ARDHI UNIVERSITY

ASSESSMENT OF AFRgeo19 GEOID MODEL BY GEOID SLOPE METHOD IN TANZANIA.

MAHIMBI DELPHINE B

BSc Geomatics

Dissertation

Ardhi University, Dar es Salaam

July, 2023

ASSESSMENT OF AFRgeo19 GEOID MODEL BY GEOID SLOPE METHOD IN TANZANIA.

MAHIMBI, DELPHINE B

A Dissertation Submitted to the Department of Geospatial Sciences and Technology in Partially Fulfilment of the Requirements for the Award of Science in Geomatics (BSc. GM) of Ardhi University

CERTIFICATION

The undersigned certify that they have read and hereby recommend for acceptance by the Ardhi University dissertation titled **"Assessment of AFRgeo19 geoid model by geoid slope method in Tanzania"** in partial fulfillment of the requirements for the award of degree of Bachelor of Science in Geomatics at Ardhi University.

……………...................................

Ms. REGINA V. PETER

Date……………………….

DECLARATION AND COPYRIGHT

I, MAHIMBI, DELPHINE B hereby declare that, the contents of this dissertation are the results of my own findings through my study and investigation, and to the best of my knowledge they have not been presented anywhere else as a dissertation for diploma, degree or any similar academic award in any institution of higher learning.

………………………………..

MAHIMBI, DELPHINE B

22769/T.2019

(Candidate)

ACKNOWLEDGEMENT

I would like to begin by thanking the Almighty God for giving me the courage and good health I needed to finish this research. My sincere thanks and obligations are extended to my main supervisor, Ms. Regina V. Peter, for her unwavering support, immeasurable guidance, pertinent materials, encouragement, comments, and criticism that helped this dissertation come to fruition, and to my lecturer, Ms. Valerie Ayubu for her counsel and contribution to it.

Also, my sincere gratitude to Dr. Prosper E. Ulotu and Ms. Regina V. Peter for their guidance in development of this research title as well as provision of necessary and sufficient references

I'd like to express my gratitude to each and every member of the Ardhi University Department of Geospatial Science and Technology personnel for their contributions and difficulties encountered during my research's intermediate presentations, which helped my dissertation come to a good conclusion.

Last but not least, I would like to express my sincere gratitude to my gorgeous family, as well as my relatives and classmates, whose contributions and advice helped me complete this dissertation successfully. I also want to thank Higher Student Education Loan Boards for supporting my education financially.

DEDICATION

I dedicate this dissertation to my loving family in a whole and my relatives in appreciation of their unceasing prayers, support, and encouragement throughout my academic career from the very first day I started grade 1. God bless you all for your unending assistance both monetarily and in the form of love, counsel, moral support, and encouragement while I was a student at Ardhi University.

ABSTRACT

The aim of this dissertation was to assess the Africa geoid model of 2019 known as AFRgeo19 by geoid slope method in Tanzania mainland. Total of 31 TPLNs benchmarks were used and were tested statistical at 95% confidence level and 13 benchmarks passed. Assessment of AFRgeo19 was based on baseline length which were categorised into three short baselines, long baselines and very long baselines with distances 0-100km for short baselines with 8 baselines, 101-200km for long baselines with 16 baselines and 201-500km for very long baselines with 51 baselines.

AFRgeo19 was assessed before accounting systematic errors and after accounting systematic errors. Systematic errors were accounted using 3-4-5-7 parametric models. The results showed that after removal of systematic errors, the values of SD and RMS decreases compared to the results before accounting systematic errors with exception of short baselines category whose SD and RMS increased after accounting for systematic errors. The results of SD and RMS for short baselines were 211.4 ppm and 205.9 ppm before accounting systematic errors and 245.3 ppm and 242.0 ppm after accounting systematic errors while results of SD and RMS for long baselines were 2.38 ppm and 2.80 ppm before accounting systematic errors and 0.68 ppm and 0.66 ppm after accounting systematic errors and results of SD and RMS for very long baselines were 22.67 ppm and 38.93 ppm before accounting for systematic errors and 0.92 ppm and 0.91 ppm after accounting systematic errors.

After obtaining the results they were contrasted with results from (Simion, 2019) research which showed that TZG17 performed better in Tanzania mainland with SD and RMS for short baselines 59.71 ppm and 58.82 ppm while SD and RMS for long baselines were 1.42 ppm and 1.39 ppm and SD and RMS for very long baselines were 1.14 ppm and 1.13 ppm after accounting systematic errors. Comparing the results of AFRgeo19 and TZG17, AFRgeo19 provides better results on 67 baselines out of 75 baselines used.

Furthermore, it is recommended to assess the Africa geoid model of 2019 with recently derived geoid models the TZG19 and EGM20. And the geoid model that performs better in Tanzania mainland should be gazetted as the National geoid model by MLHHSD to be used as the vertical datum.

Keywords: AFRgeo19, Geoid slope, Systematic errors, Geoid Height.

TABLE OF CONTENTS

CERTIFICATION ... i

DECLARATION AND COPYRIGHT ... ii

ACKNOWLEDGEMENT ... iii

DEDICATION ... iv

ABSTRACT ... v

TABLE OF CONTENTS ... vi

LIST OF FIGURES ... ix

LIST OF TABLES ... x

ACRONYMS AND ABBREVIATIONS ... xi

CHAPTER ONE ... 1

INTRODUCTION ... 1

1.1 Background ... 1

1.2 Problem Statement ... 5

1.3 Main Objective ... 5

1.4 Specific Objectives ... 5

1.5 Research Hypothesis ... 6

1.6 Scope/Limitations ... 6

1.7 Significance ... 7

1.8 Beneficiaries ... 8

1.9 Research Outline ... 8

CHAPTER TWO ... 9

LITERATURE REVIEW ... 9

2.1 Overview of Methods of Assessing Geoid Models ... 9

2.2 Geoid Slope ... 9

2.3 GPS Levelling ... 10

2.3.1 Absolute GPS Levelling 10
2.3.2 Relative GPS Levelling 11
2.4 Sea Surface Topography 11
2.5 Astrogeodetic Deflection of the Vertical 11
2.6 Gravity Anomalies 12
2.7 Selection of Geoid Slope Method 12
CHAPTER THREE 13
METHODOLOGY 13
3.1 Overview 13
3.1.1 Statistical Test 13
3.1.2 Baseline Categories 14
3.1.3 Systematic Errors Modelling 14
3.2 Data Availability 15
3.2.1 Ellipsoidal Heights and Orthometric Heights 15
3.2.2 AFRgeo19 17
3.3 Software used 17
3.3.1 Golden Surfer 17
3.3.2 MATLAB 17
3.3.3 Microsoft Office 17
CHAPTER FOUR 18
RESULT, ANALYSIS AND DISCUSSION 18
4.1 Results 18
4.1.1 Geometrical and physical Geoid 18
4.1.2 Benchmarks passed 95% Confidence Level 19
4.1.3 Assessment of AFRgeo19 before accounting systematic errors 19

4.1.3.1 Results for short baselines 20
4.1.3.2 Results for long baselines 20
4.1.3.3 Results for very long baselines 20
4.1.4 Assessment of AFRgeo19 after accounting systematic errors 21
4.1.4.1 Results of short baselines 21
4.1.4.2 Results of Long baselines 22
4.1.4.3 Results of very long baselines 23
4.2 Discussion 25
CHAPTER FIVE 27
CONCLUSION AND RECOMMENDATIONS 27
5.1 Conclusion 27
5.2 Recommendations 27
REFERENCES 28
APPENDICES 30

LIST OF FIGURES

Figure 1.1 Geoid Map of Africa 3

Figure 1.2 Gravity Data used for Ground, shipborne and Altimetry respectively........................ 4

Figure 1.3 Distribution of TPLNs Benchmarks in Tanzania 7

Figure 2.1 Geoid Slope Computations……...………………………………………………...……..9

Figure 4.1 Standard deviations and root mean squares of the baselines before accounting for systematic errors…………………………………………………………………………………21

Figure 4.2 Standard deviations and root mean squares of the baselines after accounting for systematic errors……………………………………………………………………………........24

LIST OF TABLES

Table 3.1 Categories of baselines with their respective distances and number of baselines…….14

Table 3.2 Tanzania Primary Levelling Network Benchmarks…………………………………....16

Table 4.1 Geometrical and Physical Geoid Heights……………………………………..........18

Table 4.2 Benchmarks which passed the 95% confidence level…………………………………19

Table 4.3 Statistics of short baselines before accounting systematic errors………………………20

