

ARDHI UNIVERSITY



**VALIDATION OF SHIP TRACKS AND AERIAL GRAVITY DATA
USING LATEST MARINE GRAVITY ANOMALIES.**

A Case Study: Tanzania Indian Ocean.

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BSc (Geomatics)

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VALIDATION OF SHIP TRACKS AND AERIAL GRAVITY DATA USING LATEST
MARINE GRAVITY ANOMALIES

A Case Study: Tanzania Indian Ocean.

ARRAY REHEMA K.

A Dissertation Submitted to the Department of Geospatial Sciences and Technology in
partial fulfilment of the Requirements for the Degree of Bachelor of Science in Geomatics
(BSc. Gm) of Ardhi University.

CERTIFICATION

The undersigned certify that they have read and hereby recommend for acceptance by the Ardhi University a dissertation titled “**Validation of Ship Tracks and Aerial Gravity Data Using Latest Marine Gravity Anomalies**” in partial fulfilment of the requirements for the award of degree of bachelor of science in Geomatics at Ardhi university.

.....

Ms. Regina V. Peter

(Supervisor)

Date

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I Rehema K Array declare that, the content in this dissertation are the result of my own findings through my study and investigation and to the best of my knowledge they not been presented anywhere else as a dissertation for diploma, degree or any similar academic award in any institution of higher learning.

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DEDICATION

I dedicate this dissertation to my beloved parents Mr. & Mrs. Array, my aunt Yustina, my lovely sister Neema and my brother Tumaini and my close friends for their invaluable encouragement, prayers, unlimited love, patience, support and care throughout my studies. I really appreciate you for all you have done to us. I love you, God bless you all.

ABSTRACT

This study is set out to validate ship track and aerial gravity data using latest marine gravity anomalies from three different models that is SDUST2021, DTU15 and DTU17 along the coast of Tanzania. The ship track gravity dataset contain position, height and observed surface gravity point. Satellite altimetry have the resolution of 1-arc minute with position and surface gravity anomalies. The aerial gravity dataset has the resolution of 5-arc minute with position and surface gravity.

The ship track gravity anomaly data was in GRS67, therefore transformation to GRS80 was done so that the data will be both in the same reference system as gravity anomaly from satellite altimetry is in WGS84 reference system. Validation of gravity anomaly was done by computing statistical differences of the gravity anomaly. In processing the research data and results the software used were Golden Surfer, Mat Lab and Microsoft Excel. The differences between ship track and Aerial gravity data with Gravity Anomalies (DTU15, DTU17 & SDUST2021) were computed at Ship track and aerial gravity position.

Ship track was validated against SDUST 2021, DTU15 and DTU17 at 95% confidence level and the result had the Mean, STD and RMS values which were close to each other. Ship track with SDUST2021 yielded mean value of 15.80mGal, STD of 27.63mGal and R.M.S of 39.65mGal, Ship track with DTU15 yielded mean of 15.92mGal, STD of 27.50mGal and R.M.S of 39.65, ship track with DTU17 yielded mean of 15.81mGal, STD of 27.66mGal and R.M.S of 37.20mGal.

Aerial Gravity Anomaly was validated against SDUST 2021, DTU15 and DTU17 at 95% confidence level and the result had the Mean, STD and RMS values which were close to each other. Aerial gravity anomaly with SDUST2021 yielded mean value of -19.95mGal, STD of 20.96mGal and R.M.S of 32.99mGal, aerial gravity with DTU15 yielded mean of -20.18mGal, STD of 20.89mGal and R.M.S of 32.92 and aerial with DTU17 yielded mean of -20.31mGal, STD of 20.85mGal and R.M.S of 33.02mGal.

Keywords: Validation, Aerial gravity data, Marine gravity Anomalies, Surface gravity Anomalies, 95% confidence level, RMS.

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LIST OF ABBREVIATION

ARU	Ardhi University
AOI	Area Of Interest
ARUDB	Ardhi University Database
BGI	Bureau Gravimetrique International
DTU	Technical University of Denmark
EGM	Earth Gravity Model
GOCE	Gravity Field and Steady State Ocean Circulation Explorer
GPS	Global Position System
GRACE	Gravity Recovery and Climate Experiment
GRS80	Global Reference Surface 1980
IAG	International Association of Geodesy
Mat lab	Matrix Laboratory
mGal	milli gal
MSL	Mean Sea Level
RMS	Root Mean Square
SD	Standard Deviation
WGS84	World Geodetic System 1984

CHAPTER ONE

INTRODUCTION.

1.1 Background

The computation of a gravimetric geoid model for the Tanzania region is mainly hindered by the limited availability of gravity observations. Furthermore, the quality of data is questionable since they have been gathered from different organization for various purposes and with varying accuracies over time (Ulotu, 2009). The determination of Geoid is crucial as it represents the physical figure of the Earth, and serves as the vertical Datum for orthometric heights, a widely used height system worldwide (Heiskanen & Moritz, 1967).

The Geoid is an equipotential surface of the Earth's gravity field, which coincides with the sea surface in a least square sense and extends continuously beneath the continental masses (Vaníček and Krakiwsky 1986). In physical geodesy, errors in gravity anomalies propagate into geoid models, thereby affecting the transformation parameters of GPS-derived heights. It is essential to identify and remove such errors for both disciplines (Sandwell & Smith, 1997).

There are various sources of gravity data, including Terrestrial (ground) gravity, Marine (ocean and great lakes) gravity, Satellite Gravimetry, Satellite Altimetry and Aerial gravity. In Tanzania, there are four absolute 1st order and 56-relative 2nd order reference gravity stations, Terrestrial-Marine (ocean and great lakes), 7,345 ship track point gravity data. These gravity data are full spectrum to which they are required for geoid computation but they are insufficient due to limited gravity information from terrestrial gravity in Tanzania. Although the Earth contains 30% land and 70% water bodies, establishing gravity stations on the ocean has been challenging due to instability and the difficulty in accurately determining the position on water. Marine gravity measurements from shipborne platforms are notoriously problematic, primarily due to dynamic acceleration and inaccurate vessel navigation (Featherstone et al.,1997).

Scientific and technological advancements in geodesy include dedicated satellite gravity field determination from satellites missions (satellite gravimetry). Global gravity determination from these sources began in the year 2000 with the CHAllenging Mini-satellite Payload (CHAMP) mission. The primary objective of the CHAMP mission, launched on July 15, 2000, was to determine the global gravity field of the Earth. The mission utilized the high-low satellite-to-satellite tracking (hl-SST) technique, enabling it to observe numerous GNSS satellites with a good constellation and sense high-frequency gravity field variations.

For more information on the CHAMP mission and its data, refer to Featherstone (2005). CHAMP completed its mission after approximately 10 years and re-entered the Earth's atmosphere.

In 2002 a second satellite mission by the name of Gravity Recovery and Climate Experiment (GRACE) was launched. GRACE was designed as a follow-on to CHAMP, with an overlapping period of approximately two years. The mission's primary objective was to determine different Earth's gravity field parameters, with a particular focus on high-resolution global Earth's gravity field (Hofmann-Wellenhof & Moritz, 2005). For details about the system design, operation, data acquisition techniques, and more, refer to Hofmann-Wellenhof & Moritz (2005) and Featherstone (2005).

Another mission, the Gravity field and steady-state Ocean Circulation Explorer (GOCE), was launched on March 17, 2009. Operated by the European Space Agency (ESA) as part of its core mission under the Living Planet Program, GOCE employed the satellite gravity gradiometry technique. It was designed to acquire very high-quality data to produce accurate geoid models with a resolution of less than 2 cm, as well as gravity anomalies (Hofmann-Wellenhof & Moritz, 2005). The mission concluded in 2013.

