

$$\begin{array}{ccc}
 P_A & \xrightarrow{\tilde{f}} & P_B \\
 \downarrow r_A & & \downarrow r_B \\
 A & \xrightarrow{f} & B
 \end{array}$$