## Exercise 1 - Taylor Series

Compute the Taylor series expansion up to the second order term for the following multivariate functions around a given point:

(a) $f(x) = 5x^3$ around $x_0 = 1$.
(b) $f(x, y) = x^2 \cdot y^3 + x^2$ around $x_0 = 3$, $y_0 = 2$.
(c) $f(\mathbf{x}) = x_1^3 \cdot x_2 \cdot \log(x_2)$ around $\mathbf{x}_0 = (2, 1)^\top$.
(d) $f(\mathbf{x}) = \sin(x_1) + \cos(x_2)$ around $\mathbf{x}_0 = (-\pi, \pi)^\top$.

## Exercise 2 - Eigenvalues, Eigenvectors

You are given the sets of eigevalues and eigenvectors. Compute the corresponding matrix.

(a) $\lambda_1 = 2$, $\lambda_2 = 3$, $\mathbf{v}_1 = (1, 0)^\top$, $\mathbf{v}_2 = (0, 1)^\top$.
(b) $\lambda_1 = 2$, $\lambda_2 = 3$, $\mathbf{v}_1 = (1, 1)^\top$, $\mathbf{v}_2 = (1, -1)^\top$.

## Exercise 3 - SGD with Momentum

Implement stochastic gradient descent with momentum and apply it to optimize some elementary functions in 1d and 2d.