

- Integrals of holomorphic function:

- Given  $f(z)$  a function that describes a closed curve  $\gamma$  and  $I$  is an integral of form:

$$I = \int_{\gamma} \frac{h(z)}{g(z)} dz$$

→ where  $h(z)$  and  $g(z)$  is holomorphic

- if the poles of  $g(z)$  do NOT belong INSIDE the curve  $\gamma$   
→  $I = \emptyset$

- if they do belong and  $p_0$  (pole) is simple (multiplicity 1)

→  $\text{Res} \left\{ \frac{h(z)}{g(z)}, p_0 \right\} = \lim_{z \rightarrow p_0} \left[ \frac{h(z)}{g(z)} \cdot (z - p_0) \right]$

→  $I = \pm 2\pi i \cdot \text{Res} \left\{ \frac{h(z)}{g(z)}, p_0 \right\}$   
( $+$  if  $\gamma$  COUNTERCLOCKWISE  
 $-$  if  $\gamma$  CLOCKWISE)

- If the pole has multiplicity ( $\mu > 1$ ), is complex:

→  $\text{Res} \left\{ \frac{h(z)}{g(z)}, p_0 \right\} = \frac{1}{(\mu-1)!} \cdot \lim_{z \rightarrow p_0} \left\{ \left( \frac{d}{dz} \right)^{\mu-1} \left[ \frac{h(z)}{g(z)} \cdot (z - p_0)^{\mu} \right] \right\}$

• Then if a function has more poles  $\rightarrow$  we have  $k$  "Residui"  $\text{Res}\{p_k\}$

$$\rightarrow I = 2\pi i (\text{Res}\{p_1\} + \dots + \text{Res}\{p_k\})$$

when  $\text{Res}\{ \cdot, p. \}$  are defined as written above.