HOMEWORK

(1) Provide a proof of the following result.

Lemma. Containment is a partial order on sets. In other words,
- For any set X, we have $X \subseteq X$.
- For any sets X, Y, if $X \subseteq Y$ and $Y \subseteq X$, we have $X = Y$.
- For any sets X, Y, Z, if $X \subseteq Y$ and $Y \subseteq Z$, then $X \subseteq Z$.

(2) Let $f : X \to X$ be a function and let $\sim$ be the relation given by the graph $\Gamma_f \subseteq X \times X$. In other words, $x \sim y$ if and only if $y = f(x)$.
- Show that if $\sim$ is reflexive, then $f = \mathrm{id}_X$.
- Show that if $\sim$ is symmetric, then f is an **involution**, $f^2(x) = x$ for all x.
- Show that if $\sim$ is transitive, then f is **idempotent**, $f^2(x) = f(x)$ for all x.