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DETERMINATION OF NORMAL HEIGHT OF TAREF11 CONTROL POINTS USING GEOID - QUASIGEOID SEPARATION AND VALIDATION USING GEOPOTENTIAL NUMBER

A Case Study of TAREF11 Control Points

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BSc Geomatics

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A Dissertation Submitted to the Department of Geospatial Sciences and Technology in Partially Fulfilment of the Requirements for the Award of Science in Geomatics (BSc. GM) of Ardhi University

CERTIFICATION

The undersigned certify that they have read and hereby recommend for acceptance by the Ardhi University dissertation titled "**Determination of Normal Height of TAREF11 Control Points Using Geoid - Quasigeoid Separation and Validation Using Geopotential Number, A Case Study of TAREF11 Control Points**" in partial fulfillment of the requirements for the award of degree of Bachelor of Science in Geomatics at Ardhi University.

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MS. REGINA V. PETER

supervisor

Date……………………….

DECLARATION AND COPYRIGHT

I, NASSOR, MOHAMED S. hereby declare that, the contents of this dissertation are the results of my own findings through my study and investigation, and to the best of my knowledge they have not been presented anywhere else as a dissertation for diploma, degree or any similar academic award in any institution of higher learning.

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DEDICATION

To my loving family especially my father Mr. Shaibu Amour and my mother Zuhura Mohammed. I extend my deepest appreciation for your unconditional love and continuous encouragement throughout my study. Your unwavering belief in my abilities has been a source of my strength and motivation. I am forever grateful for your support, patience and sacrifices that allowed me to pursue in my study.

ABSTRACT

Normal height refers to the height of a point above the quasigeoid. Also, it can be defined as the height of telluroid above the reference ellipsoid. It is mostly computed using geopotential number and using height anomalies extracted from the quasigeoid model. This research focus on using an alternative approach that can be used to compute normal heights using geoid-quasigeoid separation (GQGS).

This dissertation aims at determination of normal heights of TAREF11 control points over Tanzania mainland using GQGS. Firstly, geoid and quasi geoid separation distances were determined using Tenzer's explicit formula for the geoid-quasigeoid separation. Then geoidal undulations from TZG17 and EGM08 were used to obtain the height anomaly.

Later the height anomaly obtained were subtracted to the ellipsoidal height to have normal height for TAREF11 control points for both TZG17 and EGM08. The normal heights were computed for all 577 TAREF11 control points.

These normal heights computed were later on validated using geopotential number. A test was conducted at 95% confidence interval and the results showed that there is significance difference of 3.5cm between the normal height computed by using geoid-quasigeoid separation from EGM08 and by those computed by geopotential number. There was significance difference of 1.6cm by those computed using geoid to quasi-geoid separation from TZG17 and geopotential number.

KEYWORDS: GQGS, TAREF11, TZG17, EGM08, Geopotential number, Normal height, 95% Confidence level.

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LIST OF ABBREVIATION

CORS	Continuous Observation Reference Stations
GPS	Geographic Positioning System
GNSS	Global Navigation Satellite System
GRS80	Geodetic reference system of 1980
GQGS	Geoid-Quasigeoid separation
EGM08	Earth gravitational model 2008
ICGEM	International Center of Global Earth Models
ITRF	International terrestrial reference frame
RMS	Root mean square
SD	Standard deviation
TAREF11	Tanzania reference frame of 2011
TAREF14	Tanzania reference frame of 2014
TZG08	Tanzania geoidal model 2008
TZG13	Tanzania geoidal model 2013
TZG17	Tanzania geoidal model 2017
TZQ17	Tanzania Quasi Geoid Model 2017
H^N	Normal height

CHAPTER ONE

INTRODUCTION

1.1 Background

Height system is one dimensional coordinate system used to express the vertical metric distance (height) of a point above a reference surface. Height system is divided into two categories i.e. Physical heights and Geometrical heights (Sanchez, Crespi, & Tanaka, 2021). Physical height system is the height system that link the reference surface and the height to the equipotential surfaces and the plumb line of the Earth's gravity field. If the reference surface and the height do not depend on the Earth's gravity field, the height system is referred as geometric height system (Melitha, 2020).

