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STATISTICAL ANALYSIS OF GRAVITY MESUREMENTS VARIATION DUE TO NEGLECTING REFERENCE SYSTEM

A Case Study of Tanzania Gravity Database of 2021 (TGDB21)

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STATISTICAL ANALYSIS OF GRAVITY MEASUREMENTS VARIATION DUE TO NEGLECTING REFERENCE SYSTEM

A Case Study of Tanzania Gravity Database of 2021 (TGDB21)

BY

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A Dissertation Submitted to the Department of Geospatial Science and Technology (DGST) in Partial Fulfillment of the Requirement for the Award of Bachelor of Science Degree in Geomatics (BSc. GM) of the Ardhi University.

CERTIFICATION AND COPYRIGHT

The undersigned certify that they have proof read and hereby recommend for acceptance of a pre-dissertation proposal entitled **"STATISTICAL ANALYSIS OF GRAVITY MEASUREMENTS VARIATION DUE TO NEGLECTING REFERENCE SYSTEM: A case study of Tanzania Gravity Database of 2021 (TGDB21)"** and they now allow the dissertation proposal for University Examination.

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(Supervisor)	**(Supervisor)**
Date…………………………	**Date…………………………**

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I MASUNE, PATRICK AIDAN hereby declare that this dissertation is my own original work and that to the best of my knowledge, it has not been presented to any other university for a similar or any other degree award except where due acknowledgment has been made in the text.

………………………………..

MASUNE PATRICK AIDAN

23362/T.2019

(Candidate)

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DEDICATION

I dedicate this Thesis to my family, friends and TAFES-ARU family for their encouragement, prayers, boundless love, patience, support and care throughout my studies. I really appreciate you for all you have done to me. I love you and God bless you all.

ABSTRACT

Accurate gravity measurements are essential for various geodetic and geospatial applications. The reference system used in gravity measurements plays a critical role in determining the reliability and precision of the obtained results. Gravity records constitutes position (ϕ, λ, H) where an Ellipsoid is used as the reference surface for horizontal Geodetic coordinates, namely Latitude (ϕ) and Longitude (λ) and the heights has the vertical Datum. Some of the data can have the span time of hundreds of years or more. Historically, different ellipsoids have been chosen in different parts of the world in order to simplify surveying and mapping in that region and the most recent Mean Earth Ellipsoid adopted by IAG is Geodetic Reference System 1980 (GRS80) which supersedes the GRS67. The precise measurement of gravity is crucial for understanding the Earth's structure and dynamics.

This research focuses on investigating the impact of neglecting the reference system on gravity position measurements. The general comparison of geodetic coordinates between two different reference ellipsoids namely Clarke 1880 and WGS84 (assumed to be equal to GRS80) is conducted. A comprehensive analysis is conducted using a dataset of gravity measurements, with a particular emphasis on statistical techniques.

The results from this study showed that the neglect of the reference system in gravity measurements has a less significantly impacts on the observed gravity position variation. Through the analysis of data and statistical tests performed on the change of the gravity anomalies shown the following standard deviation. The Free-air gravity anomaly has high estimated error of about 6.531940506 mGal, Bouguer gravity anomaly has an estimated error of about 6.390796066 mGal and the one with the least estimated error was Gravity anomaly of about 6.33027916 mGal. From these results, the values were very small and seems to be insignificant error but their consideration are very important as it was found that neglecting the reference system introduces substantial differences in gravity measurements compared to when the reference system is properly considered.

Keywords: Gravity, Gravity anomaly, Coordinate System, Gravity Database, Statistical analysis

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ACRONYMS AND ABBREVIATIONS

ARU	Ardhi University.
BGI	International Gravity Bureau.
TGDB	Tanzania Gravity Database
DGST	Department of Geospatial Science and Technology.
DMA	US Defense Mapping Agency
DNSC	Danish National Space Center
DTU	Technical University of Denmark
ECAGN	East and Central Africa Gravity Network.
ESAMRDC	East and South Africa Mineral Resources Development Center.
GETECH	Global Exploration Technology
GOCE	Gravity field and Steady-State Ocean Circulation Explorer.
GRACE	Gravity Recovery and Climate Experiment.
GRS80	Geodetic Reference System 1980
GST	Geological Survey of Tanzania
IAG	International Association of Geodesy.
IDEMS	International DEM Service.
IGFS	International Gravity Fields Service.
IGSN71	International Gravity Standardization Network 1971.
MLHHSD	Ministry of Land, Housing and Human Settlements Development.
TPDC	Tanzania Petroleum Development Centre.

UDSM	University of Dar es salaam.
TDGB17	Tanzania Gravity Database 2017.
TDGB21	Tanzania Gravity Database 2021.
CHAMP	Challenging Minisatellite Payload
GRACE-FO	Gravity Recovery and Climate Experiment Follow-On
WGS84	World Geodetic System 1984

CHAPTER ONE

INTRODUCTION

1.0 Background of the study

Earth's gravity field has effect on almost everything within the Earth and its atmosphere. Therefore, gravity survey is one of the fundamental measurements of a nation, of which the full and accurate data are needed for various uses. One such use is the determination of the Geoid Model which has been, and still is, one of the main objectives of geodesy (Ulotu, 2009). The process of measurement of the Earth's gravitational field is called Gravimetry. Gravity (also referred to as gravitation) is the force field with which the Earth attracts other masses. At the Earth's surface, gravity is a superposition of the gravitational attraction of masses and the centrifugal acceleration due to the rotation of the Earth. Gravity measurements can be based on the analysis of satellite orbits, using sensors on dedicated gravity satellite missions, or using terrestrial techniques, specifically, airborne gravimetry, shipborne gravimetry and land-based gravimetry. The acceleration of an object under the influence of Earth's gravity, g, ranges from approximately 9.78 ms^{-2} (at the equator) to about 9.83 ms^{-2} (at the poles) .

Ground method has two types of gravity measurements absolute and relative using gravimeters, absolute gravimeters record actual gravity value of a certain position while relative gravimeters records gravity between two stations ($\Delta g = g_2 - g_1$). There are 4-absolute 1st order and 56-relative 2nd order reference gravity stations in Tanzania mainland. The gravity stations were observed using absolute and relative gravimeters respectively. Other method is space which involves dedicated satellite missions CHAMP, GRACE, GOCE and GRACE-FO. These missions' measurements made between two satellites to detects changes in gravity due to change in earth's masses. (Parker& Marobhe, 1991).

In Geophysics, The earth's gravity field and its time variation are essential in the study of fundamental earth processes such as mantle convection, plate tectonics, fluid mass transport both on the surface (e.g., ocean and atmospheric circulation, and hydrology, ice sheet) and in the core (Oba , Hart, & Moka, 2021) also the need of the geological map is of great importance in geological explorations to determine the presence of usefully minerals which are needed. Geoid model and gravity data found their important application in the mitigation of natural hazards such as floods, earthquakes, and volcanoes (Ulotu, 2016).

The value of gravity increases with the increase of the mass and it decreases with an increase of the distance between two bodies. Therefore, the value of Earth's gravity changes from one point to another depending on the mass (densities) of rocks and a distance or height from the center of the Earth. Theoretically, value of gravity is constant all over the Earth's surface but in reality, gravity values varies from place to place because the earth has the shape of a flattened sphere (not a perfect sphere), rotates, and has an irregular surface topography and variable mass distribution (Jekeli, 1999).

Gravity records constitutes position (ϕ, λ, H) where an Ellipsoid is used as the reference surface for horizontal Geodetic coordinates, namely Latitude (ϕ) and Longitude (λ) and the heights has the vertical Datum. Some of the data can have the span time of hundreds of years or more. Historically, different ellipsoids have been chosen in different parts of the world in order to simplify surveying and mapping in that region and the most recent Mean Earth Ellipsoid adopted by IAG is Geodetic Reference System 1980 (GRS80) which supersedes the GRS67. The precise measurement of gravity is crucial for understanding the Earth's structure and dynamics (Gross, 2002). However, the measurement of gravity can be affected by various factors, including the neglecting coordinate system. The gravity data that will be used, needed to be transformed to a more up-to-date and internationally adopted reference gravity field. Most of the Geodesist tends to neglect the transformation of coordinate system of these gravity data during their applications and when plotted tends to portray large displacements of sometimes around 1m displacement which can cause a point to be located on area that has high vertical difference of hundred meters added more i.e. intolerable error. So, the gravity measurements can be affected by several factors, including the choice of reference system. In particular, neglecting reference system can lead to errors in gravity measurements and results in variations in gravity position. This is because the choice of reference system can impact the determination of the local gravity field, which is used to correct for the effects of the Earth's rotation and the presence of topographic masses. Several studies have demonstrated the importance of considering the impact of reference system on gravity measurements (Andrews, 2005).

1.1 **Statement of the Research Problem**

Gravity records can have the span time of hundreds of years or more and specifically in Tanzania Gravity observation started in the 1890's and their positions were referenced to different local reference ellipsoids in early 1950's, particularly in Tanzania was Clarke 1880 ellipsoid. Different ellipsoids have been chosen in different parts of the world in order to simplify surveying and mapping in that region and the most recent Mean Earth Ellipsoid adopted by IAG is Geodetic Reference System 1980 (GRS80) which supersedes the GRS67. The gravity data to be used needs to be transformed to a more up-to-date and internationally adopted reference gravity field. Most of the Geodesist tends to neglect conversion of reference system of these gravity data during their applications and when plotted tends to portray large displacements. This research aims at investigating statistical variations in gravity position due to the neglecting reference system and to develop methods to correct for these variations for the Tanzania Gravity Database of 2021 (TGDB21).

1.2 Research Objectives

1.2.1 Main Objective

The main objective of this study is to investigate statistical variations in gravity measurements due to the neglecting reference system for the Tanzania Gravity Database 2021 (TGDB21).

