3D Mesh Classification with Graph Neural Networks

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Introduction

Aim

Being able to correctly classify faces identities.

Methodology

Training Graph Neural Networks.

Data

Using graph representation of 3D meshes.

Geometric Deep Learning - [1]

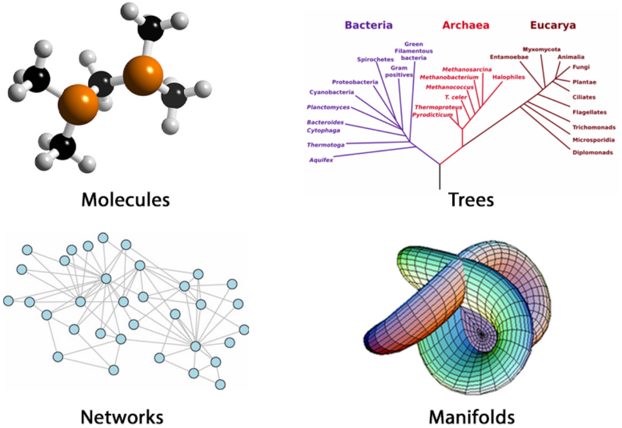


Figure 1: Geometric Deep Learning handles **non-euclidean** data

Graph Neural Network

Neural networks on graphs

- Firstly proposed by Gori *& al* [2] in 2005
- Definitive follow-up [3] in 2009
- **Idea**: use NNs to learn a node representation
- **How**: iteratively approaching the fixed point of a *dynamical system*
- Pro: Theoretical guarantees of convergence
- Con: Inefficient
- Con: Sharing parameters $\implies$ no hierarchical feature extraction
- Con: No edge information

Graph Convolution - 1

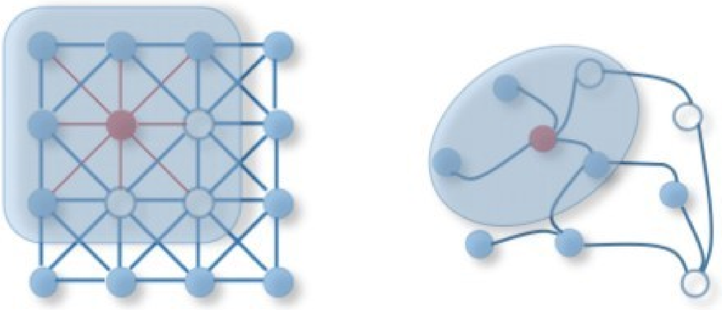


Figure 2: Different approaches for convolutional filters: images (*left*) vs. graphs (*right*)

Graph Convolution - 2

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a graph. Suppose we have a feature vector $\mathbf{x}_v \in \mathbb{R}^d$ for every node $v \in \mathcal{V}$. Given a weight matrix $\mathbf{W} \in \mathbb{R}^{f \times d}$ we define:

$$\mathbf{g}_v = \mathbf{W}\mathbf{x}_v \tag{1}$$

Defining the set of neighbors (usually first-order) of v as $\mathcal{N}_v$, given an *activation function* $\sigma(\cdot)$, the v node hidden representation $\mathbf{h}_v \in \mathbb{R}^f$ is calculated as

$$\mathbf{h}_v = \sigma\left(\sum_{u \in \mathcal{N}_v} \alpha_{vu}\mathbf{g}_u\right) \tag{2}$$

There are many possibilities on the definition of α_{vu}, one of them is interpreting as an attention weight that measures the connective strength between the node v and its neighbor u.

