ARDHI UNIVERSITY

ASSESMENT OF THE VALIDITY OF ORTHOMETRIC HEIGHT OF TAREF11 CONTROLS USING EGM08, AFRgeo2019, TZG13, TZG17 AND TZG19 GEOID MODELS AND GEOPOTENTIAL NUMBER

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A Dissertation Submitted to the Department of Geospatial Sciences and Technology in Partially Fulfilment of the Requirements for the Award of Science in Geomatics (BSc. GM) of Ardhi University

CERTIFICATION

The undersigned certify that they have read and hereby recommend for acceptance by the Ardhi University dissertation titled **"*Assessment of the Validity of Orthometric Height of TAREF11 Controls Using EGM08, AFRgeo2019, TZG13, TZG17 and TZG19 Geoid Models and Geopotential Number*"** in partial fulfillment of the requirements for the award of degree of Bachelor of Science in Geomatics at Ardhi University.

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Ms. Regina V. Peter

Supervisor

Date……………………….

DECLARATION AND COPYRIGHT

I, Chove, Lupyana .H hereby declare that, the contents of this dissertation are the results of my own findings through my study and investigation, and to the best of my knowledge they have not been presented anywhere else as a dissertation for diploma, degree or any similar academic award in any institution of higher learning.

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DEDICATION

This work is dedicated to my loving, humble and ever-supporting family. My parents Bernard Chove and Lucy Chove, they have been my pillar not only during the course of this research but also during the whole tenure of my undergraduate degree. My sister Dr. Twilumba Chove who managed to balance hectic medical school to constantly check up on me and give me the strength to carry on. God has blessed me with many things but the three of you are by far my biggest blessing.

I also dedicate this to friends and family who want to see me succeed in any capacity. None of this would have been possible without your kind words and motivation that kept me going every step of the way.

ABSTRACT

TAREF11 is the Tanzania Reference frame of 2011. It came about after the introduction of GPS technology to obtain 3D coordinates of geodetic control points which would form the reference frame. The frame has been faced with problems in orthometric heights of its monuments reflected by misclosures beyond allowable levels when using them as benchmarks for leveling.

This research was focused on with assessing the orthometric height of these control points using one global model (EGM08), one regional model (AFRgeo2019) and three local models (TZG13,TZG17 and TZG19). The heights were also compared to orthometric heights obtained by geopotential number in order to validate them.

The control points had their geoid heights extracted from the geoid models via Surfer software. The ellipsoidal heights were taken as provided from the TAREF11 report. The relation h-N=H where h is ellipsoidal height, N is geoid height and H is orthometric height was used to determine orthometric heights for each model. The EGM08 geoid model yielded orthometric heights varying from the available orthometric heights from 0.272m to 1.917m, AFRgeo2019 varied from 0.394m to 2.668m, TZG13 (modified) varied from 0m to 0.031m, TZG17 varied from 0.592m to 1.621m and TZG19 varied from 0.540m to 1.007m. TZG 13 (modified) geoid model was the model used to establish the TAREF11 network but the heights obtained from the model still differ from the heights in the report. This is due to recalculation of the geoid heights from TZG13 to the TZG13 (modified) model.

Orthometric heights were also obtained via geopotential number. Geopotential number is calculated by taking the local geopotential vertical datum (${W_0}^{LVD,TZG13}$) and subtracting the actual gravity potential obtained via GRAFLAB software from it. This difference divided by the mean gravity provides orthometric heights. The variation from the available orthometric heights ranged from 0.005m to 0.4485m. This can draw a conclusion that orthometric heights computed from geopotential number are closer to the orthometric heights from the TAREF11 report compared to the geoid models.

The heights of monuments from the ground were also taken to observe how factors like weathering could also be affecting the heights of the points.

Keywords: Orthometric heights, geoid heights, ellipsoidal heights, geopotential number

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LIST OF ABBREVIATIONS

ARC 1960	Arc Datum Geodetic Network of 1960
DORIS	Doppler Orbitography and Radiopositioning Integrated by Satellite
EGM08	Earth Gravitation Model of 2008
GNSS	Global Navigation System
GPS	Global Positioning System
ITRF	International Terrestrial Reference Frame
TAREF11	Tanzania Reference Frame 2011
TZG	Tanzania Gravimetric Model
VLBI	Very Long Baseline Interferometry
WGS	World Geodetic System

CHAPTER ONE

INTRODUCTION

1.1 Background of the research

Orthometric height is the exact distance along the curved plumb line between the geoid and point on the earth's surface (Roman, 2007). Accurate estimation of orthometric heights from ellipsoidal heights is one of the current research areas for geodesists, more so in Tanzania since it was selected to be the height system nationwide. Ellipsoidal height is the height of a point relative to the reference ellipsoid surface obtained from GNSS observations (Roman, 2007).The separation between the geoid and reference ellipsoid is referred to as geoid height or commonly geoid undulation. Knowledge of geoid undulation (N) is necessary to enable conversion of GPS derived ellipsoidal height (h) to physically meaningful orthometric height (H), commonly used in many practical applications (Kemboi & Odera, 2016).Figure 1.1 shows the relationship between the ellipsoidal heights (h) and orthometric heights (H) and geoid undulations (N). This relationship is given by equation 1.1

$$h - N = H \quad (1.1)$$

Where h is the ellipsoidal height, N is geoid undulations and H is the orthometric height.

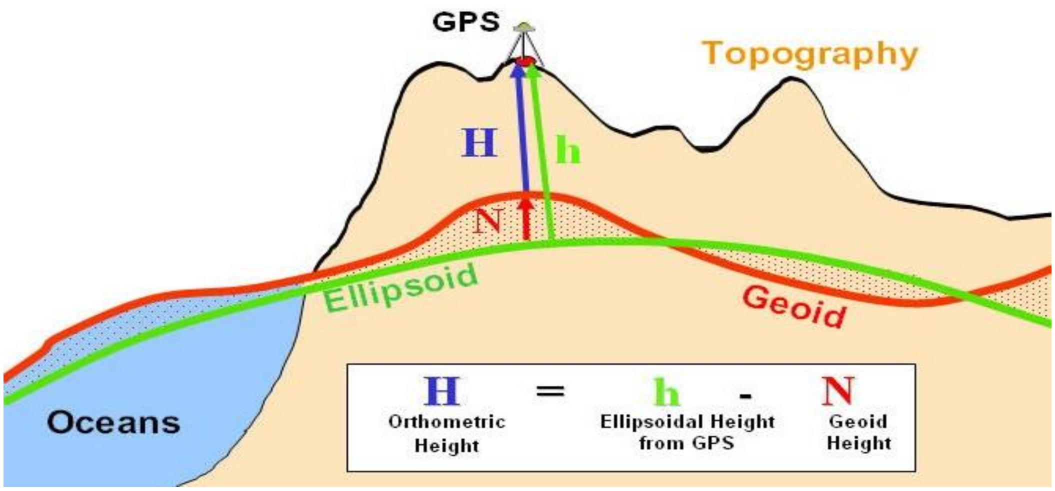

Figure 1.1 Relationship between orthometric height, ellipsoidal height and geoid height *(Amjadipavar, Rangelova, & Sideris, 2015)*

The value of geoid undulations all depend on which geoid model is being used (local, regional or global). The global geoid models are created to be used worldwide while the local and regional geoid models are developed to be a good fit in a certain locality. The local geoid models in Tanzania are; TZG07 by Olliver in 2007, TZG08 by Ulotu in 2008, TZG13 by Forsberg et al. in 2013, TZG17 by Peter in 2018 and TZG19 by Forsberg et al. in 2019. The regional geoid models in Africa are; AGP2003, and AFRgeo2019 models. The global geoid models are; EGM96 and EGM08 models. In this specific research one global model (EGM08), one regional model (AFRgeo2019) and three local models (TZG13, TZG17and TZG19) were used to assess the orthometric heights of TAREF11 control points.