1.2.2 Specific Objectives

i. To perform coordinates transformation of gravity data for Tanzania Gravity Database (TGDB) to a more up-to-date and internationally adopted reference gravity field.
ii. To compute the statistical measures and tests for the differences in gravity measurements due to neglecting reference system.

1.3 Research Questions

i. How does the choice of reference system affect the results of statistical analysis of gravity position variation?
ii. Are the observed differences statistically significant?

1.4 Significance of the Study

i. To provide a quality gravity data that will be used in the computation of the geoid, which is proposed to be a vertical datum of Tanzania.
ii. It helps in developing a more up-to-date gravity database of Tanzania, which will promote not only this research but also other scientific researches now and in the future.
iii. Also, used to show the variation of rocks densities for more geological applications.

1.5 Beneficiaries of the Study

The beneficiaries of this study are as follows;

i. The government of Tanzania (Ministry of energy minerals and ministry of land and human settlement development).
ii. Tanzania Petroleum development center (TPDC)
iii. Geodesist for computation of geoid and other parameters.
iv. Natural gases and minerals exploration companies
v. Tanzania military (JWTZ)

1.6 Expected Outcomes

The expected outcome of the study is to demonstrate statistical analysis of gravity variation due to neglecting reference system on the accuracy of gravity measurements.

1.7 Scope and Limitation

There are various regional and international gravity databases worldwide, but this study is limited to Tanzania Gravity Database of 2021 (TGDB21) which are found between 4ºN to 15ºS latitude and 26ºE to 44ºE longitude as shown in Figure 1.1 below.

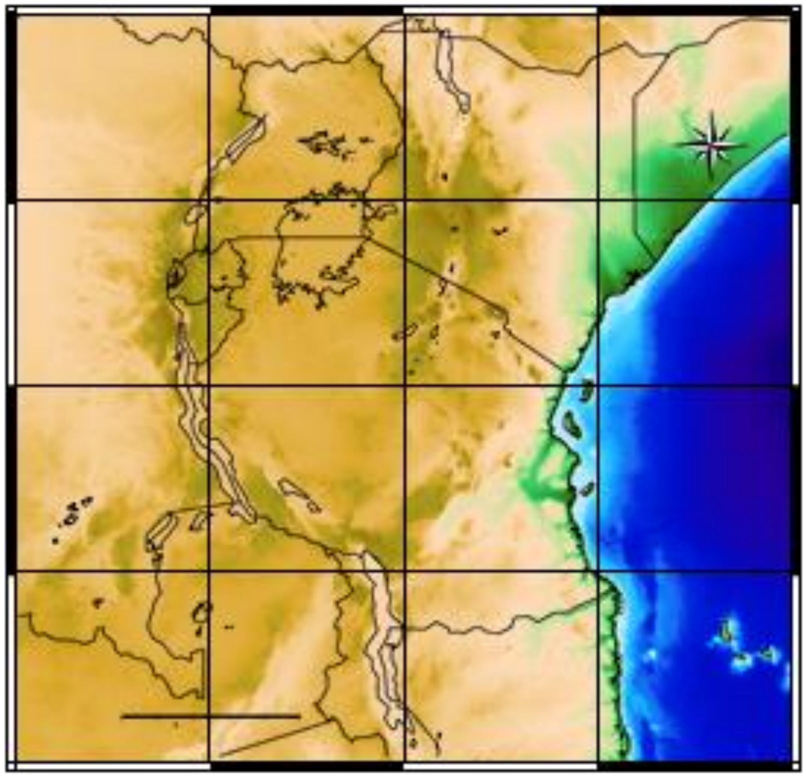


Figure 1.1: The study Area (The Land areas only)

1.8 Research Outline

This research consists of five chapters as follows;

Chapter 1: Introduction

This chapter introduces the study by explaining the background of the study, statement of the problem, main objective and specific objectives, research questions, scope of the research, significance and beneficiaries of the research. And what is ought to be done throughout the study.

Chapter 2: Literature Review

This chapter describes the basic definitions of key concepts such as Gravity and its measurements and applications followed by a brief discussion on dedicated satellite missions, Gravity Database, Gravity network, Reference system and different opinions from other literatures and already existing research on the topic.

Chapter 3: Research Methodology

This chapter details on methodology of the dissertation by describing all relevant datasets, procedures and mathematical models required to achieve the objective of the study methodology used in the study by summarizing the research approach, data collection method, sources of data, data analysis methods , data presentation and interpretation.

Chapter 4: Research Findings and Data Analysis

This chapter presents the numerical and map results of the dissertation, gives all the results from the processing of data outlined in the methodology part, it is also in this chapter were the results are analyzed and discussed.

Chapter 5: Conclusion and Recommendation

This chapter consists of conclusion and recommendations. Conclusion summarizes dissertation findings in view of the solution to dissertation problem followed by recommendations to be considered by future study related to this dissertation.

CHAPTER TWO

LITERATURE REVIEW

2.1 History of Gravity Survey in Tanzania

The first geodetic surveys in Tanzania were conducted during the German colonial period in the late 19th and early 20th centuries. These surveys were primarily focused on establishing the boundaries of German East Africa (which included present-day Tanzania, Rwanda, and Burundi) and mapping the region's natural resources (Muganda, 2010).

After Tanzania gained independence in 1961, the government established the National Mapping Agency (NMA) to oversee geodetic surveys and mapping activities in the country. The NMA was responsible for carrying out a number of large-scale mapping projects, including the First and Second Tanzanian National Mapping Projects, which were completed in the 1970s and 1980s (Muganda, 2010).

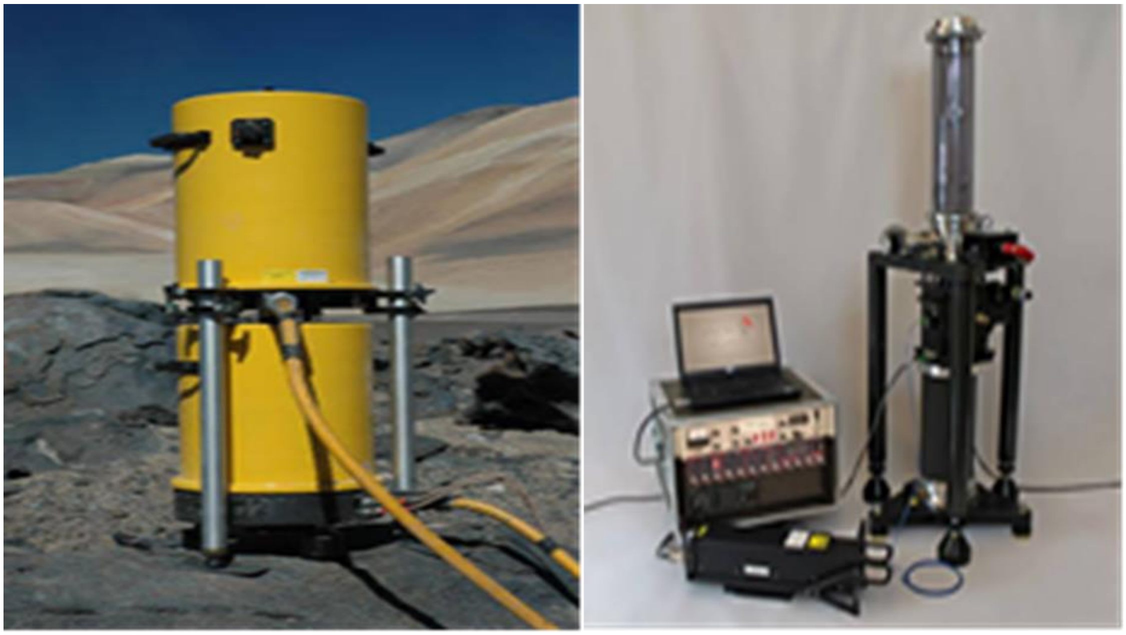
More recently, Tanzania has been involved in several international geodetic projects, including the International Earth Rotation and Reference Systems Service (IERS) and the International GNSS Service (IGS), which provide high-precision geodetic measurements and reference systems for use in a variety of applications (Muganda, 2010).

2.2 Gravity Measurements

The study and measurement of the earth's gravity field is called gravimetry. The measurement of gravity can either be absolute or relative. The instruments used to measure gravity are known as gravity meters or gravimeters, they are either absolute or relative.

a. Absolute gravity measurement

This is the determination of the total attraction at a point due to all masses. It is measured using absolute gravimeters such as A10 and F G5 − X (see Figure 2.1) which use the principle of free fall or rise and fall. Other method of absolute gravity determination includes the use of pendulum which in practice is now obsolete.



(a) A10 absolute gravimeter (b) FG5-X absolute gravimeter

Figure 2.1: Absolute gravimeters

Source: *(http://microglacoste.com,* on 15th January, 2020)

b. Relative Gravity measurement

This is the determination of the gravity difference between two stations. A relative gravimeter is used in relative gravity measurement. An example of relative gravimeter includes SCINTREX CG-6 AUTOGRAVTM pictured in Figure 2.2

Figure 2.2: Scintrex CG-6 relative gravimeter

Source: *(https://scintrexltd.com/product/cg -6-autograv gravity-meter/* on 15th January, 2020)

2.3 Gravity measurement in Tanzania

Gravity observation in Tanzania started in the 1890's and reference was to different local datums prior to arrival of the Potsdam datum in early 1950's. To unify gravity information, gravity data should be referred to a standard international datum and heights to mean sea level (MSL)/geoid. The geographical position of the land Point gravity observed during that time were referenced to local reference ellipsoid which was Clarke 1880 and very few data reductions have used GRS80 even when it was observed after 1980 (Ulotu, 2009). The gravity measurements referenced to GRS80 includes Satellite gravity data, Aerial gravity data and Shipborne gravity data while the Land Point gravity data was referenced to Clarke 1880 ellipsoid.

