## **Exercise 1 - Taylor Series**

Compute the Taylor series expansion up to the second order term for the following multivariate functions around a given point:

- $$\begin{split} &(\text{a}) \ \ f(x) = 5x^3 \ \text{around} \ x_0 = 1. \\ &(\text{b}) \ \ f(x,y) = x^2 \cdot y^3 + x^2 \ \text{around} \ x_0 = 3, \ y_0 = 2. \\ &(\text{c}) \ \ f(\mathbf{x}) = x_1^3 \cdot x_2 \cdot \log(x_2) \ \text{around} \ \mathbf{x}_0 = (2,1)^\top. \\ &(\text{d}) \ \ f(\mathbf{x}) = \sin(x_1) + \cos(x_2) \ \text{around} \ \mathbf{x}_0 = (-\pi,\pi)^\top. \end{split}$$

## Exercise 2 - Eigenvalues, Eigenvectors

You are given the sets of eigevalues and eigenvectors. Compute the corresponding matrix.

- $\begin{array}{ll} \text{(a)} \ \ \lambda_1 = 2, \ \lambda_2 = 3, \ \mathbf{v}_1 = (1,0)^\top, \ \mathbf{v}_2 = (0,1)^\top. \\ \text{(b)} \ \ \lambda_1 = 2, \ \lambda_2 = 3, \ \mathbf{v}_1 = (1,1)^\top, \ \mathbf{v}_2 = (1,-1)^\top. \end{array}$

## **Exercise 3 - SGD with Momentum**

Implement stochastic gradient descent with momentum and apply it to optimize some elementary functions in 1d and 2d.