The reference surface of most of the existing physical height systems is linked to the mean sea level determined at a tide gauge. As the mean sea level varies geographically and with time, mean sea level values used for the definition of the zero-height level in different countries may present discrepancies up to some decimeters. The traditional way to obtain physical heights is geodetic levelling in combination with gravity reductions along the so-called vertical or levelling networks. The gravity reductions are necessary to take into account the non-constant separation between equipotential surfaces due to variations of the Earth's gravity field. Depending on the gravity correction applied to levelling, the height of a point can be determined in many slightly different ways (Sanchez, Crespi, & Tanaka, 2021). There are different types of physical heights i.e orthometric heights (H), normal heights (H^N), dynamic heights (H^D) and geopotential number (Cp).

Orthometric and normal heights are widely used in the definition and realization of physical height systems, while dynamic heights are mainly employed to define the height coordinate in water bodies and construction of large pipe lines (e.g., large lakes) (Yilmaz,2008).

Physical height systems based on orthometric heights use the geoid as reference surface. The geoid is the equipotential surface of the earth's gravity potential that most closely coincides with the Mean Sea Surface in the average sense**.** Physical height systems based on normal heights use the quasi-geoid as reference surface. The quasi-geoid is close to the geoid but not an equipotential surface as the geoid is. The quasi-geoid deviates from the geoid in the same way as the normal heights deviate from the orthometric heights (Tenzer & Bagherbandi, 2013).

Since the normal heights have come up to be mostly in use. Gradually adoption of the normal height instead of orthometric height system is gaining popularity in the world mainly due to its less stringent determination process and closeness of normal heights to orthometric heights (Ulotu,2015). These two height systems are widely used since they help in process of vertical datum unification. as step towards the realization of a unified global vertical frame, computation of normal height is also needed as both orthometric and normal height are needed for realization of physical height system.

In the context of vertical datum unification, the conversion between existing height systems is essential. Since most of vertical datums are realized in the system of either normal or orthometric heights, this conversion is typically realized by means of computing the geoid-to-quasi-geoid separation, which represents the difference between the geoid and the quasi-geoid, or equivalently the difference between the normal and orthometric heights (Foroughi & Tenzer, 2017). Accurate determination of height is crucial in fields such as surveying, geodesy and mapping. The TAREF11 control points provides a benchmark for height measurements in the region, using the orthometric height which takes into account the earth gravity field. However, there are variations in the earth gravity field that cause inaccuracies in height measurements due to this problem this research will aim on determining the normal height of TAREF11 control points using geoid quasi geoid separation model. The normal height takes into account the variations on the earth gravity field. The results will provide valuable information for various applications and contribute to the development of a more accurate height reference system.

1.2 Statement of the Problem

TAREF11 network points were derived with ellipsoidal heights therefore as geomatician, we are supposed to convert the ellipsoidal height of TAREF11 to other system of height so that they can be applied in different geomatics applications. Since Tanzania uses orthometric system of height so all TAREF11 control points were converted to that system of height. In the world the most used height systems are orthometric and normal height systems, gradually adoption of normal height instead of orthometric height is on the rise and gaining popularity due to its less stringent determination process. Orthometric height takes into account the earth gravity field that led to inaccuracies in height measurements due to the variations in the earth gravity field compared to normal height that uses surface gravity anomalies that do not require advance knowledge of topographical density distribution for the downward continuation of gravity from surface to geoid (do not depend on crustal density) and closeness of normal height to orthometric height. Computation of normal height is needed as both orthometric and normal height are needed for realization of physical height system towards the realization of a unified global vertical frame.

1.3 Research Hypothesis

- The application of geoid-quasigeoid separation will yield accurate and reliable estimates of normal height for TAREF11 control points.

1.4 Research Objectives

1.4.1 Main Objective

The main objective of this study is to conduct a comparative study of normal heights on TAREF11 control points using geoid-quasigeoid separation.

1.4.2 Specific Objective

- To determine geoid-quasigeoid separation using Tenzer's explicit formula.
- To determine normal height from geoid-quasigeoid separation.
- Determination of normal height of TAREF11 control points.

1.5 Significance of the Research

The aim of this research is to determine normal height of TAREF11 control points. Many stakeholders rely on GNSS as their tool of choice, but mainly for horizontal positioning, and the number is on the increase, it is high time we reaped the advantages of GNSS 3D instead of 2D only.

1.6 Beneficiaries and Expected Users

- Geodesist
- Engineers
- Researchers
- Ardhi university students

1.7 Scope and Limitation of Research

This study will focus on determination of normal heights using geoid-quasigeoid separation on all TAREF 11 control points only.