The gravity measurement can be done on the surface of the earth, both land and marine surfaces, and it can be measured by either absolute or relative gravimeters or any other means of gravity measurements. Gravity surveys can be categorized to the following aspects:

i. Terrestrial (Land)

There are 4-absolute 1st order and 56-relative 2nd order reference gravity stations in Tanzania mainland. Involves taking gravity information from the ground. It is the most accurate source of data compared to all other sources. The gravity is measured by absolute gravimeter which involve determining the value of gravity directly. Measuring gravity by absolute method maybe of three ways which are falling body and pendulum. The terrestrial (ground) gravity observation is limited to landmasses, in national parks and most of thick forests there is no gravity. In Tanzania there are 40,000 points obtained by ground information. The land point gravity observed were in local reference ellipsoid known as Clarke 1880 ellipsoid.

ii. Marine (Shipborne)

Involves the use of ship or special boat whereby the gravimeter is mounted to obtain information. It is accurate compared to satellite sources. Terrestrial marine gravity is not accurate as ground gravity because observation in marine area is not reliable compared to land. In water, time changes due to not only ocean tides but also mass changes during observation. Observation in marine is difficult and tedious. Due to this problem it involves the use of lines whereby the intersection point is used to provide information. Therefore, observation in marine require calm weather since the ship is always tilting even though the data obtained are not always accurate due to the problems faced. In Tanzania there 7,345 points obtained by ship track.

iii. Aerial

Involves mounting a gravimeter on an airplane and records the flying height. Firstly, in 2012 aerial gravity was used to determine TZG13. Aerial gravity in Tanzania was completed in 2013 under the project between the Technical University of Denmark (DTU) (as a contractor) and the Ministry of Lands, Housing and Human Settlements Development of Tanzania; Surveys and Mapping Division (MLHHSD-SMD). Aerial gravity dumps high frequency of the actual gravity depending on the flight altitude, the dumping from very high frequency to medium frequency increases with increasing of flying altitude.

iv. Satellite Gravimetry

The dedicated satellite missions were firstly initiated. It started in year 2000, when Challenging Minisatellite Payload (CHAMP) was launched by Germany. It has a degree order 80 and it operated for 10 years. In 2002, Gravity Recovery and Climate Experiment (GRACE) which was used to determine gravity parameters globally with high resolutions. It has a degree order of 140 and operated for 5 years. In March 2009, Gravity field and Steady-state Ocean Circulation Explorer (GOCE) was launched with a degree order of 330 and operated for 5 years. In year 2018, GRACE-FO was launched due to failure of GRACE.

2.4 The Tanzania Gravity Database

In Tanzania, there is no central organization responsible for storing all gravity data collected from various surveys. Instead, each organization interested in gravity creates its own gravity database. The first gravity database in Tanzania was developed in 1968 by Sowerbutts, which was based on point gravity. In 1991, Dr. Marobhe I. from the University of Dar es Salaam (UDSM) and Mr. Parker from the East and South Africa Mineral Resource Development Centre (ESAMRDC) attempted to create a comprehensive gravity database for Tanzania in their report on the Review of Gravity Survey in Tanzania, which contained about 14,000 points of gravity data. However, there is no clear information on the creation of that database. This report is stored by the University of Dar es Salaam, ESARMDC, and TPDC. The US DMA has a database that includes data from previous compilations and some offshore reconnaissance missions, including gravity data around Tanzania's boundaries.

The first digital gravity database, TGDB08, was created in 2001 by Simon, which was later improved by Ulotu in 2009. TGDB08 covers an area of 4°N to 15°S latitude and 26°E to 44°E longitude, based on 39,677 statistically cleaned point gravity data at a 99% confidence level. The data sources for this database were Pure GGM ITG-GRACE03S, combined GGM EIGEN-CG03C, and SRTM3.1 CGIAR-CSI.

In 2017, Mrema created the next gravity database, which included satellite altimetry marine gravity data, gravity data from TGD08, and point gravity data not included in TGDB08. The latest gravity database, TGDB21, was created in 2021 at Ardhi University. This database contains 47,022-point gravity data, 48,467 aerials gridded $5' \times 5'$ gravity data, 42,562 African Geoid Project (AGP) $5' \times 5'$ gravity anomaly, and the $1' \times 1'$ free-air marine gravity anomaly cleaned at a 99% confidence interval.

The data contained in the TGDB21 are unified. The reference ellipsoid used for the gravity anomalies were the Geodetic Reference System 1980 (GRS80). The gravity data in TGDB21 were cleaned from the gross and systematic errors by the cross-validation approach. The cross-validation approach is also used to assess the spatial variation in data sampling when the simple size is not small (Ulotu, 2009). The cross-validation assess the quality of the data by removing one of the observed values and then predicts it using other values, then the first removed observation is returned and used to predict the value of another observation which will be removed by the same procedure as the first one. The differences between the predicted and observed values is used to show the variation of the data.

In Tanzania, various organizations have their own gravity databases that cater to their specific requirements and geographical areas of interest. The gravity data available in Tanzania are derived from a range of sources, such as aerial and marine gravity data, as well as land point gravity data. These data providers can be classified into two groups: those within the country and those from overseas.

The sources within the country are as mentioned below

i. The department of Geospatial Science and Technology (DGST) Ardhi University.
ii. University of Dar es Salaam the department of Geology.
iii. Ministry of Minerals and Energy of Tanzania.
iv. Tanzania Petroleum Development Center (TPDC).
v. Ministry of Lands, Housing and Human Settlements Development.
vi. Eastern and Southern African Mineral Resources Development Centre.

The sources of Gravity out of Tanzania (Oversea gravity sources).

i. The international gravimetric Bureau (BGI) which is the international association of Geodesy (IAG) tool for collecting and storing global gravity data.
ii. Gravity and Geoid for Africa (GGA).
iii. USA National Geospatial Agency (NGA).
iv. Global Exploration Technology (GETECH), provides gravity data for oil and mineral exploration.
v. Danish National Space Center (DNSC) of the Technical University of Denmark (DTU) provides satellite altimetry marine gravity data.

2.5 The Overview of the Coordinate System

A coordinate system is a mathematical framework used to specify and locate points in space. It provides a way to assign unique numerical values, called coordinates, to every point in a given space. By using coordinates, we can precisely describe the position, direction, and orientation of objects or points within that space. Coordinate systems are commonly used in various fields, including mathematics, physics, engineering and navigation. Different coordinate systems are used depending on the context and requirements of the application. Here, we'll focus on the two most common types of coordinate systems: the Cartesian coordinate system and the spherical coordinate system (Boas, 2005).

There are various types of coordinate systems, each suited for different applications. Here, we will explore the Cartesian coordinate system, polar coordinate system, and spherical coordinate system in detail:

2.5.1 **Cartesian Coordinate System**

The Cartesian coordinate system, also called the rectangular coordinate system, is named after the French mathematician René Descartes. It is widely used to represent points in both two-dimensional (2D) and three-dimensional (3D) spaces. In the 2D Cartesian coordinate system, points are located using two perpendicular axes: the x-axis (horizontal) and the y-axis (vertical). The point where these axes intersect is called the origin (0, 0). Each point in the plane is uniquely identified by its coordinates (x, y), where x represents the distance along the x-axis and y represents the distance along the y-axis. In the 3D Cartesian coordinate system, an additional axis, the z-axis (perpendicular to the x-y plane), is introduced. Points in 3D space are specified by a triplet of coordinates (x, y, z), where x, y, and z represent distances along the x, y, and z axes, respectively. The origin in 3D is the point where all three axes intersect (0, 0, 0) (Boas, 2005).

2.5.2 **Polar Coordinate System**

The polar coordinate system is an alternative coordinate system used primarily for representing points in a 2D plane. It is particularly useful when dealing with circular or radial patterns. In the polar coordinate system, points are defined using two coordinates: the radial distance (r) and the angular coordinate (θ). The radial distance represents the distance from the origin to the point, while the angular coordinate specifies the angle between a reference direction (usually the positive x-axis) and the line segment connecting the origin to the point (Kreyszig, 2011).

Polar coordinates are typically represented as (r, θ), where r is a non-negative value and θ is measured in radians or degrees. The conversion between Cartesian coordinates and polar coordinates involves trigonometric functions. The polar coordinate system provides a natural way to describe rotational or circular phenomena. It is commonly used in physics, engineering, and other fields where circular symmetry is prevalent (Kreyszig, 2011).

2.5.3 **Spherical Coordinate System**

The spherical coordinate system is a coordinate system used to represent points in 3D space. It employs three coordinates: the radial distance (r), inclination angle (θ), and azimuth angle (φ). In the spherical coordinate system, a point is specified as (r, θ, φ). The radial distance (r) is the distance from the origin to the point, inclination angle (θ) is the angle between the positive z-axis and the line segment connecting the origin and the point, and azimuth angle (φ) is the angle between the positive x-axis and the projection of the line segment connecting the origin and the point onto the x-y plane (Boas, 2005).

2.6 Reference System in Gravity Measurements

A reference system is the complete conceptual definition of how a coordinate system is formed. It defines the origin and the orientation of fundamental planes or axes of the system. It also includes the underlying fundamental mathematical and physical models. A reference frame means the practical realization of a reference system through observations. In satellite geodesy two fundamental systems are required namely Conventional Inertial Reference System (CIS) and Conventional Terrestrial Reference System (CTS). In this study the more focus will be on Conventional Terrestrial Reference System (CTS) (Seeber, 2003).

The conventional terrestrial reference system, established and maintained by the IERS, and nearly exclusively used for today's scientific and practical purposes is the *International Terrestrial Reference System* (ITRS); its realization is the *International Terrestrial Reference Frame* (ITRF). The ITRS is defined as follows (Boucher et al., 1990; McCarthy, 2000):

- it is geocentric, the center of mass being defined for the whole Earth, including oceans and atmosphere,
- the length unit is the SI meter; the scale is in context with the relativistic theory of gravitation,
- the orientation of axes is given by the initial BIH orientation at epoch 1984.0,
- the time evolution of the orientation will create no residual global rotation with regard to Earth's crust (no-net-rotation condition).

