An Introduction to Categorical Computing with Catlab

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Welcome

- This course will provide an introduction to core functionality for Catlab
- We will make reference to some other projects in AlgebraicJulia that build atop Catlab
- Our coverage will make use of scripts available here: https://github.com/UofS-CEPHIL/CatCompCMA2025
- AlgebraicJulia Installation document: https://www.cs.usask.ca/faculty/ndo885/AlgebraicJuliaInstallation.pdf

Topics Covered

- FinSet & FinFunctions, identity & compositions of them
- FinRel, relations, identity & compositions of them;
- Definition of & presentation of a category from generators & relations; working with the category
- Defining schemas
- Defining instances of C-Sets with @acset and @acset_colim
- Category of elements visualization
- Homomorphisms between C-Sets, and visualization of graph homomorphisms as examples
- Computing with universal constructions (Product, Coproduct, Pushout, pullback, Equalizer, coequalizer)
- Attributes & ACSets
- As time allows: Modular structures with structured cospans, Rewriting (no attributes)

Protocol

- Questions are welcome throughout the mini-course
- Please feel free to just ask a short clarifying point
- If you'd like to ask a more substantive question, raising hands your hand is recommended, so that the instructor can finish up any loose ends prior to answering the questions
- Absent any participant concerns, the coverage will be recorded

What is Catlab?

- Catlab is a categorical computing platform
- Catlab is based on the programming language Julia, and exploits Julia's features for domain specific languages and type-based specialization
- Catlab provides the foundation for the AlgebraicJulia ecosystem
- Catlab offers abstractions for many categorical concepts
- The core of Catlab is based on Gatlab

GATLab: Generalized Algebraic Theories & Models Thereof

- Supports diverse doctrines (ThCategory, ThSchema, ThMonoid, ThGroupoid, ThCopresheaf, ThPresheaf, FreeSymmetricMonoidalCategory, ThPointedSetCategory, etc.)
- Can create a theory within a doctrine
- Can create models of these theories that adhere to the propertiies

Example Categorical Abstractions in Catlab

- Categories (Objects, morphisms, relations)
- Presentations of categories
- C-Sets (copresheaves)
- ACSets (attributed C-Sets)
- Slice categories
- Functors
- Natural transformations between ACSets (ACSetTransformations)
- Various set-like categories (FinSet, SkelFinSet, FinRel, …)
- Various universal constructions (product, coproduct, pullbacks, pushouts, equalizers, coequalizers, etc.)
- Preorders
- Lattices
- Undirected & directed wiring diagrams
- Semi-simplicial sets & complexes
- Monoidal categories & symmetric monoidal categories
- Structured cospans
- Functorial data migrations

Example Categorical Operations in Catlab

- Composing morphisms
- Computing homomorphisms
- Computing representables
- Taking category of elements
- Computing universal constructions (product, coproduct, pullbacks, pushouts, equalizers, coequalizers, etc.)
- Performing categorical rewriting
- Assessing whether a morphism is monic, epic, etc.
- Assessing whether a transformation is natural

Catlab Resources

- Catlab GitHub site: https://github.com/AlgebraicJulia/Catlab.jl
- Catlab documentation: https://algebraicjulia.github.io/Catlab.jl/dev/
- AlgebraicJulia Repo https://github.com/AlgebraicJulia
- AlgebraicJulia information (a bit dated): https://www.algebraicjulia.org/
- Testing code for different structures e.g.,
  - https://github.com/AlgebraicJulia/Catlab.jl/blob/d07b6fc4f8c95feb52b9cc16febd49e714acb540/test/theories/Category.jl
  - https://github.com/AlgebraicJulia/Catlab.jl/blob/d07b6fc4f8c95feb52b9cc16febd49e714acb540/test/theories/Schema.jl
- Catlab channel of Julia zulip https://julialang.zulipchat.com/
- Category Theory zulip https://categorytheory.zulipchat.com/

Some AlgebraicJulia Projects

- AlgebraicRewriting.jl
- AlgebraicDynamics.jl
- StockFlow.jl
- Statecharts.jl
- Decapodes.jl
- ACSets.jl
- CombinatorialSpaces.jl
- SemiringFactorizations.jl
- DiagrammaticEquations.jl
- AlgebraicABMs.jl*
- AlgebraicPetri.jl*