$\operatorname{Hom}(P, I) \longleftarrow \operatorname{Hom}(A, I)$ with the both being quasi-isomorphisms because P is K-projective and I is K-injective. $\operatorname{Hom}(P,B)$