These specifications correspond with the IUGG resolution no. 2 adopted at the 20th IUGG General Assembly of Vienna in 1991. The orientation of axes is also called *IERS Reference Pole* (IRP) and *IERS Reference Meridian* (IRM). The realization of the ITRS, the *International Terrestrial Reference Frame* (ITRF) is formed through Cartesian coordinates and linear velocities of a global set of sites equipped with various space geodetic observing systems. If geographical coordinates (ellipsoidal latitude, longitude, and height) are required instead of Cartesian coordinates (X, Y, Z), use of the GRS80 ellipsoid is recommended.

In gravity measurements and positioning, reference systems and frames are used to define the position and orientation of a gravity sensor relative to a fixed point on the Earth's surface. A reference system is a mathematical framework that defines the coordinates used to specify positions on the Earth's surface, while a reference frame is a physical framework that defines the orientation and motion of the Earth with respect to an inertial reference system (IERS, 2003).

The most commonly used reference system for gravity measurements is the Geodetic Reference System 1980 (GRS80), which is based on a mathematical model of the Earth's shape and gravity field. The GRS80 system defines a set of coordinates known as geodetic coordinates, which are used to specify positions on the Earth's surface (Hofmann-Wellenhof et al., 2008).

In addition to the GRS80 system, there are several other reference systems and frames that are commonly used in gravity measurements and positioning, including the International Terrestrial Reference Frame (ITRF), which is a physical framework that defines the position and orientation of the Earth relative to an inertial reference system, and the World Geodetic System 1984 (WGS84), which is a mathematical model of the Earth's shape and gravity field that is widely used in navigation and positioning. The choice of reference system and frame can have a significant impact on the accuracy and precision of gravity measurements and positioning, and it is important to carefully consider these factors when designing and carrying out gravity surveys (Hofmann-Wellenhof et al., 2008).

2.6.1 **Geodetic Reference System 1980 (GRS80) Ellipsoid**

The Geodetic Reference System 1980 ellipsoid is a widely used reference ellipsoid for geodetic calculations and geodetic datums. It is a mathematical representation of the Earth’s shape and is designed to closely approximate the geoid, which is the equipotential surface of Earth’s gravity field. The GRS80 ellipsoid was adopted by the Union of Geodesy and Geophysics (IUGG) in 1979 as the reference ellipsoid for the Global Geodetic System 1980 (GSI-80). It is commonly used as the reference ellipsoid for various geodetic datums and coordinate system, including the World Geodetic System 1984 (WGS84), which is widely used in global positioning systems (GPS) and navigation applications (Moritz, 1980; IUGG, 1980).

The GRS80 ellipsoid is defined by its semi-major axis (a) and it’s flattening (f). the values for the GRS80 ellipsoid are:

Semi-major axis (a) = 6,378,137.0 meters

Flattening (f) = 1/298.257222101

These values determine the shape and size of the ellipsoid. The semi-major axis represents the equatorial radius of the ellipsoid, while the flattening parameter indicates how much the ellipsoid deviates from a perfect sphere.

The GRS80 ellipsoid is widely used in geodetic computations, such as coordinate transformations, geodetic surveying, and geodetic datum transformations. It provides a consistent and standardized reference measurements and calculations worldwide (Moritz, 1980).

2.6.2 **World Geodetic System 1984 (WGS84)**

World Geodetic System 1984 (WGS84), is a geodetic reference system widely used to represent the Earth's shape and accurately describe locations on its surface. It serves as a global standard for mapping, surveying, and positioning applications. WGS84 was developed by the U.S. Department of Defense (DoD), specifically the National Geospatial-Intelligence Agency (NGA), in partnership with the National Aeronautics and Space Administration (NASA) and other international organizations. The reference ellipsoid used in WGS84 approximates the Earth's surface, assuming an oblate spheroid shape. It is defined by two key parameters: the semi-major axis (a) and the flattening factor (f). The semi-major axis represents the equatorial radius of the ellipsoid, while the flattening factor indicates the degree of flattening compared to a perfect sphere. For WGS84, the values are approximately a = 6,378,137 meters and f = 1/298.257223563 (Hofmann-Wellenhof et al., 2008).

The WGS84 coordinate system uses latitude and longitude as the primary coordinates to specify a location on the Earth's surface. Latitude measures the distance north or south of the Equator and is given in degrees, with positive values for the Northern Hemisphere and negative values for the Southern Hemisphere. Longitude measures the distance east or west of the Prime Meridian (usually chosen as the Greenwich Meridian) and is also given in degrees, with positive values for east of the Prime Meridian and negative values for west. In addition to latitude and longitude, WGS84 also includes a vertical coordinate system called the ellipsoidal height or geodetic height. It represents the height above or below the reference ellipsoid. This height can be positive or negative, depending on whether the point is above or below the ellipsoid (NGA, 2014).

The WGS84 system is continuously refined and updated to account for the changing dynamics of the Earth. It takes into consideration factors such as plate tectonics, movements of the Earth's crust, and variations in the Earth's rotation. To maintain consistency and accuracy, the system is periodically updated with new realizations, denoted by version numbers like WGS84. It is important to note that WGS84 coordinates are not the same as Cartesian (x, y, z) coordinates, as they are based on a curved surface rather than a flat plane. However, WGS84 coordinates can be converted into Cartesian coordinates using various mathematical transformations. WGS84 is used as a standard reference system in many applications, including navigation systems, geographical information systems (GIS), and global positioning systems (GPS). It provides a consistent and globally recognized framework for representing locations on the Earth's surface (IERS, 2003).

The GRS80 ellipsoid and WGS84 ellipsoid are closely related despite the very small difference in flattening as shown in the Table 2.1.

Table 2.1: Comparison in parameters between WGS84 and GRS80

Parameters	**WGS84**	**GRS80**
Semi-major axis (a)	6,378,137.000 meters	6,378,137.000 meters
Semi-minor axis (b)	6,356,752.314 meters	6,356,752.314 meters
Flattening	1/298.257223563	1/298.257222101

2.6.3 **The Clarke 1880 Ellipsoid**

The Clarke 1880 ellipsoid, also known as Clarke's spheroid, is a reference ellipsoid commonly used as a model of the Earth's shape. It was first introduced by the British geodesist and mathematician Alexander Ross Clarke in 1880. The Clarke 1880 ellipsoid became widely adopted as a standard reference for geodetic calculations and mapping (IAG, 1980).

The Clarke 1880 ellipsoid is an oblate spheroid, meaning it is flattened at the poles and bulges at the equator. It is characterized by two parameters: the semi-major axis (a) and the flattening (f). The semi-major axis represents the equatorial radius of the ellipsoid, while the flattening quantifies the deviation from a perfect sphere. For the Clarke 1880 ellipsoid, the semi-major axis (a) is approximately 6,378,249 meters, and the flattening (f) is approximately 1/293.465. This means that the equatorial radius is larger than the polar radius by approximately 21 kilometers (IAG, 1980).

The Clarke 1880 ellipsoid served as a significant reference for geodetic surveys and cartographic work around the world. It was used as a basis for establishing geodetic datums and coordinate systems in various countries. However, over time, more accurate and refined reference ellipsoids have been developed to better model the Earth's shape, such as the GRS 80 and WGS 84 ellipsoids (Snyder, 1987).

2.7 Coordinate Transformation between different Reference Systems

The transformation involves converting between two different reference ellipsoids use a seven-parameter transformation, which involves applying shifts in the three coordinate directions (X, Y, and Z) and three rotation angles (R_X, R_Y, and R_Z), as well as a change in scale (M).

The formula for the seven-parameter transformation from another reference ellipsoid (with semi-major axis a_1 and flattening f_1) to another can be expressed as follows:

$$X' = X + dX + M \times (-R_Z \times Y + R_Y \times Z) \tag{2.13}$$

$$Y' = Y + dY + M \times (R_Z \times X - R_X \times Z) \tag{2.14}$$

$$Z' = Z + dZ + M \times (-R_Y \times X + R_X \times Y) \tag{2.15}$$

where X, Y, and Z are the geographic coordinates in the original reference ellipsoid, and X', Y', and Z' are the transformed coordinates in new reference ellipsoid. The parameters dX, dY, and dZ represent the shifts in the X, Y, and Z directions, respectively. The parameters R_X, R_Y, and R_Z represent the rotations around the X, Y, and Z axes, respectively. The parameter M represents the change in scale.

The choice of transformation method depends on the specific requirements of the application and the characteristics of the reference systems being transformed. There are various transformation methods includes Molodensky-Badekas transformation, Helmert transformation, Bursa-wolf transformation, Molodensky transformation and so many others. But in this study, Molodensky-Badekas transformation chose to be used.

The Molodensky-Badekas transformation is widely used and considered one of the best methods for coordinate transformation between different geodetic reference systems. Here are some reasons why it is often preferred:

i. *Simplicity*: The Molodensky-Badekas transformation is relatively straightforward and can be implemented with relatively simple mathematical equations. It involves a linear approximation that makes it computationally efficient.
ii. *Local Accuracy*: The transformation parameters in the Molodensky-Badekas method can be determined based on local control points or a best-fit approach, allowing for a more accurate transformation within a specific region or area of interest.
iii. *Flexibility*: The Molodensky-Badekas transformation can be applied to both horizontal (latitude and longitude) and vertical (height) coordinates simultaneously. It allows for transformations between different datums and ellipsoids, considering differences in size, shape, and orientation.
iv. *Adequate Precision*: The method provides sufficient accuracy for many geodetic applications. While it may not be suitable for high-precision applications where sub-centimeter level accuracy is required, it is often adequate for general geodetic purposes.
v. *Wide Application*: The Molodensky-Badekas transformation has been extensively used and tested in various geodetic projects worldwide. Its reliability and performance have been demonstrated in many practical applications.

