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VALIDATION OF AFRICAN GEOID MODEL OF 2019 (AFRgeo19) USING ABSOLUTE AND RELATIVE GPS/LEVELLING METHOD IN TANZANIA

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VALIDATION OF AFRICAN GEOID MODEL OF 2019 (AFRgeo19) USING ABSOLUTE AND RELATIVE GPS/LEVELLING METHOD IN TANZANIA

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A Dissertation Submitted to the Department of Geospatial Science and Technology in Partially Fulfilment of the Requirements for the Award of Bachelor of Science Degree in Geomatics (B.Sc. GM) of Ardhi University.

CERTIFICATION

The undersigned certify that they have read and hereby recommend for acceptance by Ardhi University a dissertation titled "**Validation of African Geoid Model of 2019 (AFRgeo19) using Absolute and Relative GPS/Levelling Method in Tanzania**" in partial fulfillment of the requirement for the award of Bachelor of Science Degree in Geomatics of the Ardhi University.

Signature ………………………………….

Ms. Regina V. Peter

(Supervisor)

Date ………………………………………

DECLARATION AND COPYRIGHT

I, DORCAS MLYUKA hereby declare that, the contents of this dissertation are the results of my own findings through my study and investigation, and to the best of my knowledge they have not been presented anywhere else as a dissertation for diploma, degree or any similar academic award in any institution of higher learning.

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DEDICATION

I would like to dedicate this dissertation to my lovely mother Anna Ezekiel Kiwanga who has been a great support to my life and in my whole period of study.

ABSTRACT

This dissertation aims to validate the 2019 African geoid model (AFRgeo19) using absolute and relative GPS/levelling method in Tanzania. Geometric geoid heights were used as standard against which the gravimetric geoid height was compared. The data used to evaluate the model were the orthometric and ellipsoidal height of the available 29 TPLN benchmarks and were obtained from the previous researcher (Urassa,2020). Also, the gravimetric geoid height was obtained from the geoid model (AFRgeo19).

In absolute GPS/levelling approach before accounting for systematic effects on the difference between geometric and gravimetric geoid heights, statistical analysis revealed the SD of 13.41cm and the RMS of 22.38cm over the area of interest. Also, in relative GPS/levelling approach the statistical analysis gave the SD of 0.1247ppm and the RMS of 0.1291ppm

After accounting for systematic effects through the application of a 5-parameter model to the misclosure vector, the accuracy of AFRgeo19 improved to 9.45cm. Furthermore, by removing the effects contributed by the ellipsoidal and orthometric heights using standard error estimates, the accuracy of the model further improved from 9.45cm to 8.63cm.

Comparing these results with the current geoid model of Tanzania, TZG19, obtained from previous researcher Urassa,2019 using the same methodology, the accuracy of TZG19 was determined to be 6.7cm. This indicates that TZG19 is more accurate compared to AFRgeo19.

Keywords: AFRgeo19, TZG19, absolute and relative GPS/levelling, TPLN, Gravimetric geoid height, Geometric geoid height and Systematic effects

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ABREVIATIONS AND ACRONYMS

AFRgeo19	African Geoid of 2019
AGP07	African Geoid Project of 2007
BM	Benchmark
DEM	Digital Elevation Model
DGST	Department of Geospatial Science and Technology
EGM96	Earth Gravitational model of 1996
EGM08	Earth Gravitational Model of 2008
EGM20	Earth Gravitational Model of 2020
FBM	Fundamental Benchmark
GNSS	Global Navigation Satellite System
GOCE	Gravity Field and Steady-State Ocean Circulation Explorer
GPS	Global Positioning System
GRACE	Gravity Recovery and Climate Experiment
IBM	Intermediate Benchmark
LMSL	Local mean sea level
MATLAB	Matrix Laboratory
MDT	Mean Dynamic Topography
MSL	Mean Sea Level
PGM17	Preliminary Geopotential Model 2017
SA	Satellite Altimetry
SD	Standard Deviation
SSH	Sea Surface Height
SST	Sea Surface Topography
TG-VD	Tanga Tide Gauge Vertical Datum
TPLN	Tanzania Primary Levelling Network
TZG07	Tanzania Geoid Model of 2007
TZG08	Tanzania Geoid Model of 2008

TZG13	Tanzania Geoid Model of 2013
TZG17	Tanzania Geoid Model of 2017
TZG19	Tanzania Geoid Model of 2019
VD	Vertical Datum

CHAPTER ONE

INTRODUCTION

1.1 Background

Geoid refers to an equipotential surface of the earth's gravity field that coincides with the mean sea level (MSL) in least square sense and continued under the continental masses (Vanicek, 1986). Geoid is served as the height reference surface for geodetic, geophysical and many engineering applications. So, Geoid being the vertical datum for the world height system, many investigation and studies have to be conducted in order to come up with the best geoid model which will serve as the vertical datum for the height system. The determination of geoid needs sufficient coverage of observation data related to the earth's gravity field, such as gravity anomalies (Abd-Elmotaal, 2020).

Developed countries are striving to achieve a centimeter geoid model while for developing countries the situation is different, that is they think that the situation in their areas does not allow even a few decimeters geoid models (Ulotu,2009). Tanzania has developed five geoid models each with different accuracies. These are TZG07 with 47cm accuracy by Oliver, (2007), TZG08 with 27.8cm accuracy by Ulotu, (2009), TZG13 with 10cm accuracy by Forsberg et al, (2013), TZG17 with 5cm accuracy by Peter (2017) and TZG19 with accuracy of 10cm by Forsberg et al (2019). Also, we have AGP07 and AFRgeo19 as the Regional (African) Gravimetric Geoidal Model and EGM96, EGM08 and EGM20 (coming soon) as the Global Gravimetric Geoid Model. The demand for height information from the satellite users-based positioning techniques, mostly Global Positioning System (GPS), has increased interest on determination and use of precise geoid models (Ulotu, 2009).

Different geoid models are available for Tanzania, including global geoid models like EGM08, regional geoid models like the African geoid model of 2019, and various local geoid models such as TZG07, TZG08, TZG13, TZG17, and TZG19. Numerous geodesists and researchers have made efforts to evaluate these geoid models using various methods to assess their quality, stability, and performance over Tanzania. For instance, Ntambila (2012) evaluated the EGM08 geoid model by employing sea surface topography and the GPS/levelling method, achieving an accuracy of 29.9cm. Ulotu (2013) conducted a partial evaluation of the TZG13 geoid model using the GPS levelling method, resulting in an accuracy of 10cm. Asenga (2019) evaluated the TZG17 geoid model, attaining an accuracy of 7.38cm. Furthermore, Urassa (2020) evaluated the TZG19 geoid model, which yielded an accuracy of 6.7cm.

With the exception of AFRgeo19, the accuracy and performance of various geoid models in Tanzania have been assessed. Thus, this research aims to validate the AFRgeo19 geoid model in Tanzania by utilizing GPS levelling data. The AFRgeo19 geoid model was successfully computed by Abd Elmotaaal et al. (2020) using the window remove-restore technique, resulting in reduced gravity anomalies that are small, smooth, and exhibit minimal interpolation errors, particularly in areas with significant data gaps. The reduced gravity anomalies utilized in the AFRgeo2019 geoid model demonstrate favorable statistical behavior, particularly on land, as they are centered, smooth, and possess relatively small ranges. The computed model, complete up to degree and order 2190, has stabilized the interpolation process within the data gaps (Abd-Elmotaal, 2020). So, the assessment of AFRgeo19 will be done using the TPLN benchmarks available in the area of interest.

The Tanzania Primary Levelling Network (TPLN) was designed in the 1960 and implemented between 1961 and 1964 (Mayunga, 2016). It is comprised of 53 fundamental benchmarks (FBM) made up on loops based on local Mean Sea Level. The measurements were made on land in such a way that the misclosures between forward and back leveling between successive benchmarks is less than ± 3mm√k where 'k' is the leveled distance in kilometers. The distribution of the misclosures of the levelling data in the loops was done loop-wise after the completion of observations on each loop. At present the TPLN consist of eight (8) loops namely, loop A, B, C, D, E, F, G and H. The leveled orthometric heights in the TPLN are corrected for gravity effects on the basis of the normal gravity computed by means of the International Gravity Formula, 1930 (Deus, 2007). The establishment of the TPLN was referred to tide gauge measurements at the Tanga harbor whose mean sea level was used as a reference. The value for the mean sea level (MSL) at Tanga harbor was deduced from tide gauge readings taken during a 28 months' period from August 1962 to November 1964, both months inclusive. The MSL was used to determine the elevation of the Reference Fundamental Benchmark at Maweni. The other in land benchmarks were connected to the Maweni FBM through the observations of loop 'A' and the other loops of TPLN. Figure 1.1 shows the Tanzania Primary Levelling Network (TPLN) as in 1970's

