frac{1}{2}\left(||\mathbf{x}^\star_{0,2} - \hat{\mathbf{x}}_0||^2 + ||\mathbf{a}^\star_2 - \hat{\mathbf{a}}_2||^2\right) \end{aligned} \tag{2}$$

3.2.2 Subsystem problems

First consider subsystem 1. In CO, the subsystem is tasked with minimizing inconsistencies with respect to the system problem concerning: (a) its choices for shared design variables $\mathbf{x}_0$ (denoted $\mathbf{x}_{0,1}$); and (b) its interdisciplinary coupling variables $\mathbf{a}_1$. Further, the subsystem must also satisfy its own local constraints $\mathbf{g}_1(.)$. The subsystem is free to make any choices for its local design variables $\mathbf{x}_1$, subject to these considerations. When performing its disciplinary analyses, subsystem 1 uses the copy of subsystem 2's interdisciplinary coupling variables $\hat{\mathbf{a}}_2$ supplied by the system node.

The problem formulation for subsystem 1 can be expressed as:

$$\begin{aligned} \underset{\mathbf{x}_{0,1},\mathbf{x}_1}{\text{minimize}} \quad & D_1 = \frac{1}{2}\left(||\mathbf{x}_{0,1} - \hat{\mathbf{x}}_0||^2 + ||A_1(\mathbf{x}_{0,1},\mathbf{x}_1,\hat{\mathbf{a}}_2) - \hat{\mathbf{a}}_1||^2\right) \\ \text{subject to} \quad & \mathbf{g}_1(\mathbf{x}_{0,1},\mathbf{x}_1,A_1(\mathbf{x}_{0,1},\mathbf{x}_1,\hat{\mathbf{a}}_2)) \leqslant \mathbf{0} \end{aligned} \tag{3}$$

where $A_1(.)$ is the disciplinary analysis for subsystem 1 that provides $\mathbf{a}_1$, together with any other variables needed to evaluate the local constraints (for notational simplicity, these additional variables are not shown). A solution to the subsystem 1 problem, consisting of variable settings $\mathbf{x}^\star_{0,1}$ and $\mathbf{a}^\star_1$, is returned to the system node.

There is an analogous problem for the second subsystem. Its problem formulation can be expressed as:

$$\begin{aligned} \underset{\mathbf{x}_{0,2},\mathbf{x}_2}{\text{minimize}} \quad & D_2 = \frac{1}{2}\left(||\mathbf{x}_{0,2} - \hat{\mathbf{x}}_0||^2 + ||A_2(\mathbf{x}_{0,2},\mathbf{x}_2,\hat{\mathbf{a}}_1) - \hat{\mathbf{a}}_2||^2\right) \\ \text{subject to} \quad & \mathbf{g}_2(\mathbf{x}_{0,2},\mathbf{x}_2,A_2(\mathbf{x}_{0,2},\mathbf{x}_2,\hat{\mathbf{a}}_1)) \leqslant \mathbf{0} \end{aligned} \tag{4}$$

where $A_2(.)$ is the disciplinary analysis for subsystem 2. Again, a solution to the subsystem 2 problem, consisting of variable settings $\mathbf{x}^\star_{0,2}$ and $\mathbf{a}^\star_2$, is returned to the system node.

3.3 Process

CO is a bi-level architecture, in which the system is the upper-level problem and the subsystems are lower-level problems. There is therefore a requirement to solve the subsystem problems for every candidate set of targets generated by the system problem. The sequencing works as follows:

1. Initialise the system iteration counter $t_{\text{sys}} = 0$
2. Initialise candidate target variables $\hat{\mathbf{x}}_0^{(t_{\text{sys}}=0)}$, $\hat{\mathbf{a}}_1^{(t_{\text{sys}}=0)}$ and $\hat{\mathbf{a}}_2^{(t_{\text{sys}}=0)}$
3. Execute iteration of the system problem:
   (a) For each subsystem i, solve the subsystem problem:
      i. Initialise the subsystem iteration counter $t_{\text{sub},i} = 0$

ii. Initialise candidate design variables $\mathbf{x}_{0,i}^{(t_{\text{sub},i}=0)}$ and $\mathbf{x}_{i}^{(t_{\text{sub},i}=0)}$

iii. Execute iteration of the subsystem problem:

A. Perform the disciplinary analysis to generate $\mathbf{a}_i^{(t_{\text{sub},i})}$

B. Evaluate the discrepancy function D_i and local constraints $\mathbf{g}_i(.)$

C. Generate new candidate design variables $\mathbf{x}_{0,i}^{(t_{\text{sub},i}+1)}$ and $\mathbf{x}_{i}^{(t_{\text{sub},i}+1)}$

D. Increment the subsystem iteration counter $t_{\text{sub},i} = t_{\text{sub},i} + 1$

E. If a convergence criterion is satisfied then stop the subsystem solver and return $\mathbf{x}_{0,i}^{\star} = \mathbf{x}_{0,i}^{(t_{\text{sub},i})}$ and $\mathbf{a}_i^{\star} = \mathbf{a}_i^{(t_{\text{sub},i})}$; otherwise go to A

(b) Evaluate the system objective $f(.)$ and consistency constraints $\mathbf{c}(.)$

(c) Generate new candidate target variables $\hat{\mathbf{x}}_0^{(t_{\text{sys}}+1)}$, $\hat{\mathbf{a}}_1^{(t_{\text{sys}}+1)}$ and $\hat{\mathbf{a}}_2^{(t_{\text{sys}}+1)}$

(d) Increment the system iteration counter $t_{\text{sys}} = t_{\text{sys}} + 1$

(e) If a convergence criterion is satisfied then stop the system solver; otherwise go to 3.

3.4 Strengths and weaknesses

A key strength of CO is that its decomposition reflects the organisational structures used by many engineering design companies, and this has been reflected in its use across many types of engineering design problem. A further, related, strength is the high degree of autonomy afforded to individual design teams in determining their own local designs, subject to coordination with the wider system.

The core weakness of CO is its use of equality constraints in the system problem formulation. In general, optimization algorithms struggle to satisfy equality constraints without expending a large number of function evaluations. This aspect of CO has meant that the architecture is often regarded as inefficient. To address this issue, two main measures have been proposed:

- *Relaxation of the equality constraint*, through the introduction of a small threshold. The threshold may be progressively reduced as the optimization process unfolds. The use of a threshold has been argued to be appropriate for many engineering problems, since modelled values are rarely realised precisely in practice.

- *Use of surrogate models* to approximate the disciplinary analyses or entire discrepancy functions. The surrogate models are low-complexity approximations of the often high-fidelity simulation models used for analysis. If these surrogates are sufficiently cheap and accurate, they can prevent many computationally expensive evaluations that would otherwise be required by the architecture.

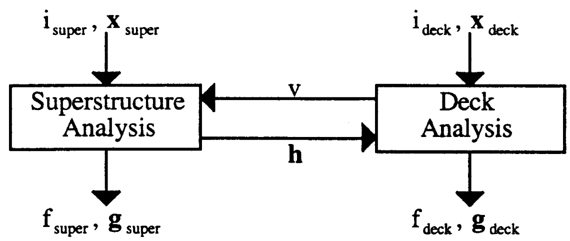

Figure 2: Bridge design problem: Design variables, objectives, constraints, and interdisciplinary couplings between the subsystems. Taken from Balling & Rawlings' *Collaborative optimization with disciplinary conceptual design*, 2000.

4 Example

4.1 Overview of the design problem

To illustrate the application of collaborative optimization, consider the design of a long-span bridge. The organisation tasked with designing the bridge operates with two design teams:

1. The *superstructure* team, who will design the towers and supporting cables for the bridge;
2. The *deck* team, who will design the deck of the bridge.

The decision problem is to find a consistent design for the bridge which minimizes the build cost, subject to constraints on the strength of the bridge. In this example, the two design teams do not share any design variables, but they do have interdisciplinary couplings resulting from the forces that the deck exerts on the superstructure and vice versa.

The overall configuration is shown in Figure 2; the notation used in the figure is introduced during the following discussion.

4.2 Subsystem 1: Superstructure

4.2.1 Shared design variables $\mathrm{x}_{0,1}$

The problem has no shared design variables, so $\mathrm{x}_{0,1} = \emptyset$.