In a previous study named the KILI2008 project, the orthometric height of Mount Kilimanjaro in Tanzania was computed. The global geoid model (EGM08) was used to provide the geoid undulations. The result was 5889.9m which is 5.1m less than the elevation obtained by director of lands and surveys in Tanzania by reciprocal trigonometric heighting (Fernandes, Msemwa, & Bos, 2009).

In another research the orthometric height of Mt. Kilimanjaro was computed using a local geoid model (TZG08) and the result was 5894.94m which is only 6cm less than the orthometric height computed by the director of lands and surveys in Tanzania (Ulotu, 2014).

The two researches aforementioned show that different geoid models will provide different values of geoid undulations at the same point. This also means that when orthometric heights are computed using the geoid models and ellipsoidal heights they will be different on each model.

The geoid undulations could also be used to validate the orthometric heights a control network. In a previous research carried out in New Zealand, GPS levelling and EGM08 geoid model were used to validate normal-orthometric height on 18 local vertical datums. The normal-orthometric heights at GPS leveling points were first converted to the normal heights. The normal to normal-orthometric height correction was then applied along the leveling lines using the leveling data. The differences between the normal-orthometric heights computed from EGM08 and those computed by GPS levelling varied ranging from 1cm to 37cm (Tenezer, Vatrt, & Abdallah, 2011).

This research aims to assess the orthometric heights in TAREF11 network further by not only using selected geoid models to obtain results but also validating these results using geopotential number in an attempt to make the network more reliable for height measurements. Geopotential height or geopotential number is a surface point's potential difference to the geoid potential. In other words, it is the difference between "A" surface point's W_A potential with "O" geoid point's W_O potential (Yilmaz, 2008). Equation 1.2 expresses the geopotential number calculation method at any point on the earth's surface.

$$C_A = W_O - W_A = -\int_O^A dW = \int_O^A g dn \qquad (1.2)$$

Although it has no distance dimension, it is the natural criterion for heights (Yilmaz, 2008). Any point has a unique geopotential number (with respect to the defined local geoid), and thus, itself, appropriately scaled, can be used as a height coordinate of the respective point (Jekeli, 2000). It is known that the disadvantage of geopotential number height system is that it is not expressed in units of length therefore, for it to have units of length scaling must be done. Scaling is done by dividing it by average gravity along the plumb line between the geoid and the observed point (Yilmaz, 2008). This results into another method of computing orthometric height. Orthometric height computed by geopotential number is given by equation 1.3

$$H = \frac{C}{\bar{g}} \qquad (1.3)$$

Where C is the geopotential number and $\bar{g}$ is the average gravity along the plumb line between the geoid and the observed point (Navratil & Unger, 2013). Previously, the geopotential number has been used to validate the orthometric heights of the Tanzania Primary Leveling Network (TPLN), in this research it will be used to validate TAREF11 control points.

1.2 Research problem

It is not known why there is a large disparity in the orthometric heights of TAREF11 control points. These disparities lead to errors in vertical measurements. When surveyors use the TAREF11 controls for levelling, the levelling network does not meet the required accuracy. This is a problem that has been noted on several occasions. The geoid models are used for comparison to obtain the best fit. Three of these models are local geoid models for Tanzania (TZG17, TZG13 and TZG19), one is a regional model for Africa (AFRgeo2019) and one is a global model (EGM08). Geopotential number is also used to obtain orthometric height which can be compared to the orthometric heights obtained from the geoid models as a means of validation.

1.3 Scope of research

The research study area is Tanzania mainland across randomly selected TAREF11 control points. Its extent is 28˚ E to 42˚E from the prime meridian and 1˚S to 12˚S of the equator. There are over 500 TAREF11 control points found in the research area. Figure 1.2 shows the study area and the points selected for the study.

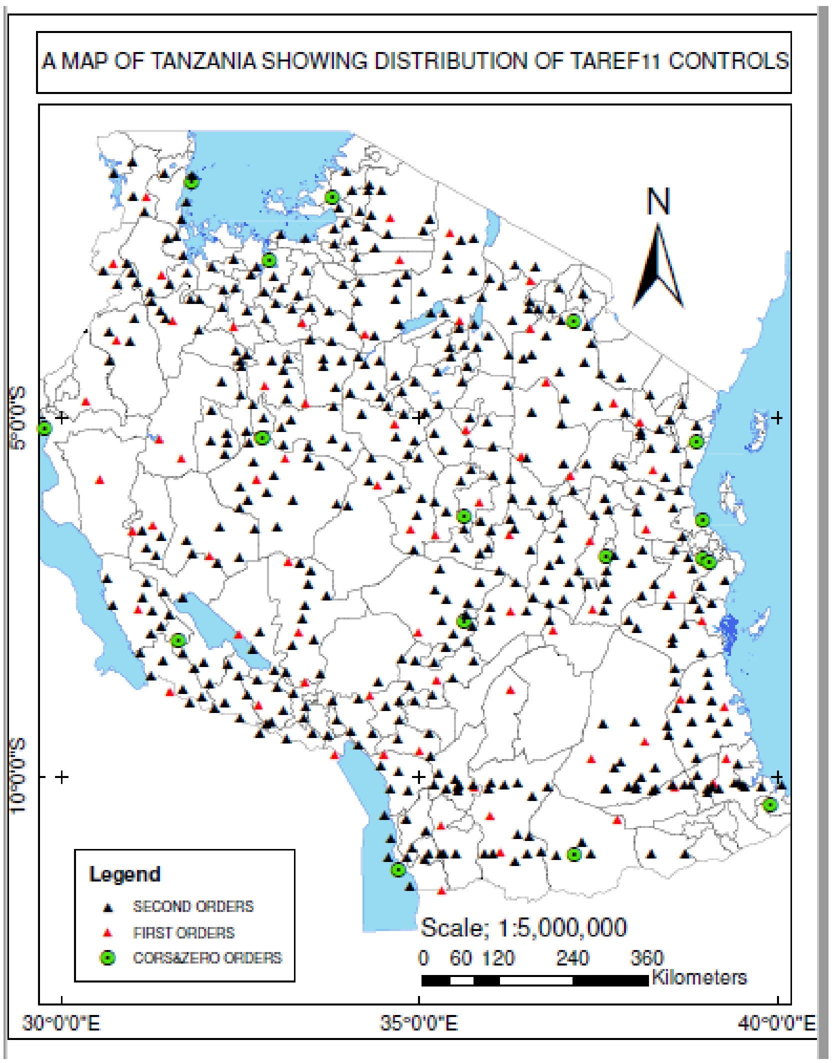

Figure 1.2 A map of Tanzania Showing distribution of TAREF11 controls

1.4 Objectives of the research

1.4.1 Main Objective

The main objective is to validate the orthometric heights of the TAREF11 control points.

1.4.2 Specific objectives

The specific objectives include the following;

- To extract the geoid heights of TAREF11 controls from the geoid models (EGM08, AFRgeo2019, TZG13, TZG17 and TZG19) and find the difference between these geoid heights and ellipsoidal heights to obtain orthometric heights.
- Determining the orthometric heights from geopotential number.

1.5 Research questions

A few questions are posed prior to the research in order to ensure the objectives are met which include;

- Are the orthometric heights similar using both methods?
- Are the geoid models used a good fit across the whole reference frame?

1.6 Significance of the research

- Tanzania is currently developing National Spatial Reference System (NSRS) so the availability of a proper height component could make TAREF11 essential in the Vertical Spatial Reference System.
- The vertical datum of Tanzania is currently the mean sea level in Tanga which has proven to be unreliable. If these network point have valid orthometric heights, a unique vertical datum can be established using these heights (Mtamakaya, 2009).

1.7 Beneficiaries of the research

The beneficiaries include; surveyors who incorporate the height dimension in their work to have a better vertical datum, geodesists who model the globe and require accurate height information to do so, geophysicists who use it to measure the rates of subsidence or land sinking.

1.8 Research Outline

(a) Chapter one

Introduces the study by explaining the background, statement of the problem, scope of the problem, objectives of the research, research questions, and significance of the research as well as the beneficiaries.

