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PERFORMANCE EVALUATION OF THE AFRgeo2019 IN TANZANIA USING GNSS/LEVELLING

A Case of 50km (Uvinza-Kigoma)

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BSc Geomatics

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A Dissertation Submitted to the Department of Geospatial Sciences and Technology in Partially Fulfilment of the requirements for the Award of Science in Geomatics (BSc. GM) of Ardhi University

CERTIFICATION

The undersigned certify that they have read and hereby recommend for acceptance by Ardhi University, a dissertation entitled "**Performance Evaluation of the AFRgeo2019 in Tanzania using GNSS/Levelling, A Case of 50km (Uvinza-Kigoma)**" in partial fulfilment of the requirements for the award of degree of Bachelor of Science in Geomatics of the Ardhi University.

Signature ………………………

Ms. REGINA V. PETER

(Supervisor)

Date ………………………

DECLARATION AND COPYRIGHT

I **MGALAWE, HUSSEIN A** declare that this dissertation report is my own original work and that to the best of my knowledge, it has not been presented to any other University for a similar or any other degree award except where due acknowledgements have been made in the text.

………………………………..

MGALAWE, HUSSEIN A

22787/T.2019

Date…………………….

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DEDICATION

To my late father Abdilah Mgalawe. Always I remember you, may Allah grant you easy in your grave also grant you a place in Paradise.

Also I dedicate this study to my beloved mother, Rehema Mtwaenzi and my uncles, Dr. Khamis Mtwaenzi and Miraji Mtwaenzi and all Mtwaenzi and Mlawa family. Your presence always gives me strength.

ABSTRACT

The geoid, being the natural mathematical figure of the earth, serves as height reference surface for geodetic, geophysical and many engineering applications. The demand for height information from the satellite users based positioning techniques, mostly Global Positioning System (GPS), has increased interest on determination and use of precise geoid models. The knowledge of the local geoid surface allows the transformation of ellipsoidal heights to physically meaningful orthometric heights which are essential in most of the geodetic applications. Thus, GPS measurements in combination with a precise geoid model are preferred in obtaining orthometric heights instead of spirit levelling measurements, which is labour-intensive and costly.

This study therefore seeks to evaluate a regional geoid model (AFRgeo2019) in Tanzania, Uvinza-Kigoma. The estimated geoid heights obtained by the AFRgeo2019 model were compared with 997 discrete geometrical heights from GPS/levelling points of a Railway profile of about 50km located at Uvinza-Kigoma, Tanzania. The evaluation began with extraction of the geoidal heights from AFRgeo2019 model, determination of the geometrical and physical geoid heights using relative method along the profile and statistical analysis on the assessment of AFRgeo2019.

The statistics of the differences between change in derived geoid heights by the geometric approach and change in corresponding geoid heights obtained from the geoid model (AFRgeo2019) suggests that, the AFRgeo2019 model is suitable to be used in Tanzania despite of the AOI being small. The measure of central tendency appears to range to millimeter and sub-millimeter accuracy where Mean, SD and RMS of their change in geoidal height differences are -0.0004m, 0.0011m and 0.0012m, which is better in the area of interest. The study concluded that, the recent regional geoid model can be applied in Kigoma, hence further validation is recommended on other parts of the country so as to be used country wise.

Keywords: GPS/levelling, Regional Gravitational Model, Geoid undulations

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ACRONYMS AND ABBREVIATIONS

AOI	Area of Interest
GGM	Global Gravity Models
GNSS	Global Navigation Satellite System
GPS	Global Position System
LMSL	Local Mean Sea Level
MDT	Mean Dynamic Topography
MSL	Mean Sea Level
MSSH	Mean Sea Surface Height
RMS	Root Mean Square
RTK	Real Time Kinematics
RTM	Residual Terrain Model
SD	Standard Deviation
TG	Tide Gauge
TGBM	Tide Gauge Benchmark
TPLN	Tanzania Primary Levelling Network
VD	Vertical Datum

CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND

The geoid, being the natural mathematical figure of the earth, serves as height reference surface for geodetic, geophysical and many engineering applications. The demand for height information from the satellite users based positioning techniques, mostly Global Positioning System (GPS), has increased interest on determination and use of precise geoid models. The knowledge of the local geoid surface allows the transformation of ellipsoidal heights to physically meaningful orthometric heights which are essential in most of the geodetic applications. Thus, GPS measurements in combination with a precise geoid model are preferred in obtaining orthometric heights instead of spirit levelling measurements, which is labour-intensive and costly. Normally, for the purpose of GPS/levelling, in the absence of a publicly available geoid model, it is beneficial to select a Gravimetric Geopotential Model which is a best fit to the local gravity field as the basis for local or Gravimetric geoid model (Kiamehr & Sjöberg , 2005).

More Gravimetric Geoid Models have now been released into the public domain, particularly those including data from the CHAMP and GRACE satellites dedicated gravimetry missions, and new gravity-field-related datasets, it is important to make validations in order to select the most appropriate geoid model (Peprah, et al., 2017).

The first attempt to compute a geoid model for Africa has been made by (Merry C, 2003) and (Merry, et al., 2005). A 5’ x 5’ mean gravity anomaly grid developed at Leeds University was used to compute the geoid model, these data set has never become available since then again. For the geoid computed by (Merry, et al., 2005), the remove-restore method, based on the EGM96 geopotential model (Lemoine , et al., 1998), was employed.

Another geoid model for Africa has been computed by (Abd-Elmotaal, et al. 2019). This geoid model employed the window remove-restore technique with the EGM2008 geopotential model, up to degree and order 2160, and a tailored reference model (computed through an iterative process), up to degree and order 2160, to fill in the data gaps.

The AFRgeo2019 gravimetric geoid model was computed in the framework of the IAG African Geoid Project. The available gravity information comprises land and sea data, the latter consisting of ship borne point data and altimetry-derived gravity anomalies along tracks. This dataset suffers from significantly large data gaps that were filled by using the EIGEN-6C4 model on a 15' x 15' grid prior to the gravity reduction scheme. The window remove-restore technique was used to generate reduced anomalies having a minimum variance to minimize the interpolation errors, especially at the large data gaps. The EIGEN-6C4 global model, complete to degree and order 2190, has served as the reference model (Abd-Elmotaal et al. 2020). The geoid undulations from the reduced gravity anomalies were computed by employing Stokes' integral with Meissl modified kernel. Finally, the restore step within the window remove-restore technique took place generating the full gravimetric geoid. In the last step, the computed geoid was scaled to the DIR_R5 GOCE satellite-only model by applying an offset and two tilt parameters. The DIR_R5 model was used because it turned out that it represents the best available global geopotential model approximating the African gravity field (Abd-Elmotaal, et al. 2015).

A direct comparison between AFRgeo2019 and the former geoid model AGP2003 clearly shows the achieved improvement, the results are as shown on Figure 1-1. Table 1-1 gives the summary of the data used and techniques in the African Geoid Project.

Table 0-1: Used data and techniques in the African Geoid Project

Geoid version	AGP2003	AFRgeo_v1.0	AFRgeo2019
Database	Leeds University	AFRGDB_v1.0 (Abd-Elmotaal, Seitz, Kühtreiber, & Heck, 2015)	AFRGDB_v2.0 and AFRGDB_v2.2 (Abd-Elmotaal, Seitz, Kühtreiber, & Heck, 2018)
Land	5’ × 5’ grided	96,472 (point values)	126,202 (grid filtering)
Ship bone	–	971,945 (point values)	148,674 (grid

			filtering)
Altimetry	–	119,249 (point values)	70,589 (grid filtering)
Underlying grid	N/A	Tailored model for Africa N_{max}= 360 on a 15'× 15' unregistered grid	EIGEN-6C4 N_{max}= 2;190 on a 15'× 15' unregistered grid
Reference model	EGM96 N_{max}= 360	Tailored model for Africa through an iterative process N_{max}= 2;160	EIGEN-6C4 N_{max}= 2;190
De-trended by	–	GOCE DIR_R5 N_{max}=280	GOCE DIR_R5 N_{max}=280
Fine DTM	GLOBE 1'× 1' (Hastings & Dunbar, 1998)	SRTM30+ 30" × 30" (Farr, T G; Rosen, P A; Caro, E; Crippen, R; Duren, R; Hensley, S; Kobrick, M; Rodriguez, E; Paller, M; Roth, L;, 2007)	AFH16S30 30" × 30" (Abd-Elmotaal et al. 2017)
Coarse DTM	GLOBE 5' × 5' (Hastings & Dunbar, 1998)	SRTM30+ 3'×3' (Farr, T G; Rosen, P A; Caro, E; Crippen, R; Duren, R; Hensley, S; Kobrick, M; Rodriguez, E; Paller, M; Roth, L;, 2007)	AFH16M03 3'×3' (Abd-Elmotaal, Makhloof, Abd-Elbaky, & Ashry, 2017)

Reduction technique	Residual Terrain Modelling (RTM) technique	Window remove-restore technique using EIGEN-6C4 till ultra-high degree (2190)	Window remove-restore technique using EIGEN-6C4 till ultra-high degree (2190)
Geoid determination technique	Molodensky integrals with original Stokes kernel, quasigeoid to geoid separation using (Rapp, 1997)	Stokes integral with original Stokes kernel, 1D-FFT	Stokes integral with (Meissl, 1971) modified kernel, solution in the space domain

As stated earlier, the first attempt to determine a geoid model for Africa "AGP2003" has been carried out by (Merry C, 2003) and (Merry, et al., 2005). Since then, the data base has been further enhanced. In particular, the calculation method, statistical combination of the various types of gravity anomalies, has been revised and further developed. This has led to a significant improvement of the African geoid model. Figure 1-1 shows the difference between the de-trended AFRgeo2019 and the AGP2003 geoid models. The light yellow pattern in Fig. 1-1 indicates differences below 1 m in magnitude. Figure 1-1 shows that the differences between the two geoids amount to several meters in the continental area, especially in East Africa. The large differences over the Atlantic Ocean arise from the fact that the AGP2003 didn't include ocean data in the solution. Figure 1-1 shows some edge effects, which are again a direct consequence of using no data outside the African continent in the AGP2003 solution. As the AFRGDB_v1.0, which has been the basis for computing the AFRgeo_v1.0, has been greatly influenced by a wrong data set in Morocco (Abd-Elmotaal , Seitz, Kühtreiber, & Heck, 2019), comparison between AFRgeo_v1.0 and the current geoid model was skipped.