Graph Attention Networks (GAT) - [4]

Inspired by the recent advances of attention models, the authors propose to define the α_{vu} coefficients as:

$$\alpha_{vu} = \mathrm{softmax}_u(e_{vu}) = \frac{\exp\left(e_{vu}\right)}{\sum_{w \in \mathcal{N}_v} \exp\left(e_{vw}\right)} \tag{3}$$

where

$$e_{vu} = a(\mathbf{g}_v, \mathbf{g}_u) = a(\mathbf{W}\mathbf{x}_v, \mathbf{W}\mathbf{x}_u) \tag{4}$$

and $a : \mathbb{R}^f \times \mathbb{R}^f \rightarrow \mathbb{R}$ is a *self-attention* mechanism: a single layer feedforward neural network with weights $\mathbf{a} \in \mathbb{R}^{2f}$. Finally:

$$\alpha_{vu} = \frac{\exp\left(\mathrm{LeakyReLU}\left(\mathbf{a}^\intercal[\mathbf{W}\mathbf{x}_v \| \mathbf{W}\mathbf{x}_u]\right)\right)}{\sum_{w \in \mathcal{N}_v} \exp\left(\mathrm{LeakyReLU}\left(\mathbf{a}^\intercal[\mathbf{W}\mathbf{x}_v \| \mathbf{W}\mathbf{x}_w]\right)\right)} \tag{5}$$

GAT: Multi-Head Attention - [4]

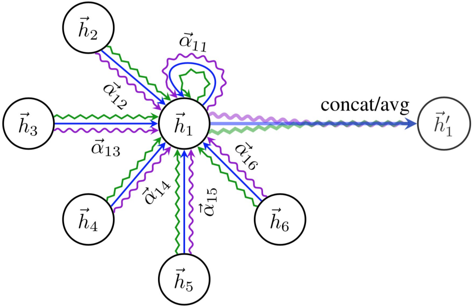


Figure 3: Multi-head attention with $H = 3$

Multi-head attention of order H: H copies (each with different parameters) of Eq. (5) $\rightarrow$ concatenated or averaged.

From Node To Graph Classification

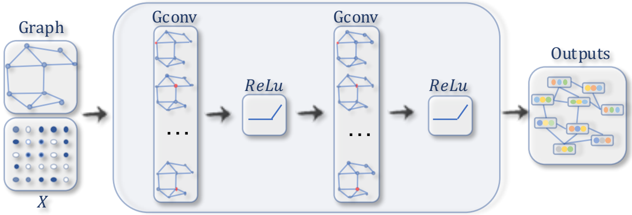


Figure 4: Learning nodes representations

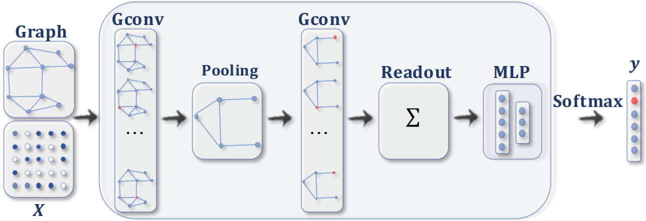


Figure 5: Graph classification

Pooling: Top-k Pooling - [5]

The top-k pooling mechanism has been applied: let $k \in [0, 1]$ and $\mathbf{H} \in \mathbb{R}^{N \times f}$ the feature matrix (with $N = |\mathcal{V}|$). Firstly

$$\mathbf{y} = \frac{\mathbf{H}\mathbf{p}}{\|\mathbf{p}\|_2} \tag{6}$$

is computed where $\mathbf{p} \in \mathbb{R}^f$ is a learnable vector of parameters, then

$$\mathbf{i} = \text{top-}k(\mathbf{y}, k) \tag{7}$$

and finally new features $\mathbf{H}'$ and adjacency matrix $\mathbf{A}'$:

$$\begin{cases} \mathbf{H}' = (\mathbf{H} \odot \sigma(\mathbf{y}))_{\mathbf{i}} \\ \mathbf{A}' = \mathbf{A}_{\mathbf{i},\mathbf{i}} \end{cases} \tag{8}$$

where, usually, $\sigma(\cdot) = \tanh(\cdot)$.