Table 4.4 Statistics of long baselines before accounting systematic errors………………………20

Table 4.5 Statistics of very long baselines before accounting systematic errors…………………21

Table 4.6 Estimates of systematic model parameters and their respective standard deviations for short baselines……………………………………………………………………………………22

Table 4.7 Statistics of short baselines after accounting systematic errors……………………….22

Table 4.8 Estimates of systematic model parameters and their respective standard deviations for long baselines…………………………………………………………………………………….23

Table 4.9 Statistics of long baselines after accounting systematic errors………………………..23

Table 4.10 Estimates of systematic model parameters and their respective standard deviations for very long baselines……………………………………………………………………………….23

Table 4.11 Statistics of very long baselines after accounting systematic errors…………………24

Table 4.12 Standard deviations and root mean squares of baselines before and after accounting for systematic errors……………………………………………………………………….………25

ACRONYMS AND ABBREVIATIONS

AFRgeo19	Africa geoid model of 2019
AGP	Africa Gravimetric Project
BM	Benchmarks
EGM	Earth Gravitation Model
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
GOCE	Gravity Field and Steady-State Ocean Circulation Explorer
MATLAB	Matrix Laboratory
MDT	Mean Dynamic Topographic
MSL	Mean Sea Level
MSSH	Mean Sea Surface Height
RMS	Root Mean Square
SD	Standard Deviation
TPLN	Tanzania Primary Levelling Network
TZG	Tanzania Gravimetric Model
VD	Vertical Datum

This page was intentionally left blank.

CHAPTER ONE

INTRODUCTION

1.1 Background

A geoid is an equipotential surface of the Earth gravity field that agrees with mean sea level in least square sense and continue under continental masses (Ulotu, 2009). Also, can be defined as a surface that best fits the mean sea level of the Earth's oceans and is defined as the mathematical description of the Earth's gravitational field in an undisturbed ocean i.e., no ocean currents no waves. A hypothetical surface called geoid serves as the standard for measuring elevations and depths. In order to create model of the Earth's geoid, geoid models must be built utilizing data from satellite measurements and some ground-based gravity data (Hofmann-Wellenhof & Moritz, 2005).

Geoid models are classified based on area coverage, there are global, regional and local geoid models examples EGM08, AFRgeo_v1.0 and TZG19 respectively. These models provide us with the geoid height or geoid undulation which are used to convert ellipsoidal heights to orthometric heights.

From the formula,

$$h = H + N \tag{1.1}$$

Where, h is ellipsoidal height, H is orthometric height and N is geoid height

Geoid finds number of applications like crustal deformation and geo-hazard mitigations, establishment and adjustment of controls, realization of ocean circulation pattern and dynamics (Ulotu, 2009).

A geoid model is a representation of the Earth's geoid, which is the shape that the surface of the Earth would take under the influence of gravity and rotation alone, without any other factors like oceans, tides, or landmasses. In simpler terms, it is the equipotential surface of the Earth's gravitational field that best approximates mean sea level. To create a geoid model, data from satellite missions and terrestrial measurements are used to determine the variations in the Earth's gravity field and to establish the reference surface. Geoid models are divided into global, regional and local geoid models.

The first global geoid model was generally attributed to the collaborative efforts of the Earth Gravitational Model 1996 (EGM96). EGM96 was developed by the National Geospatial-Intelligence Agency (NGA) of the United States and released in 1996. The EGM96 model represented the geoid with an unprecedented resolution, with grid spacing of up to 0.5 degrees (approximately 55 kilometers at the equator). It provided accurate geoid heights, which are essential for converting ellipsoidal heights to orthometric heights (Lemoine, et al., 1998). It's worth noting that after EGM96, several newer global geoid models have been released, such as EGM08 and EGM20, which continue to refine the representation of Earth's geoid with even higher accuracy and improved spatial resolution.

The first regional geoid model of Africa was AGP03 Africa Gravimetric Project of 2003 produced by (Merry et al., 2005) and data were based from EGM96 and Africa ground based data then followed by AGP07 by (Merry et al., 2007), AFRgeo19 by (Abd-Elmotaal., 2019). While for local geoid models in Tanzania, the first was Tanzania gravimetric model of 2007 (TZG07) and it was created by (Olliver, 2007) with accuracy of 47cm, then TZG08 by (Ulotu, 2008) with accuracy of 27.8cm, TZG13 by (Forsberg et al., 2013) with accuracy of 10cm, TZG17 by Peter in 2017 with accuracy of 5cm, and TZG19 by Forsberg in 2019 with accuracy of 10cm.

The Africa geoid model of 2019, AFRgeo19 is a major improvements from the previous models including the Africa geoid projects AGP03, and AFRgeo_v1.0. AFRgeo19 consists of corrections from the AFRgeo_v1.0 whose computed geoid was only compared to the AGP03, AFRgeo19 is the geoid model that provides precise Earth's gravity field such as gravity anomalies over the continent of Africa and it is based on satellite data from GOCE satellite. It provides precise geoid heights/undulations over the entire continent of Africa (Abd-Elmotaal et al., 2019). The Figure 1.1 below shows the geoid undulations of Africa.