After the launch and successful operations of the CHAMP, GRACE and GOCE missions, there has been remarkable progress in scientific fields that utilize the products of the three missions, example, geodesy and specifically physical geodesy which deals extensively with gravity data. These missions have facilitated global determination of gravity anomalies at resolution of $5' \times 5'$ mGal (Nathalie, 2011), enabling the computation of high resolution geoid models such as TZG13 (Forsberg et al., 2013).

Currently, new and improved technologies are being developed for future gravimetric satellite missions. These missions aim to achieve gravity data of very high quality, with high spectral and temporal resolution and greater accuracy compared to previous missions (Koop, 2007). The new technologies to be utilized in these missions include laser metrology, accelerometers, positioning sensing, and micro-propulsion systems. Some of these missions were expected to be launched as early as 2018 (Koop & Rummel, 2007; Canuto et al., 2017).

Additionally, the method of satellite altimetry, designed to map sea surface topography, utilizes radar techniques to measure ranges with $1' \times 1'$ resolution. The derived information, such as Sea Surface Topography, can be used to obtain mean sea surface (MSSH) and subsequently the

marine gravity by the relation $N^m = \text{MSSH} - \text{MDT}$. Satellite altimetry data are more accurate in deep water than in shallow water due to contamination from the land and part of the sea surface (Sandwell and Smith, 1997).

Despite these various sources of gravity data, precise geoid computation in Tanzania has not achieved millimeter accuracy due to biases in the data (Kimonge, 2016), (Busega and Kimboi, 2018), (Ulotu, 2009). The importance and necessity of quality terrestrial gravity data of sound distribution, for the determination of accurate precise geoid model has neither changed nor expected to be replaced in the near future. Some of the major factors which hindered Tanzania from determining its own geoid model, until it was done so from outside in 2007. Collecting, cleaning and creating a new gravity database should be of the highest priority. The available gravity data: including ship track, aerial gravity data and the gravity anomalies have been collected from the Ardhi University Database (ship track and aerial gravity data) and others have been downloaded (gravity anomalies) so that the biases can be determined and thus improve the quality of data or opting for the resurvey.

1.2 Statement of Research problem

From the previous studies, it has been proven that ship track gravity data and aerial gravity data are subjected to biases, resulting in unreliable data. However, these data have been used extensively used in geoid determination, which serves as vertical Datum for Tanzania. This calls for the users and producers of gravity database verify and identify gravity data as best as they can before use or rejection. Since there is limited auxiliary information available in Tanzania regarding the data quality, the implementation of gross error detection procedures becomes crucial. Therefore, it is important to validate the shipborne gravity data and aerial gravity data to assess their reliability. If these datasets are deemed unreliable, a repeat of the aerial gravity survey for Tanzania should be conducted.

1.3 Objective of the research

The objective of this study is to validate the ship track and aerial gravity data using gridded gravity anomalies (SDUST2021GRA & DTU15).

1.4 Significance

The output of this research can be used in computation of geoidal model of Tanzania since aerial gravity determines the medium wavelength components of the gravity signal. Also used in Tanzania Gravity Database and thus densifying the gravity information which are reliable.

1.5 Beneficiaries

This study aims at increasing the data quality which greatly improves the resolution of marine gravity field, also will benefit the:

- i. The Government on the Ministry of Land, on the department of Geodesy whereby the data will be used to update their database for further study and research.
- ii. Educational institutes such as Ardhi University, will improve the ARUDB.
- iii. Individual personnel such as students to which they will benefit from reliable gravity data.

1.6 Scope and limitation

The study area is limited within 2°N to 12°S latitude and 39°E to 43°E longitude on which it is the Tanzania Indian ocean including Isle of Pemba, Unguja and Mafia. The isles will be blanked so as to not contaminate the results on land.

1.7 Outcomes

The outcome of this study are the statistics of gravity anomalies differences from aerial gravity data and marine gravity anomaly (DTU15 &SDUST2021) and from ship tracks and marine gravity anomaly at a common point.

1.8 Description of the study area

This study covers the coast of Tanzania located on the east Tanzania and Zanzibar. It extends from longitude 26°E to 44° E and from latitude 4°N to 15°S as shown below in Figure 1-1.

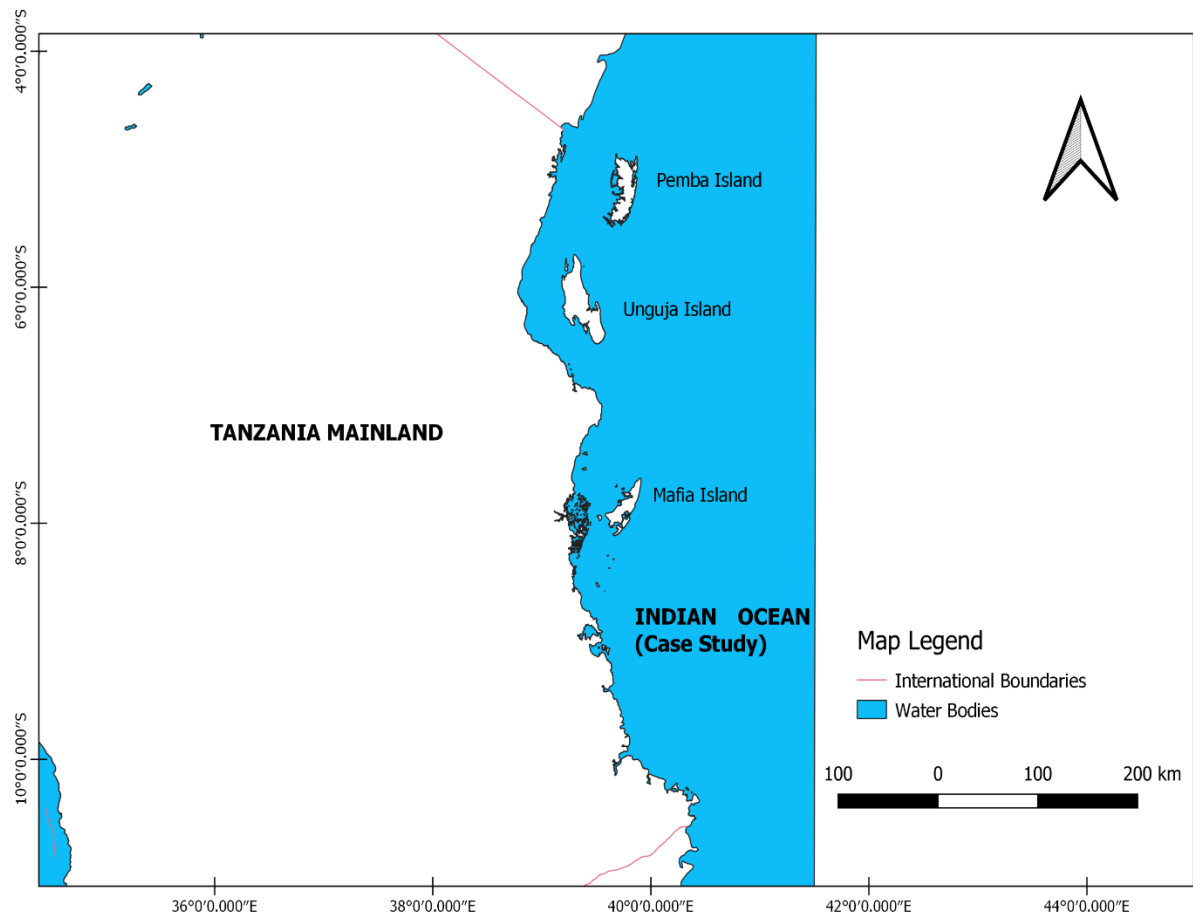


Figure 1- 1: A map showing case study (Along the coast of Tanzania)

1.9 Organization of the report

This study is devoted to validation of ship track and aerial gravity data using latest marine gravity anomalies in the coast of Tanzania organized in five chapters as follows.