Evaluation of this model has been done regional wise, its performance evaluation in Tanzania mainly has not yet been done. Therefore, for this model to be used in Tanzania, its performance evaluation has to be done so as to know if it can reliably be used in Tanzania using a profile data collected by GNSS/levelling at different areas within the boundaries of Tanzania.

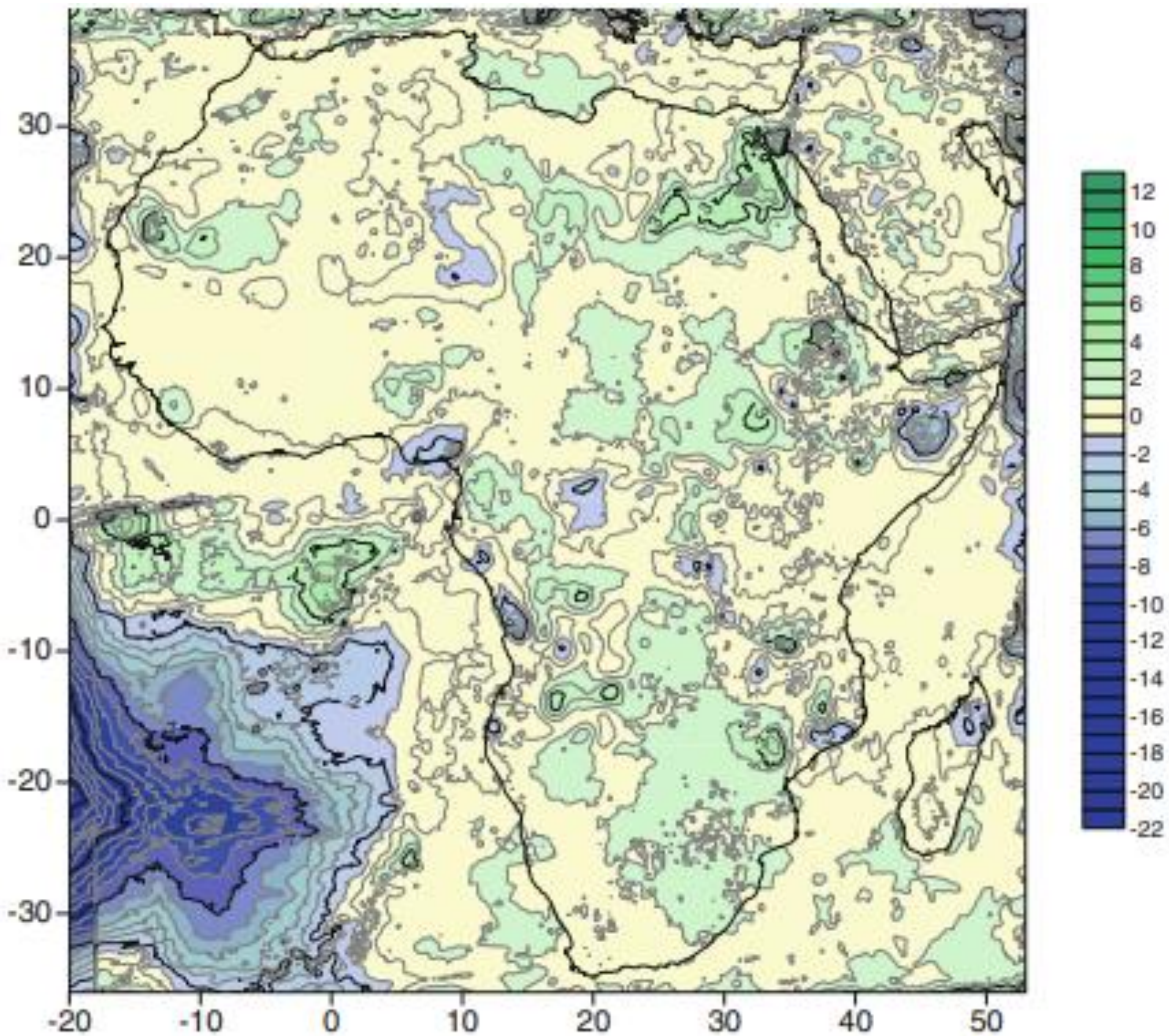

Figure 0-1 Difference between the de-trended AFRgeo2019 and the AGP2003 geoid models. Contour interval: 1 m

1.2 STATEMENT OF THE PROBLEM

The new African geoid model (AFRgeo2019) is a breakthrough in the field of gravimetry. The demand for African coverage, precise and high resolution gravity is extremely high in the field of geodesy and other geosciences in Africa. To use the new AFRgeo2019 reliably, its evaluation using globally acceptable approaches such as its evaluation using GNSS/levelling is of dominant importance and hence this research.

1.3 OBJECTIVES

1.3.1 Main Objective

The goal of this study is to evaluate AFRgeo2019 to enable its use reliably in Tanzania.

1.3.2 Specific Objectives

- To determine geometrical geoid from GNSS/levelling data of a profile within Tanzanian boundaries.
- Extraction of geoid heights from AFRgeo2019 along a profile within Tanzania boundaries.
- Comparison and validation of relative heights difference from AFRgeo2019 with those from GNSS/levelling data on a profile.

1.4 SIGNIFICANCE OF THE STUDY

Once AFRgeoid2019 is found to be reliable, it will complement the available gravity data significantly and thus removal of the sparseness and heterogeneity of gravity data in Tanzania for various applications.

1.5 BENEFITS OF THE STUDY

This study will benefit much the geoscientists especially geodesists and geomaticians e.g. for improving and updating Gravimetric geoid models, mineral prospecting & exploration and improvements of height systems in Tanzania. Also, monitoring mass and density distribution, isostasy and mantle processes in Geophysics.

1.6 STUDY AREA

The study area will be limited within 1°N to 12°S latitude and 29°E to 41°E longitude, within which Tanzania boundaries are found as shown on Figure 1:2.

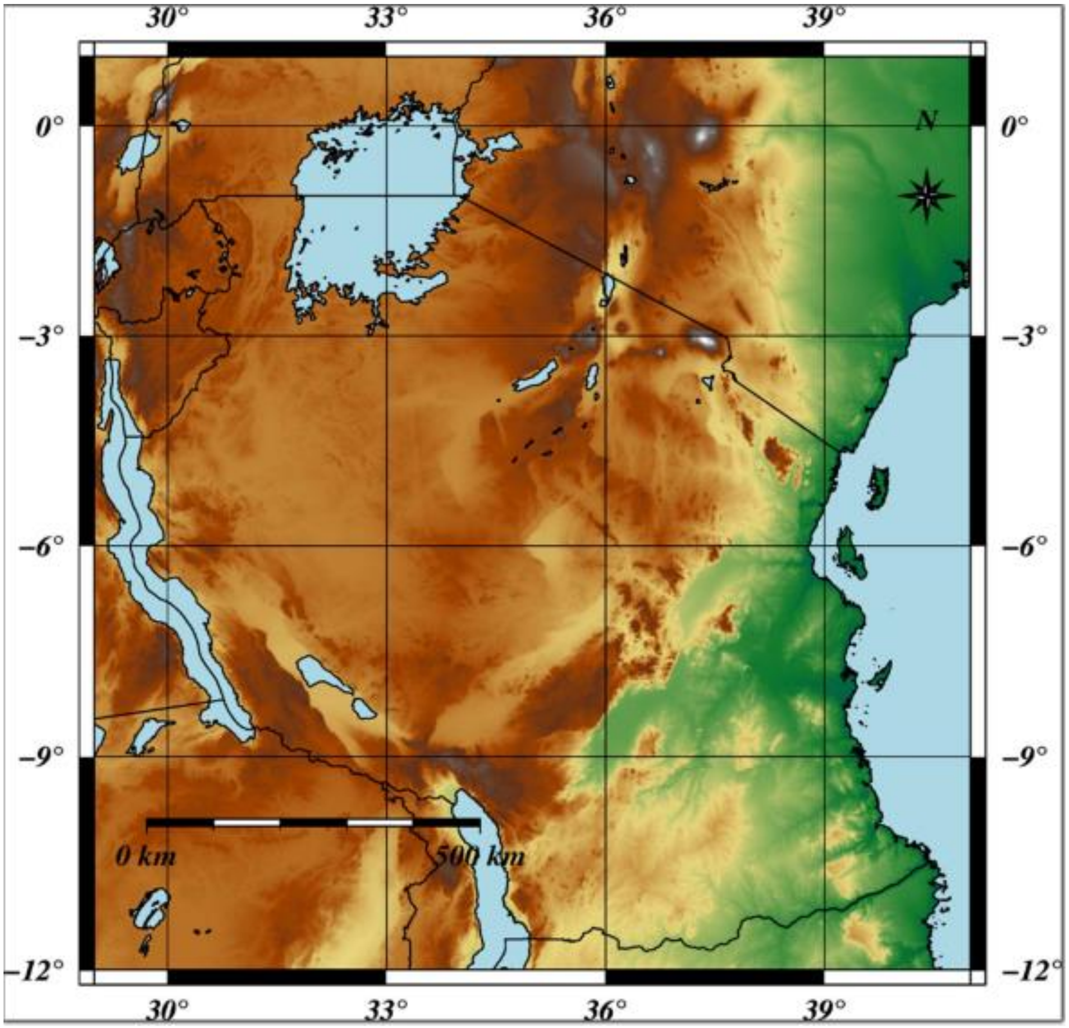

Figure 0-2 Area of study

1.7 RESEARCH QUESTIONS

i. Will African geoid model AFRgeo2019 perform better than the regional geoids models?

ii. Does relative approach in geoid model performance evaluation yield best results than any other methodology?

iii. How accurate and reliable is AFRgeo2019 in Tanzania?