(b) Chapter two

Describes the literature used to support the research. It includes literature about orthometric heights, TAREF11 and geopotential number.

(c) Chapter three

Describes the data preparation, data acquisition and methods used to acquire and process the data

(d) Chapter four

Describes the results, analysis of the results as well as discussion of the results

(e) Chapter five

Describes the challenges, recommendations for future researches and concludes the research

CHAPTER TWO

LITERATURE REVIEW

2.1 Height systems

A height system is a one dimensional coordinate system used to define the metric distance of some point from a reference surface along a well-defined path, termed simply as the height of that point. Heights indicate the vertical distances of points above reference surfaces known as reference datum. There are two classes of height systems:

- Physical height system. This is the height system that ignores earth's gravity field. These height systems do not account for fluid flow since it's not the height difference that causes fluid flow, rather it is the difference in gravity potential.
- Geometric height system. This is the height systems that follows the plumb lines of the earth's gravity field (Featherstone & Kuhn, 2006).These height systems account for fluid flow since there is always fluid flow in areas with different gravity potential. This makes geometric height systems practical for use.

Historically the most commonly used technique for the practical determination of heights is spirit levelling. This technique measures the geometric height difference between two points where the reference surface is the local horizon defined by set-up of the levelling instrument. Both staffs and the levelling instrument are aligned with the direction of the local plumb line at each respective point (Moritz, 1967). The theoretical loop closure using spirit-levelling data is route dependent (different routes will provide different heights). This will mean that there is a different misclosure depending on the route picked to perform the levelling. This research will base on the orthometric height system. There are a various height systems used in geodesy. These are geopotential number, dynamic heights, orthometric heights and normal heights.

2.1.1 Geopotential Number System

Geopotential height or geopotential number is a surface point's potential difference to the same point on the geoid. Put differently, it is the difference between "A" surface point's W_A potential with "O" geoid point's W_O potential. Although it has no distance dimension, geopotential number is the natural criterion for heights (Yilmaz, 2008). Any point has a unique geopotential number (with respect to the defined local geoid), and thus, itself, appropriately scaled, can be used as a height coordinate of the point (Jekeli, 2000). It is important to note that two points will have the same geopotential number if they are found on the same equipotential surface. The geopotential number is a good measurement of height since it does not contain many assumptions as opposed to other height systems and it is path independent. On the other hand its disadvantage is that it is not scaled into units of length. Geopotential number is given in m^2/s^2

2.1.2 Dynamic Height System

The biggest problem with geopotential number is that it is not expressed in units of length. This was solved by bringing up dynamic height as proposed by Helmert (Vaníček & Krakwisky, 1986). Dynamic heights, H^{dyn} are obtained by dividing the geopotential number C by a constant gravity value of normal gravity at $45°$ latitude as given by equation 2.1

$$H^{dyn} = \frac{C}{\gamma} \tag{2.1}$$

The value of γ is calculated as 9.80629 m/s^2 for international ellipsoid. This value divides the difference in potential between the surface point and the geoid (Geopotential number) to scale it into meters, since geopotential number, C is in m^2/s^2 and γ is in m/s^2. Its advantages over other height systems are; having a unit of length, being path independent and closer in magnitude to measured heights or height differences than geopotential number. Its disadvantage is that it lacks geometrical common sense since it does not measure the elevation of a point as measured along the gravity plumb line of that point but the gravity plumb line of the reference.

2.1.3 Orthometric Heights

Orthometric height is the distance along the actual plumb line from the equipotential surface of earth's gravity (geoid) and a particular point on the earth's surface. It can be computed by dividing the geopotential number by mean gravity.

The difficulty of measuring mean gravity within the earth makes it challenging to determine orthometric heights precisely. The determination of mean gravity works on the assumptions that firstly there is a linear behavior of gravity between the surface and the geoid and also there is a constant crustal density of 2670 kg/m^3 both of which are factually incorrect (Jekeli, 2000). Therefore, the mean gravity for determination of orthometric heights is determined by prey reduction.

Prey reduction yield actual gravity inside the earth is given by equation 2.2 where $\overline{g}$ is the mean gravity, g_p is the gravity observed at a point and H is the orthometric height.

$$\overline{g} = g(\frac{1}{2}H) = g_p + 0.0424H \tag{2.2}$$

$$H_p = \frac{C}{g + 0.0424H} \tag{2.3}$$

The orthometric height obtained by using prey reduction is known as Helmert orthometric height and is given by equation 2.3. Helmert orthometric height is based on Poincare-prey relationship for integral mean gravity,$\overline{g}$ (Heiskanen & Moritz, 1966) given by equation 2.4

$$\overline{g} = g\frac{1}{2}({}^{d\gamma}\!/_{dh})H - 2\pi K\rho H \tag{2.4}$$

Where g is the observed gravity at the topographic surface, $\frac{d\gamma}{dh}$ is the vertical free air gradient of gravity, K is the universal gravitational constant and ρ is the topographic mass density (assumed to be 2.67 g/cm^3) and H is the orthometric height.

2.1.4 Normal Height System

After introducing orthometric height and realizing that there was a problem of determining the integral mean value of actual gravity g along the actual plumb line, normal height was introduced in 1945 to overcome this problem. This also involved theoretical replacement of the earth's surface by the telluroid and the geoid by the quasi-geoid which are the surfaces involved in normal height. The telluroid is a surface obtained by point projection of a point on the earth's surface along the straight line ellipsoidal normal to points that have the same gravity potential value in the normal gravity field as the original point in the earth's gravity field (Featherstone & Kuhn, 2006).

Normal height of a point is computed by taking geopotential number dividing by average normal gravity along normal plumb line between the quasi-geoid as shown in equation 2.5

$$H^N = \frac{C}{\gamma - 0.1543H} \tag{2.5}$$

Figure 2.1 shows orthometric height and normal height as well as their relationship. Where H^N is normal height, H^O is orthometric height, h is ellipsoid height, ζ is the quasi-geoid to ellipsoid separation and N is the geoid undulation.

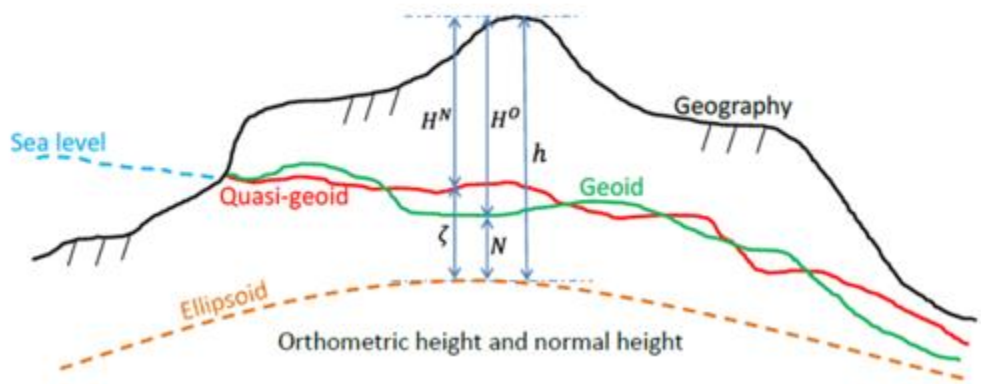

Figure 2. 1Orthometric heights and normal heights (Jekeli, 2000)

2.1.5 Ellipsoidal Height

Ellipsoidal heights are the heights referenced to a reference ellipsoid. It is the height used in GNSS positioning. Hence it can be defined as the perpendicular distance from an ellipsoidal surface to a point on the earth's surface. The difference between ellipsoidal and geoid height is the orthometric height.

