Linear Algebra

Binary Operations:

A ***binary operation*** on a set is a calculation involving two elements of the set to produce another element of the same set.

(or)

A binary operation on a set S is a mapping of the elements of the Cartesian product $S \times S$ to S.

$$* : S \times S \rightarrow S$$

(or)

An operation * on a non-empty set A is said to be a binary operation if

$a \in A$ & $b \in A$, then $a * b \in A$ **(Closure Property)**

Group:

A non empty set S with the binary operation * is said to be ***group*** if it satisfies the following conditions;

1. **Closure property:** $a \in S$ & $b \in S$, then $a * b \in S$;
2. **Associative property:** **u * (v * w) = (u * v) * w** for all u, v, w ∈ S; **(Semi group)**
3. **Identity property:** there exists an element e ∈ S such that **u * e = e * u = u** for all u ∈ S. **(Monoid)**
4. **Inverse property:** For every u ∈ S, there exists an element v ∈ S such that **u * v = v * u = e.** Then v is said to an inverse of u and denote by u^{-1}. **(Group)**
5. **Commutative property:** **u * v = v * u** for all u, v ∈ S. **(Abelian Group)**

Vector Space:

A ***vector space*** over a field F (in this entire course it is $\mathbb{R}$) is a non empty set V together with two operations vector addition '+' (just for name it is no need to usual addition) and scalar multiplication that satisfy the ten axioms listed below.

I. Abelian group under addition;

1. **Closure property**: u & $v \in V$, then $u + v \in V$;
2. **Associative property**: $\mathbf{u+(v+w) = (u+v)+w}$ for all $u,v,w \in V$;
3. **Identity property:** there exists an element $e \in V$ such that $\mathbf{u + e = e + u = u}$ for all $u \in V$;
4. **Inverse property:** For every $u \in V$, there exists an element $-u \in V$ such that $\mathbf{u + (-u) = (-u) + u = e}$. Then $-u$ is said to an additive inverse of u;
5. **Commutative property:** $\mathbf{u + v = v + u}$ for all $u, v \in V$.

II. Scalar multiplication;

6. **Closure property**: $u \in V \;\&\; \alpha \in F$, then $\alpha u \in V$;

7. **Distributive property of scalar multiplication over vector addition** : $\alpha(u+v) = \alpha u + \alpha v \quad \forall\; u, v \in V \;\&\; \alpha \in F$

8. **Distributive property of vector addition over scalar multiplication:** $(\alpha+\beta)u = \alpha u + \beta u \quad \forall\; u \in V \;\&\; \alpha, \beta \in F$

9. **Associative property:**

$$(\alpha\beta)u = \alpha(\beta u) = \alpha\beta u \quad \forall\; u \in V \;\&\; \alpha, \beta \in F$$

10. **1.u = u** for all $u \in V$.