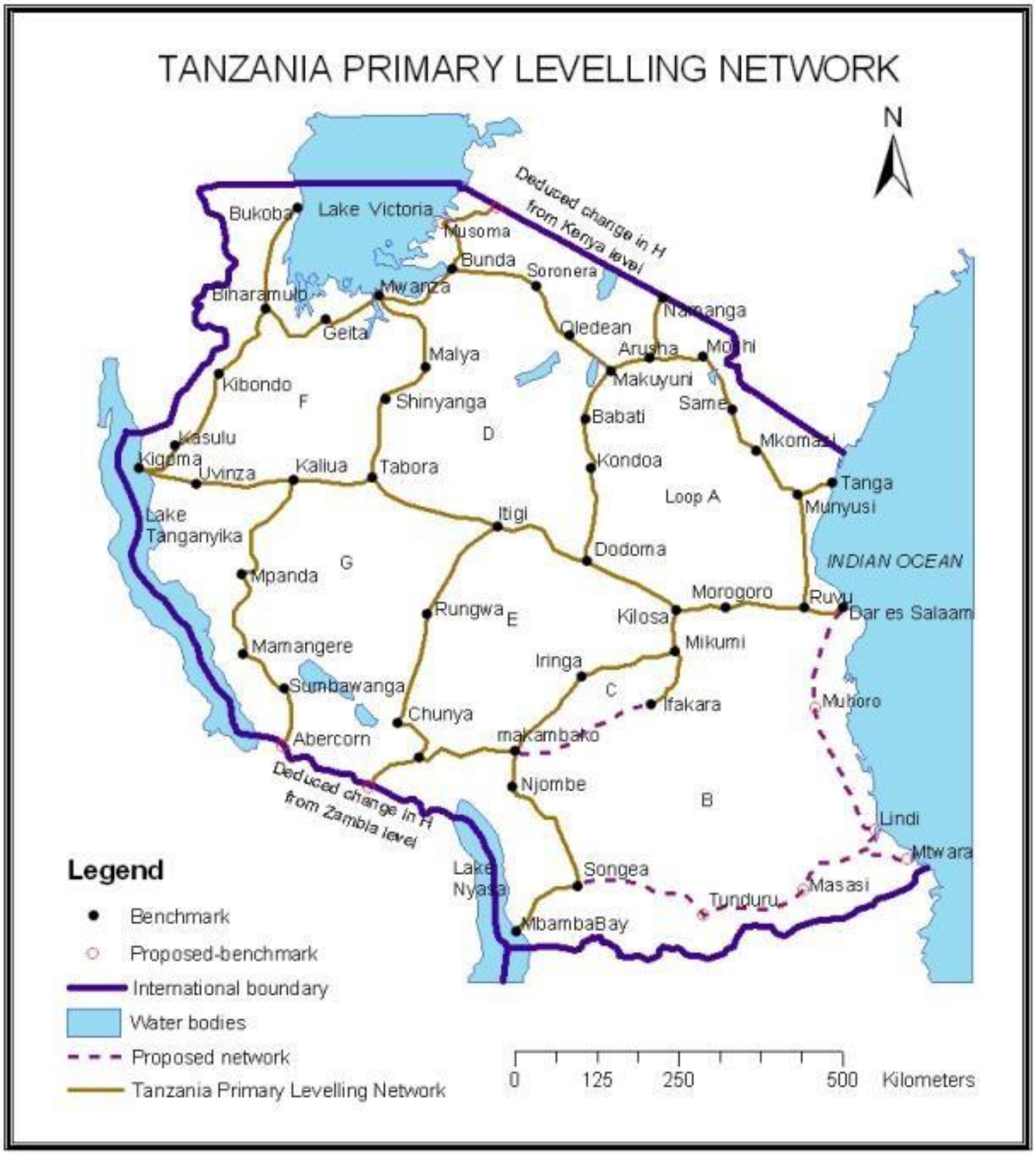

Figure 1.1: Tanzania Primary Levelling Network (TPLN) as in 1970's showing the finished loops, Fundamental Benchmarks (FBM), routes and FBMs proposed but not implemented (Ulotu,2015)

1.2 Statement of the Research Problem

Different geoid models have been evaluated in Tanzania using different methods and Tanzania is covered by various models such as EGM08 from global geoid models, AFRgeo19 from regional geoid models and we have local geoid models such as (TZG07, TZG08, TZG13, TZG17 and TZG19). Due to problems with the vertical datum like datum inconsistencies, we have to assess the accuracy of AFRgeo19 since the accuracy of all the other geoid models has been assessed in Tanzania. Thus, in this research the accuracy of AFRgeo19 will be validated in Tanzania using GPS/levelling method and it will be easy to define the most accurate geoid which will be used as the reference vertical datum in determination of orthometric heights.

1.3 Objectives

1.3.1 Main Objective

The main objective of this research is to assess the accuracy of Africa geoid model of 2019 (AFRgeo19) by using absolute and relative GPS/Levelling methods

1.4 Significance of the Research

i. This research will help to solve the problems facing the current local vertical datum including datum inconsistencies, incompatibility with the GNSS and other satellite technologies, so it is necessary to determine the accuracy of the Afrgeo19 that can be used as a reference vertical datum in determination of orthometric heights from GPS/levelling method.

ii. Due to problem facing current vertical datum for orthometric height in Tanzania, then by validating the AFRgeo19 using GPS/levelling data and comparing with the other geoid models validated previously, the best geoid model which has good performance in our country will be selected and used as the vertical datum in Tanzania.

iii. Determination of the accuracy of the Afrgeo19 will help to determine if we can use it as the reference vertical datum so that there would be a good relationship between geodetic measurement, geophysical measurement as well as oceanographic measurement in determination of the vertical position of points on the surface.

1.5 Beneficiaries

The beneficiaries of this research are the geodesist, engineers and surveyors who requires height information in their works.

1.6 Research Hypothesis

African geoid model (AFRgeo19) performs better in Tanzania than the other geoid models.

1.7 Description of the Study Area

This research is done in Tanzania mainland covering the geographical location from 1ºN to 12ºS latitude and 29ºE to 41ºN E longitude.

1.8 Scope and Limitation

Evaluation of geoid model (AFRgeo19) over the area of interest require GPS observation to be carried out over the TPLN benchmarks that are well distributed over the country. So, the study will be limited to some parts of Tanzania mainland using 29 existing TPLN benchmarks. These benchmarks will be used to evaluate the geoid model in Tanzania using absolute and relative GPS levelling methods.

1.9 Outline

This research consists of five chapters, in which the first chapter comprises of the Introduction of the study, statement of the research problem, objective of the research, significance of the research, beneficiaries, research hypothesis, description of the study area and scope and limitation. The second chapter comprises of literature review which provide the overview of the vertical datum, types of the VD (MSL VD and Geoid Model based VD) and the Geoid Model evaluation method. The third chapter comprises of the methodology which describe in detail on how the research problem is solved using GPS/levelling method and the data required for the validation of the geoid model. The fourth chapter provide in detail the results and discussion of the results. The fifth chapter which is the last chapter, provide the conclusion from the results and objective as well as the recommendations for the related future research.

CHAPTER TWO

LITERATURE REVIEW

2.1 OVERVIEW OF VERTICAL DATUM

By definition, Vertical Datum (VD) is referred as the reference surface of zero elevation relative to which heights or depths are referred to. The Mean Sea Level (MSL) has been used extensively for a long time as VD all over the World. It is realized by averaging sea water levels over a minimum period of 18.6 years at a Tide Gauge (TG) station in a coastal area (Ulotu, 2015).

For a long time, VD has been a MSL out of one or more fixed point(s) of tidal station(s). The time fixed MSL of a tidal station is referenced to a firmly monumented land point usually referred to as Tide Gauge Benchmark (TGBM). Sometimes more than one tidal station is involved in the establishment of a VD, if so, the TG levels are adjusted to give the same reading and the respective TGBMs are improved accordingly. Thereafter a network of Fundamental and Intermediate Benchmarks (FBMs and IBMs) is established on the land by geodetic levelling method based on the TGBM(s) reduced height(s), this way the MSL VD is realized on the land. MSL established from tide gauge(s) depart from the geoid by the Mean Dynamic Topography (MDT) and other coastal oceanographic effects (Ulotu, 2015). There exist different types of VD, some of them are;

i. MSL Vertical Datum
ii. Geoid Model Based Vertical Datum.

2.1.1 MSL Vertical Datum

MSL Vertical Datum is the ideal equipotential surface that could be obtained by fitting a level surface to observations of the mean level of the sea surface. It is obtained by the average reading of the tide gauge station recorded over a long period of time about 18.6 years. Establishment of a National (local) VD commenced in the 1960s. By then a model for the geoid of Tanzania did not exist. Consequently, the local mean sea level (LMSL) was established in place of the geoid surface. The LMSL was established using tide gauge measurements for 28 months only that was between August 1962 and November 1964 inclusive at the Tanga harbor. In Tanzania mainland, TG stations are found in Tanga, Dar es Salaam and Mtwara harbors. The fundamental reference benchmark is the Tanga TGBM. A dense network of ground datum markers; benchmarks (BM) of differing accuracies were monumented and referred to the Tanga TGBM from 1961 to 1969 (Ulotu,2015)

In Tanzania mainland, orthometric height is explained as the official height system, and thus its VD should be the geoid and not the MSL (Hofmann-Wallenhof, 2005). The use of MSL as a vertical datum that is equated with geoid is affected and limited by several factors such as absence of periodical updating, inability to take care of Mean Dynamic Topography (MDT) and some temporal and periodic effects.

If the observation duration does not meet the minimum requirement of 18.6 years, it displaces the MSL far from the geoid by several meters (Kileo, 2017) therefore, the heights derived from the established MSL cannot be orthometric even if they follow proper procedures in establishment.

Tanzania Vertical Datum has got some advantages as follows;

i. High relative accuracies can easily be achieved using precise levelling when complemented by gravity corrections,
ii. Levelling instrumentation is affordable, also data validation and processing are not complicated,
iii. Error tracing is easy, and formats and procedures are well established to minimize gross and systematic errors
iv. It can be improved to remove some of the datum observational and establishment deficiencies cited earlier, and
v. GPS levelling is a preferred method in the external quality assessment of a gravimetric geoid/quasi-geoid model as well as global gravity models (GGM).

