

Exercise 1 - Variance

Show that for two independent random variables, X, Y and arbitrary $a, b \in \mathbb{R}$, the following equality holds

$$\mathbf{Var}(aX + bY) = a^2 \cdot \mathbf{Var}(X) + b^2 \cdot \mathbf{Var}(Y).$$

Exercise 2 - Variance / Bias Decomposition

Let $D = \{(x_i, y_i) | i = 1 \dots n\}$ be a dataset obtained from the true underlying data distribution P , i.e. $D \sim P^n$. And let $h_D(\cdot)$ be a classifier trained on D . Show the variance bias decomposition

$$\underbrace{\mathbb{E}_{D,x,y} [(h_D(x) - y)^2]}_{\text{Expected test error}} = \underbrace{\mathbb{E}_{D,x} [(h_D(x) - \hat{h}(x))^2]}_{\text{Variance}} + \underbrace{\mathbb{E}_{x,y} [(\hat{y}(x) - y)^2]}_{\text{Noise}} + \underbrace{\mathbb{E}_x [(\hat{h}(x) - \hat{y}(x))^2]}_{\text{Bias}^2}$$

where $\hat{h}(x) = \mathbb{E}_{D \sim P^n} [h_D(x)]$ is the expected regressor over possible training sets, given the learning algorithm \mathcal{A} and $\hat{y}(x) = \mathbb{E}_{y|x} [y]$ is the expected label given x . As mentioned in the lecture, labels might not be deterministic given x . To carry out the proof, proceed in the following steps:

(a) Show that the following identity holds

$$\mathbb{E}_{D,x,y} [h_D(x) - y]^2 = \mathbb{E}_{D,x} [\hat{h}_D(x) - \hat{h}(x)]^2 + \mathbb{E}_{x,y} [\hat{h}(x) - y]^2. \quad (1)$$

(b) Next, show

$$E_{x,y} [\hat{h}(x) - y]^2 = E_{x,y} [(\hat{y}(x) - y)^2] + E_x [(\hat{h}(x) - \hat{y}(x))^2] \quad (2)$$

which completes the proof by substituting (2) into (1).

Exercise 3 - Ensembling

Download the file `exercises06-ensembling.ipynb` from quercus. It contains basic Pytorch code training a classifier on MNIST. Modify that code such that it trains an ensemble of 5-10 neural networks and computes their average prediction once trained.