The Molodensky-Badekas transformation method is a widely used method for transforming geographic coordinates between different reference ellipsoids. The method was originally developed by Boris Molodensky in 1958 and was later improved by Constantin Badekas in 1982.

The method involves applying a seven-parameter transformation to the geographic coordinates of a point in the original reference ellipsoid to obtain the corresponding coordinates in the new reference ellipsoid.

The specific values of the parameters depend on the transformation method used. One commonly used method is the Molodensky-Badekas transformation. According to the Geodetic Glossary published by the National Geodetic Survey of the United States, the parameters for this transformation from another ellipsoid to another ellipsoid are:

$$dX = -d_x \tag{2.16}$$

$$dY = -d_y \tag{2.17}$$

$$dZ = -d_z \tag{2.18}$$

$$R_X = (R - R_0) \times \sin(L) \times \cos(G) \tag{2.19}$$

$$R_Y = (R - R_0) \times \sin(L) \times \sin(G) \tag{2.20}$$

$$R_Z = (R - R_0) \times \cos(L) \tag{2.21}$$

$$M = 1 + dM/10^6 \tag{2.22}$$

where $d_x, d_y, and\ d_z$ represent the shifts in the X, Y, and Z directions, respectively, and R, L, and G represent the geographic coordinates in the original reference ellipsoid. R_0 is the radius of the GRS80 ellipsoid at the equator, and dM represents the change in scale between the two ellipsoids. The specific values of d_x, d_y, d_z, and R_0 depends on the two ellipsoids being transformed.

2.8 Gravity corrections

Gravity corrections are fundamental in geophysics and geodesy as they enable the extraction of gravity anomalies, provide insights into the Earth's subsurface, and contribute to better understanding of geodynamic processes, lithospheric structure and resource exploration. Raw gravity data are affected by a wide variety of sources of varying amplitudes, periods, and wavelengths that generally mask gravity variations of geologic or geophysical interest (Hinze, et.al., 2005). Therefore, gravity data must be reduced and corrected to minimize these effects. Firstly, the observed gravity is corrected from the instruments drift and tidal effects, the correction can be up to *0.2 mGal.* Then after the observed gravity needs to be reduced from the terrain to a certain measured surface (MSL, Geoid), for further applications in geodesy or geophysics. These reductions are as follows:

i. Latitude correction.

The latitude correction corrects for the variation in gravitational acceleration caused by the Earth's rotation and equatorial bulge. This causes changes in gravity due to latitudes i.e. gravity increases from about 9.78 ms^{-2} at the equator to about 9.83 ms^{-2} at the poles (Parker & Marobhe, 1991). The theoretical value of a gravity at a given latitude is given by obtained by the International Gravity Formula 1967 (IGF1967) and it is subtracted from the observed value. The following formula for calculating the normal gravity at GRS80 is used,

$$\gamma = 978032.7(1 + 0.0053024 sin^2\varphi - 0.0000058 sin^2 2\varphi) mgal \qquad (2.2)$$

where

γ is a normal gravity

φ is a latitude

ii. Free-air correction

This is also known as the height correction. From the newton's law, the value of gravity increases as the distance/height from the Centre of the Earth to its surface decreases. The free-air correction accounts for the change in gravitational acceleration with vertical distance above a reference surface. It assumes that the gravity field is purely vertical and that the Earth is a homogeneous and spherical object. The observed gravity g_{obs} is reduced to a mean sea level by adding a correction of 0.3086H mGal m^{-1} then we obtain the downward continuation of the gravity at the geoid g_f. The free-air correction doesn't consider the topographic masses between the Earth's surface and geiod or mean sea level. The free-air correction is given as

$$g_f = g_{obs} + 0.3086H \qquad (2.3)$$

iii. Bouguer's slab or plate correction.

The space between mean sea level (geoid) and the earth's surface is filled with weighted mass and not air as it was assumed by the Free-air correction. The Bouguer correction compensates for the gravitational effect of mass between the observation point and the reference surface. Therefore, a further correction must be made for gravitational force exerted by the weighted mass. This correction is known as the Bouguer's plate correction. The gravity reduced with a Bouguer's plate is given as follows: -

Bouguer's plate:

$$g_p = -2\pi G\rho H = -0.1119H \; mGal/m, \tag{2.4}$$

From the above equation (2.4) it is assumed that density is constant. Therefore, combination of the height effects on the equation (2.3) and equation (2.4)

Free-air: $g_f = g_{obs} + 0.3086H$ (2.5)

Therefore;

$$g^{BO} = g_{obs} + 0.1967H \tag{2.6}$$

2.9 Gravity anomaly

The gravity anomaly is the difference between the Earth's gravitational acceleration (i.e. gravitation and rotation) observed, or estimated, at some reference level, and the gravitational acceleration generated by a simple mass distribution, such as a biaxial ellipsoid of revolution, at the same or some other reference level (Hackney & Featherstone, 2003). The gravity anomaly is obtained from the gravity observed which is then reduced to the geoid (mean sea level), and the theoretical normal gravity referenced from the ellipsoid. Different types of anomalies reflect variations in the components used in defining the modeled gravity at the station (Hinze, et al., 2005). The mathematical models for anomalies changes upon the use of anomalies in geophysics and geodesy.

i. *Free-air Gravity anomaly* is obtained from the difference between the downward gravity continuation by the free-air and the theoretical normal gravity. This anomaly has its greatest use in geodesy and are frequently used in modeling and map interpretation in marine areas (Hinze, et al., 2005). In geophysics, it used to correct the Normal gravity to the Geiod.

   Δg_f = Free-air gravity anomaly

$$\Delta g_f = g_{obs} - \gamma + 0.3086H \tag{2.7}$$

   where,

γ is normal gravity.

ii. *The Bouguer Gravity anomaly* is determined from the difference in the observed gravity and the Normal gravity using the free air model plus the Bouguer and terrain model effects. This is also known as the complete Bouguer anomaly, which can be regarded as the Gravity anomaly due to the variation in the spatial distribution of rock densities (Parker & Marobhe, 1991). When the terrain correction is omitted it becomes Incomplete Bouguer anomaly

$$\Delta g^{BO} = g_{obs} - \gamma + 0.1967H \tag{2.8}$$

where,

Δg^{BO} is the Bouguer gravity anomaly

iii. *Surface Gravity anomaly* is determined by taking the difference between the observed gravity at the geiod and the normal gravity at the telluroid. Telluroid is a surface which is obtained when terrain of the Earth's surface is reduced by the geoidal height (N). The Tanzania gravity database 2008 (TGDB08) uses this approach in the creation of database. Surface gravity anomaly is used in the computation of the Gravimetric Geoidal model by using the Stoke's formula. The Figure 2.3 shows how surface anomaly can be computed on the telluroid.

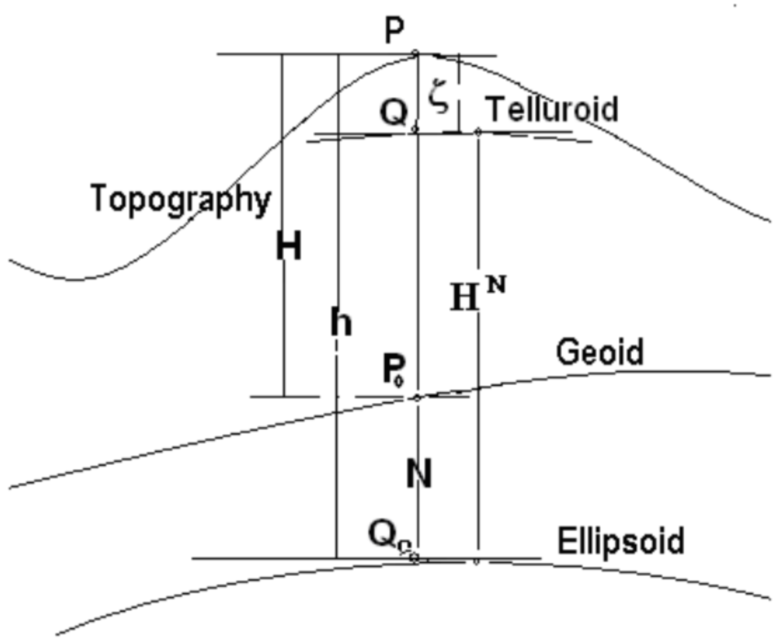


Figure 2.3: The surface anomaly Geometry (Ulotu, 2009)

From the figure 2.3, P is a point on the surface, P_0 is a point at the geoid normal to point P, H is orthometric height from P to P_0, Q and Q_0 are points on the ellipsoid and telluroid respectively. The h is ellipsoidal height and N and ς are the ellipsoidal and surface height. The surface gravity anomaly on the telluroid is computed as follows.

First the surface height is computed as

$$h_Q = H_P + \frac{\Delta g_B}{\gamma_0} H_P \tag{2.9}$$

$$\Delta g_B = \Delta g_f - 2\pi\mu H_P \tag{2.10}$$

Where,

H_P is the orthometric height

γ is the normal gravity

Δg_f is free-air anomaly

The normal gravity on the telluroid using the GRS80 is obtained as

$$\gamma_Q = \left[\gamma_0 - (0.3087691 - 0.0004398 sin^2\varphi) h_Q + 7.2125X10^{-8} {h_Q}^2\right] mgal \tag{2.11}$$

The surface gravity anomaly is given as

$$\Delta g = g_{obs} - \gamma_Q \tag{2.12}$$

2.10 Statistical Analysis

Statistical Analysis is the process of collecting and analyzing large volumes of data in order to identify trends, patterns and develop valuable insights. Statistical analysis is a scientific tool that helps collect and analyze large amounts of data to identify common patterns and trends to convert them into meaningful information. It is a method for removing bias from evaluating data by employing numerical analysis. This technique is useful for collecting the interpretations of research, developing statistical models, and planning surveys and studies. The conclusions are drawn using statistical analysis facilitating decision-making and helping on making future predictions on the basis of past trends. (Howell, 2012).