Chapter One includes the Introduction: specifying background and purpose of the study, identification of the problem, objectives, significance of the research, beneficiaries, outcomes, description of the study area.

Chapter Two provides an overview of gravity data in Tanzania, Data availability situation and short coming of gravity data in Tanzania.

Chapter Three details on the methodology of dissertation by describing all relevant datasets, procedures and mathematical models required to achieve the objective of the study.

Chapter Four gives all the results from the processing of data outlined in the methodology part, it is also in this chapter were the results are analyzed and discussed.

Chapter Five consists of conclusion and recommendations. Conclusion summarizes dissertation findings in view of the solution to dissertation problem followed by recommendation to be considered by future dissertators related to this dissertation.

CHAPTER TWO

LITERATURE REVIEW

2.1 Overview of Gravity Data in Tanzania

Tanzania is one of the few African nations with access to practically all gravity data sources. Aerial, satellite, terrestrial, and marine altimetry gravity are all included in the gravity data. However, Tanzania does not have a single public agency that collects, stores, and preserves data from local gravity surveys. In Tanzania, there are several organizations that keep track of gravity information, and some of them have even created their own gravity databases. Tanzanian gravity sources can be divided into two categories: those that originate domestically and those that originate abroad. Tanzanian sources for gravity information include the following:

- a) Ardhi University under the Department of Geospatial Sciences and Technology (DGST).
- b) University of Dar es salaam (UDSM) Department of Geology.
- c) Tanzania Petroleum Development Center (TPDC).
- d) Ministry of Minerals and Energy.
- e) Ministry of Lands, Housing and Human Settlements Development.
- f) Eastern and Southern African Mineral Resources Development Centre.

The overseas sources of Tanzania gravity data are:

- a) Bureau Gravimetrique International (BGI), which has a task of storing the gravity data from all over the world.
- b) Gravity and Geoid for Africa (GGA)
- c) USA National Geospatial Agency (NGA)
- d) Global Exploration Technology (GETECH), it provides the gravity data for Oil and mineral exploration and also for dissertation purposes.
- e) Danish National Space Centre (DNSC) of the Technical University of Denmark (DTU) which provides the satellite altimetry marine gravity data. (Busega and Kimboi,2018).

Prior to the establishment of the Potsdam datum in the early 1950s, Tanzania began to observe gravity in the 1890s, with references to a variety of local datums (Ulotu, 2009).

However, it is important to ensure that heights are measured using a unified known vertical datum and that gravity information is compared to a standard worldwide datum. Gravity measurements made after the International Gravity Standardization Network (IGSN71) was established were invariably standardized to Potsdam and IGSN71.

A network of connected gravity bases has been established in Tanzania in order to offer a unified datum for the whole country. Seven interconnected IGSN71 gravity stations plus the 40 base that Overseas Geological Surveys (OGS) established as part of the Eastern and Central Africa Gravity Net (ECAGN) between 1958 and 1961 make up a network (Ulotu, 2009).

Sowerbutts (1968) created Tanzania's first gravity database, which has since continued to advance over time. Review of Gravity Survey Tanzania, a 1991 report by Dr. Marobhe I. from the University of Dar es Salaam and Mr. Parker M. from the East and South Africa Mineral Resource Development Center (ESAMRDC), collected all known gravity data generated in Tanzania, despite the fact that some gravity surveys were not included in this report. This report attempted to construct a Tanzania gravity database at the time, however it is unclear if the database was actually produced. The University of Dar es Salaam, ESARMDC, and TPDC collectively saved around 14000 points of gravity data for the report.

As one of the current objectives of International Association of Geodesy (IAG) is to establish a global homogeneous gravity potential Vertical datum, a task assigned to Global Geodetic Observing System (GGOS) (Ulotu, 2015). The data for a reliable gravimetric geoid model for Tanzania are ready such that there are data from difference sources as follows:

2.2 Terrestrial Gravity

The gravity is observed by the use of a gravimeter which is an instrument used for measuring the local gravitational field difference of the Earth. The terrestrial gravity can be on the land or on the water and these gives the full spectrum which is essential for the determination of Geoid.

There are two types of gravimeters: relative and absolute. Absolute gravimeters measure the local gravity in absolute units, gals. Relative gravimeters compare the value of gravity at one point with another. They must be calibrated at a location where the gravity is known accurately, and then transported to the location where the gravity is to be measured. They measure the scaled difference of the gravity at the two points.

2.2.1 Absolute gravimeters

An absolute gravity instrument measures the true value of gravity each time it makes a measurement. In general, absolute gravity instruments are typically far more expensive, much bigger, take much longer to make high precision measurement and usually require more knowledge and skill to use than do relative gravity instruments. By basing the measurement on these standards, the system is inherently calibrated and will neither drift nor tare over time. (Torge, 1986).

2.2.2 Relative gravimeters

Relative determination of gravity is made using gravity meters known as gravimeters. They are portable and handier to operate, consequently they are more widely used. The accuracy of conventional gravimeters is about $+0.01 \text{ mGal}$ ($+ 10 \text{ } \mu\text{gal}$). The three major problems that can be listed against the gravimeters are: the inability to measure values of gravity directly, the need for frequent calibrations and the presence of drift due to aging of various components of the instrument (Saburi, et al.,2010).

2.2.3 Gravity networks

There are 4-absolute 1st order which are at TANZI at Ardhi University, Dar es Salaam International Airport, Mwanza Airport and in Tukuyu Mbeya, and 56- relative 2nd order reference gravity stations in Tanzania mainland (Saburi, et al., 2010) as shown in Figure 2-1 below.

The setbacks of the Terrestrial gravity (ground) is limited to the landmasses and in national parks and most of the thick forests there is no gravity information and on the Marine part the time changes due to tides and the mass changes during observation and thus becoming tedious due to ship tilting and the stations cannot be established on water meaning going back and forth.