1.8 STRUCTURE OF THE DISSERTATION

This research consist of five chapter, in which the first chapter comprises of the background of the study, previous researches, statement of the research problem, objective of the research (main objective and specific objectives), significance of the research, beneficiaries of the research and location of the study area. The second chapter comprises of literature review which gives the overview of the Vertical Datum (VD), types of the VD (MSL, Geoid model based and Quasi-geoid model based) and methods used in the assessment of the gravimetric geoid models. The third chapter provides the methodology which describes in detail how the research problem is solve using relative GPS/levelling approach. The fourth chapter gives in detail the computations, results and discussion of the results. The last chapter which is chapter five gives the conclusion of this research and recommendations for future related researches.

CHAPTER TWO

LITERATURE REVIEW

2.1 VERTICAL DATUM

By definition, Vertical Datum (VD) is referred as the reference surface where all the vertical measurements are referred to. The Mean Sea Level (MSL) has been used extensively for a long time as VD all over the World. It is realized by averaging sea water levels over a minimum period of 18.6 years at a Tide Gauge (TG) station in a coastal area (Ulotu, 2015). The time fixed MSL of a tidal station is referenced to a firmly monumented land point usually referred to as Tide Gauge Benchmark (TGBM). MSL established from tide gauge(s) depart from the geoid by the Mean Dynamic Topography (MDT) and other coastal oceanographic effects (Ulotu, 2015). There exist different types of VD, some of them are;

i. MSL Vertical Datum
ii. Geoid Model.

2.1.1 MSL Vertical Datum

MSL Vertical Datum is the ideal equipotential surface that could be obtained by fitting a level surface to observations of the mean level of the sea surface. It is obtained by the average reading of the tide gauge station recorded over a long period of time about 18.6 years. Establishment of a National (local) VD commenced in the 1960s. By then a model for the geoid of Tanzania did not exist. Consequently, the local mean sea level (LMSL) was established in place of the geoid surface. The LMSL was established using tide gauge measurements for 28 months only that was between August 1962 and November 1964 inclusive at the Tanga harbor. In Tanzania mainland, TG stations are found in Zanzibar, Tanga, Dar-es-Salaam and Mtwara harbors. The fundamental reference benchmark is the Tanga TGBM. A dense network of ground datum markers; benchmarks (BM) of differing accuracies were monumented and referred to the Tanga TGBM from 1961 to 1969 (Ulotu, 2015)

In Tanzania mainland, orthometric height is explained as the official height system, and thus its VD should be the geoid and not the MSL (Hofmann-Wellenhof & Moritz 2005). The use of MSL as a vertical datum that is equated with geoid is affected and limited by several factors such as absence of periodical updating, inability to take care of Mean Dynamic Topography (MDT) and some temporal and periodic effects.

If the observation duration does not meet the minimum requirement of 18.6 years, it displaces the MSL far from the geoid by several meters (Kileo, 2017) therefore, the heights derived from the established MSL cannot be orthometric even if they follow proper procedures in establishment.

Tanzania Vertical Datum has got some advantages as follows;

i. High relative accuracies can easily be achieved using precise levelling when complemented by gravity corrections,
ii. Levelling instrumentation is affordable, also data validation and processing are not complicated,
iii. Error tracing is easy, and formats and procedures are well established to minimize gross and systematic errors
iv. It can be improvement to remove some of the datum observational and establishment deficiencies cited earlier, and
v. GPS levelling is a preferred method in the external quality assessment of a gravimetric geoid/quasi-geoid model as well as global gravity models (GGM).

The disadvantages of the current Tanzania Vertical Datum are such as;

i. The LMSL data was observed for 28 months only instead of at least 18.6 years, and this cannot remove short- and long-term sea level variations which are periodical as well as temporal.
ii. Mean dynamic sea surface topography (MDT) was not accounted for at the Tanga TG station, making it difficult for the VD to coincide with the geoid.
iii. During spirit levelling, heights of the benchmarks of the TPLN were corrected using normal gravity instead of the actual gravity, therefore the height system of Tanzania is normal orthometric and not orthometric height system.
iv. There has not been a uniform adjustment of the TPLN. Consequently, the network has led to a lot of inconsistencies, which cannot be explained adequately. Besides, the initial raw data cannot be traced and therefore it is not possible to adjust it rigorously.
v. There is no clear documentation of how heights of trigonometric stations relate to the TPLN. In fact, it is known that some trigonometric stations existed well before TPLN.

Moreover, it is not clear in which type of height system are the heights of trigonometric network

vi. If all the TG stations are to be unified, it does not mean that a better tidal VD would be obtained. In fact, the situation could be worse.

vii. Re-observation of the entire TPLN so that rigorous adjustment can be conducted, will be very expensive, and very likely not the most relevant option in this GNSS period.

2.1.2 Geoid Model

The Geoid is essentially the real shape of the Earth, without topographic and atmospheric masses. The Geoid is defined as the equipotential surface of the Earth's gravity field which coincides with the sea surface in the absence of disturbing factors like tsunamis, ocean currents, salinities, wind, etc., and it extends through the continents (Vanicek, 1991) . Though the geoid is much smoother than the actual earth surface, unlike the ellipsoid, it is still too complicated to serve as the computational surface on which to solve geometrical problems, but it is suitable as a vertical datum. Geoid model as vertical datum, is used in GNSS/levelling where orthometric heights can be derived directly by this method as illustrated on equation (2.3). Determination of a geoid model requires extensive gravitational measurements and computations. Different attempts have been done in Tanzania in geoid model determination such as gravitational geoid models (TZG07, TZG08, TZG13, TZG17 and TZG 19) and as well as quasi-geoid (TZQ17) which employ the use of different computational approach like KTH method. There are different kinds of geoid models; these are Gravimetric Geoid Model, Quasi-Geoid Model, Hybrid Geoid Model and Geometric Geoid Model.

2.2 GRAVIMETRIC GEOID MODEL

Gravimetric geoid model is obtained from Gravimetric techniques which uses gravity measurement in its determination. Theoretically, gravimetric geoid model is a solution of Boundary Value Problems (BVP) to obtain the boundary value in the form of the geoid surface. Tanzania has coverage of different gravimetric geoid models which are TZG07, TZG08, TZG13, TZG17 and TZG 19 from local geoid models, AGP03, AGP07 (Merry, 2007) and AFRgeo2019 (Abd-Elmotaal H, 2019) from regional models and EGM08 (Pavlis, et al 2008) from global geoid models.

Tanzania Gravimetric Geoid Model of 2013 (TZG13), this model was developed by Forsberg et al, (2013). The data used to determine this model were data from airborne survey supplemented with existing survey, marine gravity data, satellite gravity data (Altimetry, GRACE and GOCE) and updated digital terrain model that covers the land and ocean parts of Tanzania. It covers 13ºS to 1ºN Latitude and 28ºE to 43ºE Longitude which is the major part of Indian Ocean offshore East of Tanzania. Computed on grid 0.02º × 0.02º resolution approximated to 2km × 2km grid. The accuracy of the model was 10cm (Forsberg et al, 2013).

Tanzania Gravimetric Geoid Model of 2017 (TZG17), this model was developed by (Peter, 2018). The model was determined from 38,483 terrestrial points gravity data from Tanzania Gravity Database of 2008 (TGDB08), airborne gravity data limited to boundaries of Tanzania, validated pure GOCE GGM (GO_CONS_GCF_2_SPW_R5) to d/o 200 and composite GGM (XGM2016) to d/o 719, combination of four validated 1" and 3" global DEM: USA SRTM-1" v3, ALOS-1" v2, MERIT 3" and SRTM-3" CGAIR v4. The model was computed on grid of 1'×1' resolution corresponding to approximately 1.8km×1.8km. It covers 12ºS to 1ºN Latitude and 29ºE to 41ºE Longitude. The accuracy attained was 5cm (Peter, 2018).

Tanzania Gravimetric Geoid Model of 2019 (TZG19), was developed by (Forsberg et al, 2019). The data used to determine this model were data measured from airborne survey supplemented with existing survey, marine gravity data, satellite gravity data (Altimetry (DTU15), GRACE and GOCE) and updated digital terrain model that covers the land and ocean parts of Tanzania. Since it is an update of TZG13 then it used the new PGM17 model which is unpublished test version of the upcoming high resolution EGM20 model. PGM17 is the major improvement of the earlier used reference models and has Tanzania airborne gravity data, GOCE and GRACE satellite data based on XGM2016 model. It has also an additional information of airborne gravity data of Mozambique and Malawi as well as the Northwest part of Tanzania. Its computation was based on the DTU-space GRAVISOFT geoid software. It covers 13ºS to 1ºN Latitude and 28ºE to 43ºE Longitude which is the major part of Indian Ocean offshore East of Tanzania. This model was computed on the grid 0.02º×0.02ºresolution approximated to 2km×2km grid. Based on the collocation error estimates from other regions with data quality and coverage, the geoid error is estimated to be on the order of 10cm for most of the region, hence the attained accuracy of the model was 10cm (Forsberg et al, 2019).