2.2 Geoid Models

Geoid is an equipotential surface of the earth's gravity field that coincides with the mean sea level in the least square sense in undisturbed ocean i.e. when there are no ocean waves, ocean currents or ice melting (Vaníček & Krakwisky, 1986). There are two main types of geoid models:

- Gravimetric Geoid model: A geoid model that attempts to model the earth by using the spectral combination of satellite gravity model, terrestrial gravity data and airborne gravity data (to fill the gravity data gaps). In this model the geoid undulations are obtained by taking the difference between ellipsoidal heights and levelled heights (orthometric heights) ,this relation gives gravimetric geoid models practical importance (Matsuo & Kuroishi, 2020). Examples of these models are TZG13 and EGM08.

- Hybrid geoid model: A geoid model that attempts to model the earth by fitting the gravimetric geoid to the geometric geoid undulations from GNSS/Leveling data. For developing the hybrid geoid model, all frequency parts (long, middle and short-frequency) of gravimetric geoid are determined using all available data with optimal remove-restore technique based on EGM2008 reference surface (Kang, Sung, & Kim, 2016). The geometrical geoid heights are computed using equation 2.6 An example of this model is GEOID09 in USA

$$N_{BM}^{GPS} = h^{GPS} - H_{BM} \tag{2.6}$$

Where N_{BM}^{GPS} is the geometric geoid height, h_{GPS} is the ellipsoidal height and H_{BM} is the ellipsoidal height.

Historically the geoid has served as the reference surface for geodetic levelling. The separation between the geoid and reference ellipsoid is referred to as geoid height or commonly geoid undulation. Knowledge of geoid undulation (N) is necessary to enable conversion of GPS derived ellipsoidal height (h) to physically meaningful orthometric height (H), commonly used in many practical applications (Kemboi & Odera, 2016).

2.2.1 EGM08 Model

The Earth's gravitational model 2008 (EGM08) is a gravitational model complete to degree and order 2160. EGM2008 also contains additional spherical harmonic coefficients extending to degree 2190 and order 2159. The geoid heights are provided every 2.5 minutes. It was released to the public by the US National Geospatial-Intelligence Agency. It was developed from satellite gravimetric data in the year 2008. For Tanzania it extends from latitudes 1°N to 12°S and longitudes 29°E to 42°E. It provides physical geoid heights on the earth's surface (Pavlis, Holmes, & Factor, 2012).

Over areas covered by high quality data, the discrepancies between EGM08 geoid undulation and independent GPS levelling value are in order of 5 to 10cm give or take.

2.2.2 AFRgeo2019 Model

The AFRgeo2019 gravimetric geoid model was computed in the framework of the IAG African Geoid Project. The available gravity information comprises land and sea data, the latter consisting of ship-borne point data and altimetry-derived gravity anomalies along tracks. This dataset suffers from significantly large data gaps that were filled by using the EIGEN-6C4 model on a 15' x 15' grid prior to the gravity reduction scheme. A direct comparison between AFRgeo2019 and the former geoid model AGP2003 clearly shows the achieved improvement. When the two geoids were compared, the differences between the two geoids amounted to several meters in the continental area, especially in East Africa. The large differences over the Atlantic Ocean arise from the fact that the AGP2003 didn't include ocean data in the solution. Also some edge effects on the continent, which are a direct consequence of using no data outside the African continent in the AGP2003 solution (Abd-Elmootal, 2020).

2.2.3 TZG13 Model

Tanzania Gravimetric Geoid Model of 2013 is the gravimetric model based on GRS80 reference ellipsoid determined from gravity data measured from airborne survey supplemented with existing survey and marine gravity data, satellite gravity data (altimetry, GRACE and GOCE) and updated digital terrain models which cover both land and ocean in Tanzania developed by Forsberg in 2013.Its boundaries are 13°S to 1°N latitude wise and 28°E to 43°E longitude wise. The model was computed on a grid of 0.02°x 0.02° resolution corresponding to about 2km by 2km. Data was collected and compiled from the various sources Ardhi University and Surveying and Mapping Division (SMD) (Mayunga, 2016).

2.2.4 TZG17 Model

Tanzania Gravimetric Geoid Model of 2017 was developed through quasi geoid by the KTH method of least squares modification of stokes formula with additive corrections. It is based on GRS80 reference ellipsoid determined from 38,483 terrestrial point gravity data from Tanzania Gravity Database of 2008 (TGDB08), airborne gravity data limited to boundaries of Tanzania .Determining gravimetric models through quasi geoid has the advantage of minimizing some terrain reductions. TZG17 geoid model was computed on a grid of 1'x 1'interval and accuracy of 5cm was attained by Peter, 2018.

2.2.5 TZG19 Model

Tanzania Gravimetric Geoid Model of 2019 is the gravimetric model based on GRS80. It is an improvement on TZG13 model. It had more updated digital terrain models in addition to all the other sources used to determine TZG13 model. It is the latest geoid model in Tanzania developed by Forsberg in 2019

2.3 Geodetic control network

A geodetic network is the framework of geodetic control points which provide coordinates on which mapping and surveys are based. Traditionally, geodetic control points are established as permanent physical monuments placed in the ground and precisely marked, located in order to be identified and documented (Baarda, 1968). The geodetic control points are established for purposes such as engineering, cadastral and hydrographic surveys. Geodesy is responsible to provide well distributed geodetic control points whose horizontal and vertical positions are known (Saburi & Ntambila, 2016). They also offer unparalleled opportunities to monitor and understand many of the rhythms of the Earth most vital to the sustainability of modern and future societies like crustal motions, sea-level, and the weather (Dokka, 2009).With the development of satellite surveying techniques, geodetic control networks could be established using Global Navigation Satellite System (GNSS). The geodetic networks are then used as reference datum from which measurements are referred. In Tanzania initially the reference datum used was the arc datum 1960, it was the local reference system which best fit the geographical area of Tanzania (Mtamakaya, 2009).

2.4 TAREF11

In Tanzania initially the reference datum used was the arc datum 1960, it was the local reference system which best fit the geographical area of Tanzania. This was proven to have several limitations which include not being compatible with current space positioning techniques (GNSS, VLBI, DORIS, etc.), the fact that it provides horizontal solutions only hence it does not suit applications that require 3-D coordinates and its mapping frame didn’t coincide with mapping frames of neighboring countries. This led to the development of the TAREF11 frame in 2010. This came after the introduction of GPS technology to obtain 3D coordinates of geodetic control points which would form the reference frame. It was formed basing on the ITRF (International Terrestrial Reference Frame) (Mtamakaya, 2009).

CHAPTER THREE

METHODOLOGY

This chapter describes in detail how the orthometric heights of TAREF11 controls are assessed and then validated. It describes how the datasets are acquired, prepared and processed by various software.

3.1 Measuring heights of TAREF11 monuments

A total of 21 points from each of the 4 categories were selected for the study; 2 CORS points, 3 zero order points, 7 first order points and 9 second order were selected for field data. The field data obtained was the height of the monuments of these points taken from the ground surface by means of a tape and a plumb bob. These points were selected basing on their distribution in the country. Figure 3.1 shows how the heights of the monuments were measured by the tape measure

Figure 3.1Measuring the height of a TAREF11 control

3.2 Extraction of geoid heights from geoid models

The geoid heights of the selected points were extracted from the five geoid models selected for the study. This was done by importing the coordinates (latitudes and longitudes) of TAREF11 points along with each geoid model on Surfer software. The software grids the geoids so that the geoid heights of the points can be interpolated. This is done for all five geoid models (EGM08, AFRgeo2019, TZG13, TZG17 and TZG19).

EGM08 and AFRgeo2019 geoid models were publically available on the International Service for the Geoid (ISG) website. TZG13, TZG17 and TZG19 models were provided by the Department of Geospatial Sciences and Technology (DGST).

3.3 Obtaining orthometric height from geoid height and ellipsoidal height

After obtaining the geoid heights via the geoid models, the orthometric heights were then obtained. This was done by subtracting the geoid heights of the control points from their ellipsoidal heights (available in the TAREF11 report). This resulted into different values of orthometric heights for every model used since they fit the earth's surface differently. This is the first method used to obtain orthometric height.