1.8 Area of Study

The research study area is Tanzania mainland across all TAREF11 control points. Its extent is 28˚ E to 42˚E from the prime meridian and 1˚ to 12˚ south of the equator as shown in Figure 1-1 below.

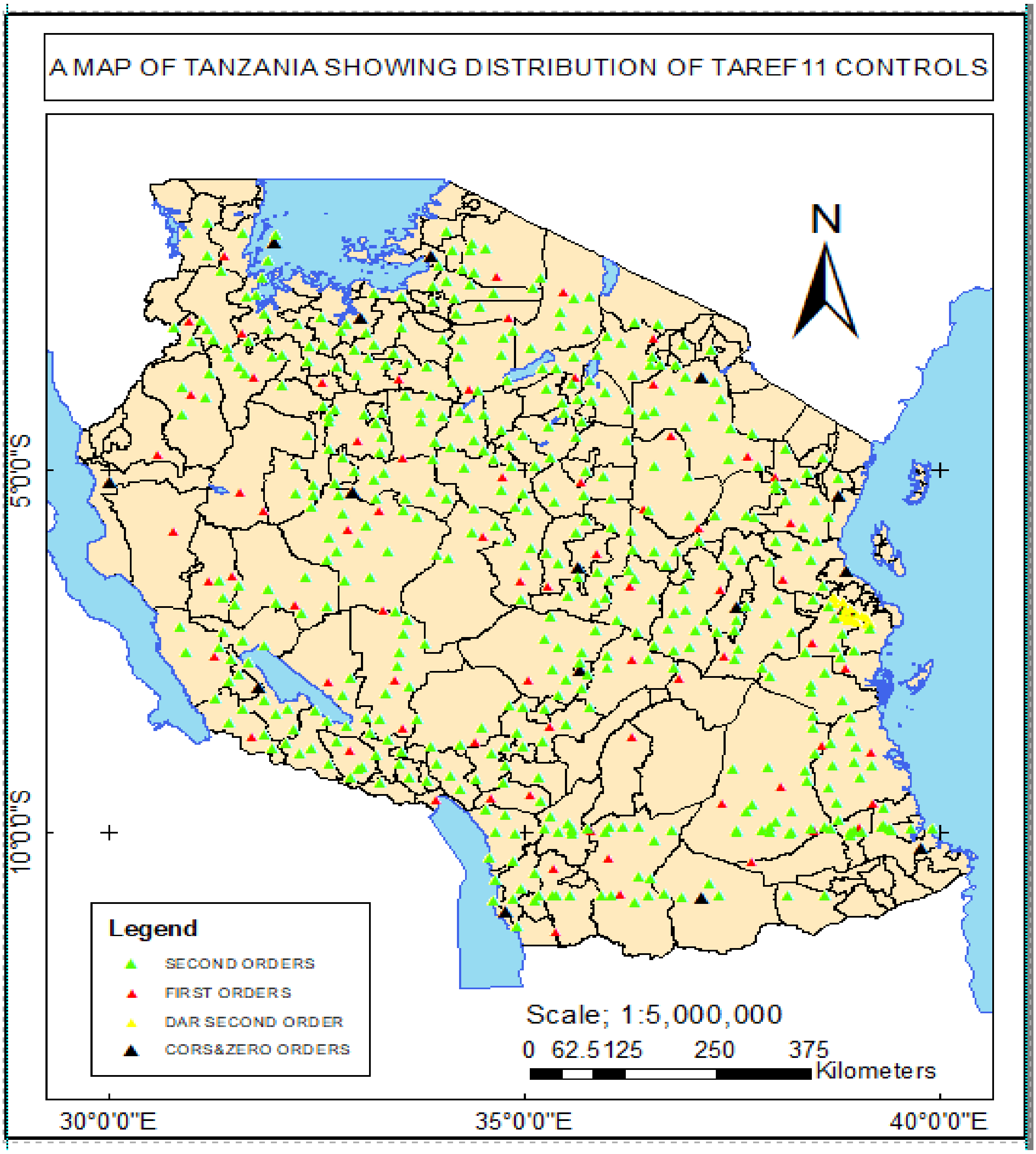

Figure 1-1: A map of Tanzania mainland showing distribution of TAREF11control points

1.9 Research Outline

This study is dedicated in determination of normal height of TAREF11 control points using geoid-quasigeoid separation and validation using geopotential number. This study consists of five chapters as listed below;

Chapter one

This chapter gives the background overview of this research, Problem Statement, objectives of the research, significance of the research, beneficiary, study area, scope and Limitation of the research.