2.10.1 **Types of Statistical Analysis**

There are different types of statistical analysis for different studies. There are two main types of statistical analysis; Descriptive and Inferential. Analysis. Descriptive statistics involves summarizing and describing data using measures such as central tendency (mean, median, mode), variability (standard deviation, range), and graphical representations (histograms, bar charts). It provides a basic overview of the data and allows for understanding its key characteristics. On the other hand, Inferential statistics is used to make inferences or draw conclusions about a population based on a sample. It involves techniques such as hypothesis testing, confidence intervals, and estimation. Inferential statistics allows researchers to generalize findings from a sample to a larger population (Rice, 2007).

CHAPTER THREE

RESEARCH METHODOLOGY

This chapter describes the methods used in data collection for the research and how the research issue will be investigated. This chapter presents the specific procedures which will be used to conceive, collect, analyses and present data. In this chapter, 2D geographical position transformations of the point gravity anomalies were transformed from Clarke 1880 ellipsoid to WGS84 ellipsoid by using Molodensky-Badekas method and quality assessment of the results were performed by using statistical measures.

3.1 Geographic Position Transformation by using Molodensky-Badekas Method

3D conformal transformations are used to convert coordinates related to the geodetic datum to another which is commonly known as Datum transformation. In such applications, the rotations between the two 3D coordinates axes are small (usually less than 1 second of arc) and certain approximations are used to simplify rotation matrices and these simplified matrices are a common feature of the Molodensky-Badekas transformations.

Molodensky-Badekas transformation is a conformal three-dimensional (3D) Cartesian transformation commonly used in many geodetic applications. It is seven-parameter transformation and it combine a scale change, three axes-rotations and three origin-shifts in a practical mathematical model of the relationships between points in two different 3D coordinate systems.

3D conformal transformations are often given in the form:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_2 = s\boldsymbol{R}_{ZYX} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_1 + \begin{bmatrix} t_X \\ t_Y \\ t_Z \end{bmatrix}_2 \tag{3.1}$$

Whereby: the subscripts $[\ \ ]_1$ and $[\ \ ]_2$ refer to the X, Y, Z Cartesian coordinates of systems 1 and 2 respectively. s is a scale factor, $\boldsymbol{R}_{XYZ}$ is a 3 X 3 rotation matrix and t_X, t_Y, t_Z are translations between the origins of the two systems measures in the directions of the system 2 coordinate axes.

The Molodensky-Badekas transformation have a modified form of equation (3.1) where;

i. The rotation matrix $\boldsymbol{R}_{XYZ}$ has the approximated form R_s where the subscript s refers to small rotation angles $\varepsilon_X, \varepsilon_Y, \varepsilon_Z$ about the coordinate axes and

$$\boldsymbol{R}_{ZYX}\boldsymbol{R}_s = \begin{bmatrix} 1 & \varepsilon_Z & -\varepsilon_Y \\ -\varepsilon_Z & 1 & \varepsilon_X \\ \varepsilon_Y & \varepsilon_X & 1 \end{bmatrix} \quad (3.2)$$

The scale factor s is expressed in the form

$$s = 1 + ds \quad (3.3)$$

Whereby ds is a small value usually expressed in ppm[4].

Then, the transformation is given in the form;

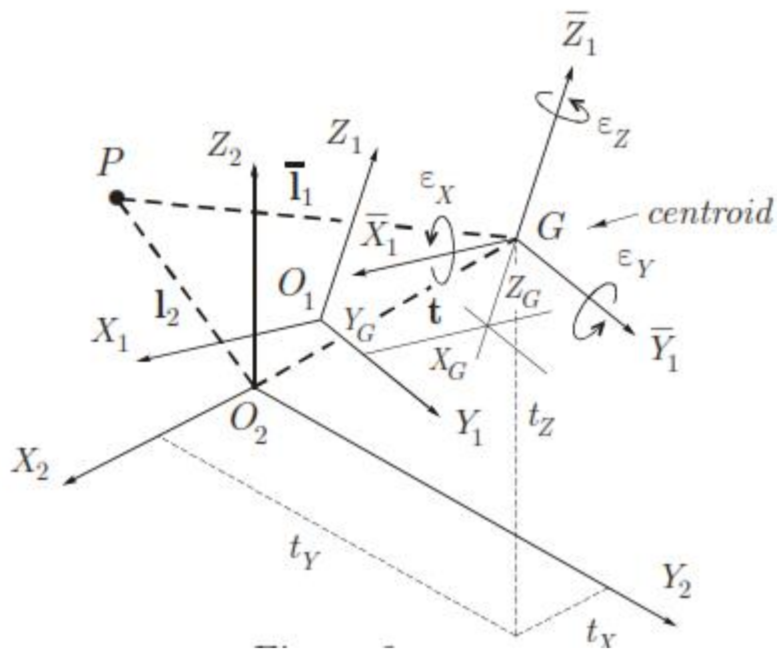


Figure 3.1: Geometry of Molodensky-Badekas transformation

Figure 3.1 shows the geometry of the Molodensky-Badekas transformation that makes use of centroid. The X, Y, Z axes of system 1 are rotated by very small angles $\varepsilon_X, \varepsilon_Y, \varepsilon_Z$ from the $\overline{X_1, Y_1, Z_1}$ axes of system 2, and the origins O_1 and O_2 of the two systems are displaced. The XYZ system is a centroidal system whose origin is at a centroid G of a set of points in system 1 and whose axes are parallel to the X,Y,Z axes of system 1. In the Figure 3.1, the centroid G is displaced from O2 by translations t_X, t_Y, t_Z measured in the directions of the X, Y, Z axes of system 2 and $t_2 = [t_X \quad t_Y \quad t_Z]_2^T$ is the position vector of the centroid.

The mathematical relationship between coordinates in both systems, this including a scale factor $s = 1 + ds$, can be developed by using vector equations, where from Figure 3.1 we may write;

$$\mathbf{I}_2 = \mathbf{t}_2 + (1 + \mathrm{ds})\mathbf{R}_s\bar{\mathbf{I}}_1 \tag{3.4}$$

Where;

$$\bar{I}_1 = \begin{bmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{bmatrix}_1 = \begin{bmatrix} X - X_G \\ Y - Y_G \\ Z - Z_G \end{bmatrix}_1 = \mathbf{I}_1 - \mathbf{g}_1 \tag{3.5}$$

$X_G Y_G Z_G$ are the coordinates of the centroid and $\mathbf{g}_1 = [X_G \quad Y_G \quad Z_G]_1^T$ is the position vector of the centroid in system 1 coordinates.

Alternatively, the Molodensky-Badekas may be written as;

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_2 = (1 + ds) \begin{bmatrix} 1 & \varepsilon_Z & -\varepsilon_Y \\ -\varepsilon_Z & 1 & \varepsilon_X \\ \varepsilon_Y & \varepsilon_X & 1 \end{bmatrix} \begin{bmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{bmatrix}_1 + \begin{bmatrix} t_X \\ t_Y \\ t_Z \end{bmatrix}_2 \tag{3.6}$$

The transformation equation between geodetic coordinates and Cartesian coordinates of point P (X_P, Y_P, Z_P) can be expressed as (Seeber,2003),

$$\begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} = \begin{bmatrix} N_P h_p cos\phi_p cos\lambda_p \\ N_P h_P cos\phi_p sin\lambda_p \\ \left[N_p(1 - e^2) + h_P\right] sin\lambda_P \end{bmatrix} \tag{3.7}$$

With;

$$N_P = \frac{a}{(1 - e^2 sin^2\phi_P)^{\frac{1}{2}}} \tag{3.8}$$

$$e^2 = 2f - f^2 = \frac{a^2 - b^2}{a^2} \tag{3.9}$$

Where, N_P is the prime vertical radius of curvature at P, and e the first eccentricity of the reference Ellipsoid.

3.2 Quality Assessment of Results

The statistics of the differences in gravity value due to variation in geographic position between the two different reference systems (i.e. Clarke 1880 Ellipsoid and WGS84 Ellipsoid) will be computed using equations presented in Table 3.1.

Table 3.1: Formulae for computing the statistics

S/N	Statistic	Formula
1.	Difference; $diff$	$\Delta g_{Clarke\ 1880} - \Delta g_{WGS84}$
2.	Mean; $\bar{X}$	$\frac{\sum diff}{n}$
3.	Standard Deviation; STD	$\sqrt{\frac{\sum (X_i - \bar{X})^2}{n-1}}$
4.	Root Mean Square; RMS	$\sqrt{\frac{\sum diff^2}{n}}$

3.3 Data Collection

The data used in this study was obtained from the Tanzania Gravity Database of 2021 (TGDB21) prepared by Nchambi Nzige in 2021 that involved the collection of gravity information collected from several sources available within Tanzania and overseas.

The current Tanzania Gravity Database TGDB21 contains terrestrial and marine point gravity data, the DTU $1' \times 1'$ satellite altimetry marine gravity data, the $1' \times 1'$ global gravity models (GGM), the African Geoid Project (AGP) $5' \times 5'$ gravity data and Aerial gravity data. Data available in the database covers the East African region at 15° = φ = 5° and 25° = λ = 44°.

TGDB21 has 95489 Point gravity data, 42562 AGP $5' \times 5'$ gravity anomaly and the $1' \times 1'$ free-air marine gravity anomaly.

3.4 Software and Programs

In this research several software compatible to gravity data were used. These are; Microsoft Office version 19, and Golden Surfer software v18.