3.4 Obtaining actual gravity potential of TAREF11 controls

The validation of the orthometric heights was done via geopotential number. To obtain geopotential number, the actual gravity potential of the surface as well as that of the geoid must be known since geopotential number is the difference of these two potentials.

Actual gravity potential is a resultant of centrifugal potential and gravitational potential. Their relationship is modeled by equation 3.1. Where W_p is the actual gravity potential of a point p, V is the gravitational potential, Z is the centrifugal potential, r is the radius of the earth, λ is the longitude of the point and θ is the latitude of the point

$$W_p(r,\theta,\lambda) = v(r,\theta,\lambda) + Z(r,\theta) \tag{3.1}$$

GRAFLAB (an open source code that runs on MATLAB software) was used to obtain the actual gravity potential of the points at the surface with an input of the 3D coordinates of the points as well as a global geopotential model (XGM2019e) of degree and order 5540.

The gravity was also obtained via GRAFLAB and aided in obtaining the orthometric height using this method. Figure 3.2 shows the graphic interface of GRAFLAB software and the parameters selected for the study.

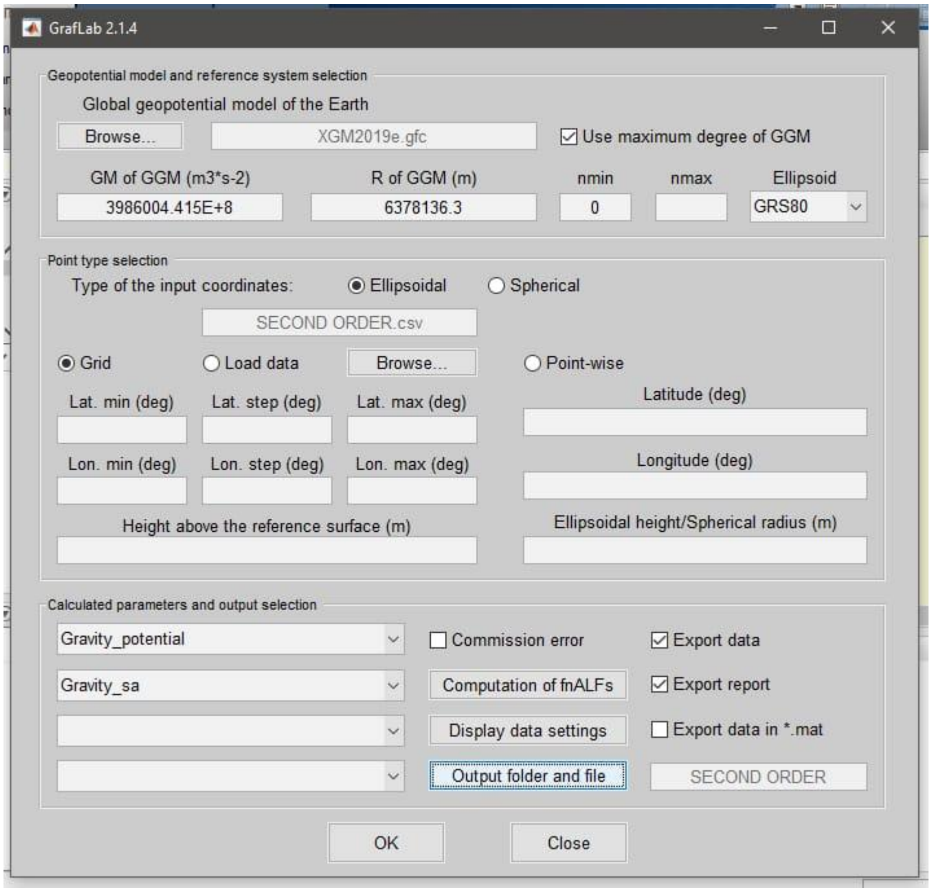

Figure 3.2 GRAFLAB software interface

3.5 Determination of Geopotential number

The local geopotential vertical datum ($W_0{}^{LVD,TZG13}$) having a value of 62636863.32 m^2/s^2 was computed using GPS and ocean levelling and TZG13 gravimetric geoid model by Masunga & Ulotu. The actual gravity potential obtained in section 3.4 was subtracted from the local geopotential vertical datum to give geopotential number (C) as given by equation 3.2

$$C = W_0^{LVD,TZG13} - W_p \tag{3.2}$$

3.6 Obtaining orthometric height from Geopotential number

The geopotential number at each point was divided by the mean gravity along the actual plumb line (g) to obtain orthometric height.

From the GRAFLAB output the gravitational acceleration (g_a) was obtained. The mean gravity was calculated by equation 3.3.

$$\bar{g} = g_a + 0.0424H \tag{3.3}$$

Where g is the mean gravity along the plumb line, g_a is the gravitational acceleration of the points in gal and H is the orthometric height of the points in km. The orthometric heights provided in the TAREF11 report were used in obtaining g.

This is the second method of obtaining orthometric height. It is used to verify the initial method (ellipsoidal height – geoid height).

3.7 Data availability and descriptions

In this research the data available were; ellipsoidal heights (h) of TAREF11 controls, geoid models (TZG13, TZG17, TZG19, AFRgeo2019 and EGM08), average gravity along the plumb line and actual gravity potentials of the TAREF11 controls and the geoid.

3.8 Software used

Throughout the research various software were used. Functions such as plotting, compiling and computing all had specific software to ensure the data was processed effectively. Table 3.1 shows all the software used for data processing.

Table 3.1 Software used for data processing

Name	Use
Microsoft Excel	Used for arranging and compiling data as well as mass computation
Microsoft Word	Used for report writing
GRAFLAB	Used for computation of gravity parameters.
Golden Surfer	Used to grid geoid data for use.
ArcGIS	Used to create map showing distribution of the points

CHAPTER FOUR

RESULT, ANALYSIS AND DISCUSSION

This chapter shows the results obtained from the research and analyses the results in order to draw conclusion from them. The section includes comparison of the different methods of obtaining orthometric heights (by geoid models and by geopotential number)

4.1 Results

4.1.1 Orthometric heights obtained via EGM08, AFRgeo2019, TZG13, TZG17 and TZG19

The orthometric heights computed via geoid models were compared to the orthometric heights from the TAREF11 report.

Table 4.1 shows the comparison between orthometric heights calculated from EGM08 model and the orthometric heights available from the TAREF11 report. The smallest difference between the two orthometric heights is 0.272m, while the largest difference is 1.917m.

Table 4.1 Orthometric heights obtained from EGM08 geoid model compared to TAREF11 report orthometric heights

S/N	STNAME	Orthometric heights (Calculated)(m)	Orthometric heights (Available)(m)	Difference in orthometric heights (m)
1	TANZ	97.870	98.456	0.586
2	DODC	1120.110	1120.544	0.434
3	T09Z	11.780	12.462	0.682
4	T14Z	1145.490	1145.843	0.353
5	T16Z	503.350	504.115	0.765
6	T14F	1252.790	1253.420	0.630
7	T24F	1302.190	1301.380	0.810
8	T32F	397.110	397.502	0.392
9	T33F	1039.090	1038.740	0.350
10	T40F	381.800	382.690	0.890
11	T53F	1097.740	1098.410	0.670
12	T59F	493.480	494.526	1.046
13	T156	1159.790	1160.955	1.165
14	T382	1250.400	1250.925	0.525
15	T395	517.980	518.541	0.561
16	T403	138.510	139.116	0.606
17	T406	161.690	162.349	0.659
18	T407	261.770	262.373	0.603
19	T518	815.100	817.017	1.917
20	T806	1193.650	1194.177	0.527
21	T895	973.450	973.178	0.271

Table 4.2 shows the comparison between orthometric heights calculated from AFRgeo2019 model and the orthometric heights available from the TAREF11 report. The smallest difference between the two orthometric heights is 0.394m, while the largest difference is 2.668m.