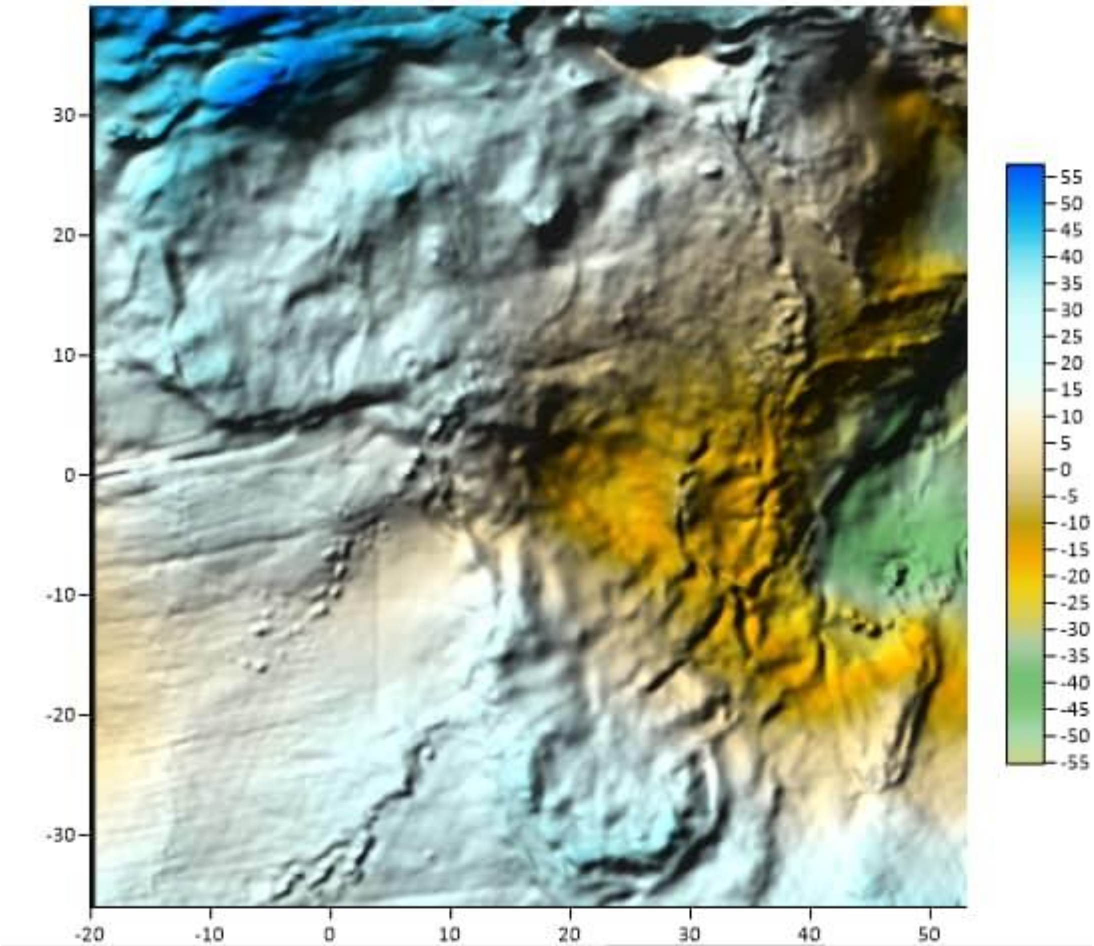

Figure 1.1 Geoid Map of Africa Scale in Meters

AFRgeo19 was created by (Abd-Elmotaal et al., 2019). He employed the window remove restore technique with the EGM08 geopotential model up to degree and order of 2160 and a tailored reference model up to degree and order of 2160 to fill in data gaps from previous geoid models. AFRgeo19 used more gravity data especially ground-based gravity data and since ground gravity data are more important than others, they were passed through numerous gross error detection process using least square prediction technique. This technique removes point from the data set if they are proven to be a gross error after examining. It is effects on the neighborhood points. A grid of 1’ x 1’ was applied to ground-based gravity data. Also, sea gravity data from shipborne and altimetry were used during creation of AFRgeo19 and a grid of 3’ x 3’ was applied. Even the shipborne and altimetry gravity data were passed through gross error detection based on the least square prediction technique which uses the nearby points to estimate the gravitational anomaly at the computational site and determines a potential error by contrasting it with a data value. The computed geoid model provided very small and smooth reduced gravity anomalies which reduces interpolation errors especially for areas with large data gaps and these reduced gravity anomalies showed good statistical behavior since they were centered, smooth and had small range. The reduced gravity data were interpolated using an unequal least squares interpolation techniques providing high precision on land data, on sea data moderate precision and lowest to underlying data (Abd-Elmotaal et al., 2018). The diagram below (Figure 1.2) shows the gravity data used to compute AFRgeo19 on ground, Oceans, and satellite altimetry

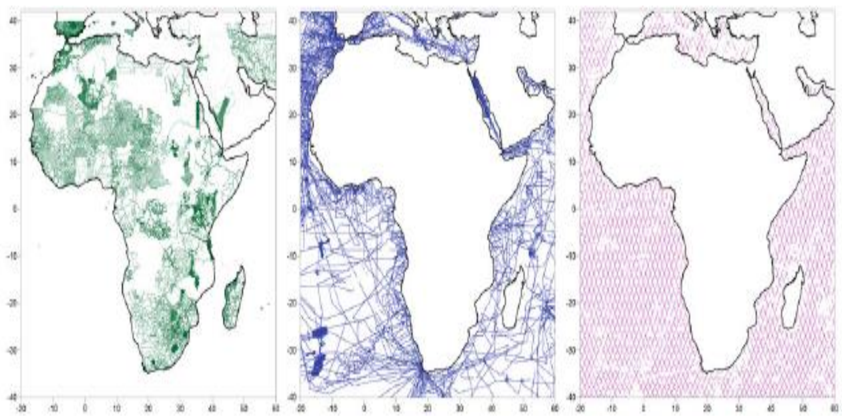

Figure 1.2 Gravity Data used for Ground, shipborne and Altimetry respectively

Source: (Abd-Elmotaal., 2019)

Geoid Models are assessed/evaluated using several methods such as GPS Levelling, gravity anomalies, sea surface topography, astrogeodetic deflection of the vertical and geoid slope method. The purpose of these methods is to check for performance, quality and stability of geoid models over an area of interest. These methods were used by several geodesists to evaluate geoid models i.e. Ntambila (2012) used sea surface topography and GPS Levelling method to assess EGM08.

The geoid slope method was used to assess the performance of AFRgeo19 gravimetric geoid model over Tanzania with the aid of Tanzania Primary Levelling Networks (TPLN) BMS, since in Tanzania large number of gravimetric models have been assessed compared to AFRgeo19 thus leaving a gap for neediness of it is evaluation across Tanzania. Also the geoid slope method in Tanzania was used by Willison, (2013) in validating AGP07, TZG08, EGM08, and TZG07 whose results were presented in terms of minimum, maximum, mean, standard deviation and RMS in which the results for those categories were not very different in magnitude for long, very long baselines. The results showed that for TZG08 results had smallest values followed by those of EGM08 and then AGP07. He concluded that TZG08 best fits the GPS/Levelling geoid for Tanzania also it was used by Simion, (2019) in validating EGM08, TZG13 and TZG17 in Tanzania with the same categories as that of Willison, the results from Simion showed that TZG17 had the smallest magnitude for all baselines followed by TZG13, then EGM08 and concluded that TZG17 produce better results in Tanzania.

1.2 Problem Statement

Due to improvements and development of satellite geodesy in recent year's neediness of high precise geoid model emerged. This is for the purpose of converting ellipsoid height to orthometric height by obtaining geoid height of respective points and fasten up observations by GPS Levelling. In Tanzania since our vertical datum (VD) is mean sea level (MSL) and is based from Tanga Tide Gauge which does not match the least requirements needed as observations were taken for 28 months instead of 18.6 years. Thus, the Tanzania primary levelling networks benchmarks miss the precision required as observations were referred to normal gravity rather than actual gravity leading to normal orthometric height and not orthometric height, as the height system of Tanzania is orthometric height and the required vertical datum for orthometric height system is geoid and not MSL (Hofmann-Wellenhof & Moritz, 2005).

Now among the developed geoid models of Tanzania and other worldwide, there is need to evaluate them so as to check their performance over Tanzania before employing them. Performance assessment of Africa geoid model of 2019 (AFRgeo19) is inevitable for it to be used in numerous applications.

1.3 Main Objective

Assessment of AFRgeo19 by geoid slope method in Tanzania to check validity of the geoid model over Tanzania.

1.4 Specific Objectives

- Extraction of physical geoid heights from AFRgeo19.
- To determine the geoid slope of AFRgeo19 and in between points of Tanzania Primary Levelling Network benchmarks.

- To obtain geoidal slope difference between geometrical and physical geoid slopes.

1.5 Research Hypothesis

It is hypothesized that AFRgeo19 geoid model will perform better in Tanzania.

1.6 Scope/Limitations

The research will be conducted over Tanzania mainland with a spatial coverage from longitude 28° to 42° east of Greenwich meridian, and from latitude 1° to 12° south of equator. The Figure 1.3 below shows the map of Tanzania Primary Levelling Network.