3.5.1 Microsoft Office version 19

This was used in writing report and performing computations. Programs which mostly used were Microsoft Word for writing report, Microsoft excel for preparation of data and computing statistics attributes such as standard deviation, root mean square and the mean , and Microsoft Access which were used to access the gravity data from the database.

3.5.2 Golden Surfer Software v18

The Golden Surfer software played a major role in this research. It was used in the production of surface maps for all three anomalies, then used for conversion of geographic positions of gravity and gravity anomalies from Clarke 1880 Ellipsoid to WGS84 Ellipsoid (assumed to be equal to GRS80 ellipsoid) and later on used for extraction of all gravity anomalies before and after geographic coordinate transformation.

CHAPTER FOUR

RESULTS AND ANALYSIS OF RESULTS

4.1 Results

The results include the surface maps produced from the gravity anomalies, the statistics of differences in gravity anomalies between the neglected reference system and the corrected reference system to highlight the variation. Also, the descriptive statistical analysis was performed. The statistics of the differences in gravity anomalies include minimum, maximum, mean and standard deviation, sample variance.

4.1.1 Production of Surface Maps

Surface maps for Gravity anomalies, Free-Air Gravity anomalies and Bouguer gravity anomalies were plotted for the given geographic coordinates considering was to be referenced to Clarke 1880 and the results are shown in the Figure 4.1, Figure 4.2 and Figure 4.3. The produced surface maps were then used for the extraction of all gravity anomalies for the point gravity data both before transformation and after transformation to determine the differences.

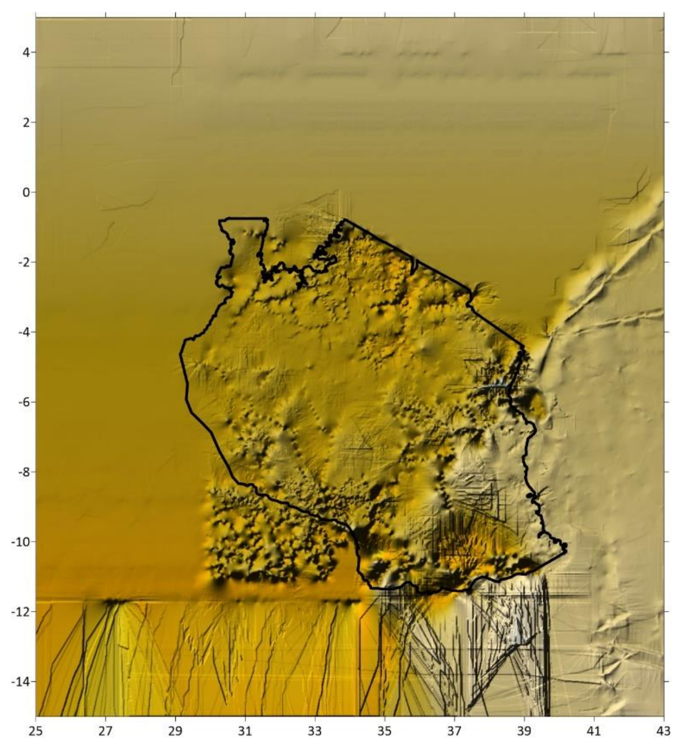


Figure 4.1: Surface map for Gravity Anomaly

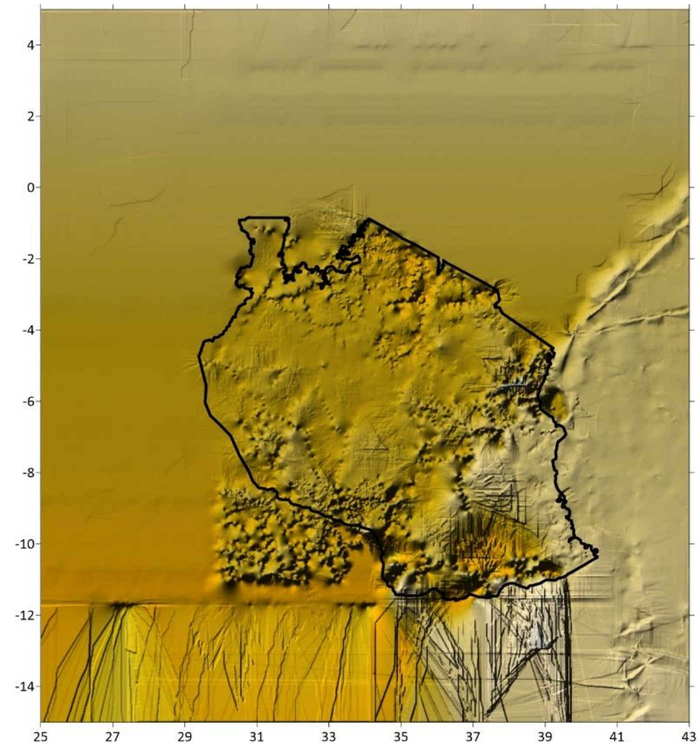


Figure 4.2: Surface map for Free-Air Gravity anomalies

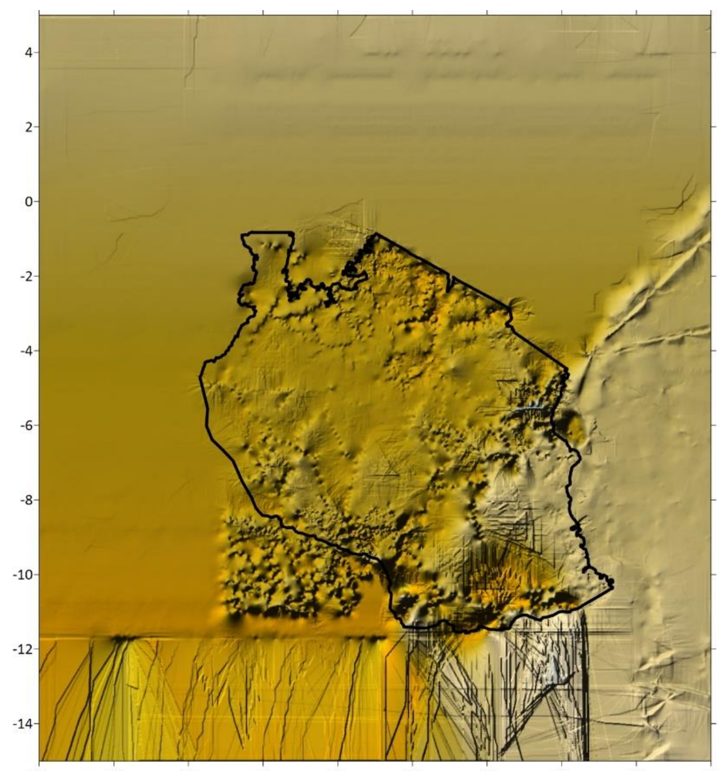


Figure 4.3: Surface map for Bouguer Gravity anomalies

4.1.2 Extraction of Gravity Anomalies from the Surface Map

<u>First Scenario</u>: When Neglecting Reference System

In this first scenario, the produced surface maps were used to extract the gravity anomalies before performing geographic coordinate transformation by using the Point gravity position and this can be used as a check by calculating the difference in gravity and the results are shown in Table 4.1, Table 4.2 and Table 4.3.

Table 4.1: Sample of extracted gravity anomalies from the surface map

Point	Latitude (dd.ddd)	Longitude (dd.ddd)	Gravity Anomaly (mGal)
PG1	-10.0055	32.7000	-475.8598
PG2	-10.0222	32.6722	-392.7989
PG3	-10.0343	32.6722	-380.4590
PG4	-10.061	32.6777	-376.4643
PG5	-10.0832	32.6832	-379.6339
PG6	-10.1000	32.6832	-381.6099
PG7	-10.1222	32.6888	-391.2360
PG8	-10.1388	32.6943	-387.4159
PG9	-10.1610	32.6943	-408.3232
PG10	-10.1610	32.7000	-393.8200
PG11	-10.1832	32.7000	-413.0074
PG12	-9.0235	32.9917	-472.0650
PG13	-9.0388	32.9763	-437.2102
PG14	-9.0555	32.9722	-440.5920
PG15	-9.0735	32.9652	-496.3508
PG16	-9.0917	32.9527	-454.6524
PG17	-9.1055	32.9485	-444.0660
PG18	-9.1082	32.9375	-441.4935
PG19	-9.1110	32.9263	-445.7999
PG20	-9.1167	32.9207	-440.9528

Table 4.2: Sample of extracted Free-Air gravity anomalies from the surface map

Point	Latitude (dd.ddd)	Longitude (dd.ddd)	Free Air Gravity Anomaly (mGal)
PG1	-10.0055	32.7000	-88.8758
PG2	-10.0222	32.6722	-5.8149
PG3	-10.0343	32.6722	6.5250
PG4	-10.061	32.6777	10.5196
PG5	-10.0832	32.6832	7.3501
PG6	-10.1000	32.6832	5.3741
PG7	-10.1222	32.6888	-4.2520
PG8	-10.1388	32.6943	-0.4319
PG9	-10.1610	32.6943	-21.3392
PG10	-10.1610	32.7000	-6.8360
PG11	-10.1832	32.7000	-26.0234
PG12	-9.0235	32.9917	-85.0810
PG13	-9.0388	32.9763	-50.2262
PG14	-9.0555	32.9722	-53.6080
PG15	-9.0735	32.9652	-109.3668

Table 4.3: Sample of extracted Bouguer gravity anomalies from the surface map

Point	Latitude (dd.ddd)	Longitude (dd.ddd)	Bouguer Gravity Anomaly (mGal)
PG1	-10.0055	32.7000	-229.1978
PG2	-10.0222	32.6722	-146.1369
PG3	-10.0343	32.6722	-133.7970
PG4	-10.0610	32.6777	-129.8023
PG5	-10.0832	32.6832	-132.9719
PG6	-10.1000	32.6832	-134.9479
PG7	-10.1222	32.6888	-144.5740
PG8	-10.1388	32.6943	-140.7539
PG9	-10.1610	32.6943	-161.6612
PG10	-10.1610	32.7000	-147.1580
PG11	-10.1832	32.7000	-166.3454
PG12	-9.0235	32.9917	-225.4030
PG13	-9.0388	32.9763	-190.5482
PG14	-9.0555	32.9722	-193.9299
PG15	-9.0735	32.9652	-249.6888

<u>Second Scenario:</u> After performing the geographic coordinate transformation

In this second scenario, the produced surface maps were used to extract the gravity anomalies after performing geographic coordinate transformation of Point gravity data (from Clarke 1880 Ellipsoid to WGS84 Ellipsoid) and the difference in gravity were calculated and the results are shown in Table 4.4 and Table 4.5.