The AFRgeo19 gravimetric geoid model was computed in the framework of the IAG African Geoid Project. The available gravity information comprises land and sea data, the latter consisting of ship borne point data and altimetry-derived gravity anomalies along tracks. The window remove restore technique was used to generate reduced anomalies having a minimum variance to minimize the interpolation errors especially at the large data gaps. The EIGEN-6C4 global model, complete to degree and order 2190, has served as the reference model. The geoid undulations from the reduced gravity anomalies were computed by employing Stokes' integral with Meissl modified kernel. The restore step within the window remove-restore technique took place generating the fully gravimetric geoid. The computed geoid was scaled to the DIR_R5 GOCE satellite-only model by applying an offset and two tilt parameters. (Abd-Elmotaal H, et al, 2019).

Earth Gravitational Model of 2008 (EGM08), this is the global geoid model complete to d/o 2159 plus some additional spherical harmonic coefficients up to degree 2190. EGM08 was developed by United state National Geospatial-Intelligence Agency (NGA) revealed a major achievement in global gravity field mapping. GRACE data (ITG-GRACE03S) 5', high resolution global DTM (DTM2006.0)30", gravity anomalies from SA with 5' data were used upon its computation. The data provides long and medium wavelengths equivalent to about 18km. The precision of the model was about ±15cm worldwide (Pavlis et al, 2008).

2.3 QUASI-GEOID MODEL AS VERTICAL DATUM

Quasi-geoid is the reference surface for normal heights. Gradually adoption of the normal height instead of orthometric height system is gaining popularity in the world mainly due to its less stringent determination process and closeness of normal heights to orthometric heights. Quasi-geoid does not deviate much from the geoid; often it is less than 1m, but the surface has no physical meaning (Ulotu, 2015). Quasi-geoid determination is attributed to a Russian geodesist (Molodensky et al. 1962). The relationship between the geoid and the quasi-geoid, i.e. geoidal height N and height anomaly ζ is expressed in the equation below;

$$N = \zeta + \left(\Delta g - \frac{2\pi\rho H}{\gamma}\right) H \tag{2.1}$$

Where:

N = Geoidal height,

ς = Height anomaly from Quasi-geoid model,

Δg = Bouguer gravity anomaly,

ρ = Constant density of continental masses,

γ = Normal gravity on the telluroid; close to the surface,

H = Elevation of the surface point.

2.4 HYBRID GEOID MODEL

When accurate and fairly well distributed benchmarks referred to appropriately determined TG based VD exist, together with precise gravimetric geoid model, a hybrid VD can be created out of the two using GPS observations on the BMs (Ulotu, 2015). Precise GPS observations on the BMs i.e. GPS/levelling enable determination of geometrical geoid height as shown on the equation below,

$$N_{BM}^{GPS} = h - H \tag{2.2}$$

Where,

N_{BM}^{GPS} is the geometrical geoidal height,

h is ellipsoidal height from precise GPS positioning and,

H is orthometric height based on the TG-VD.

From a gravimetric geoid model, we obtain N, that upon combination with N_{BM}^{GPS} over the respective region often in least squares manner, results into hybrid VD. When the above conditions are favorable, hybrid datum combines the advantages of both types of geoidal models, and that is often advantageous to the user. (Ulotu, 2015).

2.5 GEOMETRIC GEOID MODEL

This model uses ellipsoidal height determined from GPS observation together with orthometric height obtained from geodetic levelling to determine geometric geoidal height (N_{BM}^{GPS}). This can be achieved as shown in the equation (2.2). Accuracy of the established geoid depends on the distribution and the number of GPS/levelling stations, accuracy of GPS/levelling data, characteristics of the geoid in the region and the methods of interpolation. Figure 2-1 shows the relationship between ellipsoidal height h, orthometric height H and geoidal height N.

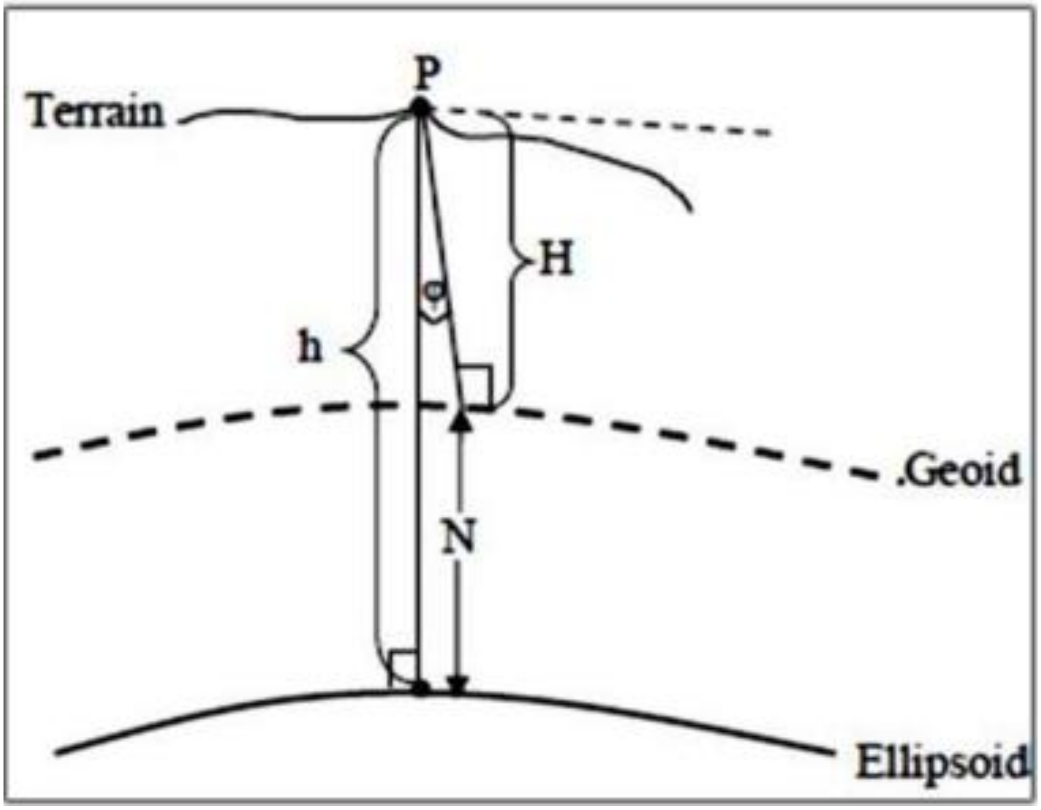

Figure 0-1: Relationship between geoidal, ellipsoidal and orthometric heights

Where;

h is the ellipsoidal height from the Ellipsoid to point P on the Terrain.

H is the Orthometric height from the Geoid to point P on the Terrain.

N is the Geoid height from the ellipsoid to the Geoid.

Φ is the deflection of Vertical.

2.6 REVIEW OF GEOID MODEL EVALUATION METHODS

In valuation of geoid model different methods are employed such as GPS/levelling technique, gravity anomalies, Astrogeodetic deflection, geoid slope and sea surface topography methods, its explanation is as below.

2.6.1 GPS/Levelling Method

This is the common method used in evaluation of geoid models**.** It involves ellipsoidal heights, h which is obtained from GPS data and orthometric heights, H which is obtained from spirit levelling. Geometrical Geoid height, N is obtained by subtracting orthometric height from ellipsoidal height. Equation (2.3) shows the illustration.

$$N^{GPS} = h - H \tag{2.3}$$

The N obtained above is used for evaluation of geoid model.

GPS/ levelling method involves two approaches which are absolute approach and relative approach.

2.6.1.1 Absolute GPS/Levelling Approach

The absolute value of ellipsoidal height, h has to be known relative to appropriate reference ellipsoid and it can be obtained through GPS measurement by geodetically connecting to ITRF (Featherstone, 2001). Geometrical geoid is obtained as by equation (2.3), then evaluation is obtained by taking the difference between the geometrical geoid (N^{GPS}) and the gravimetric geoid (N) as shown on equation (2.4).

$$\Delta N = N^{GPS} - N \tag{2.4}$$

Where; N^{GPS} is the geometrical geoidal height and

N is the gravimetric geoid height

2.6.1.2 Relative GPS Levelling Approach

Relative differences of heights between two adjacent point say $\Delta h_{i,i+1}$ and $\Delta H_{i,i+1}$ will be calculated using a mathematical relation (2.5) and (2.6) respectively. Thereafter, the relative difference in geometrical geoid height can be obtained using mathematical relation (2.7).The benefit in this method is that, any errors that are common to either end of the control baseline cancel each other on differencing (Featherstone, 2001).

$$\Delta h_{i,i+1} = h_{i+1} - h_i \tag{2.5}$$

$$\Delta H_{i,i+1} = H_{i+1} - H_i \tag{2.6}$$

$$\Delta N_{i,i+1}^{GPS} = \Delta h_{i,i+1} - \Delta H_{i,i+1} \tag{2.7}$$

Where:

H is the orthometric height of RTK GNSS/Levelling based on MSL.

$\Delta N_{i,i+1}^{GPS}$ is the relative geometrical geoid height difference between the points along a profile.

h is the ellipsoidal height of RTK GNSS/levelling.

Performance evaluation of the Gravimetric Geoid will be done by taking the difference in relative difference of geometrical geoid height and that from the Gravimetric Geoid. Mathematical relations for the evaluation is given by equation (2.8).

$$\Delta N_{i,i+1} = \Delta N_{i,i+1}^{GPS} - \Delta N_{i,i+1}^{gravimetric\ geoid} \qquad (2.8)$$

Where;

$\Delta N_{i,i+1}$ is the difference in relative heights differences about two consecutive points between geometrical geoid from GNSS/Levelling and Gravimetric Geoid points.