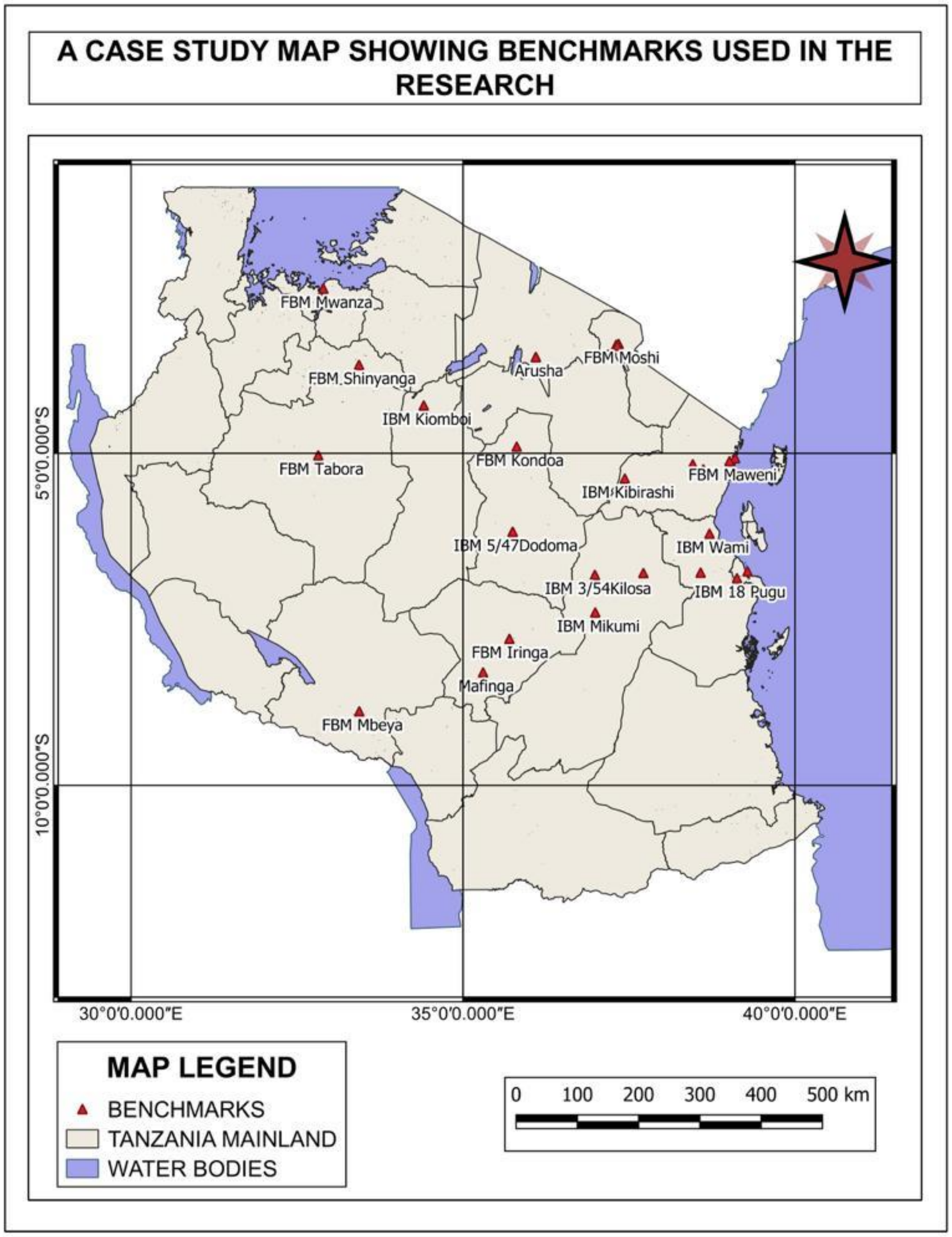

Figure 1.3 Distribution of TPLNs Benchmarks in Tanzania

1.7 Significance

Once the Africa geoid model of 2019 (AFRgeo19) is proved to be credible in Tanzania, it will provide variety of gravity data and information for numerous applications in different fields especially the field of Geodesy.

1.8 Beneficiaries

- Mostly the research will benefit Geomaticians who will use AFRgeo19 to deduce orthometric height from ellipsoid height after series of GPS Levelling also, to compute gravity data as well as in height realizations.
- Also, it will benefit Civil Engineers in construction activities as it will save time that would be used on using spirit levelling.

1.9 Research Outline

This research consists of five chapter.

i. Chapter One

   This chapter explained in details the background of the study, problem statement, main objective, specific objectives, research hypothesis, significance and beneficiaries of the research.

ii. Chapter Two

   This chapter explains in details about the literature reviews of the study.

iii. Chapter Three

   This chapter explains the details of all the procedures and methods used to obtain results.

iv. Chapter Four

   This chapter provides the results of the research.

v. Chapter Five

   This chapter explains the conclusions concluded from the research and recommendations.

CHAPTER TWO

LITERATURE REVIEW

2.1 Overview of Methods of Assessing Geoid Models

Geoid model assessment is the analysis of the quality of the geoid model, assessment of geoid model can either be internal or external. In this research geoid model was assessed externally by using GPS Levelling datasets with difficulties on separating errors from GPS levelling and geoid model and vertical motion of benchmarks (Willison, 2013).

2.2 Geoid Slope

Geoid slope method is the method that is used to assess the performance of geoid models over a given area of interest. The geoid slopes can be acquired as the ratio between change in geoid heights and the horizontal distance between points. The method enables as to check how a geoid model performs over a distance basing on the baselines between the two points. It compares geoid slopes from geoid model to that from GPS Levelling data. Advantageous of this method is that errors relating to that of baselines cancel out on differencing the geoid slopes (Simion, 2019).

Considering the Figure 2.1, with two TPLN points 1 and 2, with their delta N, horizontal distance S and slope m.

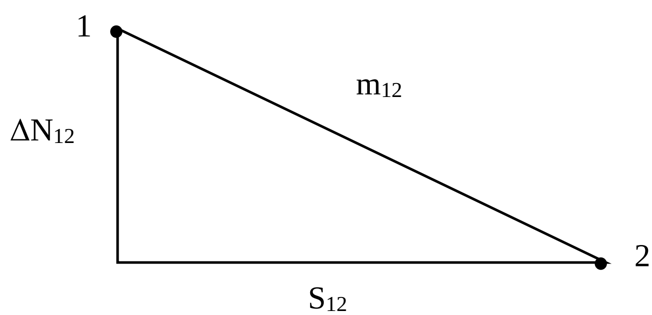

Figure 2. 1 Geoid Slope Computations

Evaluations of geoid model follows the following procedures

Procedures;

i. Determining difference in geoid heights between the two stations for both geometrical and physical geoid heights.

$$\Delta N_{12} = N_2 - N_1 \quad \text{(2.1)}$$

ii. Determining horizontal distance between the two points, by using geodetic coordinates from the formula of Puissant Inverse Problem.

$$S_{12} = M_1 \frac{\Delta\phi}{cos\alpha_{12}} (1 - \frac{3e^2 sin2\phi_1 \Delta\phi}{4(1-e^2 sin^2\phi_1)}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (2.2)$$

iii. Determining geoid slope for both geometrical and physical geoid, from relation

$$m_{12} = \frac{\Delta N_{12}}{S_{12}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.3)$$

iv. Determining slope difference between the geometrical geoid slope and physical geoid slope,

$$\Delta m = m_{12}^{G} - m_{12}^{GNSS} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

v. Determining the statistical analysts of change in slopes obtained to get the maximum, minimum, standard deviations, root mean square and mean, which are used to evaluate geoid models (Simion, 2019).

This method was used by Simion (2019) and Willison (2013) to evaluate geoid models in Tanzania.

2.3 GPS Levelling

GPS levelling is the term covering the efficient determination of physical heights (orthometric, normal, normal-orthometric) basing on the GNSS positioning technologies and the geoid. GPS provides ellipsoidal heights from reference ellipsoid while spirit levelling provides orthometric heights, the geometrical geoid is obtained by using equation 1.1

The GPS levelling involves two approaches for the evaluation of the geoid models which are Absolute GPS levelling and Relative GPS levelling, below is the short summary of the two approaches.

2.3.1 Absolute GPS Levelling

This approach requires the absolute value of ellipsoidal height to be known. In this approach evaluation is done by subtracting orthometric height from ellipsoidal height to get geoid height as given in equations 2.1 and 2.2

From, $N_{GPS} = h - H \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$2.1

Then, $\Delta N = N_G - N_{GPS} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$...2.2

The obtained values of ΔN are statistically analysed to obtain the SD and RMS (Heiskanen & Moritz, 1967).

2.3.2 Relative GPS Levelling

In this approach two points with known ellipsoidal and orthometric heights. Then computing the difference in ellipsoidal and orthometric heights, during differencing errors common to the points cancel out. Evaluation in this approach is done by subtracting difference in ellipsoidal height to difference in orthometric height get difference in geoid height over the baseline.

From, $\Delta N = \Delta h - \Delta H$ ……….………………….…………………………………...……2.3

Then, $\Delta N_G = \Delta N_A - \Delta N_B$ ………………………..……………..……..……………..2.4

Then, $\Delta N_{GPS-G} = \Delta N_{GPS} - \Delta N_G$ ……………………..……………………………..…..2.5

Statistical analysis of ΔN_{GPS-G} to check the SD and RMS of the geoid model (Featherstone, 2001). More explanations on this methods see (Featherstone, 2001).

2.4 Sea Surface Topography

In this method; geoid models are evaluated by geoid undulations and mean sea surface height (MSSH) from satellite altimetry data (Gruber, 2008).

From, $MSSH = h - R$ …………………………………………….2.6

Whereby;

h is the satellite altitude above reference ellipsoid

R is the altimeter range above sea surface

Now, $SST = MSSH - N$ ……………………………………………2.7

Whereby;

SST is the sea surface topography

The obtained sea surface topography are assessed to check the validity of the geoid model. Detailed explanations of this method see Ntambila (2012) and Gruber (2008).

2.5 Astrogeodetic Deflection of the Vertical

This method compares the physical geoidal heights and the geoidal height differences between astrogeodetic geoidal heights found from the components of deflection of the vertical (Olliver, 2007).

From, $N_A = N_B - (S_m \xi_m + S_p \eta_m)$ ……………………………........2.8

Whereby;

N_A is the geoidal height at station A

N_B is the geoidal height at station B

S_m and S_P are meridional and parallel circles distances between the stations

ξ_m and η_m are mean meridional and prime deflection of the vertical at the two stations

The meridian and prime astrogeodetic deflection of the vertical are obtained from the,

$$\varphi = \Phi - \xi \qquad (2.9)$$

$$\lambda = (\Lambda - \eta) sec\varphi \qquad (2.10)$$

Whereby;

λ and φ are geodetic latitude and longitude of a station respectively,

Λ and Φ are astronomical latitude and longitude respectively,

Detailed explanations of this method see Olliver (2007) and Willison (2013).

