HOMEWORK 3

(1) Give natural deduction proof of the formula

$$(A \leftrightarrow B) \leftrightarrow (A \to B) \wedge (\neg A \to \neg B)$$

(2) Show that $(A \to B) \to (B \to A)$ cannot be proven. Given example statements for A and B where $A \to B$ is true but $B \to A$ is not. Can you make them mathematical statements?

(3) A formula is in *conjunctive normal form* if it is written as

$$X_1 \wedge X_2 \wedge \cdots \wedge X_n$$

where each

$$X_i = Y_i^1 \vee \cdots \vee Y_i^j$$

and each Y_i^l is either $\neg A$ or A for some propositional variable A. Which of the following are in conjunctive normal form:

- $$\neg(A \vee B)$$
- $$\neg A \wedge (B \vee C)$$
- $$\neg A \vee \neg B \wedge C$$
- $$(A \wedge B) \vee C$$

(4) It is a fact that any formula in propositional logic can be rewritten in conjunctive normal form up to bi-implication. Rewrite the following formula in conjunctive normal form up to bi-implication.

- $$A \wedge (B \vee (C \wedge D))$$
- $$\neg(A \wedge (B \vee C))$$
- $$(A \to B) \vee C$$