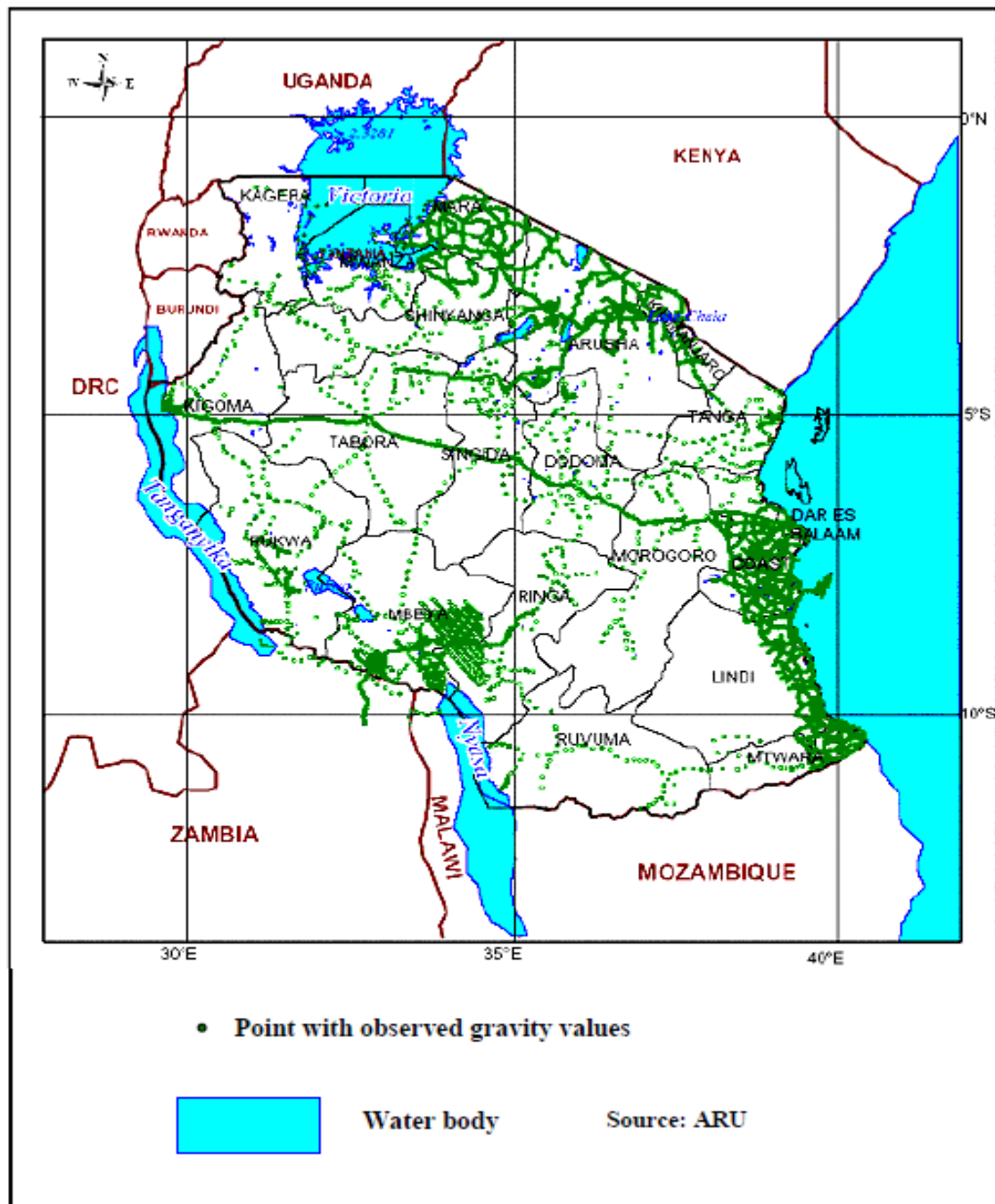


Figure 2-1: Map of Tanzania showing points with available gravity stations (Saburi, et al., 2010)

2.3 Aerial Gravity

It involves mounting a gravimeter on an airplane and records the flying height. There are limited due to sparse of data and cannot get up to 20km. Because of the flying height they lack very high frequency and provides almost full spectrum because they are 5-10 km above the ground. On downward continuation of the aerial gravity to the earth surface or the geoid the dumped frequencies cannot be restored or improved therefore aerial gravity data can only complement ground gravity but not to substitute it fully (Forsberg et al., 2013).

2.4 Satellite Gravimetry

To solve the problem of full spectrum gravity availability everywhere on Earth including marine areas, dedicated satellite mission were initiated. It started in the year 2000, when Challenging Minisatellite Payload (CHAMP) was launched by Germany which has degree order 80. In 2002 Gravity Recovery and Climate Experiment (GRACE) which was used to determine gravity parameters globally with high resolution with degree order of 140 and operated for 5 years. In March 2009, Gravity field and Steady-state Ocean Circulation Explorer (GOCE) was launched with degree order of 330 and operated for 5 years. In the year 2018, GRACE-FO was launched due to failure of GRACE (Hofmann-Wellenhof & Moritz, 2005). Satellite Gravimetry does not have the full spectrum only low and few medium frequency, currently at degree order 5540 and for a full spectrum should have d/o >21,000 because the long wavelength are good but in gravity we need the full spectrum and in turn we will get smooth geoid which is not realistic.

2.5 Satellite Altimetry

It was designed to map the sea surface Topography. It employs radar technique in measuring range. It has limited resolution that is high frequency cannot be observed only few medium and low frequencies (Ablain, et., 2016). Even if we have the best continental gravity will not be continuous as the water covers 70% of the earth.

CHAPTER THREE

METHODOLOGY

This section describes the methods used to validate the ship track data and aerial gravity data against gravity anomalies from satellite altimetry along the coast of Tanzania. The chosen validation technique involves comparing the ship track surface gravity as well as the aerial gravity with gravity derived from satellite altimetry that is DTU15, DTU17 and SDUST 2021 which are global marine gravity model. The selected method fulfils the objectives of this research which intended to validate the observed ship track gravity data and aerial gravity data using the gravity anomalies which are assumed to be superior along the coast of Tanzania.

The validation will be done following the procedure outlined below:

- i. Transformation of the surface gravity anomaly from satellite altimetry based on GRS80 and calibrated against IGNS71 using international gravity formula IGF1980.
- ii. Interpolation of satellite altimetry gridded gravity at ship track positions.
- iii. Interpolation of satellite altimetry gridded gravity at Aerial gravity gridded data.
- iv. Interpolation of aerial gravity gridded gravity at ship track positions.
- v. Calculation of statistics for the difference between ship track gravity and satellite altimetry gravity.
- vi. Calculation of statistics for the difference between aerial gravity anomalies and satellite altimetry gravity.
- vii. Statistical analysis of the differences to test the validity between the gravity data.

3.1 Data type

This section provides details on the data used in this research and the way they have been acquired and prepared. The types of data include ship borne gravity data, satellite altimetry data and aerial gravity data.

3.1.1 Ship track gravity data

The ship track point gravity data were obtained from BGI. There were approximately 10214 points gravity values that were provided within the bound the area of latitude 15° S to 5° N and longitude 26° E to 44°E, distribution of point gravity data in the area of interest, each data includes geographical coordinates (latitude and longitude), elevation, gravity, free air and simple bouguer gravity anomalies also meta data for the file. Although the BGI metadata shows that the elevations of the gravity records refer to the ocean surface, it does not explain where

the gravity was observed, and the code supplied is annotated “No information”. This is further supported by the fact that in some records, the supplied gravity anomalies can be computed accurately from the record data, but there are other instances when it is not possible. The BGI records explain clearly that the free-air gravity anomalies have been reduced to the geoid model. Given this ambiguous situation, we opted to work solely with the supplied free-air point gravity anomaly on the ocean surface after transformation from GRS67 to GRS80 according to Eq.

$$\Delta g_{\text{GRS80}} = [\Delta g_{\text{GRS67}} + (0.8316 + 0.0782 \sin^2 \varphi - 0.0007 \sin^4 \varphi)] mGals \quad (3-1)$$

Where: Δg Refers to ‘Free Air Surface Gravity Anomaly’

3.1.2 The Satellite Altimetry data (Gravity Anomaly)

The SDUST 2021 GRA was downloaded to which is the global Marine Gravity Anomaly model on grid of 1'x 1' which is established from Altimeter data of Ka-band and Ku- band altimeter satellite with coverage of 80° N to 80° S and accuracy of 2.37 mGal. Also the marine gravity model of DTU 17 (Andersen and Knudsen, 2019) released by DTU and the model SIO V30.1 (Sandwell et al., 2021) on 1'x 1' grids established from altimeter data in the similar period released by the Scripps Institution of Oceanography (SIO). The data format is that of Gravsoft package grid data, (Tscherning et al. 1994) i.e. grid data for the AOI contains a header describing the area and resolution of the grid data given by minimum and maximum latitude and longitude respectively, then grid spacings in latitude and in longitude followed by marine gravity anomalies (Andersen and Knudsen 2003). The data will be used to validate the ship track and aerial gravity data thus assumed to be superior.