2.6 Gravity Anomalies

This method compares gravity anomalies derived from terrestrial data to that from geoidal models. Difference between gravity anomalies at terrestrial and geoid derived are expressed by, (Merry, 2007).

$$\delta_{\Delta g} = \Delta g^T(\phi, \lambda) - \Delta g^G(\phi, \lambda) \text{ at a point,} \qquad (2.11)$$

Whereby;

$\delta_{\Delta g}$ is the difference in gravity anomalies

$\Delta g^T(\phi, \lambda)$ is the terrestrial gravity anomalies

$\Delta g^G(\phi, \lambda)$ is the geoid derived gravity anomalies

The results of gravity anomalies difference at a point are statistical analyzed to obtain the SD and RMS which are in turn used to evaluate the geoid models (Merry, 2007). Detailed explanations of this method see Willison (2013).

2.7 Selection of Geoid Slope Method

Geoid slope method was selected to carry out this research because it has not been used to assess AFRgeo19 geoid model in Tanzania but it was used to assess EGM08, TZG07, TZG08 and AGP07 by Willison (2013). Also it was used to assess EGM08, TZG13 and TZG17 by Simion (2019).

CHAPTER THREE

METHODOLOGY

3.1 Overview

The methodology that was used to assess the geoid model of Africa of 2019 (AFRgeo19) was geoid slope method. Geoid slope method involved comparison of geoid slopes from GPS Levelling data and from AFRgeo19 geoid model. This method determined how a geoid model performed on different baselines and it can be defined as the ratio of difference in geoid height to horizontal distance between the two points. This method comprises of several advantages such as cancellation of errors related to baseline also, it compares different geoid models of different characteristics at different spatial wavelength (Derek, 2018).

The following were the procedures followed during assessment of AFRgeo19,

- Determining geometrical geoid height difference from GPS levelling data between pairs of benchmarks by using equation 2.1
- Categorizing of baseline distances into short baselines, long baselines and very long baselines, horizontal distances were computed using equation 2.2
- Determining the geoid slope of physical geoid and geometrical geoid at each pair of two points using equation 2.3
- Determining slope difference between physical geoid slope and geometrical geoid slope using equation 2.4
- Modelling of systematic errors, to reduce datum inconsistences on the change in physical geoid heights using equations 3.2, 3.3, 3.4 and 3.5 for 3-4-5-7 parameters respectively
- Determining the geoid slope differences by dividing the residuals with respective baselines distance after accounting systematic errors

$$\Delta m = \frac{v}{s_{12}} \quad \text{(3.1)}$$

- Determining the statistical attributes maximum, minimum, mean, RMS and SD for geoid slope differences obtained before and after accounting for systematic errors.

3.1.1 Statistical Test

Conduction of statistical test at 95% confidence level by normal distribution test on obtained geoid height differences for AFRgeo19 geoid model on each benchmark. The test showed that 13 benchmarks out of 31 benchmarks obtained passed the test at 95%. But the research was undertaken by using all benchmarks as the 13 benchmarks were not enough to produce enough baselines.

3.1.2 Baseline Categories

Benchmarks were put into pairs and their respective horizontal distance between each pair were categorized into short, long and very long baselines with distance of 0-100km, 101-200km and 201-500km respectively. The obtained baselines were,

Table 3.1 Categories of baselines with their respective distances and number of baselines

BASELINES	LENGTH (Km)	No. BASELINES
SHORT	0-100	8
LONG	101-200	16
VERY LONG	201-500	51

3.1.3 Systematic Errors Modelling

After obtaining the change in geoid heights systematic errors which usually exist in three heights data set (N, H and h) was modelled to absorb if not eliminate all datum inconsistences on this research parametric models of 3-4-5-7 were used. Appendix X, XI, XII and XIII shows the codes for 3, 4, 5 and 7 parametric models respectively.

The following are the four parametric models,

3-parametric model equation

$$a_{ij}x = [(cos\varphi_j cos\lambda_j) - (cos\varphi_i cos\lambda_i)]x_1 + [(cos\varphi_j sin\lambda_j) - (cos\varphi_i sin\lambda_i)]x_2 +$$

$$[(sin\varphi_j) - (sin\varphi_i)]x_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.2)}$$

4-parametric model equation

$$a_{ij}x = [(cos\varphi_j cos\lambda_j) - (cos\varphi_i cos\lambda_i)]x_1 + [(cos\varphi_j sin\lambda_j) - (cos\varphi_i sin\lambda_i)]x_2 +$$

$$[(sin\varphi_j) - (sin\varphi_i)]x_3 + [sin^2\varphi_j - sin^2\varphi_i]x_4 \ldots\ldots\ldots\ldots\ldots\ldots \text{(3.3)}$$

5-parametric model equation

$$a_{ij}x = [(cos\varphi_j cos\lambda_j) - (cos\varphi_i cos\lambda_i)]x_1 + [(cos\varphi_j sin\lambda_j) - (cos\varphi_i sin\lambda_i)]x_2 +$$

$$[(sin\varphi_j)-(sin\varphi_i)]x_3 + [(sin^2\varphi_j) - (sin^2\varphi_i)]x_4 + x_5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.4)}$$

7-parametric model equation

$$a_{ij}x = (\delta\lambda_{ij})x_1 + (\delta\varphi_{ij})x_2 + (\delta\lambda_{ij}\delta\varphi_{ij})x_3 + (\delta\varphi_{ij}^2)x_4 + (\delta\lambda^2_{ij})x_5 + (\delta\varphi_{ij}^3)x_6 + (\delta\lambda^3_{ij})x_7 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.5)}$$

Where

$\delta\lambda_{ij} = \lambda_j - \lambda_i$ in radian and $\delta\varphi_{ij} = \varphi_j - \varphi_i$ in radian

φ and λ are horizontal geodetic network coordinates

The above equations can be expressed in matrix form as

$$Ax_{ij}+v_{ij} = \Delta N_{ij} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.6)}$$

Where

A is the design matrix

ΔN_{ij} is the misclosure vector between the gravimetric geoidal heights and geometrical geoid heights at each baseline

v is residual vector of each baseline (Fotopulos, Kotsakis, & Sideris, 2003).

3.2 Data Availability

The data set required in this research were ellipsoidal heights (h) of the benchmarks, orthometric heights (H) and geoid model (AFRgeo19).

3.2.1 Ellipsoidal Heights and Orthometric Heights

From the ellipsoidal heights obtained from GPS observations over the TPLN benchmark points. In this research ellipsoidal heights were used to compute geometrical geoid heights. Orthometric heights were obtained through series of precise levelling from Tanga Tide Gauge on TPLN benchmarks. Orthometric heights have been published by Tanzania Ministry of Lands, Housing and Human Settlements Development (MLHHSD) (Simion, 2019). These heights respectively with ellipsoidal heights were used to compute geometrical geoid heights.

Table 3.2 Benchmarks of Tanzania Primary Levelling Network with ellipsoidal and orthometric heights

S/N	BM-D	Latitude (degree)	Longitude (degree)	Heights (m)	
				Orthometric	Ellipsoidal
1	FBM Makuyuni	-3.553	36.097	1070.213	1051.145
2	IBM Arusha	-3.551	36.097	1068.955	1050.619
3	Arusha	-3.553	36.097	1068.955	1050.800
4	IBM 18 Pugu	-6.882	39.126	103.670	76.688
5	FBM Dar	-6.778	39.285	10.513	-16.709
6	IBM 5/47Dodoma	-6.184	35.746	1130.975	1110.378
7	IBM 5/46Dodoma	-6.183	35.747	1132.704	1112.557
8	FBM Kondoa	-4.902	35.809	1392.666	1373.695
9	IBM 5/47Dodoma	-6.184	35.749	1131.927	1112.815
10	FBM Iringa	-7.794	35.699	1549.545	1532.884
11	Mafinga	-8.300	35.302	1836.387	1823.027
12	IBM A25/1Moshi	-3.367	37.309	798.010	780.118
13	IBM A24/50Moshi	-3.382	37.309	793.346	775.354
14	FBM Moshi	-3.355	37.343	809.793	792.215
15	IBM A24/51Moshi	-3.379	37.323	805.414	788.416
16	IBM 3/54Kilosa	-6.831	36.986	490.330	469.000
17	IBM 2/77Morogoro	-6.804	37.716	493.729	471.976
18	FBM Mwanza	-2.523	32.898	1139.667	1122.052
19	FBM Kwala	-6.798	38.578	81.073	54.951
20	IBM Wami	-6.212	38.712	14.442	-8.058
21	FBM Shinyanga	-3.670	33.434	1123.194	1103.618
22	IBM Kiomboi	-4.281	34.409	1595.003	1575.412
23	FBM Tabora	-5.032	32.820	1236.751	1218.728
24	IBM Korogwe	-5.166	38.461	300.032	276.375
25	IBM Kibirashi	-5.382	37.437	1129.579	1110.109
26	Tanga	-5.120	39.014	64.244	37.447
27	IBM A29/7Tanga	-5.074	39.099	19.646	-7.203
28	FBM Maweni	-5.120	39.014	63.237	37.046
29	FBM Mnyusi	-5.243	38.619	243.999	220.654
30	IBM Mikumi	-7.398	36.992	519.925	499.150
31	FBM Mbeya	-8.886	33.438	1782.839	1770.901

