University of Moratuwa

Faculty of Engineering

EN4563 – Robotics

Robotics Mini Project: Kinematic Analysis of a Robot Arm

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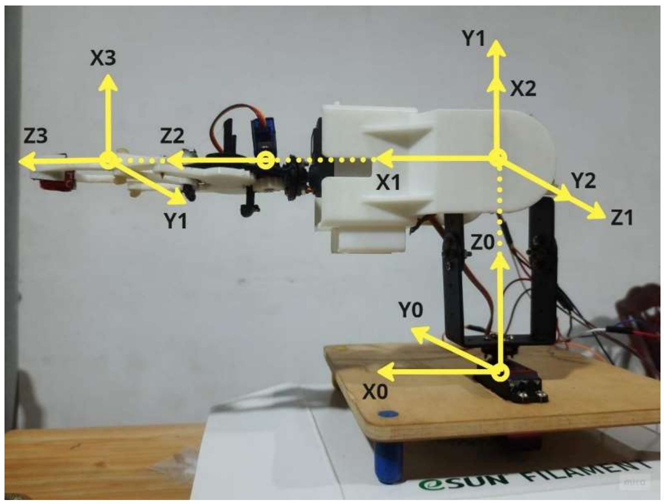
Introduction

The Denavit-Hartenberg (DH) table provides a fundamental depiction of the geometry of a three-degree-of-freedom (DOF) robot arm. The three rows, each representing a joint, that make up the DH parameters (a, α, d, and θ) describe the spatial configuration of the arm's linkages. The forward kinematics are calculated by multiplying the transformation matrices for every joint after the DH table. These matrices represent the translation and rotation between each link. The more difficult issue of inverse kinematics involves calculating joint angles based on the intended pose of the end-effector. In addition, the Jacobian matrix is essential for dynamic control since it connects joint and end-effector velocities. Pick and place tasks include defining specified end-effector postures and using inverse kinematics to calculate the relevant joint angles to manipulate and position objects precisely. Understanding the kinematics and dynamics of a 3-DOF robot arm in the context of a pick and place operation is made possible by this thorough examination.



Denavit-Hartenberg (DH) Table

Link	Link Length (a_i)	Link Twist (α_i)	Link Offset (d_i)	Joint Angle (θ_i)
1	0	90	115 mm (l_1)	θ_1^*
2	0	90	0	90 + θ_2^*
3	0	0	238.84 mm (d_3)	θ_3^*

Note: * is for variables

Forward Kinematics

Transformatin Matrices,

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} -S_2 & 0 & C_2 & 0 \\ C_2 & 0 & S_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The end effector, T_3^0 = $\begin{bmatrix} -C_1S_2C_3+S_1S_3 & C_1S_2S_3+S_1C_3 & C_1C_2 & C_1C_2d_3 \\ -S_1S_2C_3-C_1S_3 & S_1S_2S_3-C_1C_3 & S_1C_2 & S_1C_2d_3 \\ C_2C_3 & -C_2S_3 & S_2 & S_2d_3+l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Forward kinematics testing - 01:

$$\begin{aligned} \theta_1 = \theta_2 = \theta_3 = 0 \\ C_1 = C_2 = C_3 = 1 \\ S_1 = S_2 = S_3 = 0 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Position vector = [238.84 0 115]

Rotation matrix = $R_{(x,-90)}R_{(z,-180)}$

Forward kinematics testing - 02:

$$\begin{aligned} \theta_1 = \theta_2 = 90, \theta_3 = 0 \\ C_1 = C_2 = 0, C_3 = 1 \\ S_1 = S_2 = 1, S_3 = 0 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & d_3+l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Position vector = [0 0 353.84]

Rotation matrix = $R_{(Z,-90)}$

Inverse Kinematics

In Inverse Kinematics we focuses on figuring out a robotic system's joint configurations that are required to reach a specific end-effector pose.

End effector position – x, y, z

$$d_3cos\theta_1\, cos\theta_2 = x$$

$$d_3sin\theta_1 cos\theta_2 = y$$

$$d_3sin\theta_2 + l_1 = z$$

Solution for those equations

$$\theta_1 = tan^{-1}(\frac{y}{x})$$

$$\theta_2 = cos^{-1}\left(\frac{\sqrt{x^2+y^2}}{d_3}\right)$$

Manipulator Jacobian

$$t_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad t_1^0 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \quad t_2^0 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \quad t_3^0 = \begin{bmatrix} C_1 C_2 d_3 \\ S_1 C_2 d_3 \\ S_2 d_3 + l_1 \end{bmatrix}$$

$$z_0^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1^0 = \begin{bmatrix} S_1 \\ -C_1 \\ 0 \end{bmatrix} \quad z_2^0 = \begin{bmatrix} C_1 C_2 \\ S_1 C_2 \\ S_2 \end{bmatrix} \quad z_3^0 = \begin{bmatrix} C_1 C_2 \\ S_1 C_2 \\ S_2 \end{bmatrix}$$

$The\ Jacobian\ Matrix,$

$$J = \begin{bmatrix} -S_1 C_2 d_3 & -C_1 S_2 d_3 & 0 \\ C_1 C_2 d_3 & S_1 S_2 d_3 & 0 \\ 0 & S_1 S_2 C_2 d_3 + {C_1}^2 C_2 d_3 & 0 \\ 0 & S_1 & C_1 C_2 \\ 0 & -C_1 & S_1 C_2 \\ 1 & 0 & S_2 \end{bmatrix}$$

Pick and Place Task

When we are given position of end effector (x,y,z) , joint angles can be calculated using inverse kinematics equations defined above

Position of picking in world coordinate frame: $[158\ \ 135\ \ -9]^T$

$$\theta_1 = 40\,,\quad \theta_2 = 122\quad ,\theta_3 = 0$$

Position of placing in world coordinate frame: $[\ 176\ \ -205\ \ 120]^T$

$$\theta_1 = 130\,,\quad \theta_2 = 45\quad ,\theta_3 = 180$$