The disadvantages of the current Tanzania Vertical Datum are such as;

i. The LMSL data was observed for 28 months only instead of at least 18.6 years, and this cannot remove short- and long-term sea level variations which are periodical as well as temporal.
ii. Mean dynamic sea surface topography (MDT) was not accounted for at the Tanga TG station, making it difficult for the VD to coincide with the geoid.
iii. During spirit levelling, heights of the benchmarks of the TPLN were corrected using normal gravity instead of the actual gravity, therefore the height system of Tanzania is normal orthometric and not orthometric height system.

iv. There has not been a uniform adjustment of the TPLN. Consequently, the network has led to a lot of inconsistencies, which cannot be explained adequately. Besides, the initial raw data cannot be traced and therefore it is not possible to adjust it rigorously (John & Deus 2009).
v. There is no clear documentation of how heights of trigonometric stations relate to the TPLN. In fact, it is known that some trigonometric stations existed well before TPLN. Moreover, it is not clear in which type of height system are the heights of trigonometric network
vi. If all the TG stations are to be unified, it does not mean that a better tidal VD would be obtained. In fact, the situation could be worse.
vii. Re-observation of the entire TPLN so that rigorous adjustment can be conducted, will be very expensive, and very likely not the most relevant option in this GNSS period.

2.1.2 Geoid Model Based Vertical Datum

Geoid is an equipotential surface of the Earth's gravity field that best fits the mean sea surface in a least squares sense, and this means that it is the VD for orthometric height system which is also the official height system in Tanzania (Ulotu, 2015). The quality of a geoid model depends much on the available gravity data; quality, density and distribution on the surface of the earth or on the geoid and methodology used. In determination of precise local or regional geoid model, a fully coverage of all types of data or information must be taken in an integrated solution, also its determination requires extensive gravitational measurements and computations. Geoid model is the best to be used as the vertical datum for the orthometric height system, however its surface is somehow irregular (Kamugisha, 2019). Some of the geoid-based VD are as explained below**.**

i. Gravimetric Geoid Model

Gravimetric geoid model is obtained from Gravimetric techniques which uses gravity measurement in its determination. Theoretically, gravimetric geoid model is a solution of Boundary Value Problems (BVP) to obtain the boundary value in the form of the geoid surface. Tanzania has coverage of different gravimetric geoid models which are EGM08 from global geoid models, AGP07 & AFRgeo2019 from regional models, TZG07, TZG08, TZG13, TZG17 and TZG 19 from local geoid models (Pavlis et al, 2008, Merry, 2007, Abd-Elmotaal,2020, Oliver, 2007, Ulotu, 2009, Forsberg et al, 2013, Peter, 2018 and Forsberg et al, 2019)

Earth Gravitational Model of 2008 (EGM08), this is the global geoid model complete to d/o 2159 plus some additional spherical harmonic coefficients up to degree 2190. EGM08 was developed by United state National Geospatial-Intelligence Agency (NGA) revealed a major achievement in global gravity field mapping. It used the data like, GRACE data (ITG-GRACE03S)5', high resolution global DTM (DTM2006.0)30", gravity anomalies from SA with 5'. The data provides long and medium wavelengths equivalent to about 18km. the precision of the model was about ±15cm worldwide (Pavlis et al, 2008)

African Geoid Model of 2019 (AFRgeo2019), The AFRgeo19 gravimetric geoid model was computed in the framework of the IAG African Geoid Project. The available gravity information comprises land and sea data, the latter consisting of shipborne point data and altimetry-derived gravity anomalies along tracks. The window remove restore technique was used to generate reduced anomalies having a minimum variance to minimize the interpolation errors especially at the large data gaps. The EIGEN-6C4 global model, complete to degree and order 2190, has served as the reference model. The geoid undulations from the reduced gravity anomalies were computed by employing Stokes' integral with meissl modified kernel. The restore step within the window remove-restore technique took place generating the fully gravimetric geoid. The computed geoid was scaled to the DIR_R5 GOCE satellite-only model by applying an offset and two tilt parameters (Abd-Elmotaal et al,2020).

Tanzania Gravimetric Geoid Model of 2013 (TZG13), was developed by Forsberg et al, (2013). The data used to determine this model were data from airborne survey supplemented with existing survey, marine gravity data, satellite gravity data (Altimetry, GRACE and GOCE) and updated digital terrain model that covers the land and ocean parts of Tanzania. It covers 13ºS to 1ºN Latitude and 28ºE to 43ºE Longitude which is the major part of Indian ocean offshore East of Tanzania. Computed on grid 0.02º × 0.02º resolution approximated to 2km × 2km grid. The accuracy of the model was 10cm (Forsberg et al, 2013).

Tanzania Gravimetric Geoid Model of 2017 (TZG17), was developed by Peter, (2018). The model was determined from 38,483 terrestrial points gravity data from Tanzania Gravity Database of 2008 (TGDB08), airborne gravity data limited to boundaries of Tanzania, validated pure GOCE GGM (GO_CONS_GCF_2_SPW_R5) to d/o 200 and composite GGM (XGM2016) to d/o 719, combination of four validated 1" and 3" global DEM: USA SRTM-1" v3, ALOS-1" v2, MERIT

3" and SRTM-3" CGAIR v4. The model was computed on grid of 1'×1' resolution corresponding to approximately 1.8km×1.8km. It covers 12ºS to 1ºN Latitude and 29ºE to 41ºE Longitude. The accuracy attained was 5cm (Peter, 2018).

Tanzania Gravimetric Geoid Model of 2019 (TZG19), developed by Forsberg et al, (2019). The data used to determine this model were data measured from airborne survey supplemented with existing survey, marine gravity data, satellite gravity data (Altimetry (DTU15), GRACE and GOCE) and updated digital terrain model that covers the land and ocean parts of Tanzania. Since it is an update of TZG13 then it used the new PGM17 model which is unpublished test version of the upcoming high resolution EGM20 model. PGM17 is the major improvement of the earlier used reference models and has Tanzania airborne gravity data, GOCE and GRACE satellite data based on XGM2016 model of pail et al. It has also an additional information of airborne gravity data of Mozambique and Malawi as well as the Northwest part of Tanzania. Its computation was based on the DTU-space GRAVISOFT geoid software. It covers 13ºS to 1ºN Latitude and 28ºE to 43ºE Longitude which is the major part of Indian ocean offshore East of Tanzania. This model was computed on the grid 0.02º×0.02ºresolution approximated to 2km×2km grid. Based on the collocation error estimates from other regions with data quality and coverage, the geoid error is estimated to be on the order of 10cm for most of the region, hence the attained accuracy of the model was 10cm (Forsberg et al, 2019)

ii. **Quasi-geoid Model as Vertical Datum**

Quasi-geoid is the reference surface for normal heights. Gradually adoption of the normal height instead of orthometric height system is gaining popularity in the world mainly due to its less stringent determination process and closeness of normal heights to orthometric heights. Quasi-geoid does not deviate much from the geoid; often it is less than 1m, but the surface has no physical meaning (Ulotu, 2015). Quasi-geoid determination is attributed to a Russian geodesist (Molodensky et al, 1962). The relationship between the geoid and the quasi-geoid, i.e. geoidal height N and height anomaly ζ is expressed in equation 2.1 below;

$$N = \zeta + \left(\Delta g - \frac{2\pi\rho H}{\gamma}\right) H \quad (2.1)$$

where:

N = Geoidal height,

ς = Height anomaly from Quasi-geoid model,

Δg = Bouguer gravity anomaly,

ρ = Constant density of topographical masses,

γ = Normal gravity on the telluroid; close to the surface,

H = Elevation of the surface point.

iii. Hybrid Geoid Model

When accurate and fairly well distributed benchmarks referred to appropriately determined TG based VD exist, together with precise gravimetric geoid model, a hybrid VD can be created out of the two using GPS observations on the BMs (Ulotu, 2015). Precise GPS observations on the BMs i.e. GPS/levelling enable determination of geometrical geoidal height as shown in equation 2.2 below,

$$N_{BM}^{GPS} = h - H \tag{2.2}$$

where,

N_{BM}^{GPS} is the geometrical geoidal height,

h is ellipsoidal height from precise GPS positioning and,

H is orthometric height based on the TG-VD.

From a gravimetric geoid model, we obtain N, that upon combination with N^{GPS}_{BM} over the respective region often in least squares manner, results into hybrid VD. When the above conditions are favorable, hybrid datum combines the advantages of both types of geoidal models, and that is often advantageous to the user (Ulotu, 2015).

iv. Geometric Geoid Model

This model uses ellipsoidal height determined from GPS observation together with orthometric height obtained from geodetic levelling to determine geometric geoidal height (N^{GPS}_{BM}). This can be achieved as shown in the equation (2.2).

Accuracy of the established geoid depends on the distribution and the number of GPS/levelling stations, accuracy of GPS/levelling data, characteristics of the geoid in the region and the methods of interpolation. Figure 2.1 shows the relationship between ellipsoidal height h, orthometric height H and geoidal height N.

