$$x^2 + 2\sqrt{3}xy - y^2 = 3$$
$$\Rightarrow (x + \sqrt{3}y)^2 - 4y^2 = 3$$

Let $\begin{cases} u = x + \sqrt{3}y \\ v = 2y \end{cases}$

Then $u^2 - v^2 = 3 \qquad (1)$

$$x^2 + y^2 = (u - \frac{\sqrt{3}}{2}v)^2 + \frac{1}{4}v^2 = u^2 - \sqrt{3}uv + v^2 \qquad (2)$$

Transform Eq.(1) into parametric equations:

$$\begin{cases} u = \sqrt{3}\sec t \\ v = \sqrt{3}\tan t \end{cases}$$

Eq.(2) becomes:

$$\frac{3}{\cos^2 t} - \frac{3\sqrt{3}\sin t}{\cos^2 t} + \frac{3\sin^2 t}{\cos^2 t}$$
$$= \frac{3 - 3\sqrt{3}\sin t + 3\sin^2 t}{1 - \sin 2t} \qquad (3)$$

Let $q = \sin t$, Eq.(3) becomes:

$$f(q) = \frac{3 - 3\sqrt{3}q + 3q^2}{1 - q^2}$$
$$f'(q) = \frac{-3\sqrt{3}q^2 + 12q - 3\sqrt{3}}{(1 - q^2)^2}$$

For numerator:

$$\frac{12}{6\sqrt{3}} > 1$$

Roots of the numerator are $q_1 = \frac{1}{\sqrt{3}} < 1$, $q_2 = \sqrt{3} > 1$

Thus $f(q)$ decreases at $(-1, \frac{1}{\sqrt{3}}]$, increases at $(\frac{1}{\sqrt{3}}, 1)$

The minimum of $f(q)$, or $x^2 + y^2$ is $f(\frac{1}{\sqrt{3}}) = \frac{3}{2}$