Table 4.2 Orthometric heights obtained from AFRgeo2019 geoid model compared to TAREF11 report orthometric heights

S/N	STNAME	Orthometric heights (Calculated)(m)	Orthometric heights (Available)(m)	Difference in orthometric heights(m)
1	TANZ	98.062	98.456	0.394
2	DODC	1119.400	1120.544	1.144
3	T09Z	11.514	12.462	0.948
4	T14Z	1144.429	1145.843	1.414
5	T16Z	503.047	504.115	1.067
6	T14F	1252.412	1253.420	1.008
7	T24F	1300.205	1301.380	1.175
8	T32F	396.837	397.502	0.665
9	T33F	1037.149	1038.740	1.591
10	T40F	381.447	382.690	1.243
11	T53F	1096.854	1098.410	1.556
12	T59F	492.127	494.526	2.399
13	T156	1159.355	1160.955	1.600
14	T382	1250.053	1250.925	0.872
15	T395	517.419	518.541	1.122
16	T403	138.499	139.116	0.617
17	T406	161.383	162.349	0.966
18	T407	261.915	262.373	0.458
19	T518	814.350	817.017	2.668
20	T806	1192.714	1194.177	1.463
21	T895	971.940	973.178	1.238

Table 4.3 shows the comparison between orthometric heights calculated from TZG13 model (modified) and the orthometric heights available from the TAREF11 report. The smallest difference between the two orthometric heights is 0m, while the largest difference is 0.031m.

TZG13 (modified) model was the model used to establish the network which is why there are very small differences between the two orthometric heights. The differences are a result of recalculation of the TZG13 modified model which was done by subtracting 1.05m from the original TZG13 model geoid heights to account for mean dynamic topography. In initial procedure to obtain the TZG13 (modified) model not exactly 1.05m was subtracted at every point, rather the value that was subtracted depended on the geographical position of the point. 1.05m was just the average difference between the geoid heights of the two models.

Table 4.3 Orthometric heights obtained from TZG13 geoid model compared to TAREF11 report orthometric heights

S/N	STNAME	Orthometric heights (Calculated)(m)	Orthometric heights (Available)(m)	Difference in orthometric heights (m)
1	TANZ	98.456	98.456	0
2	DODC	1120.544	1120.544	0.001
3	T09Z	12.462	12.462	0
4	T14Z	1145.843	1145.843	0
5	T16Z	504.115	504.115	0
6	T14F	1253.416	1253.420	0.004
7	T24F	1301.380	1301.380	0
8	T32F	397.501	397.502	0.001
9	T33F	1038.756	1038.740	0.016
10	T40F	382.674	382.690	0.016
11	T53F	1098.379	1098.410	0.031
12	T59F	494.529	494.526	0.003
13	T156	1160.957	1160.955	0.002
14	T382	1250.927	1250.925	0.002
15	T395	518.539	518.541	0.002
16	T403	139.105	139.116	0.011
17	T406	162.351	162.349	0.002
18	T407	262.361	262.373	0.012
19	T518	817.022	817.017	0.005
20	T806	1194.173	1194.177	0.004
21	T895	973.179	973.178	0.001

Table 4.4 shows the comparison between orthometric heights calculated from TZG17 model and the orthometric heights available from the TAREF11 report. The smallest difference between the two orthometric heights is 0.592m, while the largest difference is 1.621m

Table 4.4 Orthometric heights obtained from TZG17 geoid model compared to TAREF11 report orthometric heights.

S/N	STNAME	Orthometric heights (Calculated)(m)	Orthometric heights (Available)(m)	Difference in orthometric heights (m)
1	TANZ	97.730	98.456	0.726
2	DODC	1119.750	1120.544	0.794
3	T09Z	11.665	12.462	0.797
4	T14Z	1145.011	1145.843	0.832
5	T16Z	502.952	504.115	1.162
6	T14F	1252.496	1253.420	0.924
7	T24F	1300.788	1301.380	0.592
8	T32F	396.903	397.502	0.599
9	T33F	1037.725	1038.740	1.015
10	T40F	381.372	382.690	1.318
11	T53F	1097.536	1098.410	0.874
12	T59F	492.905	494.526	1.621
13	T156	1159.839	1160.955	1.116
14	T382	1250.061	1250.925	0.864
15	T395	517.592	518.541	0.949
16	T403	138.342	139.116	0.774
17	T406	161.344	162.349	1.006
18	T407	261.558	262.373	0.815
19	T518	816.115	817.017	0.902
20	T806	1193.342	1194.177	0.835
21	T895	972.491	973.178	0.687

Table 4.5 shows the comparison between orthometric heights obtained from TZG19 model and the orthometric heights obtained from the TAREF11 report. The smallest difference between the two orthometric heights is 0.540m, while the largest difference is 1.007m

Table 4.5 Orthometric heights obtained from TZG19 geoid model compared to TAREF11 report orthometric heights

S/N	STNAME	Orthometric heights (Calculated)(m)	Orthometric heights (Available)(m)	Difference in orthometric heights (m)
1	TANZ	97.832	98.456	0.624
2	DODC	1119.916	1120.544	0.628
3	T09Z	11.869	12.462	0.593
4	T14Z	1145.184	1145.843	0.659
5	T16Z	503.348	504.115	0.766
6	T14F	1252.633	1253.420	0.787
7	T24F	1300.840	1301.380	0.540
8	T32F	396.871	397.502	0.631
9	T33F	1038.141	1038.740	0.599
10	T40F	381.909	382.690	0.781
11	T53F	1097.764	1098.410	0.646
12	T59F	493.549	494.526	0.977
13	T156	1160.157	1160.955	0.798
14	T382	1250.145	1250.925	0.781
15	T395	517.755	518.541	0.786
16	T403	138.471	139.116	0.645
17	T406	161.639	162.349	0.710
18	T407	261.741	262.373	0.632
19	T518	816.441	817.017	0.577
20	T806	1193.498	1194.177	0.679
21	T895	972.172	973.178	1.007

4.1.2 Orthometric heights obtained using geopotential number

Table 4.6 shows the comparison between orthometric heights calculated from geopotential number and the orthometric heights available from the TAREF11 report. The smallest difference between the two orthometric heights is 0.005m, while the largest difference is 0.450m

Table 4.6 Orthometric heights obtained from geopotential number compared to TAREF11 report orthometric heights

S/N	STNAME	Orthometric heights (Calculated)(m)	Orthometric heights (Available)(m)	Difference in orthometric heights (m)
1	TANZ	98.801	98.456	0.345
2	DODC	1120.783	1120.544	0.239
3	T09Z	12.688	12.462	0.226
4	T14Z	1146.099	1145.843	0.256
5	T16Z	504.171	504.115	0.057
6	T14F	1253.605	1253.420	0.185
7	T24F	1301.763	1301.380	0.383
8	T32F	397.848	397.502	0.346
9	T33F	1039.033	1038.740	0.293
10	T40F	382.695	382.690	0.005
11	T53F	1098.682	1098.410	0.272
12	T59F	494.466	494.526	0.060
13	T156	1161.029	1160.955	0.074
14	T382	1251.048	1250.925	0.123
15	T395	518.688	518.541	0.147
16	T403	139.367	139.116	0.250
17	T406	162.472	162.349	0.123
18	T407	262.672	262.373	0.299
19	T518	817.347	817.017	0.329
20	T806	1194.526	1194.177	0.349
21	T895	973.628	973.178	0.450

4.1.3 Heights of the monuments from the ground

The heights of the monuments from the ground were taken to account for factors like deformation by weathering. Table 4.7 shows the heights of TAREF11 monuments from the ground.