Readout: Global Attention - [6]

Suppose we want to describe our graph $\mathcal{G}$ with a vector $\mathbf{p}_{\mathcal{G}} \in \mathbb{R}^p$. Let $g : \mathbb{R}^f \to \mathbb{R}^p$ a neural network, then a possible solution is

$$\mathbf{p}_{\mathcal{G}} = \sum_{v \in \mathcal{V}} \alpha_v^p g(\mathbf{h}_v) \tag{9}$$

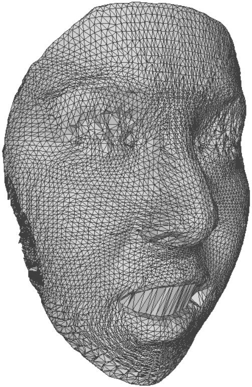
where $\alpha_v^p \in [0, 1]$ is a global attention coefficient for the node v calculated by another neural network f as

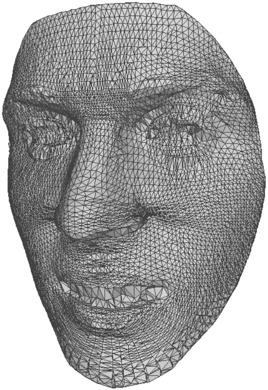
$$\alpha_v^p = \text{softmax}(f(\mathbf{H}))_v \tag{10}$$

where the softmax is applied across nodes, and not across features.

Data: Bosphorus & FRGC

1. Bosphorus [7]:
   - 105 identities
   - 2889 meshes
   - *mesh / id* ≈ 28
   - Every mesh has $\sim$ 6k nodes, $\sim$ 40k edges $\sim$ 13k faces
2. FRGC [8]:
   - 466 identities
   - 3948 meshes
   - *mesh / id* ≈ 8
   - Same structure as Bosphorus





Network Architecture & Hyperparameters

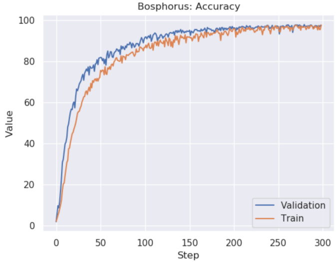
#	Layer
1	$\texttt{GAT}(3, 32, H = 2)$
2	`InstanceNorm`
3	$\texttt{Top-}k$
4	$\texttt{GAT}(64, 128, H = 2)$
5	`InstanceNorm`
6	$\texttt{Top-}k$
7	$\texttt{GAT}(256, 512, H = 2)$
8	`InstanceNorm`
9	$\texttt{GlobalAttention}(512, 512)$
10	`Dropout`
11	$\texttt{Dense}(512, 512)$
12	`Dropout`
13	$\texttt{Dense}(512, 256)$
14	`Dropout`
15	$\texttt{Dense}(256, C)$

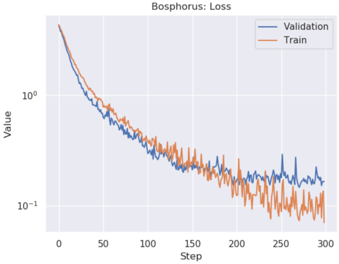
Pre-processing

- Norm vertices to $[0, 1]^3$
- Filtering out identities with number of meshes: $N_m < T_{low} \lor N_m > T_{up}$

- 70% train / 30% test split
- 3-Fold cross validation
- 300 epochs
- Batch size: 16
- Starting learning rate: $\lambda = 10^{-4}$
- Top-k with $k = 0.3$
- Dropout $p = 0.5$