3.1.3 Limiting Satellite Altimetry Data to the ocean part

Marine gravity anomalies were obtained from the DTU15 and SDUST2021GRA altimetry data. Since the DTU15 1' x 1' and SDUST 2021GRA 1' x 1' are from altimetry, its quality in the land areas is usually not to the standard needed. Therefore, we started by blanking land altimetry data (i.e. limiting DTU15 and SDUST2021GRA to ocean part) so that it does not mix up and compromises the entire gravity data quality in the subsequent processes of merging sea and land gravity data.

The procedures were as follows;

1. The Indian Ocean coastline for Tanzania mainland from 4° north to 15° south and the boundaries of the three isles where the DTU15 1' × 1' grids cover (i.e. Unguja, Pemba, Mafia and Comoros) were extracted from the African shape file. The shape file was downloaded from <http://www.maplibrary.org>. In Global Mapper software the coordinates of the vertices of the coastline and the isles' boundaries were extracted systematically in clockwise direction closing to the opening vertices. The coordinates were then copied and pasted to the Surfer's worksheet and saved as BLN Golden Software Blanking (.bln).

2. The blanking files were used to blank the DTU15 1' × 1' grids in systematic way as follows: First the DTU15 1' × 1' grids were blanked outside the Indian Ocean coastline for the AOI's mainland boundary line.

Secondly all the isles were blanked-in from the grid obtained from the first step in logical way one after another to finally obtain the grid with no land coverage.

3.1.4 Aerial gravity data

Tanzania is now fully covered by 5' x 5' surface aerial gravity anomaly of about 5mGal precision as of Dec. 2013. Acquisition of the data took place between 2012 and 2013 and the pattern are as shown in Figure 3-1 below. Air gravity data acquisition technique is much affected by weather condition which can cause aircraft turbulence. Therefore, data collection was planned to take place when the temperatures were most stable and that was just after midnight until around 08.00am. Generally, weather condition was good and hence data quality was almost guaranteed. A few times they experienced some difficulties of plane instability, where turbulence was a big problem, the areas were re-flown. Validation of data was conducted and they were satisfied with the results (Forberg & Olsesen, 2013). The airborne data used was converted into points and downward continued to the surface by a software called Gravsoft and there were about 15,111 points which were confined to the area of study.

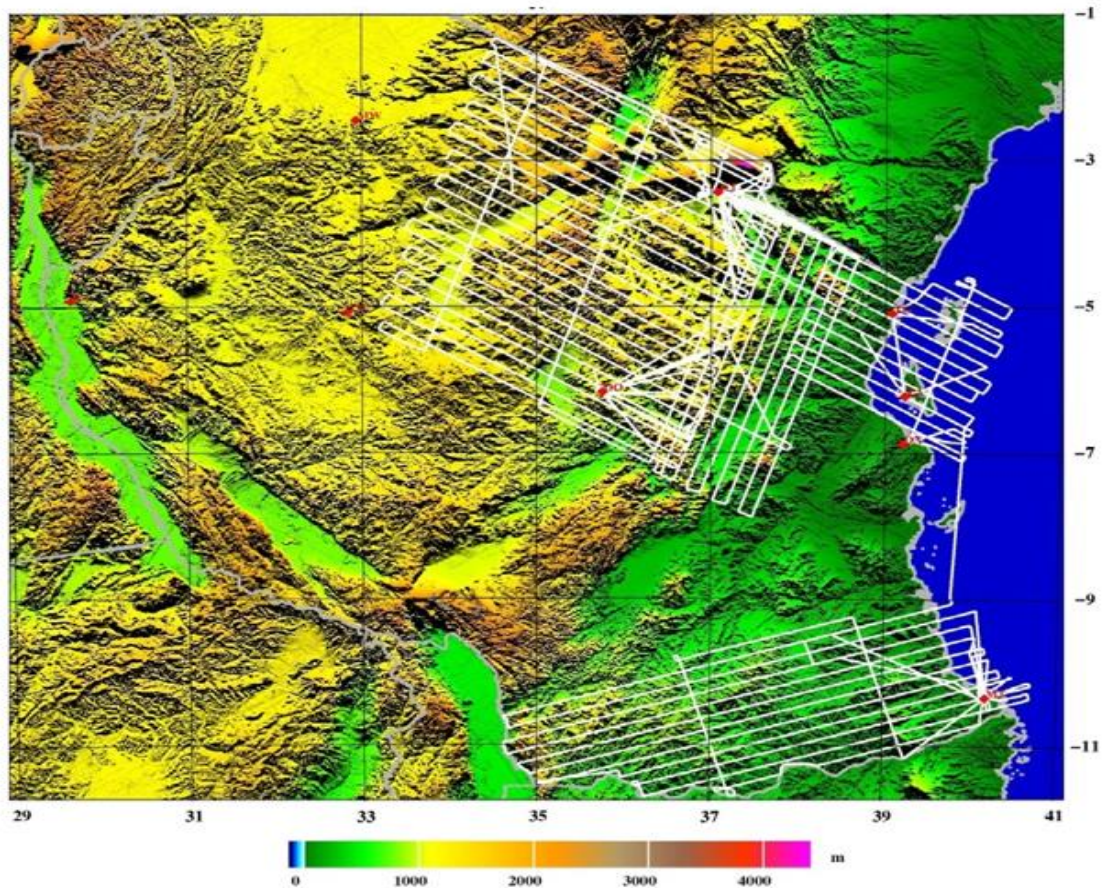


Figure 3-1: Aerial Ground track pattern (Olesen, 2013)

3.2 Cross Validation (XV)

Visual inspection and cross validation to the point gravity data were performed to ensure that the data were clean that is free from gross and systematic errors. According to (Abdalla, 2009) among three useful gridding algorithms (Kriging, Inverse Distance Weighting and Nearest Neighbor), it is Kriging algorithm that gives minimum residuals thus this algorithm was used in cross validation of point gravity data to detect and remove outliers according to a particular confidence level.

XV process can be explained as follows: Given known values at N observation locations in the original data set, XV assesses the relative quality of the N data by removing one observation at a time from the data set, and use the remaining data in the neighborhood and the specified

algorithm in this matter Kriging, to predict a value at the removed observation location (Ulotu, 2009).

3.3 Software used

In this study, various software were used such as:

- a. Golden Surfer Software It was used in gridding, merging and conversion of grid formats, extraction of data and computations of statistics.
- b. Global Mapper it was used convert tiff file format to grid format.
- c. Mat lab was used to compute the ship track gravity anomaly in GRS80 reference system.
- d. Microsoft Excel It was used in arranging data and computation of statistics
- e. Microsoft Word in Report writing.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Results

This chapter describes the results obtained and the discussion of results. The computations and results under consideration are those related to the methodology and data described in Chapter 3.

4.1.1 Interpolation of gravity at ship track positions using satellite altimetry

The satellite altimetry gravity was provided in grid value of 1' x 1' along the coast of Tanzania. To get the predicted gravity along ship track position, Kriging method was used through Surfer version 7. Kriging as an interpolation method required values at the sought points using residual approach (Bohling,2005). Table 4-1, Table 4-2 and Table 4-3 below shows the samples of the interpolated gravity from DTU15, SDUST2021 and DTU17 at ship track position.