4.2.2 Local design variables x_1

The local design variables for the superstructure team are, collectively, the choice of cabling concept (fan, harp, semi-harp, and suspension) i_{super}, together with the cable diameters and tower geometry $\mathrm{x}_{\text{super}}$. These variables are shown in Figure 3.

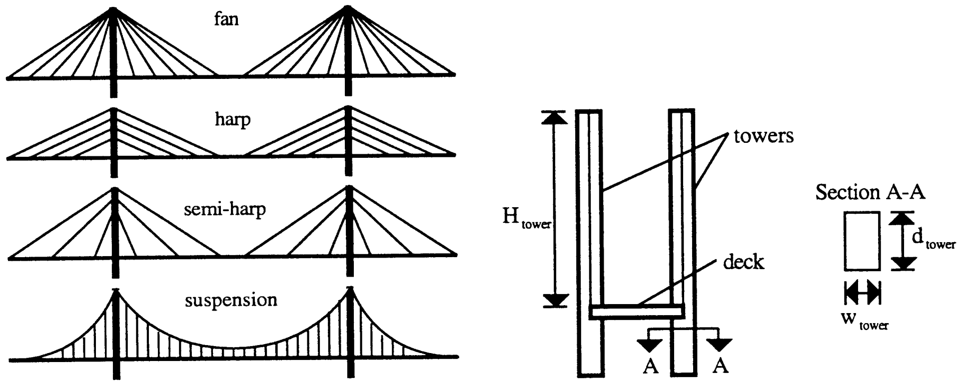

Figure 3: Bridge design problem subsystem 1: Superstructure design concepts and associated design variables. Taken from Balling & Rawlings' *Collaborative optimization with disciplinary conceptual design*, 2000.

4.2.3 Interdisciplinary coupling variables $\mathbf{a}_1$ and disciplinary analysis $A_1(\mathbf{x}_{0,1}, \mathbf{x}_1, \hat{\mathbf{a}}_2)$

The superstructure generates a horizontal force on the deck at the location of each cable. Since the cables may incline by different amounts, a vector coupling variable (denoted $\mathbf{h}$) is needed to represent the force at each cable location. To work out the horizontal forces, the superstructure needs to know the vertical force (denoted v) exerted on the cables due to the mass of the deck. These forces therefore depend on decisions taken in the deck subsystem and consequently, according to the CO architecture, are provided to the superstructure subsystem as a copy $\hat{\mathbf{a}}_2$—denoted here as v'.

4.2.4 Discrepancy function D_1 and local constraints $\mathbf{g}_1(.)$

According to CO, the superstructure subsystem should aim to minimize a discrepancy function d_{super} based on the deviation of the shared variables and interdisciplinary coupling variables from targets. The bridge example departs slightly from the standard CO formulation here by having the subsystem also attempt to meet a target for build cost, in addition to considering how far the horizontal forces are from target. The build cost of the superstructure acts as a local objective, which needs to be calculated as part of the disciplinary analysis.

Subsystem 1 also has local inequality constraints, denoted $\mathbf{g}_{\text{super}}(.)$, to be satisfied, relating to requirements for axial/flexural and shear strengths at the base of the tower, and also cable strength, under two load cases.

