HOMEWORK

(1) Prove that
$$\frac{d^n}{dx^n}(xe^x) = (n+x)e^x$$
for all $n \in \mathbb{N}$.

(2) Let $a, r \in \mathbb{R}$ with $r \neq 1$. Prove that
$$\sum_{k=0}^{n} ar^k = \frac{a(1 - r^{n+1})}{1 - r}$$
for all $n \in \mathbb{N}$.

(3) Let $f : X \to X$ be a function. Prove that for any $m \in \mathbb{N}$ and any $n \geq m$ that
$$f^n(X) \subseteq f^m(X)$$

(4) Prove the following formula for binomial coefficients: If $n \geq k$,
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$