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1. (a) Let $q_1, ..., q_n, ...$ be a list of distinct rational numbers. Consider $\bigcap_{n=1}^{n=\infty} A_n = 0$. Based on the proof for part b, **Q** - $\{q_n\} = 0$ does not create a contradiction because both **Q** and all $\{q_n\}$ are countable. So there is no contradiction when the infinite intersection of all the countable stuff is empty.
(b) In the reals, $x_1, ..., x_n, ...$ is a countable list of distinct real numbers, and the collection of all x_n is countable. If some $B_n \neq 0$, then any b $\in B_n$ will be in any B_k where k $\leq$ n. Thus, if $\bigcap_{n=1}^{n=\infty} B_n = 0$, some $B_k = 0$. So suppose $B_k = 0$. Then $\mathbf{R} - \{x_k\} = 0$, so $\{x_k\} = \mathbf{R}$. However, **R** is uncountable and all $\{x_n\}$ are countable, so this is a contradiction. Therefore $\bigcap_{n=1}^{n=\infty} B_n \neq 0$.

2. Let A be obstructive in **R**. Let x $\in$ **R**, and let $N_r(x)$ be a neighborhood of x with r > 0. Consider y $\in N_r(x), y \neq x$. Then because A is obstructive in **R**, there is some a $\in$ A such that x<a<y. Then a $\in N_r(x)$ and $a \neq x$ so x is a limit point of A. Thus A is thick in **R**.
Now let A be thick in **R**. Let x, y $\in$ **R** be given. WLOG, let x $<$ y. Consider $N_r(x)$ where r = d(x, y). Because x is a limit point of A, there is some point, $a \in N_r(x)$ such that $a \neq x, a \in A$. Thus x<a<y. Thus A is obstructive in X. QED

3. Attempt to establish an equivalence relation from J to B where $b_N \in B$

if $b_N = \sum$

Then for N ≥ 2, $b_N = a_1 \times b_1 + a_2 \times (-\sqrt{b_2}) + ...$ and $b_N = a_1 \times b_1 + a_2 \times (\sqrt{b_2}) + ...$ Thus J $\rightarrow$ B is not a 1-1 function, so B is not countable. QED

4. Let M be a nonempty set bounded in R^k, and let $\delta > 0$ be given. Then because M is bounded, M is a subset of some k-cell, call it K. Then K is compact. Construct an open cover of K by letting G = $\{G_x | G_x = N_\delta(x), x \in R^k\}$. Because K is compact, there is a finite subcover, $\{\bigcup G_x\}$, of G over K, and because M is a subset of K, M is a subset of $\{\bigcup G_x\}$. QED

5. Let M be a nonempty set bounded in X. Let X be a metric space and let $\delta > 0$ be given. Since X is a metric space, $\overline{M}$ is closed in X. Consider M'. Let x $\in$ M'. Then d(x,m) $<$ r where m $\neq$ x, m $\in$ M, and m $\in N_r(x)$ for all r $>$ 0. Since M is bounded in X, there exists some N such that d(p, x) $\leq$ N for all p $\in$ M. Thus d(p,m) $\leq$ d(p,x) + d(x,m) = N + r. Therefore $\overline{M}$ is also bounded in X. Since $\overline{M}$ is closed and bounded, it is compact in X. Construct an open cover of $\overline{M}$, G = $\{G_x | G_x = N_\delta(x), x \in R^k\}$. Then because $\overline{M}$ is compact, and M $\subseteq \overline{M}$, there is a finite subcover of G over $\overline{M}$ which also covers M. QED

6. Let K be some open subset of **R**. Consider G, a collection of disjoint open sets, g, in **R**. Suppose $\bigcup_{g \in G} g = K$. Then $K^c = \bigcap_{g \in G} g^c$.

7. Let A and B be nonempty, disjoint, open sets. Assume for contradiction that

A $\bigcup$ B is connected. Then either A $\bigcap \overline{B}$ is nonempty or B $\bigcap \overline{A}$ is nonempty. WLOG, consider x $\in A \bigcap \overline{B}$. Then x $\in$ A and x $\in$ $\overline{B}$. Since A is open, there is some neighborhood, $N_r(x) \subset A$ (r>0). Since A and B are disjoint, this neighborhood contains no points of B. Therefore x $\notin$ B' and since x $\notin$ B, x $\notin$ $\overline{B}$. Thus x $\notin$ A $\bigcap \overline{B}$. =><= So A and B must not be connected. QED

8. Let K be compact in X where X is a metric space. Let n $\in$ Z^+ be given. Let $G_n = \{N_r(x)|g \in X, r = 1/n\}$. Then any G_n is an open cover of K, because if k $\in$ K, k $\in$ X, so any $N_r(k) \in G_n$ contains k, for any n. Since K is compact, there is a finite subcover of G_n over K. QED

9. Let A and B be nonempty sets in an ordered space, S. Let x = sup A and y = inf B, and let a$\leq$b for all a$\in$A and all b$\in$B. Suppose for contradiction that x>y. WLOG, consider x. Since x>y and y$\leq$ $b \in B$, there are two cases: $y < x \leq b$ or $y \leq b \leq x$ $(x \neq y)$. Let $y < x \leq b$ (for all $b \in B$). Then by definition of infimum, inf B = x. This contradicts the assumption that inf B = y, y $\neq$ x. Now consider y $\leq$ $b \leq x$. Then there are two cases, b<x for all b$\in$B, or $c \leq x \leq$d for some arbitrary c and d in B. Let b<x for all b in B. Then a$\leq$ $b < x$ for all a in A and all b in B, so by definition of supremum, sup A is some b. This contradicts the assumption that sup A = x. Now consider $c \leq x \leq$d. Then because inf B = y, y $\leq$ c. Since y $\neq$ x, either y = c < x or y < c $\leq$ x. Let y = c. Then since c < x and c $\geq$ a for all a in A, sup A = c = y, which contradicts the assumption that sup A = x, x $\neq$ y. Now let y < c $\leq$ x. Then sup A = c, which contradicts the assumption that sup A = x. Now that I've dispensed with the last of these pesky cases, I have proved that sup A $\leq$ inf B. QED