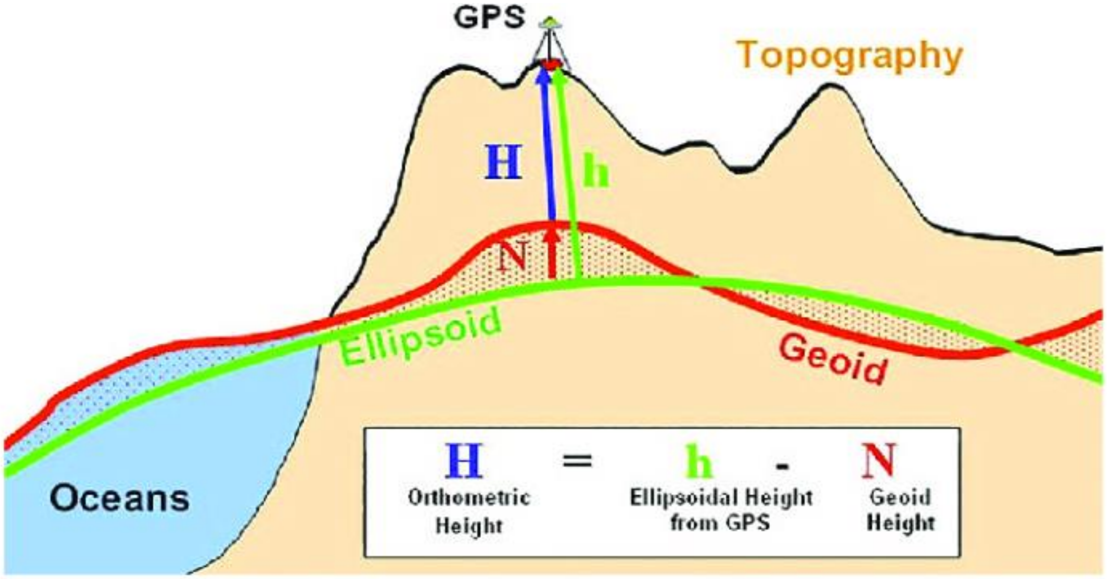

Figure 2.1: Relationship between geoid, ellipsoid and orthometric heights (Amjadiparvar,2015)

2.2 Review of Geoid Model Evaluation Methods

In valuation of geoid model different methods are employed such as GPS/levelling technique, gravity anomalies, astrogeodetic deflection, geoid slope and sea surface topography methods, as explained below.

2.2.1 GPS/levelling Method

This is the common method used in evaluation of geoid models**.** It involves ellipsoidal heights, h which is obtained from GPS data and orthometric heights, H which is obtained from spirit levelling. Geoid height, N is obtained by subtracting orthometric height from ellipsoidal height as given by equation 2.3

$$N_{GPS} = h - H \tag{2.3}$$

The N obtained above is used for evaluation of geoid model. GPS/ levelling method involves two approaches which are absolute approach and relative approach.

a. **Absolute GPS/Levelling Approach**

The absolute value of ellipsoidal height, h has to be known relative to appropriate reference ellipsoid and it can be obtained through GPS measurement by geodetically connecting to ITRF (Featherstone,2001). In this approach the evaluation of the geoid model is be done by subtracting orthometric height (H) from the ellipsoidal height (h) to points that covers the area of interest to obtain the geoid height N_{GPS} as in the equation 2.3 above.

Then the difference between the geometrical geoidal height N_{GPS} and the gravimetric geoid height N_G is obtained as;

$$\Delta N = N_{GPS} - N_G \tag{2.4}$$

Where; N_{GPS} is the geometrical geoidal height and N_G is the gravimetric geoid height

b. **Relative GPS Levelling Approach**

In relative method, the ellipsoidal height (h) and orthometric height (H) of the same point at their respective location must be known. The benefit in this method is that, any errors that are common to either end of the control baseline cancel each other on differencing (Featherstone, 2001). In this approach the evaluation of geoid models is be done by subtracting difference in orthometric height from ellipsoidal height difference to a number of points that cover the area of interest to get difference in geoid separation (geoid height) over the base line.

The geoid height difference for GPS is given by the equation (2.5) below;

$$\Delta N_{GPS} = (h_A - h_B) - (H_A - H_B) = \Delta h - \Delta H \tag{2.5}$$

Difference of geoid height between two selected benchmarks for geoid model is computed as (2.6) below;

$$\Delta N_G = N_A - N_B \tag{2.6}$$

Difference in geoid height between GPS data and geoid model is computed as in the equation (2.7) below;

$$\Delta N_{GPS-G} = \Delta N_{GPS} - \Delta N_G \tag{2.7}$$

Accuracy of gravimetric geoid model in relative sense between ΔN_G and ΔN_{GPS} is calculated as the mean difference over the mean baseline length in parts per million (ppm) using equation (2.8).

$$ppm = \left|\frac{\Delta N_{GPS-G}}{D}\right| \tag{2.8}$$

Where; D is the baseline distance in km

2.2.2 Astrogeodetic Deflection of the Vertical

This is among the methods of geoid evaluation that involves comparison of geoidal height differences between the Astrogeodetic geoidal heights obtained from the components of the deflection of the vertical and the geoid height from geoid model (Olliver, 2007). The geoid height difference can be computed as in the equation below;

$$N_A = N_B - (S_m \xi_m + S_P \eta_m) \tag{2.9}$$

Where; N_A and N_B are geoid heights at station A and B respectively,

ξ_m and η_m are mean meridional and prime deflection of the vertical at station A and B,

S_P and S_m meridional and parallel circles distances between station A and B.

The meridian and prime vertical astrogeodetic deflection of the vertical (ξm, η_m) are determined from the following equations (Hofmann-Wallenhof, 2005).

$$\varphi = \Phi - \xi \tag{2.10}$$

$$\lambda = (\Lambda - \eta)\, cos\varphi \tag{2.11}$$

Where; φ and λ are geodetic latitude and longitude values of a station referred to GRS80 respectively, Φ and Λ are astronomical latitude and longitude respectively.

The evaluation of geoid model is done through analysis of the magnitudes of the overall minimum, maximum, mean differences and standard deviation (SD) of the geoid heights differences from different geoid models and those between respective points along the astrogeodetic profile. The use of this method in geoid evaluation is more reasonable, when differences of geoid heights are compared instead of actual heights because it minimizes the systematic errors by differencing the geoidal heights along the astrogeodetic profile. The problem associated with this technique is that,

Φ and Λ are usually not available especially in Tanzania, they are very seldom to be determined nowadays on national control networks.

2.2.3 Gravity Anomalies Method

This method involves comparison between gravity anomalies at terrestrial data points and the geoid model derived gravity anomalies. The difference between gravity anomalies at terrestrial data point and the geoid derived gravity anomalies are expressed using equation (2.12) (Merry, 2007)

$$\delta_{\Delta g} = \Delta g^{T} (\varphi, \lambda) - \Delta g^{G} (\varphi, \lambda) \tag{2.12}$$

Where; $\delta_{\Delta g}$ is the difference in gravity anomalies at point (φ, λ),

$\Delta g^{T} (\varphi, \lambda)$ is the terrestrial gravity anomaly at point (φ, λ),

$\Delta g^{G} (\varphi, \lambda)$ is the geoid derived gravity anomaly at point (φ, λ).

The method was used by Merry, (2007) to evaluate EGM08 for Africa and it involved comparison between measured gravity anomalies of AGP07 computed in a 5' × 5' grid of gravity anomalies and EGM08 gravity anomalies. The standard deviation of 9.3mgal accuracy was attained

2.2.4 Geoid Slope Method

This method involves comparison of geoid slopes from GPS/levelling data and geoid slope from geoid models. It helps to determine how the geoid models perform over different distances. Depending on the actual baseline length, baselines are categorized into different distances from shortest to longest in Kilometers. The advantage of this method is that, in differencing of geoid slopes the errors related to the baseline cancel out.

Let two GPS stations A and B with the horizontal distance D_{AB} and slope distance S_{AB} between them. Evaluation of geoid model using this method involves the following procedures;

i. Difference in geoidal height (ΔN_{AB}) between two station A and B from gravimetric geoid models and GPS/levelling is determined using equation (2.13)

$$\Delta N = N_B - N_A \tag{2.13}$$

ii. Determining horizontal distance, D_{AB} between stations A and B using join computation.

iii. Geoid slopes for both geoid models and GPS points (S_{AB}) is computed using equation (2.14) below;

$$S_{AB} = \frac{\Delta N_{AB}}{D_{AB}} \tag{2.14}$$

iv. Determine slope difference between slope from geoid model (S_{AB}^{G}) and slope from GPS data (S_{AB}^{GPS}) using equation (2.15)

$$\Delta S = S_{AB}^{G} - S_{AB}^{GPS} \tag{2.15}$$

Statistical testing of ΔS is conducted and the one with smallest standard deviation is considered as the best geoid model over an area of interest.

The method was used by Simion, (2019) to evaluate EGM08, TZG13, TZG17 and TZG19 in Tanzania.

2.2.5 Sea Surface Topography Method

Over the ocean surfaces, geoid models are evaluated through their geoidal undulations and Mean Sea Surface Height (MSSH) derived from satellite Altimetry data. MSSH is obtained from the difference between satellite Altitude, h and their Altimetry range, R using equation (2.16) (Gruber, 2008)

$$MSSH = h - R \tag{2.16}$$

Where; MSSH = Mean Sea Surface Height derived from satellite altimetry data,

h = Satellite altitude above the reference ellipsoid (WGS84) and

R = An altimeter range above the sea surface.

Sea Surface Topography (SST) can be obtained by subtracting geoid height N, from SSH using equation (2.17) (Gruber,2008).

$$SST = MSSH - N \tag{2.17}$$

Sea Surface Topography solutions are inspected visually in order to check and assess how realistic are from the model depicts oceanographic features of water bodies (Grummer, 2008). SST is used to study oceans tides, circulation and amount of heat that the ocean holds. The observations are used to predict short term changes in weather and long-term climate pattern, also are used to map the oceanographic features such as oceanic currents and temperature variation.