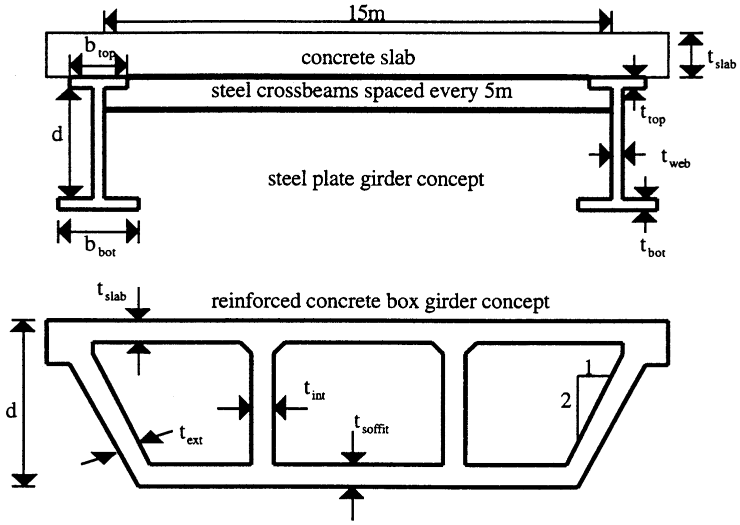

Figure 4: Bridge design problem subsystem 2: Deck design concepts and associated design variables. Taken from Balling & Rawlings' *Collaborative optimization with disciplinary conceptual design*, 2000.

4.3 Subsystem 2: Deck

4.3.1 Shared design variables $\mathbf{x}_{0,2}$

The problem has no shared design variables, so $\mathbf{x}_{0,2} = \varnothing$.

4.3.2 Local design variables $\mathbf{x}_2$

The deck subsystem has local design variables relating to the choice of deck concept i_{deck}—steel plate girder or reinforced concrete box girder—and corresponding geometry variables $\mathbf{x}_{\text{deck}}$. The deck design variables are shown in Figure 4.

4.3.3 Interdisciplinary coupling variables $\mathbf{a}_2$ and disciplinary analysis $A_2(\mathbf{x}_{0,2}, \mathbf{x}_2, \hat{\mathbf{a}}_1)$

As mentioned in Section 4.2.3, the deck generates vertical force on the superstructure. The force is equal across the superstructure, due to the prismatic geometry of the deck and (pre-determined) equal spacing of the cables, so can be represented as a scalar—here denoted v. The deck will also be subjected to horizontal forces that depend on cable angles arising from the chosen cabling concept. Since the choice of concept is unknown to the deck team, these forces are provided as copies $\hat{\mathbf{a}}_1$—here denoted as $\mathbf{h}'$.

The disciplinary analysis provides the vertical force v, together with the other forces needed to evaluate the local constraints.

4.3.4 Discrepancy function D_2 and local constraints $\mathbf{g}_2(.)$

The deck subsystem attempts to minimize the deviation of its interdisciplinary coupling variable v from a target. Like the superstructure subsystem, it also attempts to minimize deviation from a target build cost, which places a requirement on the disciplinary analysis to compute the build cost of the deck. The overall discrepancy function is denoted d_{deck}.

In the deck subsystem, local constraints $\mathbf{g}_{\text{deck}}(.)$ relate to a variety of flexural strengths, shear strengths and deflections.

4.4 System problem

The system problem for the bridge manages the targets for the interdisciplinary coupling variables: $\mathbf{h}'$ and v'. There are no shared design variables to manage, and the system problem does not concern itself with the subsystem design variables $\mathbf{x}_{\text{super}}$ and $\mathbf{x}_{\text{deck}}$, these being left to the individual teams.

Unlike in a conventional CO architecture, the system node in this example does not directly evaluate the objective to be minimized—in this case, total build cost for the bridge. Rather, it decomposes the total cost into targets for the superstructure and deck subsystems to meet, given by f'_{super} and f'_{deck} respectively. This approach is close in spirit to the MDO architecture known as *analytical target cascading* (ATC) that is used widely in the automotive industry.

The system problem aims to minimize the sum of the subsystem build cost targets, i.e. $f'_{\text{super}} + f'_{\text{deck}}$. To produce a consistent bridge design, the discrepancy terms must be reduced to zero in each of the subsystems. The system problem therefore includes equality constraints $d_{\text{super}} = 0$ and $d_{\text{deck}} = 0$.

4.5 Special considerations

The decision to optimize build cost through the use of cost targets in the subsystems has knock-on consequences for the CO architecture. Since it is no longer possible for the system node to know whether a positive discrepancy term reflects failure to meet the consistency target or failure to meet the cost target, the architecture may not converge on a consistent overall design. To mitigate this issue, each subsystem is allowed to deviate from both its own interdisciplinary coupling variable copy *and* the copy of the coupling variable for the other subsystem. For example, the superstructure problem includes a variable v'' that represents its own assumption concerning the vertical force v. Deviation from the copy is then also included in the discrepancy function—i.e. by including the term $(v' - v'')^2$ in d_{super}. Another option, used in ATC, would be to incorporate penalty weights for the different terms of the discrepancy equation, where the weights are gradually updated to emphasise the consistency term as the optimization process progresses.