Table 4.7 The heights of TAREF11 monuments from the ground

S/N	STNAME	Location	Heights from ground (m)	Order
1	TANZ	Ardhi University	2	CORS
2	DODC	Dodoma	2	CORS
3	T09Z	Tanga	1.34	Zero
4	T14Z	Mwanza	1.4	Zero
5	T16Z	Morogoro	1.475	Zero
6	T14F	Arusha	1.005	First
7	T24F	Mishoma	0.9	First
8	T32F	Kabuku	0.95	First
9	T33F	Kalilankulukulu	0.9	First
10	T40F	Dakawa	0.91	First
11	T53F	Rujewa	0.9	First
12	T59F	Kyela	0.9	First
13	T156	Uyui	0.19	Second
14	T382	Gairo	0.122	Second
15	T395	Mlandizi	0.192	Second
16	T403	Kibaha	0.192	Second
17	T406	Lukose	0.19	Second
18	T407	Kisarawe	0.178	Second
19	T518	Kabwe	0.194	Second
20	T806	Mbarali	0.19	Second
21	T895	Mpepai	0.19	Second

4.2 Discussion

The orthometric heights calculated from each geoid model were compared with the orthometric height from the TAREF11 report to compare how well each geoid model fits on the reference frame.

From the results it can be seen that the TZG13 model provides the best fit because it was used to establish the network. Furthermore the orthometric heights calculated from local models (TZG13, TZG17 and TZG19) have smaller disparities in comparison to the orthometric heights from regional (AFRgeo2019) and global (EGM08) geoid models.

The comparison between the TAREF11 report orthometric heights and orthometric heights computed by geopotential number was done as a validation method. The results had differences of up to 0.450m. The large difference is a result of using spherically approximated gravity values at each point when calculating mean gravity.

Also a 99% confidence test was applied to the resulting differences of each model to remove outliers. Table 4.8 shows the results for a 99% confidence test conducted on each geoid model.

Table 4.8 99% confidence test for each model

Geoid Model	EGM08	AFRgeo2019	TZG13	TZG17	TZG19
Number of points that passed the 99% confidence test	14	11	17	12	8

With the two methods yielding different results, it can be noted that using geopotential number produced orthometric heights which were much closer to the available TAREF11 report orthometric heights in comparison to using the selected geoid models. This is mainly due to the fact that the geopotential number was computed using a local geopotential vertical datum which also used TZG13 geoid model.

CHAPTER FIVE

CONCLUSION, CHALLENGES AND RECCOMENDATIONS

This chapter explains the findings of this research basing on the results obtained, draws conclusions, outlines the challenges and provides recommendations for future researches.

5.1 Conclusion

From the results obtained it was determined that the orthometric heights were not completely accurate mainly due to some inaccuracies of the geoid models in covering some parts of Tanzania as well as using spherically approximated values of gravity when computing orthometric heights by geopotential number. When finally obtaining the height, severe weathering is also a factor, this affects the ellipsoidal height while performing levelling, and hence the orthometric height is also affected. Point T382 in Gairo is a good example of this.

Also from the five geoid models used; EGM08, AFRgeo2019, TZG13, TZG17 and TZG19, the closest orthometric heights to the ones in the report were the heights from TZG13 (modified) model due to the fact that this is the model that was used to establish the TAREF11 network. It did not yield the exact same results due to recalculation of the model from TZG13 model.

From the results it can also be determined that the geoid models were a good fit in some areas, while in other areas they were very inaccurate. A 99% confidence test was also conducted on the results from each model to remove outliers.

The orthometric heights were also compared to orthometric height obtained by geopotential number which was found to have differences varying from 0.05m up to 0.449m which was a better result than all the models apart from TZG13 (the model used to establish the network).

5.2 Challenges

- Some of the monuments of the points were removed due to reasons like construction and not replaced. E.g. Chalinze (T404) and Lukole (T393)

5.3 Recommendations

- There should be more TAREF11 points established to replace the weathered points that provide inaccurate heights.

- The orthometric heights of TAREF11 should also be validated by GPS levelling which is a better method of validation.

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APPENDICES

This section shows the data used throughout this research, these data include the following:

- Geodetic coordinates of TAREF11 control points
- Heights of TAREF11 monuments from the ground
- Geoid heights, ellipsoidal heights and orthometric heights for each geoid model
- Gravity and geopotential from graflab software

APPENDIX I: Geodetic coordinates of TAREF11 controls selected for this study.

s/n	Station	Latitude	Longitude	Ellipsoidal height
1	TANZ	-6.76559	39.20792563	70.4607
2	DODC	-6.16964	35.74830403	1099.9705
3	T09Z	-5.08776	39.11662974	-15.9371
4	T14Z	-2.447	32.92251841	1127.8713
5	T16Z	-6.74847	37.80492904	480.3136
6	T14F	-3.43763	36.70670341	1234.5921
7	T24F	-5.61465	30.47756463	1288.1
8	T32F	-5.50006	38.48510476	372.8097
9	T33F	-6.36742	30.9403036	1024.53
10	T40F	-6.50106	37.56872342	359.4782
11	T53F	-8.74455	34.3807643	1083.1621
12	T59F	-9.60934	33.8733165	477.27
13	T156	-4.73992	32.83758681	1141.8445
14	T382	-6.13929	36.87270541	1231.9704
15	T395	-6.93586	37.40342713	495.5241
16	T403	-6.75073	38.99366489	111.6222
17	T406	-6.8426	38.17213332	136.9734
18	T407	-6.92213	39.06240444	235.1539
19	T518	-7.05534	30.57021633	800.6763
20	T806	-8.81855	33.83782111	1179.5992
21	T895	-11.1283	35.17394534	956.1214

APPENDIX II: Heights of TAREF11 monuments from the ground

S/N	STNAME	Location	Heights from ground
1	TANZ	Ardhi University	2
2	DODC	Dodoma	2
3	T09Z	Tanga	1.34
4	T14Z	Mwanza	1.4
5	T16Z	Morogoro	1.475
6	T14F	Arusha	1.005
7	T24F	Mishoma	0.9
8	T32F	Kabuku	0.95
9	T33F	Kalilankulukulu	0.9
10	T40F	Dakawa	0.91
11	T53F	Rujewa	0.9
12	T59F	Kyela	0.9
13	T156	Uyui	0.19
14	T382	Gairo	0.122
15	T395	Mlandizi	0.192
16	T403	Kibaha	0.192
17	T406	Lukose	0.19
18	T407	Kisarawe	0.178
19	T518	Kabwe	0.194
20	T806	Mbarali	0.19
21	T895	Mpepai	0.19

APPENDIX III: Geoid heights, ellipsoidal heights and orthometric heights for each geoid model

EGM 08

S/N	STNAME	Location	Ellip Ht	Geoid Ht	Orthometric Ht (Calculated)
1	TANZ	Ardhi University	70.4607	-27.4977	97.87
2	DODC	Dodoma	1099.971	-19.3163	1120.11
3	T09Z	Tanga	-15.9371	-27.3492	11.78
4	T14Z	Mwanza	1127.871	-16.4432	1145.49
5	T16Z	Morogoro	480.3136	-22.6251	503.35
6	T14F	Arusha	1234.592	-17.7126	1252.79
7	T24F	Mishoma	1288.1	-12.009	1302.19
8	T32F	Kabuku	372.8097	-23.9107	397.11
9	T33F	Kalilankulukulu	1024.53	-12.5053	1039.09
10	T40F	Dakawa	359.4782	-21.8514	381.8
11	T53F	Rujewa	1083.162	-13.5796	1097.74
12	T59F	Kyela	477.27	-14.7361	493.48
13	T156	Uyui	1141.845	-17.4022	1159.79
14	T382	Gairo	1231.97	-17.9842	1250.4
15	T395	Mlandizi	495.5241	-21.7894	517.98
16	T403	Kibaha	111.6222	-26.7682	138.51
17	T406	Lukose	136.9734	-24.2986	161.69
18	T407	Kisarawe	235.1539	-26.6496	261.77
19	T518	Kabwe	800.6763	-13.572	815.1
20	T806	Mbarali	1179.599	-13.0061	1193.65
21	T895	Mpepai	956.1214	-15.7141	973.45