Source: (Ardhi University Department of Geospatial Science and Technology)

3.2.2 AFRgeo19

AFRgeo19 geoid model was used to provide the physical geoid height which was used to assess the geoid model by obtaining the geoid slope difference of AFRgeo19 geoid slope to that of GPS geoid slope.

3.3 Software used

All of the software used during execution of this research are described in this section

3.3.1 Golden Surfer

This software was used to extract physical geoid heights in AFRgeo19 geoid model by using function of grids by residuals.

3.3.2 MATLAB

To calculate the 3- 4-5-7 parameters and their related SD, MATLAB software was employed. This MATLAB package was also used to determine the residuals of each baseline.

3.3.3 Microsoft Office

Microsoft Office was used to write reports and run calculations. Microsoft Word and Microsoft Excel were the two most often used programs for generating reports and computing statistical properties including standard deviation, root mean square, and the minimum and maximum values of geoid slope differences. Additionally, Microsoft Excel was utilized to create Figure 4.1 and Figure 4.2.

CHAPTER FOUR
RESULT, ANALYSIS AND DISCUSSION

This section benevolences the results acquired with respect to methodologies described in the prior chapter

4.1 Results

In this section the AFRgeo19 geoid model was evaluated using geoid slope method and results obtained during execution of the research were presented in tables.

4.1.1 Geometrical and physical Geoid

Table 4.1 below shows the obtained geometrical and physical geoid heights computed using equation 1.1 for geometrical geoid heights.

Table 4.1 Geometrical and Physical Geoid Heights

S/N	BM-D	Latitude (degree)	Longitude (degree)	N-GPS (m)	N-AFRgeo19 (m)
1	FBM Makuyuni	-3.553	36.097	-19.068	-18.393
2	IBM Arusha	-3.551	36.097	-18.336	-18.393
3	Arusha	-3.553	36.097	-18.155	-18.393
4	IBM 18 Pugu	-6.882	39.126	-26.982	-26.969
5	FBM Dar	-6.778	39.285	-27.222	-27.847
6	IBM 5/47Dodoma	-6.184	35.746	-20.597	-19.324
7	IBM 5/46Dodoma	-6.183	35.747	-20.147	-19.320
8	FBM Kondoa	-4.902	35.809	-18.971	-18.223
9	IBM 5/47Dodoma	-6.184	35.749	-19.112	-19.315
10	FBM Iringa	-7.794	35.699	-16.661	-16.018
11	Mafinga	-8.300	35.302	-13.360	-14.535
12	IBM A25/1Moshi	-3.367	37.309	-17.892	-17.903
13	IBM A24/50Moshi	-3.382	37.309	-17.992	-18.026
14	FBM Moshi	-3.355	37.343	-17.578	-17.821
15	IBM A24/51Moshi	-3.379	37.323	-16.998	-18.017
16	IBM 3/54Kilosa	-6.831	36.986	-21.330	-20.464
17	IBM 2/77Morogoro	-6.804	37.716	-21.753	-22.332
18	FBM Mwanza	-2.523	32.898	-17.615	-16.445
19	FBM Kwala	-6.798	38.578	-26.122	-25.788
20	IBM Wami	-6.212	38.712	-22.500	-26.433
21	FBM Shinyanga	-3.670	33.434	-19.576	-18.686
22	IBM Kiomboi	-4.281	34.409	-19.591	-18.823
23	FBM Tabora	-5.032	32.820	-18.023	-17.515

24	IBM Korogwe	-5.166	38.461	-23.657	-23.164
25	IBM Kibirashi	-5.382	37.437	-19.470	-19.089
26	Tanga	-5.120	39.014	-26.797	-26.476
27	IBM A29/7Tanga	-5.074	39.099	-26.849	-27.131
28	FBM Maweni	-5.120	39.014	-26.191	-26.476
29	FBM Mnyusi	-5.243	38.619	-23.345	-24.019
30	IBM Mikumi	-7.398	36.992	-20.775	-20.519
31	FBM Mbeya	-8.886	33.438	-11.938	-11.808

4.1.2 Benchmarks passed 95% Confidence Level

Table 4.2 shows the results of benchmarks that passed the statistical test at 95% the results were obtained by using Microsoft excel and the student t distribution Table and 13 benchmarks passed at 95% confidence level.

Table 4.2 Benchmarks which passed the 95% confidence level

BM-D	Latitude (degree)	Longitude (degree)	N-GPS (m)	N-AFRgeo19 (m)	ΔN (m)	outliers
IBM Arusha	-3.550	36.096	-18.336	-18.393	-0.057	FALSE
Arusha	-3.553	36.096	-18.155	-18.393	-0.238	FALSE
IBM 18 Pugu	-6.882	39.126	-26.982	-26.969	0.013	FALSE
FBM Dar	-6.778	39.285	-27.749	-27.847	-0.098	FALSE
IBM 5/47Dodoma	-6.184	35.749	-19.112	-19.315	-0.203	FALSE
IBM A25/1Moshi	-3.367	37.308	-17.892	-17.903	-0.011	FALSE
IBM A24/50Moshi	-3.381	37.308	-17.992	-18.026	-0.034	FALSE
FBM Moshi	-3.355	37.343	-17.578	-17.821	-0.243	FALSE
Tanga	-5.120	39.013	-26.797	-26.476	0.321	FALSE
IBM A29/7Tanga	-5.074	39.098	-26.849	-27.131	-0.282	FALSE
FBM Maweni	-5.120	39.013	-26.191	-26.476	-0.285	FALSE
IBM Mikumi	-7.398	36.991	-20.775	-20.519	0.256	FALSE
FBM Mbeya	-8.885	33.437	-11.938	-11.808	0.130	FALSE

4.1.3 Assessment of AFRgeo19 before accounting systematic errors

AFRgeo19 was assessed before accounting systematic errors first and below were the outcomes for the short baselines, long baselines and very long baselines.

4.1.3.1 Results for short baselines

This category had 8 baselines which were assessed, their geoid slope differences by the statistical attributes of RMS, SD, max, min and mean of the obtained results. Appendix IV shows the results of the geoid slope differences of each of the 8 baselines, and the below Table 4.3 shows the statistical attributes of the change in geoid slope for the short baselines and Figure 4.1 shows the graphical representation of the statistical attributes.

Table 4.3 Statistics of short baselines before accounting systematic errors

Attributes	Values
MAX	5.7748E-04
MIN	-5.7334E-05
MEAN	5.7122E-05
SD	2.1145E-04
RMS	2.0587E-04

4.1.3.2 Results for long baselines

This category had 16 baselines which were evaluated their geoid slope differences by the statistical attributes of RMS, SD, max, min and mean of the obtained results. Appendix V shows the results of the geoid slope differences of each of the 16 baselines, and the below Table 4.4 shows the statistical attributes of the change in geoid slope for the short baselines and Figure 4.2 shows the graphical representation of the statistical attributes.

Table 4.4 Statistics of long baselines before accounting systematic errors

Attributes	Values
MAX	1.6656E-06
MIN	-6.4269E-06
MEAN	-1.5970E-06
SD	2.3780E-06
RMS	2.8021E-06

4.1.3.3 Results for very long baselines

This category had 51 baselines which were evaluated their geoid slope differences by the statistical attributes of RMS, SD, max, min and mean of the obtained results. Appendix VI shows the results of the geoid slope differences of each of the 51 baselines, and the below Table 4.5 shows the statistical attributes of the change in geoid slope for the short baselines and Figure 4.3 shows the graphical representation of the statistical attributes.