Bosphorus

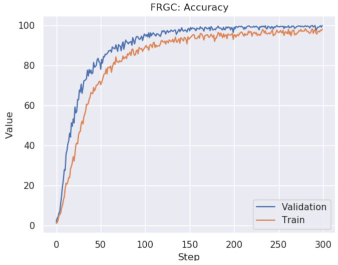


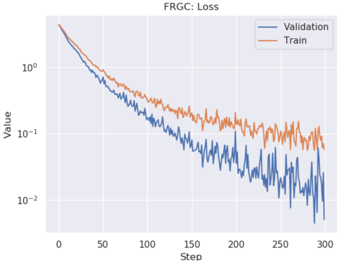


- Filtered out identities with $N_m > 34$ meshes $\rightarrow$ **1294** training and **555** testing examples
- **76** identities left
- $mesh \,/\, id \approx 24$

Dataset	Split	Accuracy
Bosphorus	Train	99.4 %
Bosphorus	**Test**	**90.6 %**

FRGC





- Filtered out identities with $N_m < 16$ meshes $\rightarrow$ **997** training and **472** testing examples
- **78** identities left
- $mesh\ /id \approx 18$

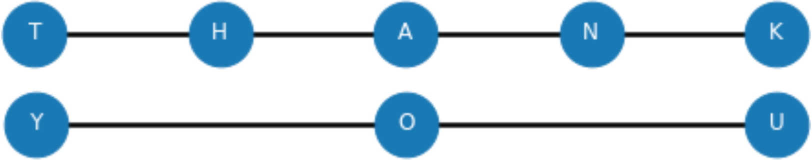
Dataset	Split	Accuracy
FRGC	Train	100 %
FRGC	**Test**	**92.1 %**

Conclusions & Further Developments

- CNN-like Graph Neural Network on 3D meshes
- Con: Did not reach SOTA performances
- Pro: Easily customizable model
- Could be a **baseline** for future architectures

Code

https://github.com/w00zie/3d_face_class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Vertex Normalization

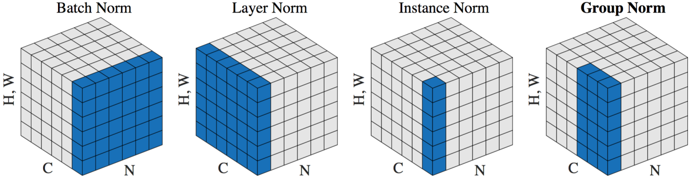
Let $\mathbf{x}_v \in \mathbb{R}^d$ the vector of initial features for node $v \in \mathcal{V}$.
Let $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_N]^\intercal \in \mathbb{R}^{N \times d}$ the feature matrix of the whole graph, being $N = |\mathcal{V}|$. For each node we define a mapping from $\mathbb{R}^d \to [0, 1]^d$ as following:

$$
\begin{aligned}
M &= \max_{i \in \{1,\ldots,N\}} \max_{j \in \{1,\ldots,d\}} \mathbf{X}_{ij} \\
m &= \min_{i \in \{1,\ldots,N\}} \min_{j \in \{1,\ldots,d\}} \mathbf{X}_{ij} \\
\alpha &= \frac{1}{M - m} \\
\mathbf{x}_v^n &= \alpha(\mathbf{x}_v - m\mathbf{1})
\end{aligned}
$$

where $\mathbf{1} = [1, \ldots, 1]^\intercal \in \mathbb{R}^d$. It is possible to show that this transformation scales the distances by a factor α^2:

$$
d(\mathbf{x}_v^n, \mathbf{x}_u^n) = \|\mathbf{x}_v^n - \mathbf{x}_u^n\|_2 = \alpha^2 \|\mathbf{x}_v - \mathbf{x}_u\|_2 = \alpha^2 d(\mathbf{x}_v, \mathbf{x}_u)
$$

Instance Normalization - [9]



Normalization is applied on each element $\mathbf{h}_v$ of every batch $\mathcal{B}$:

$$\mathbf{h}_v = \frac{\mathbf{h}_v - \mathbb{E}[\mathbf{h}_v]}{\mathrm{Var}[\mathbf{h}_v] + \epsilon} \odot \gamma + \beta$$

where $\mathbf{h}_v, \gamma, \beta \in \mathbb{R}^f$ and $\odot$ is the element-wise multiplication.