From the geoid evaluation methods, GPS/levelling method will be used in this research to evaluate the 2019 Africa Geoid Model (AFRgeo19).

CHAPTER THREE

METHODOLOGY

This chapter explains in details the methods used in evaluation of the geoid model, procedures and data description. In this thesis both absolute and relative GPS/levelling methods are used to validate AFRgeo19 model.

3.1 Evaluation of Geoid Model using GPS/levelling Method

This involves ellipsoidal heights, h which will be obtained from GPS data and orthometric heights, H which will be obtained from spirit levelling. Geoid height, N is obtained using the equation (2.3).

The N obtained will be used for evaluation of geoid model. GPS/ levelling method involves two approaches which are absolute approach and relative approach and both methods will be implemented in this research.

i. **Absolute GPS Levelling Approach**

In this approach the evaluation of the geoid model is be done by subtracting orthometric height, H from the ellipsoidal height, h to points that covers the area of interest to obtain the geoid height, $N_{GPS.}$ Then ΔN is obtained as the difference between the geometrical geoidal height, N_{GPS} and the gravimetric geoid height, N_G as shown in equation (2.4). The values of ΔN are used in statistical testing.

ii. **Relative GPS Levelling Approach**

In relative approach the evaluation of geoid models is done by subtracting difference in orthometric height from ellipsoidal height difference to a number of points that cover the area of interest to get difference in geoid separation (geoid height) over the base line. The geoid height difference for GPS will be given as in the Equation (2.5) . Difference between two selected benchmarks for the geoid model is obtained as in the Equation (2.6). Difference in geoidal height between GPS data and geoid model is computed as in Equation (2.7).

3.1.1 Statistical Testing of the Data

Statistical testing of computed difference in geoidal height is done by using normal distribution under 95% confidence level. Statistical testing helps to eliminate outliers and blunders in data sets.

The measurement outside the selected high percentage range are rejected using equation 3.0 below. (Crawshaw & Chamber, 2001)

$$P(-1.96\sigma \leq \mu \leq 1.96\sigma) \ = \ 0.95 \qquad (3.1)$$

Where; σ = Standard deviation and

μ= Point estimate of the population

3.1.2 Modelling Systematic Errors in Absolute Approach

The presence of random error and systematic error in ΔN values causes equation (2.4) in section 2 to not function well. These errors are due to datum inconsistences, long wave systematic errors, distortion in orthometric height datum due to over constrained adjustment of the levelling network and result of various geodynamic effects. In order to reduce the effect of systematic errors when evaluating geoid models then systematic parameters models are tested in order to fit the geoid to a set of GPS/levelling points through Least Square Adjustment (LSA). Modelling of the differences between geometrical geoid height and gravimetric geoid height enables fulfillment of the relation below (Ntambila, 2012)

$$\Delta N = N^{GPS} \ - \ N^{G} = h_i - H_I - N_I = a_i^T x + \varepsilon_i \qquad (3.2)$$

Where; x is the vector of unknown parameters, a_i is the vector of unknown coefficients and ε_i is the residual random noise term.

The parametric model ($a_i^T X$) is supposed to describe the systemic errors and datum inconsistence inherent indifferent height datasets. Parametric model varies in form of and complexity depend on number of factors. Basing on this study, 3-,4-,5-,7- parametric models are tested. The model equations are as below (Ulotu, 2009);

3-parametric model equation is given by,

$$a_i x = (cos\varphi_i cos\lambda_i)x_1 + (cos\varphi_i sin\lambda_i)x_2 + (sin\varphi_i)x_3 \qquad (3.3)$$

4-parametric model equation is given by,

$$a_i x = (cos\varphi_i cos\lambda_i)x_1 + (cos\varphi_i sin\lambda_i)x_2 + (sin\varphi_i)x_3 + x_4 \qquad (3.4)$$

5-parametric model equation is given by,

$$a_i x = (cos\varphi_i cos\lambda_i)x_1 + (cos\varphi_i sin\lambda_i)x_2 + (sin\varphi_i)x_3 + (sin^2\varphi_i)x_4 + x_5 \quad (3.5)$$

7-parametric model equation is given by,

$$a_i x = (cos\varphi_i cos\lambda_i)x_1 + (cos\varphi_i sin\lambda_i)x_2 + (sin\varphi_i)x_3 + \left(\frac{cos\varphi_i\, sin\varphi_i cos\lambda_i}{W_i}\right)x_4 + \left(\frac{cos\varphi_i\, sin\varphi_i sin\lambda_i}{W_i}\right)x_5 + \left(\frac{sin^2\varphi_i}{W_i}\right)x_6 + x_7 \quad (3.6)$$

But;

$$W_i = \sqrt{(1 - e^2 sin^2\varphi_i)}$$

Where; φ *and* λ are horizontal geodetic network coordinates and e is the first eccentricity of the reference ellipsoid.

The equations above can be expressed in matrix form as;

$$AX = \Delta N - v \quad (3.7)$$

Where; A is the design matrix, ΔN is the misclosure vector between gravimetric geoidal heights and the geometrical geoidal heights and v is the residual vector. The equation (3.6) above can be evaluated using Least square method basing on least square criterion $v * v = minimum$. Least Square method gives the best estimates for the parameters ($\hat{X}$), residuals ($\hat{v}$) and variance-covariance matrices of the estimated parameters $C_{\hat{X}}$ (Ulotu,2009).

$$\hat{X} = (A^T A)^{-1} A^T \Delta N \quad (3.8)$$

$$\hat{v} = (I - A(A^T A)^{-1} A^T)\Delta N \quad (3.9)$$

$$C_{\hat{X}} = \hat{\sigma}_O^2 (A^T A)^{-1} \quad (3.10)$$

Where; I is an identity matrix

The adjusted residual vector $\hat{v}$ should show the level of absolute agreement between gravimetric and GPS levelling geoid model, the disagreement between the geoid models is a function of errors that involved in height systems, i.e. errors of gravimetric geoid model, GPS heights and orthometric heights including levelling. The fit between the three systems is expressed by a posterior unit weight variance factor $\hat{\sigma}_O^2$ computed from (Ulotu, 2009).

$$\hat{\sigma}_O^2 = \frac{\hat{v}^T \hat{v}}{n-m} \tag{3.11}$$

Where n is the number of known points (GPS points) and m is the number of estimated parameters (unknown parameters).

Statistical analysis of the residuals obtained from equation (3.8) are done for the particular parametric model and the one with smallest SD is selected as the best parametric model for modeling systematic errors.

3.2 Data Required for Validation of the Geoid Model

Different types of data are required for the validation of the model such as ellipsoidal heights and orthometric heights of the TPLN benchmarks.

3.2.1 Ellipsoidal Heights of TPLN Benchmarks from GPS Observation

A total of 29 ellipsoidal heights are used to evaluate the geoid model. Ellipsoidal heights of 29 benchmarks were made available in this research by previous researcher Urassa, (2020).

3.2.2 Orthometric Heights of TPLN Benchmarks from Spirit Levelling

The orthometric heights for the available 29 benchmarks used in this research were made available by the previous researcher Urassa, (2020). Table 3.1 shows the longitude, latitude, ellipsoidal height and orthometric heights of 29 benchmarks used for evaluation of geoid model.

Table 3. 1 Ellipsoidal heights and orthometric heights of benchmarks

POINT NAME	λ (d.ddd)	φ (d.ddd)	h (m)	H (m)
FBM+B2:H33 TABORA	32.820	-5.032	1218.728	1236.751
FBM MWANZA	32.898	-2.523	1122.052	1139.667
FBM SHINYANGA	33.434	-3.670	1103.618	1123.194
FBM MBEYA	33.438	-8.886	1770.901	1782.839
FBM KIOMBONI	34.409	-4.281	1575.412	1595.003
IRINGA	35.302	-8.300	1823.027	1836.387
FBM IRINGA	35.699	-7.794	1532.884	1549.545

IBM5/47 DODOMA	35.746	-6.184	1110.378	1130.975
FBM KONDOA	35.809	-4.902	1373.447	1391.263
FBM MAKUYUNI	36.097	-3.553	1051.145	1070.213
IBM3/54 KILOSA	36.986	-6.832	469.000	490.330
IBMA25/1 MOSHI	37.309	-3.367	780.118	798.010
IBMA24/50 MOSHI	37.309	-3.382	775.354	793.346
IBMA24/51 MOSHI	37.323	-3.379	787.442	806.156
FBM MOSHI	37.343	-3.356	792.215	809.793
IBM KIBIRASHI	37.437	-5.382	1110.109	1129.579
IBM MOROGORO2/77	37.716	-6.804	471.976	493.729
IBM KOROGWE	38.461	-5.166	276.375	300.032
FBM KWALA	38.579	-6.798	56.344	79.983
FBM MNYUSI	38.619	-5.252	220.654	243.999
IBM WAMI	38.712	-6.212	-8.058	14.442
FBM MAWENI	39.014	-5.120	36.853	63.237
IBM TANGA A/29/7	39.099	-5.074	-7.203	19.646
IBM18 PUGU	39.126	-6.882	76.688	103.670
FBM DAR	39.285	-6.778	-15.510	12.239
FBM BABATI	35.749	-4.207	1320.931	1338.874
MWANZA	32.897	-2.523	1122.480	1139.667
TABORA	32.820	-5.032	1218.335	1236.751
TGBM	39.107	-5.041	-23.819	3.317

3.2.3 Geoid Heights for TPLN Benchmarks from the Geoid Model AFRgeo19

The geoid heights for TPLN benchmarks were extracted from the gridded AFRgeo19 model.