Table 4.5 Statistics of very long baselines before accounting systematic errors

Attributes	Values
MAX	3.0138E-07
MIN	-6.8300E-05
MEAN	-3.1807E-05
SD	2.2673E-05
RMS	3.8931E-05

Before accounting for systematic errors the values of SD and RMS for long baselines were lower than those from short and very long baselines. The Figure below shows the trend of SD and RMS for short, long, and very long baselines.

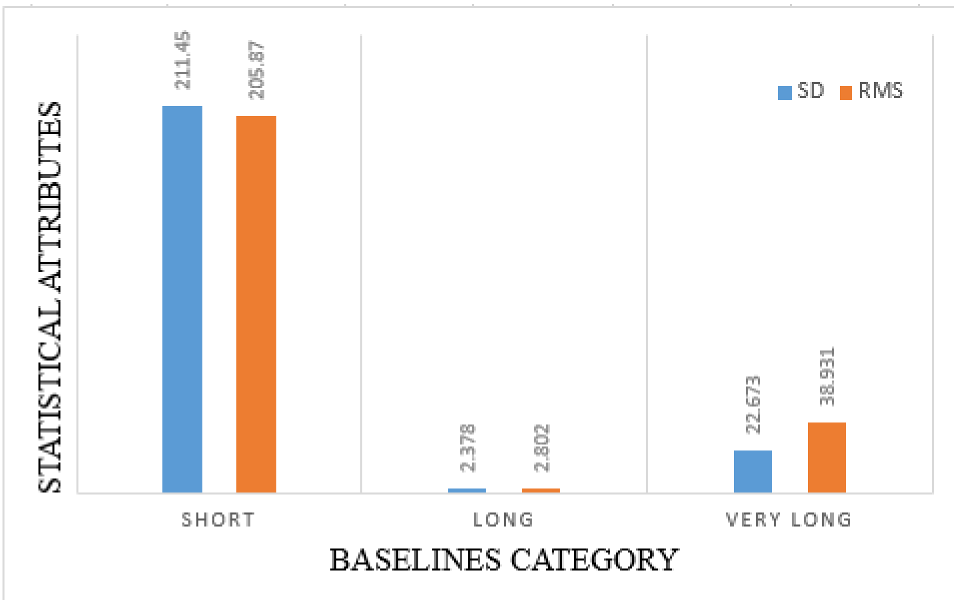

Figure 4.1 Standard deviations and root mean squares of the baselines before accounting for systematic errors

4.1.4 Assessment of AFRgeo19 after accounting systematic errors

After removal of inconsistencies in data set, AFRgeo19 was assessed by using the residuals of parametric model with minimum values were divided by respective baselines distances to obtain change in geoid slope. Then statistical attributes of change in geoid slope were derived.

4.1.4.1 Results of short baselines

The Table 4.6 show the result for 3-4-5 parametric model, 7 parametric model was not used because of lacking enough number of baselines to make the matrix more usable. And the Figure

4.4 shows the computed results of statistical attributes. While Table 4.7 shows the statistical attributes.

Table 4.6 Estimates of systematic model parameters and their respective standard deviations for short baselines

ADJUSTED PARAMETERS	3		4		5	
	Value(m)	S.D(m)	Value(m)	S.D(m)	Value(m)	S.D(m)
X1	-434.844	670.567	287.704	493.898	215.703	511.911
X2	-4689.535	5529.037	976.294	4032.542	424.288	4156.510
X3	3718.097	4042.855	-2343.786	3249.097	-1810.061	3409.657
X4			-11100.886	3423.299	-10384.590	3719.301
X5					-0.025	0.054

Table 4.7 Statistics of short baselines after accounting systematic errors

Attributes	Values
MAX	0.0006831
MIN	-0.0000366
MEAN	0.0000768
SD	0.0002453
RMS	0.0002420

4.1.4.2 Results of Long baselines

The Table 4.8 show the result for 3-4-5-7 parametric model, the residuals of the parametric model with minimum values were used to obtain the geoid slope differences. And the Table 4.9 shows the computed results of geoid slope differences statistical attributes.

Table 4.8 Estimates of systematic model parameters and their respective standard deviations for long baselines

ADJUSTED PARAMETER	3		4		5		7	
	Value(m)	S.D(m)	Value(m)	S.D(m)	Value(m)	S.D(m)	Value(m)	S.D(m)
X1	-33.056	35.186	15.208	17.666	-4.717	12.864	185.959	66.358
X2	-565.933	400.462	-545.835	187.461	-450.347	129.323	-729035.04	308678.290
X3	437.926	314.719	992.217	164.582	860.008	115.983	-14509.178	5191.504
X4			2771.816	367.036	2485.510	258.141	737343.368	310531.877
X5					-0.302	0.070	4457.356	1831.231
X6							-70236.112	31285.427
X7							1.407	0.731

Table 4.9 Statistics of long baselines after accounting systematic errors

Attributes	Values
MAX	1.1784E-06
MIN	-1.4991E-06
MEAN	-4.5131E-09
SD	6.7777E-07
RMS	6.5627E-07

4.1.4.3 Results of very long baselines

The Table 4.10 show the result for 3-4-5-7 parametric model, also the residuals of parametric model with minimum values were used to compute the geoid slope differences shown on Appendix IX. And Table 4.11 shows the computed results of geoid slope differences statistical attributes.

Table 4.10 Estimates of systematic model parameters and their respective standard deviations for very long baselines

ADJUSTED PARAMETER	3		4		5		7	
	Value(m)	S.D(m)	Value(m)	S.D(m)	Value(m)	S.D(m)	Value(m)	S.D(m)
X1	30.227	8.365	18.110	9.937	14.663	10.547	6.383	13.717
X2	110.902	81.510	-39.600	106.797	-31.556	106.287	1981.849	5625.202
X3	-98.454	66.109	-1.436	78.893	-9.831	78.781	3062.385	673.204
X4			-131.266	63.361	-140.101	63.576	-4488.52	5397.972
X5					-0.120	0.131	-1756.74	461.451
X6							417.936	605.379
X7							-0.143	0.115

Table 4.11 Statistics of very long baselines after accounting systematic errors

Attributes	Values
MAX	1.9302E-06
MIN	-2.2236E-06
MEAN	-4.4025E-08
SD	9.1677E-07
RMS	9.0880E-07

After accounting for systematic errors the results of the baselines SD and RMS decreases compared to the results before accounting systematic errors with exception on short baselines category. The graph below shows the trend of SD and RMS on short, long and very long baselines after accounting systematic errors.

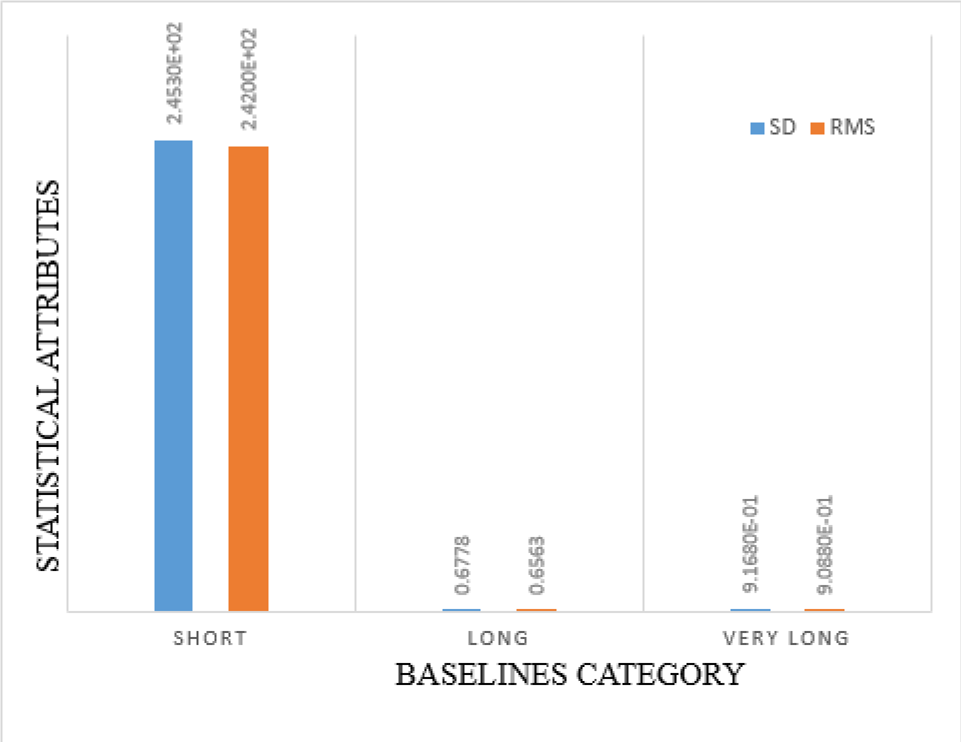

Figure 4. 2 standard deviations and root mean squares of the baselines after accounting for systematic errors

4.2 Discussion

From the above Tables and graphical representations of statistical attributes for both before and after accounting for systematic errors on the geoid slope differences, the following summary can be expressed in Table 4.12 with the followings discussions.