$\Delta N_{i,i+1}^{GPS}$ is the relative difference of geometrical geoid heights about two consecutive points on a profile.

$\Delta N_{i,i+1}^{gravimetric\ geoid}$ is the relative difference of physical geoid heights about two consecutive points on a profile in gravimetric geoid model.

i = 1, 2, 3, 4…

2.6.1.3 Performance assessment of the Gravimetric Geoid

Statistical analysis will be used to assess the relative performance of the Gravimetric Geoid, i.e. mean, Root mean square and standard deviation and will be computed using equations (2.9), (2.10) and (2.11).

a) The mean of relative height differences will be computed as;

$$\Delta N_{i,i+1} mean = \frac{1}{n} \sum_j^n \Delta N_{i,i+1} \qquad (2.9)$$

Where;

n is the number of grid points under consideration.

j is 1, 2, 3…

b) Standard deviation (SD) of relative height difference is computed as;

$$SD = \sqrt{\frac{\sum_j^n (\Delta N_{i,i+1} - \Delta N_{i,i+1} mean)^2}{n-1}} \qquad (2.10)$$

c) The root mean square (RMS) value of relative heights differences in elevation is given as;

$$RMS = \pm \sqrt{\frac{\sum_j^n (\Delta N_{i,i+1})^2}{n}} \qquad (2.11)$$

2.6.2 Astrogeodetic Deflection of the Vertical

This is among the methods of geoid evaluation that involves comparison of geoid height differences between the Astrogeodetic geoid heights obtained from the components of the deflection of the vertical and the geoid height from geoid model (Olliver, 2007). The geoid height difference can be computed as in the equation below;

$$N_i = N_{i+1} - (S_m \xi_m + S_p \eta_p) \tag{2.12}$$

Where; N_i and N_{i+1} are geoid heights at station i and $i+1$ respectively,

$\xi_m$and η_p are mean meridianal and prime deflection of the vertical at station i and $i+1$,

$S_m$and S_p meridianal and parallel circles distances between station i and $i+1$.

The meridian and prime vertical astrogeodetic deflection of the vertical (ξ_m ,η_p) are determined from the following equations (Hofmann-Wellenhof & Moritz, 2005).

$$\xi = \Phi - \varphi \tag{2.13}$$

$$\eta = (\Lambda - \lambda) \cos\varphi \tag{2.14}$$

Where; φ and λ are geodetic latitude and longitude values of a station referred to GRS80 respectively, Φ and Λ are astronomical latitude and longitude respectively.

The evaluation of geoid model is done through analysis of the magnitudes of the overall minimum, maximum, mean differences and standard deviation (SD) of the geoid heights differences from different geoid models and those between respective points along the astrogeodetic profile.

The use of this method in geoid evaluation is more reasonable, when differences of geoid heights are compared instead of actual heights because it minimizes the systematic errors by differencing the geoidal heights along the Astrogeodetic profile. The problem associated with this technique is that, Φ and Λ are usually not available especially in Tanzania, they are very seldom to be determined nowaday (Hofmann-Wellenhof & Moritz, 2005) on national control networks.

2.6.3 Geoid Slope Method

This method involves comparison of geoid slopes from GPS/levelling data and geoid slope from geoid models. It helps to determine how the geoid models perform over different distances. Depending on the actual baseline length, baselines are categorized into different distances from shortest to longest in Kilometers. The advantage of this method is that, in differencing of geoid slopes the errors related to the baseline cancel out.

Let two GPS stations i and $i+1$ with the horizontal distance $D_{i,i+1}$ and slope distance $S_{i,i+1}$ between them. Evaluation of geoid model using this method involves the following procedures;

i. Difference in geoidal height ($\Delta N_{i,i+1}$) between two station i and $i+1$ from gravimetric geoid models and GPS/levelling is determined using equation (2.15)

$$\Delta N_{i,i+1} = N_i - N_{i+1} \tag{2.15}$$

ii. Determining horizontal distance, $D_{i,i+1}$ between stations i and $i+1$ using join computation.

iii. Geoid slopes for both geoid models and GPS points ($S_{i,i+1}$) is computed using equation (2.16) below;

$$S_{i,i+1} = \left| \Delta N_{i,i+1} / D_{i,i+1} \right| \tag{2.16}$$

iv. Determine slope difference between slope from geoid model ($S_{i,i+1}$) and slope from GPS data ($S_{i,i+1}^{GPS}$) using equation (2.17)

$$\Delta S = S_{i,i+1} - S_{i,i+1}^{GPS} \tag{2.17}$$

With ΔS, assessment can be done on it where statistical values such as mean, SD and RMS are obtain to analyze how two models depart from one another.

2.6.4 Sea Surface Topography Method

Over the ocean surfaces, geoid models are evaluated through their geoidal undulations and Mean Sea Surface Height ($MSSH$) derived from satellite Altimetry data. $MSSH$ is obtained from the difference between satellite Altitude, h and their Altimetry range, R using equation (2.18) (Gruber, 2008)

$$MSSH = h - R \tag{2.18}$$

Where; $MSSH$ = Mean Sea Surface Height derived from satellite altimetry data,

h = Satellite altitude above the reference ellipsoid (WGS84) and

R = Altimeter range above the sea surface.

Sea Surface Topography (SST) can be obtained by subtracting geoid height N, from SSH using equation (2.19) (Gruber, 2008).

$$SST = MSSH - N \tag{2.19}$$

Sea Surface Topography solutions are inspected visually in order to check and assess how realistic are from the model depicts oceanographic features of water bodies (Gruber, 2008). SST is used to study oceans tides, circulation and amount of heat that the ocean holds. The observations are used to predict short term changes in weather and long term climate pattern, also are used to map the oceanographic features such as oceanic currents and temperature variation.

2.7 HYPOTHESIS TESTING

A hypothesis is a statement about the parameters of a distribution. A test of a hypothesis is a rule that, based on the sample values, leads to a decision to accept or reject the null hypothesis. Normally, a test statistic is computed from the sample values (observations) and from the specification of the null hypothesis. If the test statistic falls within a critical region, the null hypothesis is rejected otherwise it is accepted.

However, the null hypothesis is that the differences have a normal distribution with mean, φ and varianceσ^2. The sample mean, ΔN_{mean} and sample variance, S^2 are tested to see if they really belong to normal distribution ΔN (φ, σ2). For statistical testing, the assumption made is that the population mean, φ and variance, σ^2 are normally distributed. Thus, in order to see if the sample mean δ_{mean} and variance S^2 are within the confidence interval of the population mean, φ and variance σ^2 from which the sample is drawn, the following hypothetical statistical tests was used:

- Let n_j be the change in geoid undulation differences from the recent geoid models such that (j =1, 2, 3... n) with estimated statistics, $\Delta N_{\ mean}$ and S. Then, the sample mean δ_{mean} has a T-distribution function given by Equation (2.20).

$$\frac{\Delta N_{\ mean} - \varphi}{S/\sqrt{n}} = t_{n-1} \qquad (2.20)$$

Where (n-1) is the degree of freedom, the equal to sign (=) indicates that the right-hand side is distributed with respect to left hand side. Thus at 95% confidence level, the interval of population mean, φ should be given by Equation (2.21).

$$\Delta N_{\ mean} - t_{0.95,n-1}\frac{S}{\sqrt{n}} \leq \varphi \geq \Delta N_{\ mean} + t_{0.95,n-1}\frac{S}{\sqrt{n}} \qquad (2.21)$$

CHAPTER THREE

METHODOLOGY

This chapter presents the methods and procedures applied in this study. It details on how Performance Evaluation of AFRgeo2019 was performed. In this study, the relative GPS/Levelling approach using a profile of 50km in length located in Uvinza-Kigoma was used in performance evaluation of AFRgeo2019.

3.1 RELATIVE GPS/LEVELLING

Relative GPS/Levelling employs the use of two consecutive points say i and $i+1$ whose geodetic coordinates are known (φ, λ, h, H). In performing assessment using this method, different procedures must be adhered to; these procedures are Determination of geometrical geoid height difference between points ($\Delta N_{i,i+1}^{GPS}$), Determination of AFRgeo2019 geoid height difference between points ($\Delta N_{i,i+1}^{AFRgeo2019}$) and Comparison of the ($\Delta N_{i,i+1}^{AFRgeo2019}$) against ($\Delta N_{i,i+1}^{GPS}$).

3.1.1 Determination of geometrical geoid height difference between points ($\Delta N_{i,i+1}^{GPS}$)

In order to determine the difference in geometrical geoid height between points, the difference in ellipsoidal height and orthometric height between these points were determined first as given by equation (2.5) and (2.6) respectively. Thereafter, geometrical geoid height difference between the same points was then calculated using equation (2.7). $\Delta N_{i,i+1}^{GPS}$ was then later used in the assessment of the AFRgeo2019 model.

3.1.2 Determination of AFRgeo2019 geoid height difference between points ($\Delta N_{i,i+1}^{AFRgeo2019}$)

Determination of AFRgeo2019 geoid height difference was done by using equation (3.1) but the geoid heights of the specific points must be extracted first from AFRgeo2019 grid model. Extraction of geoid heights from AFRgeo2019 involved several procedures including; gridding of AFRgeo2019 then extraction of the geoid heights follows as illustrated below.

$$\Delta N_{i,i+1}^{AFRgeo2019} = N_{i+1}^{AFRgeo2019} - N_{i}^{AFRgeo2019} \tag{3.1}$$

3.1.2.1 Gridding of the AFRgeo2019 and Extraction of Geoid Heights of Respective Points

The AFRgeo2019 was downloaded from www.isgeoid.polimi.it in isg format (.isg) as shown in Table 3-1 hence it was converted into a suitable format as shown on Table 3-2 that could be interpreted by Golden Surfer Software to grid. Then a gridded file of the AFRgeo2019 was created for the extraction of geoid heights (N) of the sample point data. With the latitude and longitude of individual point with zero elevation, geoid heights of such points were obtained using residual function.