3.3 Devices and Software's Used

The software's used in this study are Golden suffer software, MATLAB and Microsoft Excel and Microsoft Word. Golden suffer version 15 software is used to interpolate gravimetric geoid height from geoid models. MATLAB software is used to compute values of different variables including adjusted parameters and residuals when modelling systematic errors in the data sets. Microsoft Excel spread sheet is used in different calculations of the given datasets and Microsoft Word is used for report writing.

CHAPTER FOUR

RESULTS AND DISCUSSION

This chapter mainly deals with the results obtained in this research from both methods used. Also, it involves critical discussion of the results from absolute and relative approach, and comparison of the best method for the assessment of the model.

4.1 RESULTS

Basing on methodology described in chapter three, the following are the results obtained and will be mainly represented in Tables.

4.1.1 Extracted Gravimetric Geoid heights from Geoid Model

Figure 4.1 shows the gridded AFRgeo19 model where the geoidal heights were extracted.

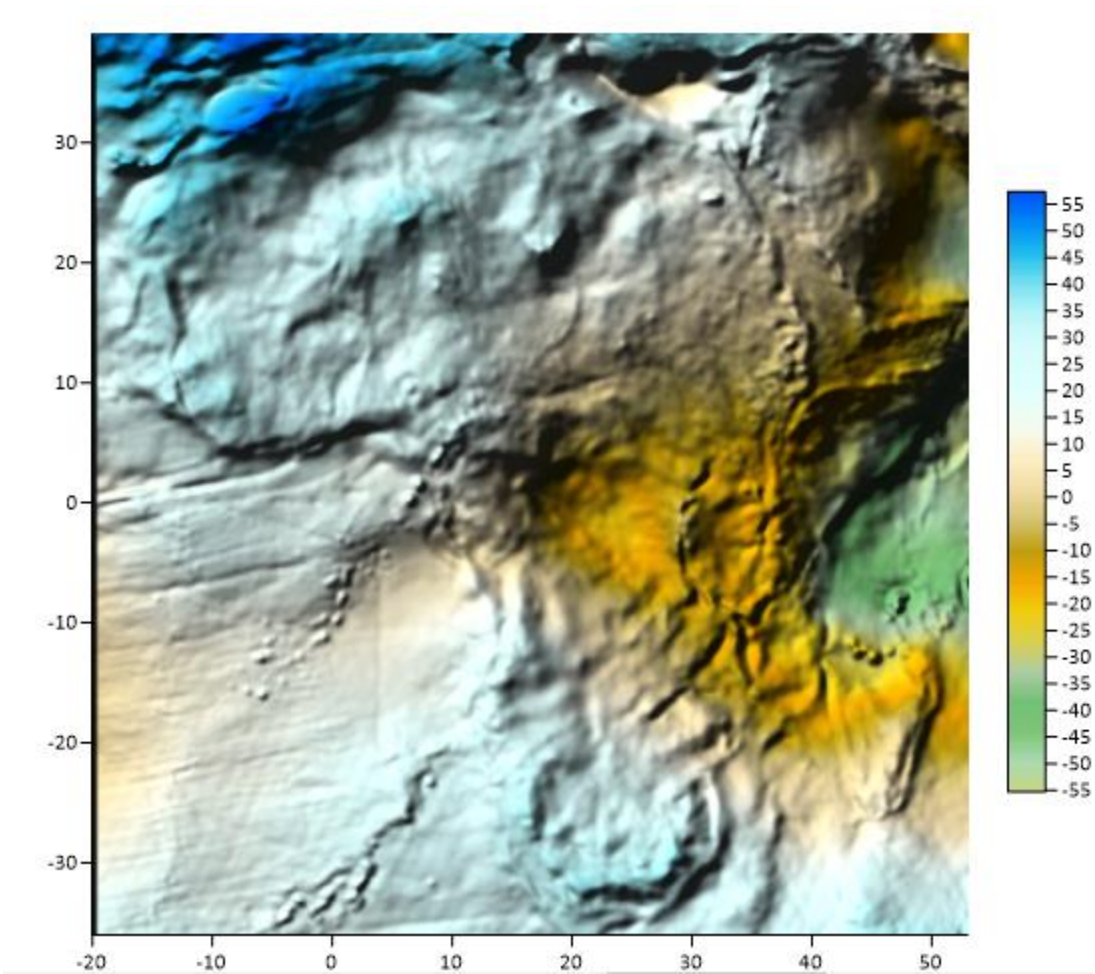

Figure 4.1: The gridded AFRgeo19 model.

Gravimetric geoid heights from AFRgeo19 were extracted from the gridded model and tabulated in Table 4.1

Table 4. 1 Extracted Gravimetric Geoid Heights from AFRgeo19

POINT NAME	λ (d.ddd)	φ (d.ddd)	$N^{AFRgeo19}$ (m)
FBM+B2:H33 TABORA	32.820	-5.032	-17.515
FBM MWANZA	32.898	-2.523	-16.445
FBM SHINYANGA	33.434	-3.670	-18.686
FBM MBEYA	33.438	-8.886	-11.808
FBM KIOMBONI	34.409	-4.281	-18.823
IRINGA	35.302	-8.300	-14.535
FBM IRINGA	35.699	-7.794	-16.018
IBM5/47 DODOMA	35.749	-6.184	-19.315
FBM KONDOA	35.809	-4.902	-18.223
FBM MAKUYUNI	36.097	-3.553	-18.393
IBM3/54 KILOSA	36.986	-6.832	-20.464
IBMA25/1 MOSHI	37.309	-3.367	-17.903
IBMA24/50 MOSHI	37.309	-3.382	-18.026
IBMA24/51 MOSHI	37.323	-3.379	-18.017
FBM MOSHI	37.343	-3.356	-17.821
IBM KIBIRASHI	37.437	-5.382	-19.089
IBM MOROGORO2/77	37.716	-6.804	-22.332
IBM KOROGWE	38.461	-5.166	-23.164
FBM KWALA	38.579	-6.798	-25.788
FBM MNYUSI	38.619	-5.252	-24.056
IBM WAMI	38.712	-6.212	-26.433
FBM MAWENI	39.014	-5.120	-26.476
IBM TANGA A/29/7	39.099	-5.074	-27.131
IBM18 PUGU	39.126	-6.882	-26.969
FBM DAR	39.285	-6.778	-27.848
FBM BABATI	35.749	-4.207	-17.984

MWANZA	32.897	-2.523	-16.445
TABORA	32.820	-5.032	-17.515
TGBM	39.107	-5.041	-27.094

4.1.2 Statistical testing of GPS Levelling points

The difference ΔN between geometrical geoid height N^{GPS} and gravimetric geoid height N^{G} were computed using equation (2.4) and tabulated in Table 4.2 below

Table 4. 2 Geoid Height Difference (ΔN)

POINT NAME	λ (d.ddd)	φ (d.ddd)	N^{GPS} (m)	$N^{AFRgeo19}$ (m)	ΔN
FBM+B2:H33 TABORA	32.820	-5.032	-18.023	-17.515	-0.508
FBM MWANZA	32.898	-2.523	-17.615	-16.445	-1.170
FBM SHINYANGA	33.434	-3.670	-19.576	-18.686	-0.890
FBM MBEYA	33.438	-8.886	-11.938	-11.808	-0.130
FBM KIOMBONI	34.409	-4.281	-19.591	-18.823	-0.768
IRINGA	35.302	-8.300	-13.360	-14.535	1.175
FBM IRINGA	35.699	-7.794	-16.661	-16.018	-0.643
IBM5/47 DODOMA	35.749	-6.184	-19.112	-19.315	0.203
FBM KONDOA	35.809	-4.902	-17.816	-18.223	0.407
FBM MAKUYUNI	36.097	-3.553	-19.068	-18.393	-0.675
IBM3/54 KILOSA	36.986	-6.832	-21.330	-20.464	-0.866
IBMA25/1 MOSHI	37.309	-3.367	-17.892	-17.903	0.011
IBMA24/50 MOSHI	37.309	-3.382	-17.992	-18.026	0.034
IBMA24/51 MOSHI	37.323	-3.379	-18.714	-18.017	-0.697
FBM MOSHI	37.343	-3.356	-17.578	-17.821	0.243
IBM KIBIRASHI	37.437	-5.382	-19.470	-19.089	-0.381
IBM MOROGORO2/77	37.716	-6.804	-21.753	-22.332	0.579
IBM KOROGWE	38.461	-5.166	-23.657	-23.164	-0.493
FBM KWALA	38.579	-6.798	-26.122	-25.788	-0.334
FBM MNYUSI	38.619	-5.252	-23.553	-24.056	0.503
IBM WAMI	38.712	-6.212	-22.500	-26.433	3.933

FBM MAWENI	39.014	-5.120	-26.797	-26.476	-0.321
IBM TANGA A/29/7	39.099	-5.074	-26.849	-27.131	0.282
IBM18 PUGU	39.126	-6.882	-26.982	-26.969	-0.013
FBM DAR	39.285	-6.778	-27.749	-27.848	0.099
FBM BABATI	35.749	-4.207	-17.943	-17.984	0.041
MWANZA	32.897	-2.523	-17.187	-16.445	-0.742
TABORA	32.820	-5.032	-18.416	-17.515	-0.901
TGBM	39.107	-5.041	-27.136	-27.094	-0.042

ΔN from the Table (4.2) are incorporated with outliers, random errors and systematic errors. So, the outliers should be removed and selecting data with less probability of having random and systematic errors by using the normal distribution under 95% confidence level. Out of 29 GPS levelling points, 7 benchmarks were qualified for the evaluation of the model as shown in the Table 4.3.

