HOMEWORK 1

(1) Here is another (gruesome) logic puzzle by George J. Summers, called "Murder in the Family."

Murder occurred one evening in the home of a father and mother and their son and daughter. One member of the family murdered another member, the third member witnessed the crime, and the fourth member was an accessory after the fact.
 (a) The accessory and the witness were of opposite gender.
 (b) The oldest member and the witness were of opposite gender.
 (c) The youngest member and the victim were of opposite gender.
 (d) The accessory was older than the victim.
 (e) The father was the oldest member.
 (f) The murderer was not the youngest member.
Which of the four—father, mother, son, or daughter—was the murderer?

Solve this puzzle, and write a *clear* argument using a deduction table to demonstrate your reasoning.

(2) Using the mnemonic F (Father), M (Mother), D (Daughter), S (Son), K (Murderer), V (Victim), W (Witness), A (Accessory), O (Oldest), Y (Youngest), we can define propositional variables like FK (Father is the murderer), DV (Daughter is the victim), etc. Notice that only the son or daughter can be the youngest, and only the mother or father can be the oldest.

With these conventions, the first clue can be represented as

$$FA \lor SA) \to (MW \lor DW)) \land ((MA \lor DA) \to (FW \lor SW))$$

in other words, if the father or son was the accessory, then the mother or daughter was the witness, and vice-versa. Represent the other five clues in a similar manner.

Representing the fourth clue is tricky. Try to write down a formula that describes all the possibilities that are not ruled out by the information.

(3) Write down a natural deduction proof for $A \land (B \land C) \vdash (A \land B) \land C$.

(4) Write down a natural deduction proof for $Q \vdash (Q \to R) \to R$.

(5) Write down a natural deduction proof for $A \lor B \to B \lor A$.