Table 4-1: Samples of Gravity Anomaly from DTU15 at ship track position

S/No	Longitude (d.ddd)	Latitude (d.dddd)	dg_DTU15 (mGal)
1.	40.246	-4.2275	-4.82715
2.	40.247	-4.224	-5.29023
3.	40.247	-4.255	-1.40885
4.	40.248	-4.1969	-9.76751
5.	40.248	-4.2667	-0.3536
6.	40.248	-4.5933	-8.12883
7.	40.249	-4.2205	-5.66472
8.	40.25	-4.217	-6.09
9.	40.252	-4.2135	-6.53476
10.	40.252	-4.59	-9.3162

Table 4-2: Samples of Gravity Anomaly from SDUST2021 in ship track position

S/No	Longitude (d.ddd)	Latitude (d.dddd)	dg_sdust2021 (mGal)
1.	40.246	-4.2275	-5.751383165
2.	40.247	-4.224	-5.960625903
3.	40.247	-4.255	-3.411952154
4.	40.248	-4.1969	-9.873319273
5.	40.248	-4.2667	-2.312724811
6.	40.248	-4.5933	-10.52653337
7.	40.249	-4.2205	-6.146813366
8.	40.25	-4.217	-6.415537028
9.	40.252	-4.2135	-6.849632446
10.	40.252	-4.59	-11.43792165

Table 4-3: Samples of Gravity Anomaly from DTU17 in ship track position

S/No	Longitude (d.ddd)	Latitude (d.dddd)	dg_DTU17 (mGal)
1.	40.246	-4.2275	-7.4652
2.	40.247	-4.224	-7.96044
3.	40.247	-4.255	-3.1896
4.	40.248	-4.1969	-12.6653
5.	40.248	-4.2667	-1.60253
6.	40.248	-4.5933	-6.50942
7.	40.249	-4.2205	-8.33541
8.	40.25	-4.217	-8.796
9.	40.252	-4.2135	-9.21424
10.	40.252	-4.59	-7.6268

4.1.2 Interpolation of gravity at aerial gravity positions using satellite altimetry

The satellite altimetry gravity was provided in grid value of 1' x 1' along the coast of Tanzania. To get the predicted gravity along aerial gravity position, Kriging method was used through Surfer version 7. Kriging as an interpolation method required values at the sought points using residual approach (Bohling,2005). Table 4-4, Table4-5 and Table4-6 below shows the samples of the interpolated gravity from DTU15, SDUST2021 and DTU17 at ship track position.

Table 4-4: Samples of Gravity Anomaly from DTU15 in Aerial gravity position

S/No	Longitude (d.dd)	Latitude (d.dd)	dg_DTU15 (mGal)
1.	40.95	-2.55	55.0875
2.	40.7	-2.6	38.525
3.	40.75	-2.6	41.875
4.	40.8	-2.6	46.2125
5.	40.85	-2.6	50.2
6.	40.9	-2.6	52.725
7.	40.95	-2.6	56.95
8.	40.6	-2.65	29.3625
9.	40.65	-2.65	33.3625
10.	40.7	-2.65	38.15
11.	40.75	-2.65	42.725
12.	40.8	-2.65	47.4125

Table 4-5: Samples of Gravity Anomaly from SDUST2021 in Aerial gravity position.

S/No	Longitude (d.dd)	Latitude (d.dd)	dg_SDUST2021 (mGal)
1.	40.95	-2.55	56.55699
2.	40.7	-2.6	41.46878
3.	40.75	-2.6	43.1475
4.	40.8	-2.6	44.69629
5.	40.85	-2.6	51.07788
6.	40.9	-2.6	54.30136
7.	40.95	-2.6	57.39963
8.	40.6	-2.65	25.89024
9.	40.65	-2.65	31.28584
10.	40.7	-2.65	38.1391
11.	40.75	-2.65	44.34177

Table 4- 6: Samples of Gravity Anomaly from DTU17 in Aerial gravity position

S/No	Longitude (d.dd)	Latitude (d.dd)	dg_DTU17 (mGal)
1.	40.95	-2.55	54.3
2.	40.7	-2.6	37.5
3.	40.75	-2.6	41.4
4.	40.8	-2.6	45.2
5.	40.85	-2.6	48.7
6.	40.9	-2.6	51.75
7.	40.95	-2.6	55.65
8.	40.6	-2.65	28.8
9.	40.65	-2.65	33.4
10.	40.7	-2.65	38.15
11.	40.75	-2.65	42.9

4.1.3 Evaluation of ship track gravity data using satellite altimetry data

The difference between the observed ship track gravity anomaly and the grid interpolated gravity anomaly data from satellite altimetry (SDUST2021GRA, DTU15 & DTU17) are shown as Δg_1 , Δg_2 and Δg_3 respectively. On the basis of the values of Δg_1 , Δg_2 & Δg_3 the sum, minimum, maximum, mean, standard deviation (SD) and root mean Square are given in Table 4-7, Table 4-8 and Table 4-9 respectively.

Table 4-7: Statistics of the difference between gravity anomaly from Ship track and the predicted gravity anomaly from SDUST2021 in mGal.

Attribute	Population	Sum	Minimum	Maximum	Mean	STD	R.M.S
Δg_1	8680	137157.08	-54.92	81.39	15.80	27.63	39.65

Where $\Delta g_1 = dg_{SDUST2021} - dg_{SHIPTRACK}$ (4-1)

Table 4-8: Statistics of the difference between gravity anomaly from Ship track and the predicted gravity anomaly from DTU15 in mGal.

Attribute	Population	Sum	Minimum	Maximum	Mean	STD	R.M.S
Δg_2	8677	138202.73	-54.60	81.01	15.92	27.50	39.65

Where $\Delta g_2 = dg_{DTU15} - dg_{SHIPTRACK}$ (4-2)

Table 4-9: Statistics of the difference between gravity anomaly from Ship track and the predicted gravity anomaly from DTU17 in mGal.

Attribute	Population	Sum	Minimum	Maximum	Mean	STD	R.M.S
Δg_3	8684	137294.51	-54.85	81.39	15.81	27.66	37.20

Where $\Delta g_3 = dg_{DTU17} - dg_{SHIPTRACK}$ (4-3)

4.1.4 Evaluation of aerial gravity data using Satellite Altimetry

The difference between the observed aerial gravity and the grid interpolated gravity anomaly data from satellite altimetry (SDUST2021GRA, DTU15 & DTU17) are shown as Δg_4 , Δg_5 & Δg_6 respectively. On the basis of the values of Δg_4 , Δg_5 & Δg_6 the minimum, maximum, mean, standard deviation (SD) and root mean Square are given in Table 4-10, 4-11 and Table 4-12 respectively.

Table 4-10: Statistics of the difference between gravity anomaly from Aerial and the predicted gravity anomaly from SDUST2021 in mGal.

Attribute	Population	Sum	Minimum	Maximum	Mean	STD	R.M.S
Δg_4	14365	-286618.02	-70.60	27.6123595	-19.95	20.96	32.99

Where $\Delta g_4 = dg_{SDUST2021} - dg_{AERIAL}$ (4-4)

Table 4-11: Statistics of the difference between gravity anomaly from Aerial and the predicted gravity anomaly from DTU15 in mGal.

Attribute	Population	Sum	Minimum	Maximum	Mean	STD	R.M.S
Δg_5	14359	-289889.40	-70.33	27.20	-20.18	20.89	32.92

Where $\Delta g_5 = dg_{DTU15} - dg_{AERIAL}$ (4-5)

Table 4-12: Statistics of the difference between gravity anomaly from Aerial and the predicted gravity anomaly from DTU17 in mGal.