Table 4. 3 Selected Benchmarks for Evaluating AFRgeo19

POINT NAME	λ (d.ddd)	φ (d.ddd)	N^{GPS} (m)	$N^{AFRgeo19}$ (m)	ΔN (m)
IBM5/47 DODOMA	35.749	-6.184	-19.112	-19.315	0.203
FBM MOSHI	37.343	-3.356	-17.578	-17.821	0.243
IBM TANGA A/29/7	39.099	-5.074	-26.849	-27.131	0.282
FBM DAR	39.285	-6.778	-27.749	-27.848	0.099
FBM BABATI	35.749	-4.207	-17.943	-17.984	0.041
IBM18 PUGU	39.126	-6.882	-26.982	-26.969	-0.013
FBM MBEYA	33.438	-8.886	-11.938	-11.808	-0.130

4.1.3 Evaluation of the Model Using Absolute GPS/levelling before accounting for Systematic errors

After the removal of outliers from the geoid height difference (ΔN) then statistical analysis of the remaining ΔN values were done and the results were tabulated in Table (4.4).

Table 4. 4: Statics of geoid heights difference

	ΔN
Minimum	-0.1302
Maximum	0.2849
Mean	0.1043
Standard deviation	0.1341

From Table 4.4, the model produces the Standard deviation (SD) value of 0.1341 before accounting of systematic effects.

4.1.4 Modelling of the Systematic Effects

Modelling is done to remove systematic errors on the selected benchmarks from Table 4.3. The parametric model ($a_i^T x$) was used to estimate errors as in the equation (3.1) The values of the adjusted parameters ($\hat{X}$) as obtained from Eq. (3.7) were tabulated in Table 4.5

Table 4. 5 Adjusted Parameters for AFRgeo19 Geoid Model

MODEL	3-PARAMETER		4-PARAMETER		5-PARAMETER		7-PARAMETER	
Adjusted parameter	Value (m)	SD ±(m)	Value (m)	SD ±(m)	Value (m)	SD ±(m)	Value (m)	SD ±(m)
X_1	-0.866	0.794	64.538	59.088	-78.494	147.326	-4691.588	3290.691
X_2	1.481	0.976	49.557	43.440	-59.490	111.696	-3524.307	2482.993
X_3	1.181	1.327	-8.086	8.474	-16.725	11.716	38998.525	26726.372
X_4			-81.590	73.704	-148.918	140.738	-30790.965	21043.460
X_5					97.933	184.752	-22929.253	15765.136
X_6							3096.184	2157.596
X_7							5886.715	4135.870

The statistics of the residuals ($\hat{v}$) of the tested parameters as obtained in equation (3.8) were tabulated in Table 4.6.

Table 4. 6 Statistics of the Residuals for AFRgeo19 Model

ATTRIBUTE	3-PARAMETER (m)	4-PARAMETER (m)	5-PARAMETER (m)	7-PARAMETER (m)
Minimum	-0.1329	-0.1443	-0.1224	-0.0840
Maximum	0.1687	0.1368	0.1623	0.1172
Mean	0.0000	0.0000	0.0000	0.0000
RMS	0.1932	0.1518	0.1624	0.0616
Standard deviation	0.1115	0.1029	0.0945	0.0646

From Table 4.6 above, results from 7-parameter model provide best fitting residual with minimum value of standard deviation compared to 3-, 4-, 5- and 7-parameter. Despite the results of 7-parameter being better but due to small size of the area with GPS/Levelling data, compared to the AFRgeo19 geoid model, low distribution coverage and small density, the standard deviations of 7-parameter model results are out of range. Therefore, 5-parameter model was qualified to model the systematic effects inherited from all the three height systems (ellipsoidal height, orthometric height and geoid height). The statistical analysis of the residual after accounting systematic error by 5-parameter model were shown in Table 4.7 below.

Table 4. 7 Statistics of the Residual of AFRgeo19 Model after accounting for Systematic Errors by 5-Parameter Model

Attribute	AFRgeo19 (m)
Minimum	-0.1224
Maximum	0.1623
Mean	0.0000
RMS	0.1624
Standard deviation	0.0945

From Table 4.7, the model produces the SD value of 0.0945 and the RMS of 0.1624 after accounting for systematic effects.

4.1.5 Accuracy Estimation Gravimetric of Geoid Model

This is used to estimate the accuracy of the model after modelling systematic effect from the misclosure vector ΔN by the suitable parametric model. The accuracy estimates of the gravimetric geoid model can be propagated from the combined system accuracy σ_C^2 (Peter,2018), as in equation 4.1.

$$\sigma_N = \sqrt{\sigma_c^2 - \sigma_h^2 - \sigma_H^2} \tag{4.1}$$

The accuracy estimate of the geoid model is very much dependent on the ability to assign weights to the observations or systems involved and also on how far the systematic effects have been removed. In turn, the reliability of systematic models to correctly account for the effects, is not only or always a function of the model type and the number of parameters involved, but also depends much on the GPS/Levelling network accuracy, distribution, coverage and density.

Upon the removal of systematic effects from the misclosure vector by 5-parameter model, the accuracy of AFRgeo19 was 9.15cm. Since the accuracy obtained incorporates residuals from all the height systems then the removal of the effect contributed by other two height systems (ellipsoidal and orthometric heights) was done using the standard error estimates given by equation (4.1). The adjustment carried out during the GPS observations on benchmarks for GPS levelling data in 2008 gave the accuracy of ellipsoidal height as $\sigma_h = 0.028m$ (Ulotu,2009). Assessment of the achievable accuracy of the TPLN using its difference with Satellite altimetry mean sea surface established for over 19 years gave the accuracy of orthometric height as $\sigma_H = 0.012m$ (Kileo,2017).

With the above information of the estimates of the standard deviation of the ellipsoidal and orthometric heights, the accuracy estimate of AFRgeo19 is 8.63

4.1.6 Relative GPS levelling Method

In this method the difference in ellipsoidal height and the difference in orthometric height of the points at their location must be known according to equation (2.5). Geometric geoid heights and gravimetric geoid heights difference between two benchmarks were computed using equations

(2.5) and (2.6). Then, the difference (ΔN) between GPS and geoid model as in equation (2.7) were tabulated in Table (4.8).

Table 4. 8 The Difference (ΔN) between GPS and AFRgeo19 Geoid Model

POINT NAME	Δh (m)	ΔH (m)	ΔN^{GPS} (m)	$\Delta N^{AFRgeo19}$ (m)	$\Delta N^{GPS} - \Delta N^{AFRgeo19}$ (m)
FBM MBEYA					
	-52.126	-53.548	1.422	2.7277	-1.3057
IRINGA					
	290.143	286.842	3.301	1.4826	1.8184
FBM IRINGA					
	420.069	417.618	2.451	3.297	-0.846
IBM5/47 DODOMA					
	-260.88	-260.739	-0.141	-1.0923	0.9513
FBM KONDOA					
	904.695	902.336	2.359	2.2412	0.1178
IBM3/54 KILOSA					
	-2.976	-3.399	0.423	1.8682	-1.4452
IBM MOROGORO2/77					
	415.632	413.746	1.886	3.4562	-1.5702
FBM KWALA					
	64.402	65.541	-1.139	0.6451	-1.7841
IBM WAMI					
	-84.746	-89.228	4.482	0.5351	3.9469
IBM18 PUGU					
	93.397	-89.228	0.24	0.8789	-0.6389
FBM DAR					
	-1126.818	-1119.066	-7.752	-8.7582	1.0062
IBM KIBIRASHI					
	889.663	885.58	4.083	4.9664	-0.8834

FBM MNYUSI					
	227.649	224.353	3.296	3.0756	0.2204
IBM TANGA A/29/7					
	-283.578	-280.386	-3.192	-3.9674	0.7754
IBM KOROGWE					
	300.194	296.715	3.479	3.9302	-0.4512
TGBM					
	-60.672	-59.92	-0.752	-0.6176	-0.1344
FBM MAWENI					
	-743.265	-734.773	-8.492	-8.5732	0.0812
IBMA25/1 MOSHI					
	4.764	4.664	0.1	0.1228	-0.0228
IBMA24/50 MOSHI					
	-12.088	-12.068	-0.994	0.0086	-1.0026
IBMA24/51 MOSHI					
	-4.773	-4.379	0.58	-0.1966	0.7766
FBM MOSHI					
	-258.93	-260.42	1.49	0.5722	0.9178
FBM MAKUYUNI					
	-269.786	-268.661	-1.125	-0.4094	-0.7156
FBM BABATI					
	-254.481	-256.129	1.648	0.8396	0.8084
FBM KIOMBONI					
	1538.559	1531.766	6.793	7.6532	-0.8602
FBM MAWENI					
	-1066.765	-1059.957	-6.808	-7.7908	0.9828
FBM SHINYANGA					
	-18.434	-16.473	-1.961	-2.2403	0.2793
FBM MWANZA					
	-96.283	-97.084	0.801	1.0693	-2.2683

TABORA					

The relative difference between geoid heights from GPS and from the model in parts per million (ppm) as given by equation (2.8) is expressed in Table 4.9 from the agreement between the relative values of the gravimetric and GPS levelling derived geoid heights.

Table 4. 9 The Accuracy of the AFRgeo19 Model in Relative sense between Gravimetric Geoid Height and the derived Geoid Height from GPS levelling points.