1) Before accounting for systematic errors the values of SD and RMS 211.4 ppm and 205.8 ppm for short baselines category have far better results compared to after accounting systematic errors which were 245.3 ppm and 242.0 ppm for SD and RMS respectively.
2) Before accounting systematic errors long and very long baselines have higher values of SD and RMS 2.378 ppm and 2.802 ppm for long and 22.673 ppm and 38.931 ppm for very long baselines respectively compared to after accounting for systematic errors which were 0.678 ppm and 0.656 ppm for long and 0.917 ppm and 0.909 ppm for very long baselines.
3) Before accounting for systematic errors long baselines had better results compared to short and very long baselines, long baselines SD and RMS were 2.378 ppm and 2.802 ppm.
4) Also after accounting for systematic errors long baselines provided better results for SD and RMS than those from short and very long baselines, the values of SD and RMS for long baselines were 0.678 ppm and 0.656 ppm.

Table 4.12 Standard deviations and root mean squares of baselines before and after accounting for systematic errors

BASELINES CATEGORY	ATTRIBUTES	ppm		ppm
SHORT	SD	211.4	<	245.3
	RMS	205.8	<	242
LONG	SD	2.378	>	0.678
	RMS	2.802	>	0.656
VERY LONG	SD	22.673	>	0.917
	RMS	38.931	>	0.909
		BEFORE	< OR >	AFTER

From the results obtained by (Simion, 2019) which showed that the Tanzania geoid model of 2017 (TZG17) performs better in Tanzania compared to EGM08 and TZG13. The results of SD and RMS were 59.71 ppm and 58.82 ppm for short baselines, 1.44 ppm and 1.39 ppm for long baselines and 1.14 ppm and 1.13 ppm for very long baselines respectively for TZG17. On contrasting the two results of TZG17 and AFRgeo19, theAfrica geoid of 2019 performed better than TZG17 on total of 67 baselines out of 75 baselines present.

CHAPTER FIVE
CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The main objective of this research was to assess Africa geoid model of 2019 in Tanzania by geoid slope method as it was developed to provide gravity undulations over the whole continent thus hypothesized that the geoid model performs well enough in Tanzania.

From the discussion of the results obtained it was concluded that the geoid model of Africa of 2019 performed better in Tanzania as it had SD of 0.678 ppm and 0.917 ppm and RMS of 0.656 ppm and 0.909 ppm for 67 benchmarks long and very long baselines respectively out of 75 benchmarks.

5.2 Recommendations

The following recommendations were made basing on results and conclusions obtained,

1) Assessing AFRgeo19 by using other methods of geoid model validation to check it is validity over Tanzania mainland, to see if it performs better on other methods as well.
2) Assessing the more recently Tanzania regional geoid model TZG19 by geoid slope method and comparing it is results with the results from AFRgeo19 so as to find out which model performs better over Tanzania to be gazetted as the National geoid model.
3) MLHHSD is suggested to make observation and produce more benchmarks especially on the southern part of Tanzania which have fewer number of benchmarks.

REFERENCES

Abdalla, A. & Fairhead,D. (2011). Determination of a gravimetric geoid model of Sudan using the KTH Method. *Journal of African Earth Sciences, 60*(4), 213-221.

Abd-Elmotaal, H. ,Seitz, K., Kuhtreiber,N. & Heck,B. (2016). Establishment of the gravity database AFRGDB_V1.0 for the African Geoid IGFS 2014. *Proceedings of the 3rd International Gravity Field Service (IGFS), Shanghai, China, June 30-July 6,2014* (pp. 131-138). Springer International Publishing.

Abd-Elmotaal, H. ,Seitz, K., Kuhtreiber,N. & Heck,B. (2019). AFRGDB_V2.0: the gravity database for the geoid determination in Africa. *In International Symposium on Advancing Geodesy in a Changing World: Proceedings of the IAG Scientific Assembly, Kobe, Japan, July 30-August 4, 2017* (pp. 61-70). Springer International Publishing.

Abd-Elmotaal, H. ,Seitz, K., Kuhtreiber,N. & Heck,B. (2019). AFRgeo_v1.0: a geoid model for Africa. *KIT scientific working papers,* 125.

Featherstone, E. W. (2001). Absolute and relative testing of gravimetric geoid models using GPS and orthometric height data. *Computers and Geosciences, 27*(7), 807-814.

Forsberg, R., Olesen, A. V., Nielsen, E., & Einarsson, I. (2016). Airborne gravimetry for geoid and GOCE. *In IGFS 2014: Proceedings of the 3rd International Gravity Field Service (IGFS), Shanghai, China, June 30-July 6, 2014* (pp. 27-38). Springer International Publishing.

Fotopulos, G., Kotsakis, C., & Sideris, M. G. (2003). How accurately can we determine orthometric height differences from GPS and geoid data. *Journal of surveying engineering, 129*(1), 1-10.

Gruber, T. (2009). *Evaluation of EGM08 gravity field by means of GPS levelling and sea surface topography solutions.* Germany: Technical University Munich.

Heiskanen, w. & Moritz, H. (1967). *Physical Geodesy.* San Fransisco: W. H. Freeman Company. Retrieved from https://doi.org/10.1007/BF02525647

Hofmann-Wellenhof, B. & Moritz, H. (2006). *Physical Geodesy*. New York, USA: Springer Science and Business Media.

Kotsakis, C. & Katsambalos, K. (2010). Evaluation of EGM08 based on GPS and levelling height in Greece. *In Gravity, Geoid and Earth Observation: IAG Commission 2: Gravity Field, Chania, Crete, Greece, 23-27 June, 2008* (pp. 481-488). Springer Berlin Heidelberg.

Lemoine, F. G., Kenyon, S. C., Factor, J. K., Trimmer, R. G., Pavlis, N. K., & Chinn, D. S. (1998). The development of the joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) geopotential model EGM96. *Earth Resources and Remote Sensing, 1*(60).

Merry , C. L., Blitzkow, D., Fashir, H. H., John, S., Podmore, F., & Fairhead, J. D. (2005). A preliminarly geoid model for Africa. *In A Window on the Future of Geodesy: Proceedings of the International Association of Geodesy IAG General AssemblySapporo, Japan June 30-July 11, 2003* (pp. 374-379). Springer Berlin Heidelberg.

Merry, C. (2007). An updated geoid model for Africa. *In Presented, Symposium G, (Vol. 2).*

Ntambila, D. (2012). *Validation of recent geoid models for Tanzania.* M.sc Geomatics, Ardhi University, Department of geospatial science, Dar es salaam.

Olliver, J. G. (2007). The gravimetric geoid model of Tanzania. *Survey review, 39.*(305), 212-225.

Simion, J. M. (2019). *Evaluation of EGM08, TZG13 and TZG17 geoid models by using geoid slope method.* BSc. Geomatics, Ardhi University, Department of Geospatial Science, Dar es salaam, Tanzania.

Ulotu, P. (2009). Geoid Model For Tanzania From Sparse And Varying Gravity Data Density By The KTH Method. *A PhD Thesis Of The Division Of Geodesy, Loyal Institute Of Technology (KTH) Stockholom Sweeden*.

Van Westrum, D., Ahlgren, K., Hirt, C., & Guillaume, S. (2021). A geoid slope validation survey (2017) In the rugged terrain of Colorado, USA. *Journal of Geodesy, 95*, 1-19.

Willson, D. (2013). *Geoid model validation using geoid slopes.* B.Sc. Geomatics, Ardhi University., Department of geospatial science., Dar-es-Salaam.

APPENDICES