Table 0-1: AFRgeo2019 grid data in isg format

(°)/(°)	-20	-19.75	-19.5	-19.25	-19	-18.75	-18.5	-18.25	-18	-17.75
39	51.867	51.47	51.16	50.917	50.795	50.773	50.791	50.837	51.196	51.595
38.75	51.704	51.506	51.392	50.609	50.287	50.175	49.942	50.169	50.469	50.918
38.5	51.704	50.995	50.491	50.163	50.197	50.112	50.018	50.112	50.218	50.112
38.25	51.704	50.11	50.024	50.061	50.335	50.38	50.568	50.647	50.267	49.734
38	51.704	49.444	49.508	49.774	50.093	50.475	51.021	51.266	50.7	49.813
37.75	51.704	49.249	49.101	49.375	49.632	50.227	50.803	51.057	50.776	50.123
37.5	51.704	48.458	48.25	48.433	49.174	49.976	50.216	50.193	50.1	49.899
37.25	51.704	48.131	47.998	48.379	49.177	49.55	49.161	48.946	48.635	48.468
37	51.704	47.701	47.114	47.072	47.349	47.524	47.566	47.402	47.319	47.285
36.75	51.704	46.186	45.793	45.656	45.575	45.605	45.774	45.765	45.704	45.923
36.5	51.704	44.378	44.246	44.157	43.963	43.893	43.99	44.004	44.096	44.413
36.25	51.704	42.965	42.827	42.828	42.844	42.905	43.105	43.475	43.634	43.83
36	51.704	42.469	42.412	42.339	42.411	42.487	42.736	43.13	43.316	43.853
35.75	51.704	41.883	42.083	42.213	42.319	42.416	42.681	43.145	43.726	44.348
35.5	51.704	41.916	42.165	42.385	42.385	42.503	42.834	43.366	44.058	44.652
35.25	51.704	41.872	42.041	42.283	42.481	42.783	43.132	43.529	43.977	44.557
35	51.704	41.61	41.856	42.176	42.471	42.765	43	43.383	43.91	44.653
34.75	51.704	41.433	41.746	42.157	42.485	42.639	42.892	43.328	43.802	44.426
34.5	51.704	41.211	41.616	42.034	42.294	42.659	42.861	43.155	43.354	43.805
34.25	51.704	40.937	41.397	41.889	42.318	42.669	42.879	42.911	43.021	43.341
34	51.704	40.841	41.548	41.935	42.303	42.686	42.876	42.949	43.144	43.4
33.75	51.704	40.901	41.604	42.185	42.588	42.893	43.162	43.414	43.666	43.843
33.5	51.704	40.886	41.694	42.237	42.865	43.468	43.843	44.149	44.487	44.538
33.25	51.704	40.88	41.85	42.582	43.339	43.93	44.436	44.991	45.271	45.262
33	51.704	41.022	41.961	42.717	43.476	44.071	44.7	45.392	45.641	45.876
32.75	51.704	40.744	41.597	42.281	42.87	43.363	43.879	44.547	45.101	45.668
32.5	51.704	40.244	40.98	41.501	41.852	42.107	42.373	42.643	43.217	43.693
32.25	51.704	39.697	40.329	40.71	40.918	40.994	41.116	41.353	41.62	41.881

Table 0-2: AFRgeo2019 data in csv format

Longitude (°)	Latitude (°)	N (m)
-20	39	51.867
-19.75	39	51.47
-19.5	39	51.16
-19.25	39	50.917
-19	39	50.795
-18.75	39	50.773
-18.5	39	50.791
-18.25	39	50.837
-18	39	51.196
-17.75	39	51.595
-17.5	39	51.596
-17.25	39	51.278
-17	39	51.086
-16.75	39	50.457
-16.5	39	49.983
-16.25	39	49.833
-16	39	49.545
-15.75	39	49.502
-15.5	39	49.637
-15.25	39	49.779
-15	39	49.999
-14.75	39	49.654
-14.5	39	49.442
-14.25	39	49.776
-14	39	50.594

3.1.3 Comparison of the ($\Delta N^{AFRgeo2019}$) against (ΔN^{GPS})

This comparison assessment involves two models i.e. the $\Delta N^{AFRgeo2019}$ and the ΔN^{GPS} where the height differences between them are computed along a profile located at Uvinza-Kigoma. Before data processing sorting of a profile data was necessary so as to have consecutive point data; the difference between the two models was performed through the equation (2.8).

Then data skewing followed as it is very important to have good data for the assessment therefore, outliers were removed from the sample data to remain with the clean ones. Data skewing took place in 95% confidence level and outliers were detected and removed.

A mathematical relation (2.21) was used to skew data within 95% confidence with degree of freedom (n-1) as can be shown on Table 3-3 and Table 3-4 as it was before and after removal of outliers respectively.

Table 0-3: Difference between change in geoid heights of GPS/Levelling to that of AFRgeo2019 geoid models before removal of outliers

ΔN(GPS) (m)	ΔN(AFRgeo2019) (m)	ΔN (m)
-	-	-
-0.0003	0.0011	-0.0014
0.0006	0.0011	-0.0005
-0.0006	0.0011	-0.0017
0.0006	0.0011	-0.0005
-0.0007	0.0011	-0.0018
0.0003	0.0011	-0.0008
-0.0002	0.0011	-0.0013
0.0006	0.0011	-0.0005
0	0.0011	-0.0011
-0.0007	0.0011	-0.0018
0.0002	0.0011	-0.0009
0.0003	0.0011	-0.0008
-0.0007	0.0011	-0.0018
0.0004	0.0011	-0.0007
0.0004	0.0011	-0.0007
0	0.0011	-0.0011
1E-04	0.0011	-0.001
-0.0003	0.0011	-0.0014
-0.0005	0.0011	-0.0016
0.0006	0.0011	-0.0005
0	0.0011	-0.0011
-0.0004	0.0011	-0.0015
0.0006	0.0011	-0.0005
-2.672	-0.504	-2.1681
-2.27374E-13	0.0008	-0.0008
-0.001	0.0008	-0.0018
1.138	0.0008	1.13723
-2.27374E-13	0.0008	-0.0008
-1.137	0.0008	-1.1378
0	0.0008	-0.0008
-2.27374E-13	0.0008	-0.0008
2.27374E-13	0.0008	-0.0008

Table 0-4: Difference between change in geoid heights of GPS/Levelling to that of AFRgeo2019 geoid models after detecting outliers

ΔN(GPS) (m)	ΔN(AFRgeo2019) (m)	ΔN (m)	OUTLIERS
-	-	-	-
-0.0003	0.0011	-0.0014	FALSE
0.0006	0.0011	-0.0005	FALSE
-0.0006	0.0011	-0.0017	FALSE
0.0006	0.0011	-0.0005	FALSE
-0.0007	0.0011	-0.0018	FALSE
0.0003	0.0011	-0.0008	FALSE
-0.0002	0.0011	-0.0013	FALSE
0.0006	0.0011	-0.0005	FALSE
0	0.0011	-0.0011	FALSE
-0.0007	0.0011	-0.0018	FALSE
0.0002	0.0011	-0.0009	FALSE
0.0003	0.0011	-0.0008	FALSE
-0.0007	0.0011	-0.0018	FALSE
0.0004	0.0011	-0.0007	FALSE
0.0004	0.0011	-0.0007	FALSE
0	0.0011	-0.0011	FALSE
1E-04	0.0011	-0.001	FALSE
-0.0003	0.0011	-0.0014	FALSE
-0.0005	0.0011	-0.0016	FALSE
0.0006	0.0011	-0.0005	FALSE
0	0.0011	-0.0011	FALSE
-0.0004	0.0011	-0.0015	FALSE
0.0006	0.0011	-0.0005	FALSE
-2.672	-0.504	-2.1681	TRUE
-2.27374E-13	0.0008	-0.0008	FALSE
-0.001	0.0008	-0.0018	FALSE
1.138	0.0008	1.13723	TRUE
-2.27374E-13	0.0008	-0.0008	FALSE
-1.137	0.0008	-1.1378	TRUE
0	0.0008	-0.0008	FALSE
-2.27374E-13	0.0008	-0.0008	FALSE
2.27374E-13	0.0008	-0.0008	FALSE

The TRUE value means it is an outlier while for the FALSE values are not outliers, hence are good to perform assessment of the model.

The differences of the geoid models computed were done at Railway centerline points in which the following statistical analysis were determined.

- Maximum difference
- Minimum difference
- Mean differences
- Standard deviation (SD) and
- Root mean square (RMS) of the differences.

These statistical analysis of these geoid height differences was done using Microsoft Excel 2019 software based from equation (2.9) to (2.11), the analysis is used to judge at what value or amount these two geoid models depart from one another.

3.2 DATA AND SOFTWARE USED

3.2.1 Data Used

In this research there are two types of data that have been used upon the assessment of AFRgeo2019, these data include;

- AFRgeo2019 grid data which was first downloaded in isg file format then transformed to a grid file.
- The GPS/levelling from a profile located in Uvinza-Kigoma. A profile has 50km length where only 997 rail way centerline points were used which are consecutive to one another.

