(1) Let $\phi : X \to X'$ be an isomorphism in a category $\mathcal{C}$. Show that

$$\mathrm{Hom}(X', Y) \to \mathrm{Hom}(X, Y)$$
$$\psi \mapsto \psi \circ \phi$$

is a bijection. Show that it induces an isomorphism $h_X \cong h_{X'}$.

(2) In the proof that a fully-faithful and essential surjective functor is an equivalence, we constructed what we claimed was the inverse. Show that it is well-defined and indeed the inverse functor.

(3) Prove that $F \vdash G$ if and only if there exists $\eta : \mathrm{Id} \to GF$ and $\eta : FG \to \mathrm{Id}$ satsifying

$$F \xrightarrow[F(\eta)]{} FGF \xrightarrow[\epsilon_F]{} F$$

with the arc labeled 1 over the composite.

and

$$G \xrightarrow[\eta_G]{} GFG \xrightarrow[G(\epsilon)]{} G$$

with the arc labeled 1 over the composite.

(4) Complete the proof of tensor-hom adjunction. Show that the claimed inverse natural transformation is well-defined and indeed the inverse.