Table 4.4: Sample of gravity anomalies extracted from transformed Point gravity position

Point	Longitude (dd.ddd)	Latitude (dd.ddd)	Gravity Anomaly (mGal)	Change in Gravity Anomaly (mGal)
PG1	32.7007	-10.0074	-461.1474	-14.7125
PG2	32.6729	-10.0241	-390.9674	-1.8314
PG3	32.6729	-10.0362	-380.2894	-0.1696
PG4	32.6784	-10.0629	-376.2083	-0.2560
PG5	32.6839	-10.0851	-379.8082	0.1743
PG6	32.6839	-10.1019	-383.6477	2.0379
PG7	32.6895	-10.1241	-390.1515	-1.0845
PG8	32.6950	-10.1407	-387.5199	0.1039
PG9	32.6949	-10.1629	-409.1307	0.8075
PG10	32.7007	-10.1629	-395.3718	1.5518

Table 4.5: Sample of Free Air gravity anomalies extracted from transformed Point gravity position

Point	Longitude (dd.ddd)	Latitude (dd.ddd)	Free Air Anomaly (mGal)	Change in Free Air Gravity Anomaly (mGal)
PG1	32.7007	-10.0074	-74.1634	-14.7125
PG2	32.6729	-10.0241	-3.9834	-1.8314
PG3	32.6729	-10.0362	6.6946	-0.1696
PG4	32.6784	-10.0629	10.7757	-0.2560
PG5	32.6839	-10.0851	7.1758	0.1743
PG6	32.6839	-10.1019	3.3363	2.0379
PG7	32.6895	-10.1241	-3.1675	-1.0845
PG8	32.6950	-10.1407	-0.5359	0.1039
PG9	32.6950	-10.1629	-22.1467	0.8075
PG10	32.7007	-10.1628	-8.3878	1.5518

Table 4.6: Sample of Bouguer gravity anomalies extracted from transformed Point gravity position

Point	Longitude (dd.ddd)	Latitude (dd.ddd)	Bouguer Gravity Anomaly (mGal)	Change in Bouguer Gravity Anomaly (mGal)
PG1	32.7007	-10.0074	-214.4854	-14.7125
PG2	32.6729	-10.0241	-144.3054	-1.8314
PG3	32.6729	-10.0362	-133.6274	-0.1696
PG4	32.6784	-10.0629	-129.5463	-0.2560
PG5	32.6839	-10.0851	-133.1462	0.1743
PG6	32.6839	-10.1019	-136.9857	2.0379
PG7	32.6895	-10.1241	-143.4895	-1.0845
PG8	32.6950	-10.1407	-140.8579	0.1039
PG9	32.6950	-10.1629	-162.4687	0.8075
PG10	32.7007	-10.1629	-148.7098	1.5518
PG11	32.7007	-10.1851	-177.3223	10.9790
PG12	32.9924	-9.0255	-223.2560	-2.1471
PG13	32.9770	-9.0408	-192.0896	1.5415
PG14	32.9729	-9.0575	-195.3386	1.4086

4.1.3 Statistical Summary for the Differences in Gravity Anomalies

The statistical measures were performed on the differences between the gravity anomalies before coordinates and gravity anomalies after coordinate transformation and their results are shown in the Table 4.7, Table 4.8 and Table 4.9.

Table 4.7: Statistics for the Change in Gravity Anomaly

Change in Gravity Anomaly	
Statistical Measure	**mGal**
Mean	-0.4358
Standard Deviation	6.3303
Sample Variance	40.0724
Minimum	-100.8476
Maximum	531.5238
Count	39179
Confidence Level(95.0%)	0.0627

Table 4.8: Statistics for the Change in Free-Air Gravity Anomaly

Change in Free-Air Gravity Anomaly	
Statistical Measure	**mGal**
Mean	0.4259
Standard Deviation	6.5319
Sample Variance	42.6662
Minimum	-531.5238
Maximum	100.8476
Count	39179
Confidence Level(95.0%)	0.0647

Table 4.9: Statistics for the Change in Bouguer Gravity Anomaly

Change in Bouguer Gravity Anomaly	
Statistical Measure	**mGal**
Mean	-0.4295
Standard Deviation	6.3908
Sample Variance	40.8423
Minimum	-100.8476
Maximum	531.5238
Count	39179
Confidence Level(95.0%)	0.0633

4.2 Analysis and Discussions of the Results

According to the International Association of Geodesy (IAG, 2011), the variation of 1 mGal in the Earths gravitational field can cause an error in the determination of the geoid at the level of centimeters. The exact value of error magnitude depends on various factors, such as the measurement technique, the geographical location, and the local gravity field characteristics. The estimation of this error commonly performed by using the formula known as the "Free-air correction." This correction relates the difference in gravity measurements to the difference in geoid heights.

The free air correction can be expressed approximately as follows:

$$\Delta h = \left(\frac{\Delta g}{\gamma}\right) \times k \tag{4.1}$$

Whereby;

Δh = is the difference in geoid heights caused by a variation in gravity,

Δg= is the variation in gravity (mGal)

γ= is the average gravity acceleration (approximately 980.665 cm/s^2)

k= is a correction factor that depends on the measurement location and technique.

Using this formula, for $\Delta g = 1$ mGal (0.001cm/s^2), we can estimate the approximate Δh or error in geoid height caused by this variation. For the conservative value of k = 0.1 cm/mGal then;

$$\Delta h = \left(\frac{0.001}{980.665}\right) \times 0.1 \approx 1.02 \times 10\hat{}(-6)\ cm = 0.000102\ cm.$$

Therefore, a variation of 1 mGal can result in an error of approximately 0.000102 cm in the determination of the geoid height.

Then, from the estimation above, the error was applied to the standard deviation obtained in the change of the gravity anomalies and their results is shown in the Table 4.10.

Table 4.10: Estimated error in gravity anomalies

S/N	Anomaly	Standard Deviation (mGal)	Estimated Error (mm)
1	Gravity Anomaly	6.33027916	0.0064569
2	Free-Air Gravity Anomaly	6.531940506	0.0066626
3	Bouguer Gravity Anomaly	6.390796066	0.0065186

From the results above, it is shown that Change Free-Air Gravity Anomaly has higher standard deviation indicates greater variability in the gravity measurements, suggesting that neglecting the reference system contribute much in the computed Free-Air Gravity Anomaly compared to Bouguer Gravity Anomaly and Gravity Anomaly with a lower standard deviation suggests that neglecting the reference system has less impact on the gravity measurements.

From these results, the values were very small and seems to be insignificant error but their consideration are very important as it was found that neglecting the reference system introduces some differences in gravity measurements compared to when the reference system is properly considered.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The paramount importance of this study was to investigate statistical variations in gravity position due to the neglecting reference system for the Tanzania Gravity Database 2021 (TGDB21). The geographic coordinates transformation of gravity data was performed by using Molodensky-Badekas transformation method and statistical analysis were also performed.

Considering the objectives of the study and analyses conducted in this research results which were obtained in chapter four, the following conclusion can be drawn:

i. The neglect of the reference system in gravity measurements has a less significantly impacts on the observed gravity. Through the analysis of data and statistical tests performed on the change of the gravity anomalies shown the following standard deviations. The Free-air gravity anomaly has high estimated error of about 6.531940506 mGal, Bouguer gravity anomaly has an estimated error of about 6.390796066 mGal and the one with the least estimated error was Gravity anomaly of about 6.33027916 mGal. From these results, the values were very small and seems to be insignificant error as represented in Table 4.10 but their consideration are very important as it was found that neglecting the reference system introduces some differences in gravity measurements compared to when the reference system is properly considered.

ii. The small amount of error obtained from this study may be contributed with the gravity resolution, because the gravity resolution of the gravity anomalies used in this study was $1' \times 1'$ equal to $1.8 \times 1.8\ km$, so real value of error can be obtained with higher resolution.

The findings from this research emphasize the importance of properly accounting for the reference system in gravity measurements. Neglecting the reference system can lead to inaccurate and unreliable gravity positions, which can have implications for geospatial applications such as geodesy, mapping, and surveying. It is crucial to consider the reference system as an integral part of gravity measurements to ensure accurate and consistent results.

5.2 Recommendations

The gravity records play a crucial role in promoting economic and social development. It has various applications such as mineral and hydrocarbons exploration, monitoring crustal deformation, mitigating geo-hazards, and calculating geoid models, which are proposed as a new vertical reference point. Tanzania is at the forefront of technological and economic progress, making the availability of a more up-to-date and internationally adopted reference gravity field, and comprehensive and reliable gravity database essential for supporting these advancements. Therefore, the followings are recommended to enhance the gravity data situation in Tanzania:

i. The next researcher should check for the effect of neglecting reference ellipsoid in term of ellipsoidal and orthometric heights.

ii. The next research should analyze the error determination on the effect of neglecting reference system using higher resolution gravity anomalies data.

iii. Further research could explore the specific sources of variation introduced by neglecting the reference system and investigate potential mitigation strategies to minimize these effects.

iv. It is pertinent for other researchers to validate the findings of this study by using other models such as the Bursa-Wolf transformation, or Helmert transformation methods.

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