Table 0-5: Sample of a profile data used

	NORTHINGS	EASTINGS	H (m)	DESCRIPTION	Latitude (°)	Longitude (°)	h (m)
1	9523796.95	188132.561	1386.908	RAIL WAY CENTERLINE	-4.30309	30.19061	1375.794
2	9523758.345	188165.067	1391.811	RAIL WAY CENTERLINE	-4.30344	30.1909	1380.696
3	9523720.145	188197.155	1396.977	RAIL WAY CENTERLINE	-4.30379	30.19119	1385.863
4	9523681.736	188229.084	1400.82	RAIL WAY CENTERLINE	-4.30414	30.19147	1389.705
5	9523643.564	188261.078	1404.252	RAIL WAY CENTERLINE	-4.30448	30.19176	1393.138

	NORTHINGS	EASTINGS	H (m)	DESCRIPTION	Latitude $(^\circ)$	Longitude $(^\circ)$	h (m)
7	9523566.625	188325.348	1407.377	RAIL WAY CENTERLINE	-4.30518	30.19234	1396.263
8	9523528.473	188357.482	1407.579	RAIL WAY CENTERLINE	-4.30553	30.19262	1396.464
9	9523490.189	188389.462	1406.868	RAIL WAY CENTERLINE	-4.30587	30.19291	1395.754
10	9523451.764	188421.259	1405.837	RAIL WAY CENTERLINE	-4.30622	30.1932	1394.723
11	9523413.226	188453.607	1403.792	RAIL WAY CENTERLINE	-4.30657	30.19349	1392.677
12	9523375.08	188485.605	1401.834	RAIL WAY CENTERLINE	-4.30692	30.19377	1390.72
13	9523336.734	188517.739	1401.898	RAIL WAY CENTERLINE	-4.30726	30.19406	1390.784
14	9523298.387	188549.74	1404.266	RAIL WAY CENTERLINE	-4.30761	30.19435	1393.151
15	9523259.664	188581.665	1406.608	RAIL WAY CENTERLINE	-4.30796	30.19463	1395.494
16	9523221.688	188614.022	1408.576	RAIL WAY CENTERLINE	-4.30831	30.19492	1397.462
17	9523183.314	188645.979	1409.628	RAIL WAY CENTERLINE	-4.30865	30.19521	1398.514
18	9523144.839	188678.087	1409.211	RAIL WAY CENTERLINE	-4.309	30.1955	1398.097
19	9523106.294	188710.372	1408.722	RAIL WAY CENTERLINE	-4.30935	30.19579	1397.608
20	9523067.967	188741.919	1408.742	RAIL WAY CENTERLINE	-4.3097	30.19607	1397.627
21	9523029.778	188774.405	1408.864	RAIL WAY CENTERLINE	-4.31005	30.19636	1397.75
22	9522991.431	188806.183	1409.024	RAIL WAY CENTERLINE	-4.31039	30.19664	1397.91
23	9522952.916	188838.537	1409.848	RAIL WAY CENTERLINE	-4.31074	30.19693	1398.733
24	9522914.635	188870.394	1410.449	RAIL WAY CENTERLINE	-4.31109	30.19722	1399.335

3.2.2 Software Used

Software used under this research were;

- Golden Surfer software for gridding of the model and extraction of geoid heights.
- Microsoft Excel for arranging data and simple computation.
- Civil 3D for filtering the profile data so as to have consecutive point data.
- Microsoft World for report writing.

CHAPTER FOUR

RESULTS AND DISCUSSION OF RESULTS

4.1. RESULTS

This chapter describes the results obtained and the discussion of results. The computations and results under consideration are those related to the methodology and data described in Chapter 3.

4.1.1 Gridded AFRgeo2019

The grid file of the AFRgeo2019 was created for the coverage of all African continent from 39^0N to 36^0S latitude and from 20^0W to 53^0E longitude, where Microsoft excel was used to arrange the model data from isg format to a suitable format (.csv) that Golden Surfer software was able to interpret and grid the model. The results of the gridded AFRgeo2019 is shown in Figure 4-1**.**

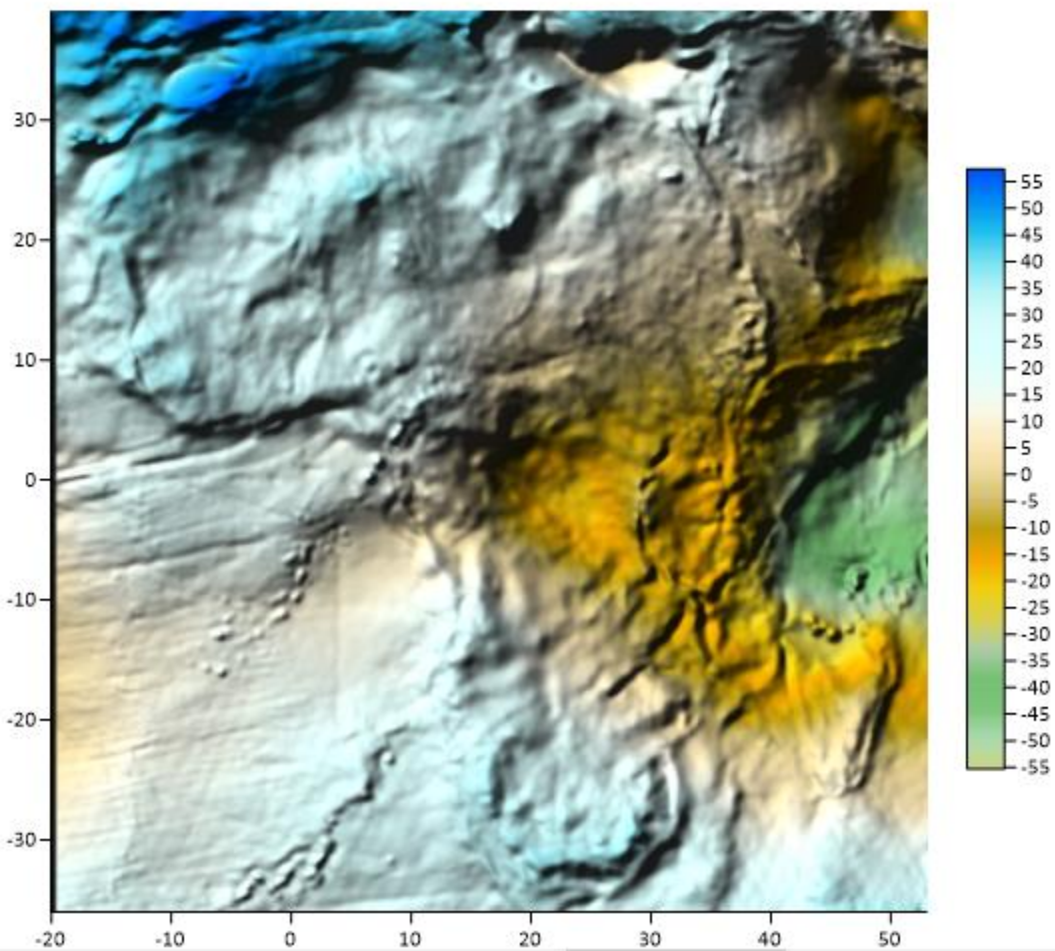

Figure 0-1: A map showing a gridded AFRgeo2019

4.1.2 Geometrical Geoid Heights and Physical Geoid Heights

Some of 973 geometrical geoid heights consecutive railway centerline points were obtained using equation (2.7) and the physical geoid heights extracted for all the geoid models are presented in Table 4-1

Table 0-1: Sample of the data with their geometrical and physical geoid heights

Latitude (°)	Longitude (°)	N(AFRgeo2019) (m)	ΔH (m)	Δh (m)	ΔN(GPS) (m)	ΔN(AFRgeo2019) (m)
-4.30309	30.19061	-11.5791	-	-	-	-
-4.30344	30.1909	-11.5802	4.903	4.9027	-0.0003	0.0011
-4.30379	30.19119	-11.5813	5.166	5.1666	0.0006	0.0011
-4.30414	30.19147	-11.5824	3.843	3.8424	-0.0006	0.0011
-4.30448	30.19176	-11.5835	3.432	3.4326	0.0006	0.0011
-4.30483	30.19205	-11.5846	2.303	2.3023	-0.0007	0.0011
-4.30518	30.19234	-11.5858	0.822	0.8223	0.0003	0.0011
-4.30553	30.19262	-11.5869	0.202	0.2018	-0.0002	0.0011
-4.30587	30.19291	-11.588	-0.711	-0.7104	0.0006	0.0011
-4.30622	30.1932	-11.5891	-1.031	-1.031	0	0.0011
-4.30657	30.19349	-11.5902	-2.045	-2.0457	-0.0007	0.0011
-4.30692	30.19377	-11.5913	-1.958	-1.9578	0.0002	0.0011
-4.30726	30.19406	-11.5924	0.064	0.0643	0.0003	0.0011
-4.30761	30.19435	-11.5935	2.368	2.3673	-0.0007	0.0011
-4.30796	30.19463	-11.5946	2.342	2.3424	0.0004	0.0011
-4.30831	30.19492	-11.5957	1.968	1.9684	0.0004	0.0011
-4.30865	30.19521	-11.5968	1.052	1.052	0	0.0011
-4.309	30.1955	-11.598	-0.417	-0.4169	1E-04	0.0011
-4.30935	30.19579	-11.5991	-0.489	-0.4893	-0.0003	0.0011
-4.3097	30.19607	-11.6001	0.02	0.0195	-0.0005	0.0011

4.1.3 Difference between change in geometrical height to that of AFRgeo2019 between consecutive points

Ellipsoidal height differences and orthometric height difference of the consecutive points at their location must be known basing on equation (2.5) and (2.6) respectively. GPS derived geoid heights and AFRgeo2019 geoid height difference between two consecutive points were computed by Equation (2.7) and (3.1) then the difference between them was computed and outliers were removed then tabulated in Table 4-2.