Attribute	Population	Sum	Minimum	Maximum	Mean	STD	R.M.S
Δg_6	14339	-291294.36	-70.39	27.09	-20.31	20.85	33.027

Where $\Delta g_6 = dg_{DTU17} - dg_{AERIAL}$ (4-6)

4.2 Discussion of results

The differences between ship track and Aerial gravity data with Gravity Anomalies (DTU15, DTU17 & SDUST2021) that were computed at Ship track and aerial gravity position. The gravity difference between SDUST 2021, DTU15 and DTU17 was validated at 95% confidence level and had the Mean, STD and RMS values which were close to each other. Ship track with SDUST2021 yielded mean value of 15.80mGal, STD value of 27.63mGal and R.M.S of 39.65mGal, Ship track with DTU15 yielded mean of 15.92mGal, standard deviation of 27.50mGal and R.M.S of 39.65, ship track with DTU17 yielded mean of 15.81mGal, standard deviation of 27.66mGal and R.M.S of 37.20mGal.

The gravity difference between Aerial Gravity Anomaly with SDUST 2021, DTU15 and DTU17 was validated at 95% confidence level and had the Mean, STD and RMS values which were close to each other. Aerial gravity anomaly with SDUST2021 yielded mean value of -19.95mGal, STD value of 20.96mGal and R.M.S of 32.99mGal, aerial gravity with DTU15 yielded mean of -20.18mGal, STD value of 20.89mGal and R.M.S of 32.92 and aerial with DTU17 yielded mean of -20.31mGal, standard deviation of 20.85mGal and R.M.S of 33.02mGal.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The study is undertaken for the purpose of validating the ship track and aerial gravity data using grid gravity anomaly from satellite altimetry data. The study has examined if the gravity differences are significant and if it is possible to use the aerial gravity data and ship track gravity data for the geophysical but most especially for the geodetic activities in Tanzania.

According to the results obtained and discussion in chapter 4 and the objective of this study, the following conclusions can be made:

- a) The gravity dataset consisted of 10215 surface ship track point gravity values. Data was reduced due to lack of height information and was shown by symbol “9999” which represent no height data. Also other places the height observed was unrealistic due to the height observed (up to 39meters). Surface ship track was reduced to 9246 and 969 data were rejected. The aerial gravity data was downward continued to the sea surface where there were about 15,111 data used in this study.
- b) The computed standard deviation and RMS of difference results from Δg_1 given in Table 4-7 are close to that of Δg_2 in Table 4-8 and that of Δg_3 in Table 4-9 and one can conclude that there is a good agreement of ship track and SDUST2021 as well as that of DTU15 and DTU17 .

The computed standard deviation and RMS of difference results from Δg_4 given in Table 4-10 are close to that of Δg_5 in Table 4-11 and that of Δg_6 in Table 4-12 and one can conclude that there is a good agreement of Aerial gravity data and SDUST2021 as well as that of DTU15 and DTU17.

According to the validation performed on the ship track and aerial gravity anomaly against marine gravity anomalies. The statistics of the differences are close to each other that is the ship track are in good agreement with the satellite altimetry as well as the Aerial gravity data with Satellite Altimetry. Therefore, these data can now be used for its intended purpose which is gravimetric geoid model computation of Tanzania. Also, it can be used for the other geodetic, Geomatics and geological purposes.

5.2 Recommendations

Based on the results and conclusions presented in this study, the following recommendation can be made the aerial gravity data have shown to have agreement with the Satellite Altimetry data and the 2012-2013 Aerial survey is deemed reliable thus effort should be made to create a more reliable gravity anomaly database.

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APPENDICES

A. MATLAB script to compute surface gravity anomalies from ground gravity data

```
% Program name:

%           Ground gravity to surface gravity anomaly.

% This program has been written by Michael, M. and Busega,
H. (Ardhiuniversity,2018).

% This program computes surface gravity anomaly from ground gravity
onsurface of the earth.

% The input file should not contain header info but have columns arranged

% in form of: phi lambda H g

% Normal gravity gamma_GRS80 is based on GRS80 reference ellipsoid.

% Gravity is in mGal unit.

% We start with loading text file and then assigning vectors, phi,
lambda,

H, g

%
clear
clc
load reference_Gravit.dat;
mat = reference_Gravit;
phi = mat(:,2); % latitude
lambda = mat(:,3); % longitude
H = mat(:,4); % Orthometric height
g = mat(:,5); % Point gravity

% Then computation of notmal gravity (gamma_grs) reffered to GRS80
% is initialized.

gamma_GRS80 = 978032.67715*(1+0.0052790414*(sind(phi).^2)+...
0.0000232718*(sind(phi).^4)...
+
0.0000001262*(sind(phi).^6)+0.0000000007*(sind(phi).^8))
% mgal

% We proceed with computation of Free Air Gravity Anomalies
(delta_g_air).

delta_g_air = g + 0.3086*H - gamma_GRS80 %mgal

% Ellipsoidal height on telluroid (h_roid) is computed as follows.
% Compute first mu which is the product of gravitational costant and
% topographic density.
mu = 2670*6.67408*10^-11*10^5;
C = delta_g_air.*H - 2*pi*mu*H.^2;
h_roid = H + C./gamma_GRS80 ; % m
%
% Normal gravity on telluroid (gamma_roid) is now computed as follows.
```

```

gamma_roid = gamma_GRS80-(0.3087691-0.0004398*(sind(phi)).^2).*h_roid+
...
7.2125*10^-8*h_roid.^2; % mgal
%
% Finally, surface gravity anomalies (delta_g_surf) are computed as
% follows.
delta_g_surf = g - gamma_roid; % mgal

%
data=zeros(length(phi),8);
k=0;
for i=1:length(phi)
    k=k+1;
    data(k,:)= [phi(i) lambda(i) g(i) gamma_GRS80(i) delta_g_air(i)
...
h_roid(i) gamma_roid(i) delta_g_surf(i)];
end
%STATISTICAL ANALYSIS
N=length(data(:,3));
Min=min(data(:,3));
Max=max(data(:,3));
Mean=mean(data(:,3));
STD=std(data(:,3));
RMS=sqrt(sum(data(:,3).^2)/N);
S_OUTPUT=[Min Max Mean STD RMS];
%    THANK YOU FOR USING THIS PROGRAM.

```

B. Sample data of Ship track gravity Anomalies from BUREAU GRAVIMETRIQUE INTERNATIONAL

BGI DATA EXTRACTION

REQUEST : latitude =-15.00000000/ 4.00000000 longitude =26.00000000/44.00000000
country=000

source	lat(deg) validity	long(deg)	alt(m)	g(mgal)	FreeAir	Bouguer	
61021408	-5.4611	39.2976		502.1	978034.83	-43.78	-9.21 1
61021408	-5.4372	39.3094		605.4	978025.63	-52.58	-10.89 1
65320070	-6.525	39.31167	57	978064.9	-50.7	-46.77	1
61021408	-5.414	39.3211	746	978019.03	-58.78	-7.41	1
61021408	-5.3918	39.3327	870	978014.96	-62.48	-2.57	1
65320070	-6.55333	39.335	67	978056	-60.1	-55.49	1
61021408	-5.3695	39.3442		99999.9	978013.89	-63.18	9999.99 1
61021408	-5.3473	39.3558	730	978016.11	-60.58	-10.31	1
65320070	-6.58167	39.36	77	978052.1	-64.5	-59.2	1
61021408	-5.3251	39.3673	749.7	978014.94	-61.38	-9.76	1
61021408	-5.3029	39.3789	763.8	978011.88	-64.08	-11.49	1
61021408	-5.6047	39.3798	356	978046.11	-34.99	-10.48	1
65320070	-6.61167	39.38333	200	978044.2	-73	-59.23	1
61021408	-5.2647	39.3955	787.3	978021.55	-53.77	0.45	1
61021408	-5.2461	39.4035	789	978020.54	-54.47	-0.14	1
65320070	-6.64167	39.40833	292	978043.3	-74.6	-54.49	1
61021408	-5.631	39.4084	382.7	978044.97	-36.59	-10.24	1
61021408	-5.2274	39.4114	789	978020.44	-54.27	0.06	1
61021408	-5.6407	39.419	392	978044.94	-36.79	-9.8	1
61021408	-5.2088	39.4194	789	978020.43	-53.97	0.36	1
61021408	-5.1902	39.4273	789	978020.53	-53.57	0.76	1
61021408	-5.6505	39.4297	406.3	978045.91	-35.99	-8.01	1
61021407	-5.4496	39.4343		99999.9	978005.44	-72.98	9999.99 1

