

CSC 372, Spring 2025

# High-order functions and scope

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# Plan

- **Announcements**

- SA3 MT1 review is posted and due Wednesday Feb 12th
- Simon Peyton Jones joining us via zoom on Thursday Feb 13th
- LA1 parser/shapes2svg is posted and due Friday Feb 14<sup>th</sup>

- **Last time**

- TopHat Questions from ICA4 and ICA5
- Recursive descent parsing
- Polymorphism in SML

- **Today**

- Questions for Simon
- High order functions
- Moved scope to last third of class when covering Chapel

# TopHat Questions

- 10 minutes of a Simon video,  
<https://simon.peytonjones.org/darwin-codes/>, starting at game  
of life 28:15 to 38
- Questions to ask Simon
- Some terminology Simon uses in assigned podcast

# Outline for rest of today

- Polymorphic Types in SML
- More SML info
- High-order functions in SML

# Different kinds of Polymorphism

- **Ad-hoc Polymorphism (Overloading & Coercion)**
  - Function/operator overloading (e.g., + for ints and reals).
  - Implicit type conversions (e.g., int to float in C/C++).
- **Parametric Polymorphism**
  - Functions and data structures operate on any type.
  - Example: fun identity  $x = x$  in SML ('a -> 'a)
  - Implemented as generics in languages like Java and Rust.
- **Subtype Polymorphism (Inheritance and Subtyping)**
  - Objects of a class can be used where a superclass is expected.
  - Enables method overriding and dynamic dispatch.
  - Example: Animal superclass, Dog subclass.

# Pattern Matching with Polymorphism

```
datatype 'a tree = Child
                | Parent of 'a * 'a tree * 'a tree

fun inOrder Child = []
  | inOrder
```

- Questions

- Finish the rest of the above?
- How is polymorphism different than overloading?

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# Tuple Pattern Matching

- Question

- Indicate what the variables in the pattern will be bound to.

```
val (x,y) = (1,2) (* answer: x=1, y=2 *)
```

```
val (n,xs) = (3,[1,2,3]) (* answer: n=3, xs=[1,2,3] *)
```

```
val (x::xs) = [1,2,3] (* answer: x=1, xs=[2,3] *)
```

```
val (_::xs) = [1,2,3] (* answer: xs=[2,3] *)
```

```
val (_::xs) = [3] (* answer: xs=[] *)
```



# Case Expressions also use pattern matching

- **case expression**

```
fun length xs =  
  case xs  
  of []          => 0  
   | (x::xs)    => 1 + length xs
```

- **At top level, fun is better than case**

```
fun length []      = 0  
  | length (x:xs) = 1 + length xs
```

# Case works for any datatype

- At top level, **fun** is better than **case**

```
fun toStr t =  
  case t  
    of Leaf => "Leaf"  
       | Node(v,left,right) => "Node"
```

- Question: how do we rewrite above case using fun?

```
fun toStr ??
```

- Order of clauses matters

```
fun take n (x::xs) = x :: take (n-1) xs
  | take 0 xs = []
  | take n [] = []
(* what goes wrong? *)
```

- Gotcha - overloading

```
- fun plus x y = x + y;
> val plus = fn : int -> int -> int
- fun plus x y = x + y : real;
> val plus = fn : real -> real -> real
```

- Gotcha - equality types, `'a` is equality type var

```
- (fn (x,y) => x=y);
> val 'a it = fn : 'a * 'a -> bool
```

# Outline for rest of today

- Polymorphic Types in SML
- More SML info
- **High-order functions in SML**

# Higher-Order Functions

- Goal: start with functions on elements, end up with functions on lists
  - Generalizes to sets,
  - arrays,
  - search trees,
  - hash tables, ...
- Goal: Capture common patterns of computation or algorithms
  - `exists` (example: is there a number?)
  - `all` (example: is everything a number?)
  - `filter` (example: take only the numbers)
  - `map` (example: add 1 to every element)
  - `foldr` (general: can do all of the above and more)

## List search: **exists**

- Algorithm encapsulated: linear search
- Example: Is there an even element in the list?

```
fun exists p [] = ???
```

```
fun exists p (x::xs) = ???
```

```
(* What are some example calls to exists? *)
```

## List search: `all`

- Algorithm encapsulated: linear checking
- Example: Is every element in the list even?

```
fun all p [] = ???
```

```
fun all p (x::xs) = ???
```

```
(* What are some example calls to all? *)
```

## List search: **filter**

- Algorithm encapsulated: linear filtering
- Example: Given a list of numbers, return only the even ones

```
fun filter p [] = ???
```

```
fun filter p (x::xs) = ???
```

```
(* What are some example calls to filter? *)
```

- What are the restrictions on **p** for **exists**, **all**, and **filter**?



# Defining filter

```
-> fun filter p [] = []  
    | filter p (x::xs) =  
        if p x then x :: filter p xs  
        else filter p xs;
```

```
-> val test1 = filter (fn n => n > 0) [1, 2, ~3,  
~4, 5];  
??
```

```
-> val test2 = filter (fn n => n <= 0) [1, 2,  
~3, ~4, 5];  
??
```

## List search: **map**

- “Lifting” functions to lists
- Algorithm encapsulated: transform every element
- Example: square every number of a list

```
fun map f [] = ???
```

```
fun map f (x::xs) = ???
```

# Defining map

```
-> fun map f [] = []  
    | map f (x::xs) = f x :: map f xs;  
  
-> val test1 = map (fn n => n*n) [1, 2, ~3, ~4,  
5];  
??  
  
-> val test2 = map (fn n => n mod 2) [1, 2, ~3,  
~4, 5];  
??
```

# **foldr**: the universal list function

- **foldr** takes two arguments
  - `plus`: how to combine elements with running results
  - `zero`: what to do with the empty list
- **Example: foldr plus zero [a b]**

<b>a</b>	<b>::</b>	<b>b</b>	<b>::</b>	<b>[]</b>
<b>v</b>		<b>v</b>		<b>v</b>
<b>plus</b>	<b>a</b>	<b>(plus</b>	<b>b</b>	<b>zero)</b>

# The universal list function: fold

```
-> fun foldr plus zero [] = zero
    |   foldr plus zero (x::xs) =
        plus x (foldr plus zero xs);
```

```
-> val sum = foldr (fn x => fn y => x+y) 0 [1,
2, 3, 4];
??
```

```
(* How is this different than the builtin foldr?
*)
```

# Studying for the midterm

- Implement each of the following using foldr
  - exists
  - all
  - filter
  - map
- Feel free to post possible answers on piazza