AFRgeo2019

S/N	STNAME	Location	Ellip Ht	Geoid Ht	Orthometric Ht (Calculated)
1	TANZ	Ardhi University	70.4607	-27.4977	97.9583949
2	DODC	Dodoma	1099.971	-19.3163	1119.286786
3	T09Z	Tanga	-15.9371	-27.3492	11.41210582
4	T14Z	Mwanza	1127.871	-16.4432	1144.314525
5	T16Z	Morogoro	480.3136	-22.6251	502.9387285
6	T14F	Arusha	1234.592	-17.7126	1252.304716
7	T24F	Mishoma	1288.1	-12.009	1300.10904
8	T32F	Kabuku	372.8097	-23.9107	396.7204137
9	T33F	Kalilankulukulu	1024.53	-12.5053	1037.035313
10	T40F	Dakawa	359.4782	-21.8514	381.3296483
11	T53F	Rujewa	1083.162	-13.5796	1096.74168
12	T59F	Kyela	477.27	-14.7361	492.0061003
13	T156	Uyui	1141.845	-17.4022	1159.246672
14	T382	Gairo	1231.97	-17.9842	1249.954606
15	T395	Mlandizi	495.5241	-21.7894	517.3134904
16	T403	Kibaha	111.6222	-26.7682	138.3903634
17	T406	Lukose	136.9734	-24.2986	161.2719702
18	T407	Kisarawe	235.1539	-26.6496	261.8035465
19	T518	Kabwe	800.6763	-13.572	814.2483203
20	T806	Mbarali	1179.599	-13.0061	1192.605295
21	T895	Mpepai	956.1214	-15.7141	971.8355295

TZG13 modified

S/N	STNAME	Location	Ellip Ht	Geoid Ht	Orthometric Ht (Calculated)
1	TANZ	Ardhi University	70.4607	-27.995	98.4557
2	DODC	Dodoma	1099.971	-20.573	1120.5435
3	T09Z	Tanga	-15.9371	-28.399	12.4619
4	T14Z	Mwanza	1127.871	-17.972	1145.8433
5	T16Z	Morogoro	480.3136	-23.801	504.1146
6	T14F	Arusha	1234.592	-18.824	1253.4161
7	T24F	Mishoma	1288.1	-13.28	1301.38
8	T32F	Kabuku	372.8097	-24.691	397.5007
9	T33F	Kalilankulukulu	1024.53	-14.226	1038.756
10	T40F	Dakawa	359.4782	-23.196	382.6742
11	T53F	Rujewa	1083.162	-15.217	1098.3791
12	T59F	Kyela	477.27	-17.259	494.529
13	T156	Uyui	1141.845	-19.112	1160.9565
14	T382	Gairo	1231.97	-18.957	1250.9274
15	T395	Mlandizi	495.5241	-23.015	518.5391
16	T403	Kibaha	111.6222	-27.483	139.1052
17	T406	Lukose	136.9734	-25.378	162.3514
18	T407	Kisarawe	235.1539	-27.207	262.3609
19	T518	Kabwe	800.6763	-16.346	817.0223
20	T806	Mbarali	1179.599	-14.574	1194.1732
21	T895	Mpepai	956.1214	-17.058	973.1794

TZG17

S/N	STNAME	Location	Ellip Ht	Geoid Ht	Orthometric Ht (Obtained)
1	TANZ	Ardhi University	70.4607	-27.2003	97.66100295
2	DODC	Dodoma	1099.971	-19.7046	1119.675149
3	T09Z	Tanga	-15.9371	-27.5339	11.59681679
4	T14Z	Mwanza	1127.871	-17.0637	1144.934951
5	T16Z	Morogoro	480.3136	-22.5661	502.8797126
6	T14F	Arusha	1234.592	-17.8324	1252.424494
7	T24F	Mishoma	1288.1	-12.6243	1300.724324
8	T32F	Kabuku	372.8097	-24.0152	396.8249246
9	T33F	Kalilankulukulu	1024.53	-13.1192	1037.649193
10	T40F	Dakawa	359.4782	-21.816	381.2941776
11	T53F	Rujewa	1083.162	-14.2988	1097.460887
12	T59F	Kyela	477.27	-15.5541	492.8241372
13	T156	Uyui	1141.845	-17.9221	1159.766584
14	T382	Gairo	1231.97	-18.0254	1249.995821
15	T395	Mlandizi	495.5241	-21.9977	517.5217888
16	T403	Kibaha	111.6222	-26.6473	138.2694786
17	T406	Lukose	136.9734	-24.2963	161.2697291
18	T407	Kisarawe	235.1539	-26.3304	261.4843048
19	T518	Kabwe	800.6763	-15.3716	816.0478956
20	T806	Mbarali	1179.599	-13.6708	1193.270044
21	T895	Mpepai	956.1214	-16.3001	972.4214575

TZG19

S/N	STNAME	Location	Ellip Ht	Geoid Ht	Orthometric Ht (Calculated)
1	TANZ	Ardhi University	70.4607	-27.268	97.7287
2	DODC	Dodoma	1099.971	-19.833	1119.8035
3	T09Z	Tanga	-15.9371	-27.704	11.7669
4	T14Z	Mwanza	1127.871	-17.198	1145.0693
5	T16Z	Morogoro	480.3136	-22.926	503.2396
6	T14F	Arusha	1234.592	-17.934	1252.5261
7	T24F	Mishoma	1288.1	-12.644	1300.744
8	T32F	Kabuku	372.8097	-23.945	396.7547
9	T33F	Kalilankulukulu	1024.53	-13.497	1038.027
10	T40F	Dakawa	359.4782	-22.314	381.7922
11	T53F	Rujewa	1083.162	-14.489	1097.6511
12	T59F	Kyela	477.27	-16.158	493.428
13	T156	Uyui	1141.845	-18.204	1160.0485
14	T382	Gairo	1231.97	-18.076	1250.0464
15	T395	Mlandizi	495.5241	-22.125	517.6491
16	T403	Kibaha	111.6222	-26.74	138.3622
17	T406	Lukose	136.9734	-24.555	161.5284
18	T407	Kisarawe	235.1539	-26.476	261.6299
19	T518	Kabwe	800.6763	-15.663	816.3393
20	T806	Mbarali	1179.599	-13.79	1193.3892
21	T895	Mpepai	956.1214	-15.946	972.0674

APPENDIX IV: Gravity and geopotential from graflab software

St/Name	Location	Gravity	Gravity Potential	Geopotential number	Mean gravity	Orthometric height (Calculated)
TANZ	Ardhi University	9.781	62635897	966.348	9.781	98.801
DODC	Dodoma	9.777	62625904	10958.870	9.778	1120.782
T09Z	Tanga	9.780	62636739	124.092	9.780	12.688
T14Z	Mwanza	9.777	62625658	11205.650	9.777	1146.099
T16Z	Morogoro	9.779	62631933	4930.617	9.780	504.171
T14F	Arusha	9.776	62624607	12256.610	9.777	1253.605
T24F	Mishoma	9.777	62624135	12728.070	9.778	1301.763
T32F	Kabuku	9.779	62632972	3890.911	9.780	397.848
T33F	Kalilankulukulu	9.778	62626704	10159.630	9.778	1039.033
T40F	Dakawa	9.780	62633121	3742.640	9.780	382.695
T53F	Rujewa	9.778	62626120	10743.510	9.778	1098.682
T59F	Kyela	9.779	62632028	4835.536	9.779	494.466
T156	Uyui	9.777	62625512	11351.650	9.777	1161.029
T382	Gairo	9.778	62624630	12232.990	9.778	1251.048
T395	Mlandizi	9.779	62631791	5072.500	9.779	518.688
T403	Kibaha	9.781	62635500	1363.095	9.780	139.367
T406	Lukose	9.781	62635274	1589.039	9.780	162.472
T407	Kisarawe	9.780	62634294	2569.069	9.780	262.672
T518	Kabwe	9.778	62628871	7992.244	9.778	817.347
T806	Mbarali	9.777	62625183	11680.120	9.778	1194.526
T895	Mpepai	9.779	62627341	9521.994	9.779	973.628