POINT NAME	$\Delta N^{GPS} - \Delta N^{AFRgeo19}$ (m)	DISTANCE (km)	AFRgeo19 (ppm)
FBM MBEYA			
	-1.3057	215.1	0.0061
IRINGA			
	1.8184	71.16	0.0256
FBM IRINGA			
	-0.846	179.13	0.0047
IBM5/47 DODOMA			
	0.9513	142.74	0.0067
FBM KONDOA			
	0.1178	251	0.0005
IBM3/54 KILOSA			
	-1.4452	80.68	0.0179
IBM MOROGORO2/77			
	-1.5702	95.22	0.0165
FBM KWALA			
	-1.7841	66.83	0.0267
IBM WAMI			
	3.9469	87.42	0.0451
IBM18 PUGU			
	-0.6389	21.006	0.0304

FBM DAR			
	1.0062	256.7	0.0039
IBM KIBIRASHI			
	-0.8834	131.64	0.0067
FBM MNYUSI			
	0.2204	56.74	0.0039
IBM TANGA A/29/7			
	0.7754	71.42	0.0109
IBM KOROGWE			
	-0.4512	72.97	0.0062
TGBM			
	-0.1344	13.601	0.0099
FBM MAWENI			
	0.0812	271.5	0.0003
IBMA25/1 MOSHI			
	-0.0228	1.579	0.0144
IBMA24/50 MOSHI			
	-1.0026	1.5968	0.6279
IBMA24/51 MOSHI			
	0.7766	3.4674	0.2240
FBM MOSHI			
	0.9178	140.08	0.0066
FBM MAKUYUNI			
	-0.7156	82.22	0.0087
FBM BABATI			
	0.8084	148.9	0.0054
FBM KIOMBONI			
	-0.8602	518.8	0.0017
FBM MAWENI			
	0.9828	639.3	0.0015

FBM SHINYANGA			
	0.2793	140.8	0.0020
FBM MWANZA			
	-0.2683	279.1	0.0010
TABORA			

Statistical analysis of the relative values of the gravimetric and geometric derived geoid height (ppm) were computed and tabulated in Table 4.10.

Table 4. 10 Statistics for Relative values of the Gravimetric and Geometric derived Geoid Height

Attribute	AFRgeo19 (ppm)
Minimum	0.0003
Maximum	0.6279
Mean	0.0413
RMS	0.1291
Standard deviation	0.1247

4.2 DISCUSSION

This part discusses the results obtained from computations of the data and the methodology described in chapter three. The results obtained from absolute and relative GPS levelling methods are discussed below;

4.2.1 Results from Absolute GPS/Levelling Method

Before accounting for systematic effects in ΔN which was computed as the difference between gravimetric geoidal height and geometric geoidal height, the statistical analysis of ΔN for AFRgeo19 was 13.41cm SD and 12.91cm RMS.

Upon the removal of the systematic effects from the misclosure vector by the 5-parameter model the accuracy estimate for AFRgeo19 was 9.45cm.

After the removal of systematic effects contributed by the other two systems of height (ellipsoidal height and orthometric height) using the standard error estimates the accuracy of the geoid model improved from 9.45cm to 8.63cm. As the standard deviation decrease upon the removal of systematic errors, it shows that there is agreement of AFRgeo19 geoid model with GPS/levelling data

4.2.2 Results from Relative GPS/levelling Method

Statistical analysis of the relative values between geometric and gravimetric derived geoidal height in ppm shows that AFRgeo19 has the standard deviation (SD) of 0.1247ppm and the RMS of 0.1291ppm. In this the systematic errors present in the model are cancelled upon differencing unlike in absolute method.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

The main objective of this study was to validate the African Geoid Model of 2019 using absolute and relative GPS/levelling methods in Tanzania and was successfully achieved. The validation was done using 29 TPLN benchmarks that were distributed over the area of interest, only 7 benchmarks were qualified for the validation of the model. From the results obtained in Chapter Four and the objective of the study, the following conclusion can be made;

- AFRgeo19 has the accuracy of 8.63cm by using Absolute GPS/levelling method and the accuracy of 0.1247ppm by relative GPS/levelling method.
- Upon comparing the evaluation of the current geoid model of Tanzania which is TZG19 from previous researcher by using GPS/levelling method, TZG19 had the accuracy of 6.7cm (Urassa,2020).

Therefore, TZG19 still qualify to be used as the National geoid model for Tanzania since it is more accurate than the AFRgeo19.

5.2 RECOMMENDATION

Basing on the results and conclusion from this research, several areas has been identified for future work in order to improve the evaluation of AFRgeo19 model and using GPS Levelling in Tanzania. Thus, the following recommendations are made for this research as listed below;

- To use more benchmarks covering the rest of the Tanzania Primary Levelling Network (TPLN) particularly the western part of Tanzania.
- To collect GPS/levelling data for the rest of FBMs and IBMs all over the whole Tanzania so that performance of the model is established over a large area.
- Evaluation of AFRgeo19 model should be done using ocean levelling for oceanic part of Tanzania (Indian Ocean) so as to complete the validation.

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APPENDICES

APPENDIX 1

MATLAB scripts for Adjusted parameter computations for 3-, 4-, 5- and 7- Parameter for AFRgeo19

```
Data=load('TPLNBM.txt');
Long = Data(:,1); % Longitude in decimal degree
Lat = Data(:,2); % Latitude in decimal degree
Delta_N_AFRgeo19 = Data(:,3); % Is the difference between GPS derived geoidal
height and gravimetric geoid height
X = degtorad(Long); % Converting longitude in decimal degree to radian
Y = degtorad(Lat); % Converting latitude in decimal degree to radian
I = eye(length(Data(:,1))); % I is an identity matrix
E = cos(Y);
F = cos(X);
G = sin(X);
H = sin (Y);
A = [E.*F E.*G H]; % A is the first design matrix
N = A'*A; % N is the normal equation
X19 = inv(N)*A'*Delta_N_AFRgeo19; % Adjusted parameters for AFRgeo19
v19 = (I-(A*inv(N)*A'))*Delta_N_AFRgeo19; % adjusted residuals for AFRgeo19
dof = 8; % degree of freedom for 3-par with 11 observation
aprv19 = (v19'*v19)/dof; % posterior variance for AFRgeo19
cx19 = aprv19*inv(N); % Variance covariance matrix
```

```
Data=load('TPLNBM.txt');
Long = Data(:,1); % Longitude in decimal degree
Lat = Data(:,2); % Latitude in decimal degree
Delta_N_AFRgeo19 = Data(:,3); % Is the difference between GPS derived geoidal
height and gravimetric geoid height
X = degtorad(Long); % Converting longitude in decimal degree to radian
Y = degtorad(Lat); % Converting latitude in decimal degree to radian
I = eye(length(Data(:,1))); % I is an identity matrix
E = cos(Y);
F = cos(X);
G = sin(X);
H = sin (Y);
K = ones(1,11);
A = [E.*F E.*G H K']; % First design matrix
N = A'*A; % Normal equation
X19 = inv(N)*A'*Delta_N_AFRgeo19; % Adjusted parameters for AFRgeo19
v19 = (I-(A*inv(N)*A'))*Delta_N_AFRgeo19; % Adjusted residuals for AFRgeo19
dof = 7; %  dof is the degree of freedom for 4-par with 11 observation
aprv19 = (v19'*v19)/dof; % A posterior variance for AFRgeo19
CX19 = aprv19*inv(N); % variance covariance matrix
```

```
Data=load('TPLNBM.txt');
Long = Data(:,1); % Longitude in decimal degree
Lat = Data(:,2); % Latitude in decimal degree
Delta_N_AFRgeo19 = Data(:,3); % Is the difference between GPS derived geoidal
height and gravimetric geoid height
X = degtorad(Long); % Converting longitude in decimal degree to radian
Y = degtorad(Lat); % Converting latitude in decimal degree to radian
I = eye(length(Data(:,1))); % I is an identity matrix
E = cos(Y);
F = cos(X);
G = sin(X);
H = sin (Y);
K = ones(1,15);
a = 6378137; % a is the semi major axis
b = 6356752; % b is the semi minor axis
e = (a^2-b^2)/(a^2); % first eccentricity
S = (1-e*(H.^2)).^0.5;
A = [E.*F E.*G H (E.*F.*H)./(S) (E.*H.*G)./(S) (H.^2)./S K']; % First design
matrix
N = A'*A; % N is normal equation
X19 = inv(N)*A'*Delta_N_AFRgeo19; % Adjusted parameters for AFRgeo19
v19 = (I-(A*inv(N)*A'))*Delta_N_AFRgeo19; % Adjusted residuals for AFRgeo19
dof = 4; %  dof is the degree of freedom for 5-par with 11 observation
aprv19 = (v19'*v19)/dof; % A posterior variance for AFRgeo19
CX19 = aprv19*inv(N); % variance covariance matrix
```

APPENDIX 2

Adjusted residuals of AFRgeo19 after modelling of systematic effects

0.1687	0.0642	0.0842	-0.0023
-0.1280	-0.1026	-0.0702	-0.0840
-0.1048	-0.0805	-0.0486	-0.0368
0.1024	0.1296	0.1623	0.1172
0.1318	0.1355	0.0913	0.1095
-0.0606	-0.0569	-0.1011	-0.0831
0.1257	0.1368	0.0832	-0.0243
-0.0278	-0.0287	0.0006	-0.0076
-0.0348	-0.0744	-0.1224	0.0041
-0.1329	-0.1443	-0.0891	0.0071
-0.0401	0.0213	0.0098	0.0002