Table 0-2: The difference between ΔN^{GPS} and $\Delta N^{AFRgeo2019}$ with no outliers

ΔN(GPS) (m)	ΔN(AFRgeo2019) (m)	ΔN (m)
-0.0003	0.00113	-0.0014
0.0006	0.00111	-0.0005
-0.0006	0.00111	-0.0017
0.0006	0.00111	-0.0005
-0.0007	0.00112	-0.0018
0.0003	0.00111	-0.0008
-0.0002	0.00111	-0.0013
0.0006	0.00111	-0.0005
0	0.00111	-0.0011
-0.0007	0.00112	-0.0018
0.0002	0.00111	-0.0009
0.0003	0.00111	-0.0008
-0.0007	0.00111	-0.0018
0.0004	0.00111	-0.0007
0.0004	0.00111	-0.0007
0	0.00111	-0.0011
1E-04	0.00111	-0.001
-0.0003	0.00111	-0.0014
-0.0005	0.0011	-0.0016
0.0006	0.00112	-0.0005
0	0.0011	-0.0011
-0.0004	0.00111	-0.0015
0.0006	0.0011	-0.0005
-2.274E-13	0.00077	-0.0008
-0.001	0.00077	-0.0018
-2.274E-13	0.00077	-0.0008

0	0.00077	-0.0008
-2.274E-13	0.00076	-0.0008
2.2737E-13	0.00076	-0.0008
-2.274E-13	0.01357	-0.0136
0	0.00076	-0.0008
0	0.00075	-0.0008
0	0.00075	-0.0008
0	0.00076	-0.0008
2.2737E-13	0.00075	-0.0008
0	0.00076	-0.0008
0	0.0143	-0.0143
-2.274E-13	0.00075	-0.0008
2.2737E-13	0.00075	-0.0008
-0.001	0.00075	-0.0017
0.001	0.00075	0.00025
0	0.00075	-0.0007
0	0.00075	-0.0007
-0.001	0.00075	-0.0017
0.001	0.00075	0.00025
-2.274E-13	0.00074	-0.0007
2.2737E-13	0.00075	-0.0007
0	0.00075	-0.0007
0	0.00075	-0.0007
0	0.00074	-0.0007
-2.274E-13	0.00074	-0.0007
0	0.00074	-0.0007
0	0.00074	-0.0007
0	0.00072	-0.0007
0	0.00072	-0.0007
0	0.00057	-0.0006
2.2737E-13	0.00012	-0.0001
-2.274E-13	0.00067	-0.0007
0	0.00067	-0.0007
2.2737E-13	0.00064	-0.0006

Statistical analysis between the relative values of the GPS derived geoid heights and AFRgeo2019 geoid heights were computed and tabulated in Table 4-3.

Table 0-3: Statistical values of ΔN in meters

Attribute	MAX	MIN	MEAN	SD	RMS
ΔN	0.0025	-0.0143	-0.0004	0.0012	0.0012

From the Table 4-2**,** the deviation of AFRgeo2019 from GPS/levelling derived geoid is presented graphically in Figure 4-2.

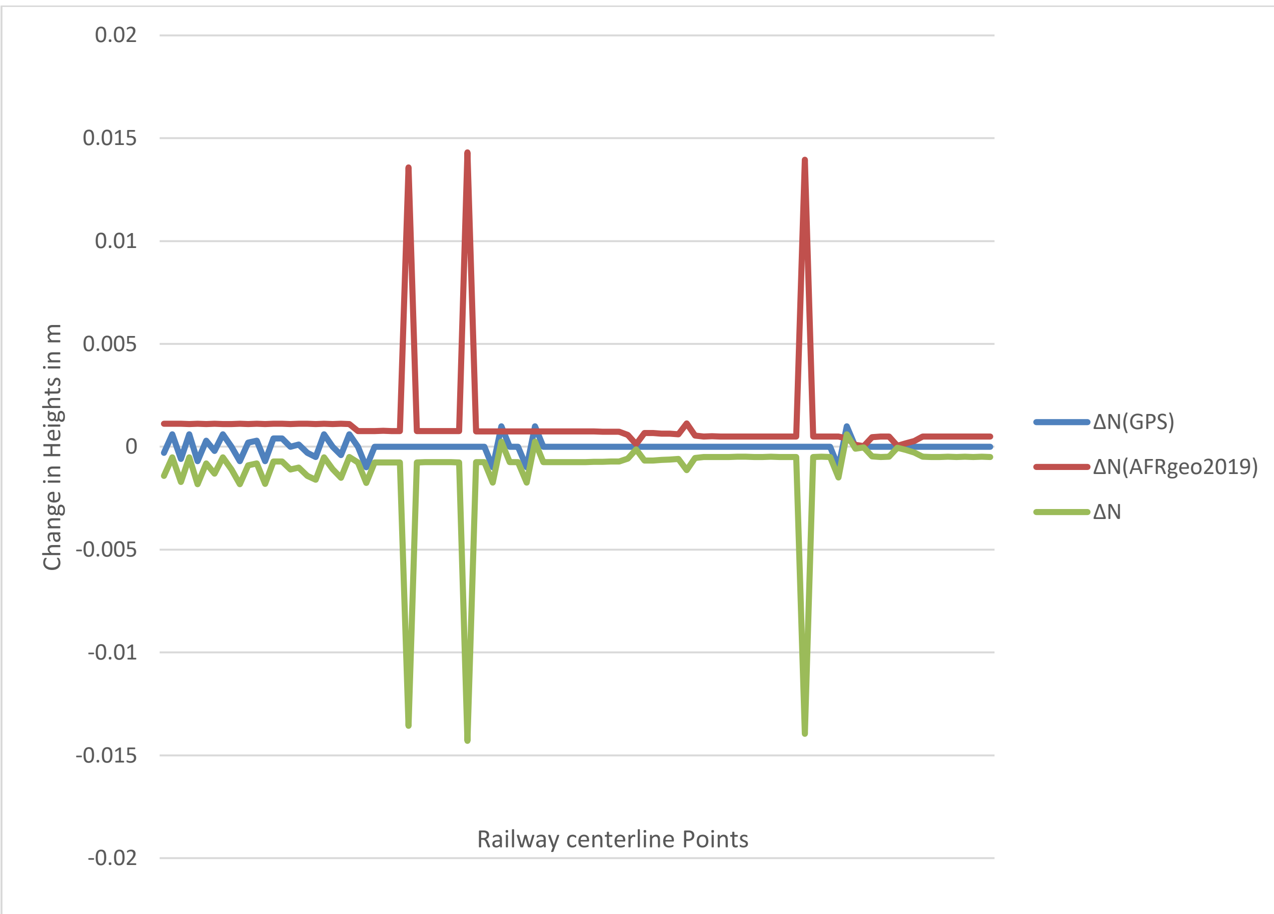

Figure 0-2: A graph representing sample of difference between ΔN^{GPS} and $\Delta N^{AFRgeo2019}$

4.2 DISCUSSION OF RESULTS

Table 4.3 summarizes the results obtained after the difference between change in geoid height derived from GPS and the change in geoid height as computed from the AFRgeo2019. The computations of AFRgeo2019 provided the least values of Mean, STD and RMS which were -0.00043m, 0.001166m and 0.001242m when compared with 973 GPS points of railway centerline at 95% confidence level over the AOI respectively. This shows that AFRgeo2019, provided best results since the measure of the central tendency on the difference between GPS derived geoid heights and those from AFRgeo2019 vary for millimeter up to sub-millimeter accuracy.

For the case of GPS railway centerline points, geoid heights differences between GPS derived geoid and AFRgeo2019 were examined through statistics at 95% confidence level since the sample size (n = 997) was large enough to assume a normal distribution at 95% confidence level for the mean. The outliers were first rejected basing on the confidence limits at 95% confidence level and remained with (n=973) points suitable for the assessment. The computations of ΔN had the these values of Mean, STD and RMS which were -0.0004 m, 0.384 m and0.3839 m at GPS railway centerline points before rejecting 2.4% of the data which is equal to 24 values out of 997, AFRgeo2019.

Therefore, AFRgeo2019 computed over AOI performs better and hence it can be used in Kigoma.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1. CONCLUSION

The evaluation of AFRgeo2019 in Tanzania was successfully achieved as the model provided the best results on the location where it was used. The results were based on statistics of the differences between geometrical geoid heights as have been derived from GPS/levelling and physical geoid heights from AFRgeo2019. AFRgeo2019 provided the least values of Mean, STD and RMS which were -0.0004m, 0.0012m and 0.0012m when compared with 973 GPS points of railway centerline at 95% confidence level over the AOI respectively. Although the new Regional model used different kind of gravity data such as ground gravity data and ship born gravity data, there still exist a large gravity information gap on some of the areas where interpolation of such information was done to obtain mean gravity of a particular square during computation of the model which is just an approximation of the actual gravity on that area. Thus the magnitudes of the statistics of the differences between the geometrical geoid heights from GPS/levelling and AFRgeo2019, i.e. Mean, STD and RMS could significantly decrease if the area will be fully covered with gravity information.

5.2. RECOMMENDATION

From the results obtained and conclusion of this study the following are recommended,

1) Next researchers should use more than one profile at different locations in Tanzania to have high confidence on using the model as the AOI under this research is very small compared to Tanzania.
2) Efforts should be made on all the data that are collected using GNSS technique, should also be exported in the form (φ, λ, h, H) as they are useful for research purposes.
3) Next researcher should perform assessment on the methods to be used between relative method and geoid slope method when assessing these regional and local geoid models since both use relative points when assessing the model.

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