65320070	-6.67333	39.435 349	978040.8	-77.8	-53.77	1
61021408	-5.1715	39.4353	788	978022.53	-51.27	2.99 1
61021408	-5.6603	39.4403	415.3	978047.08	-34.99	-6.39 1
61021408	-5.1529	39.4432	797.6	978024.53	-48.97	5.96 1
61021407 1	-5.4766	39.4452	99999.9	978011.6	-67.28	9999.99
61021407	-5.3321	39.4477	808	978005.86	-70.58	-14.94 1
61021408	-5.6701	39.4509	428	978050.16	-32.09	-2.62 1
61021408	-5.1342	39.4512	775.4	978027.73	-45.47	7.92 1
61021407 1	-5.3015	39.4551	99999.9	978001.75	-74.18	9999.99
61021407	-5.5037	39.4561	827	978015.76	-63.58	-6.64 1
61021408	-5.1156	39.4591	805	978030.83	-42.07	13.36 1
65320070	-6.705	39.46 378	978041.6	-77.7	-51.67	1
61021408	-5.6799	39.4616	446.3	978051.13	-31.29	-0.56 1
61021407	-5.271	39.4624	99999.9	977996.65	-78.77	9999.99 1
61021407 1	-5.5307	39.4669	99999.9	978018.82	-60.99	9999.99
61021408	-5.0969	39.467 799	978030.83	-41.77	13.25	1
61021407	-5.2405	39.4697	821	977991.55	-83.37	-26.84 1
61021408	-5.6897	39.4722	461.3	978051.1	-31.49	0.28 1
61021408	-5.0783	39.475 769.5	978028.74	-43.57	9.42	1
61021407	-5.21	39.4771	99999.9	977987.45	-86.97	9999.99 1
61021407 1	-5.5577	39.4778	99999.9	978022.99	-57.29	9999.99
61021408	-5.0583	39.4814	746	978033.72	-38.26	13.1 1
61021408	-5.6995	39.4829	477	978052.18	-30.59	2.25 1
61021407 1	-5.1795	39.4844	99999.9	977985.36	-88.57	9999.99
65320070	-6.73667	39.48667	390	978045.2	-74.7	-47.84 1
61021408	-5.0373	39.4867	725	978033.69	-37.96	11.96 1
61021407	-5.5848	39.4887	677	978024.96	-55.79	-9.17 1

61021407	-5.149	39.4917	818	977984.37	-89.07	-32.74	1	
61021408	-5.0164	39.492	699.7	978034.76	-36.56	11.62	1	
61021408	-5.7093	39.4935	504.5	978050.15	-32.79	1.95	1	
61021408	-4.9955	39.4973	684	978038.74	-32.26	14.84	1	
61021407	-5.1185	39.499	99999.9	977983.28	-89.67	9999.99		1
61021407	-5.6118	39.4995	645.4	978027.13	-54.09	-9.65	1	
61021407	-6.6092	39.5001	373	978057.21	-43.04	-17.35	1	
61021408	-4.9745	39.5026	664	978041.91	-28.76	16.96	1	
61021408	-5.7191	39.5041	534.3	978046.03	-37.09	-0.3	1	
61021407	-5.088	39.5064	99999.9	977982.29	-90.17	9999.99		1
61021408	-4.9536	39.508	651.6	978042.99	-27.36	17.51	1	
65320070	-6.76833	39.51167	366	978046.6	-73.9	-48.7	1	
61021407	-5.0575	39.5137	714	977982.31	-89.66	-40.5	1	
61021407	-6.5899	39.5182	356.7	978067.51	-32.34	-7.78	1	
61021408	-4.9119	39.5185	652	978050.34	-19.36	25.53	1	
61021407	-5.027	39.521	719.7	977983.33	-88.16	-38.6	1	
61021407	-5.6666	39.5216	609	978034.9	-47.29	-5.36	1	
61021408	-4.8911	39.5238	653.3	978052.43	-16.96	28.02	1	
61021407	-4.9964	39.5283	701.1	977984.35	-86.66	-38.38	1	
61021408	-4.8704	39.529	708	978054.31	-14.76	34	1	
61021407	-5.6973	39.5338	613.7	978036.94	-45.79	-3.53	1	
61021408	-4.8496	39.5343	632.6	978054.49	-14.26	29.3	1	
61021407	-6.5706	39.5364	343.6	978075.82	-23.64	0.02	1	
65320070	-6.8	39.53833	346	978051.1	-70.1	-46.27	1	
61021408	-4.8288	39.5395	614	978055.48	-12.96	29.32	1	
61021407	-4.9452	39.5407	616.7	977990.26	-79.96	-37.5	1	
65320070	-5.62333	39.54333	726	978029.5	-69	-19.01	1	
61021408	-4.8081	39.5448	612	978056.37	-11.75	30.39	1	
61021407	-5.728	39.546	640.4	978038.99	-44.29	-0.19	1	
61021407	-4.9201	39.5468	610.7	977992.27	-77.56	-35.51	1	

61021408	-4.7873	39.55	551.9	978059.46	-8.35	29.65	1
61021407	-4.895	39.5529	580	977995.39	-74.06	-34.12	1
61021408	-4.7666	39.5553	509	978061.65	-5.85	29.2	1
61021407	-4.8699	39.5589	620.1	977995.4	-73.66	-30.96	1
61021408	-4.7458	39.5605	476	978064.64	-2.55	30.22	1
65320070	-6.83167	39.56333	346	978056.3	-65.6	-41.77	1
61021407	-4.8448	39.565	695	977995.32	-73.36	-25.5	1
61021408	-4.725	39.5658	434.6	978068.83	1.95	31.88	1
61021407 2	-6.8454	39.5662	99999.9	978068.65	-36.55	9999.99	
61021408	-4.7043	39.571	408.7	978069.93	3.35	31.49	1
61021407	-5.7894	39.572	732.6	978037.59	-46.8	3.65	1
61021407	-6.5289	39.5738	338	978095.17	-3.43	19.84	1
61021407 2	-6.8534	39.5747	99999.9	978067.72	-37.65	9999.99	
61021408	-4.6835	39.5763	402	978067.82	1.55	29.23	1
61021408	-4.6627	39.5816	422.7	978064.62	-1.35	27.76	1
65320070	-5.61667	39.585	862	978028.4	-70	-10.64	1
61021407	-5.8202	39.5859	826.3	978040.65	-44.3	12.6	1
61021408	-4.642	39.5868	437.4	978062.62	-3.05	27.07	1
65320070	-6.86333	39.58833	346	978058.9	-63.7	-39.87	1
61021408	-4.6212	39.592	427	978060.52	-4.85	24.56	1
61021408	-5.7313	39.5923	796.7	978036.75	-46.59	8.27	1
61021407	-6.5064	39.5931	334	978102.51	4.37	27.37	1
61021407	-4.7726	39.5946	566.5	977998.14	-69.45	-30.44	1
61021408	-4.6004	39.5973	426	978058.42	-6.65	22.68	1