To reduce the number of coupling variables in the problem, the vector of horizontal forces $\mathbf{h}$ can be replaced with a parameterised function based on distance of the cable attachment from the tower, which requires only two variables rather than the

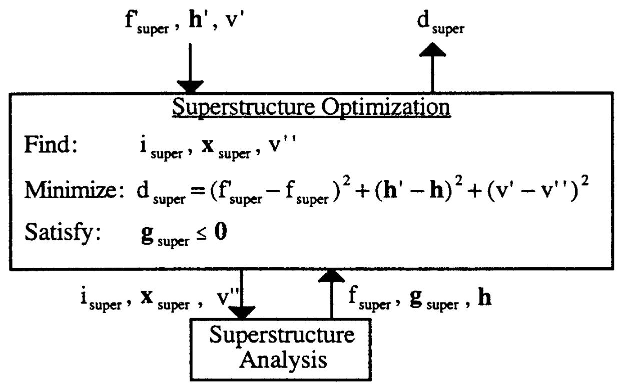

Figure 5: Modified version of the superstructure subsystem problem. Taken from Balling & Rawlings' *Collaborative optimization with disciplinary conceptual design*, 2000.

10-element force vector. Use of such parameterised functions is a common way of reducing the complexity of the interactions in an MDO problem.

5 Overview of MDO architectures

There are a large number of MDO architectures available. They can be classified into two fundamental types, which vary in the approach used for dealing with shared design variables and interdisciplinary coupling variables: *individual discipline feasible* (IDF) MDO and *multidisciplinary feasible* (MDF) MDO. The specific choice of MDO architecture will ultimately depend on how the design teams have been organised and the amount of computing resource that is available to each team.

5.1 Individual discipline feasible architectures

Individual discipline feasible architectures use copies of the coupling variables and propagate these copies to the individual teams, who are working on their designs in parallel. Over time, the architectures work towards delivering an overall design that is fully consistent (or *feasible*, from the perspective of the physics that is being modelled and any shared aspects of the design)—i.e. the copies are updated over the course of the optimization process until all teams agree on their values. A disadvantage of the IDF approach is that, if the optimization process is terminated early, then there is no guarantee that the current candidate design will be consistent, or consistently modelled, across teams—with potentially serious consequences for the validity of the objective and constraints assessments.

Collaborative optimization is an example of an IDF architecture. Other examples include analytical target cascading (mentioned in Section 4.4) and *quasi-separable decomposition* (QSD).

5.2 Multidisciplinary feasible architectures

Multidisciplinary feasible architectures impose (or attempt to impose) consistent coupling variables at each stage of the optimization process. An advantage of this approach is that consistent designs are generated by the architecture, even if the optimization process is terminated earlier than anticipated.

Classical MDF approaches organise the disciplinary analyses in a strict sequence, where—given a fixed design—coupling variables are fed from the output of one team to the input of the next. This sequence is iterated until the teams agree on a consistent set of coupling variables. Since the work of real-world engineering design teams tends not to be organised according to such a sequential schedule, more modern MDF approaches allow for more concurrent working. This is achieved by using surrogate models within each design team that aim to predict the interdisciplinary couplings arising from other teams' analyses *in response to* the choices/analyses of the design team. In this way, the design team does not have to wait for the 'true' interdisciplinary coupling variable inputs before getting on with their work. Clearly, the success of these methods will depend on the accuracy of the predictions made by the surrogate model.

Examples of MDF architectures include *concurrent subspace optimization* (CSSO), *bilevel integrated system synthesis* (BLISS), and *asymmetric subspace optimization* (ASO).

6 Further reading

- Martins J.R.R.A., Lambe A.B. Multidisciplinary design optimization: A survey of architectures. *AIAA Journal* 2013;51:2049–2075.
- Balling R., Rawlings M.R. Collaborative optimization with disciplinary conceptual design. *Structural and Multidisciplinary Optimization* 2000;20:232–241.