Chapter two

This chapter provides an overview of the TAREF11, Physical height system which consist of (normal heights, geopotential numbers and orthometric heights), geoid-quasigeoid separation and global geopotential models (GGMs).

Chapter three

This chapter describes the methodologies used to achieve the objectives, mathematical models and detailed description of all datasets required to achieve the objectives of the study.

Chapter four

This chapter present the results and discussion of the results.

Chapter five

This chapter present conclusion and recommendations from the conducted research. conclusion gives the summary of the research findings in view of the solution to research problem and attaining the main objectives while recommendation explain follow up after the research findings.

CHAPTER TWO

LITERATURE REVIEW

2.1 Overview of the TAREF11

Tanzania reference frame of 2011 (TAREF11) is a current geodetic network on which its computation has completed based on the International Reference Frame of 2014 (ITRF14). TAREF11 comprises of 6 continuous observation reference stations (CORS), 16 zero order points, 72 first order points, and 525 second order point as shown in Figure 1-1. The system comprises of only ellipsoidal height. TAREF11 is being updated from ITRF 2008 to ITF2014 and it has 4 levels of accuracy (Mayunga & Mtamakaya, 2015).

The TAREF11 is the geodetic network that is well distributed throughout Tanzania ready for consumer. It is denser in populated areas and high investment places and plans for its densification and epochal updates are in place. It is based on WGS84 geocentric ellipsoid and thus GNSS compatible and tallies well with TZG08, TZG13 and TZG19 gravimetric geoid models which are referenced to GRS80.The system comprises of only ellipsoidal height and have been only converted to orthometric height since it the height system of Tanzania. This system needs to have the physical height system such as normal height because these two heights system are widely used since, they help in process of vertical datum unification. as step towards the realization of a unified global vertical frame, computation of normal height is also needed as both orthometric and normal height are needed for realization of physical height system.

2.2 Physical Height Systems

These coordinate systems are called physical height systems as the height determination depends on the level surfaces and the plumb line of Earth's gravity field (Sanchez, Crespi, & Tanaka, 2021). Physical height system links the reference surface and the height to the equipotential surfaces and the plumb line of the Earth's gravity field (Melitha,2020). These are height systems based on the earth's gravity field particularly determined using geodetic levelling technique that measure the distance between two equipotential surface of the earth's gravity field and provide height along the curved plumb line. The physical height system is referred to the geoid which is the most common used reference surface for establishment of the vertical datum. From the definition, geoid is an equal-potential surface of the earth's gravity field that approximately coincide with the MSL in a least square sense.

The traditional way to obtain physical heights is geodetic levelling in combination with gravity reductions along the so-called vertical or levelling networks. The gravity reductions are necessary to take into account the non-constant separation between equipotential surfaces due to variations of the Earth's gravity field. Depending on the gravity correction applied to levelling, different types of physical heights are distinguished: orthometric heights system, normal heights system, dynamic heights system and geopotential number.

2.2.1 Geopotential number (c)

Geopotential height or geopotential number is a surface point's potential difference to the geoid potential. Put differently, it is the difference between surface point's W_A potential with geoid point's W_O potential. The difference can be obtained as in equation below;

$$C_A = W_O - W_A = -\int_0^A dW = \int_0^A g dn \tag{2.1}$$

C_A is known as the geopotential number and is path independent, W_O is the geoid potential, W_A is a point potential, dW is the potential difference in differential meaning, dn is the height difference in differential meaning and g is gravity. The geopotential number is measured in kgal*m and C_A is equal in the same equipotential surface. Although it has no distance dimension, it is the natural criterion for heights (Yilmaz, 2008).

One of the equal potential surfaces of the earth is the geoid. For a point on the surface of the earth, the geopotential number can be obtained by simply finding the difference between the potential on the geoid and that on the equal potential surface on which the point lies on the surface of the earth.

Fluid flows from points with higher potential energy to points with lower potential energy. As the fluid flows the potential energy is converted to kinetic energy. Therefore, geopotential numbers govern fluid flow hence becoming the physically meaningful and conceptually sensible heights. Thus, spirit levelling height difference data needs to be corrected for gravity to reduce inconsistence that may cause levelling loop to have misclosures. Geopotential numbers have some advantages to be used in physical applications, for example they have dimensions of length squared divided by time squared. The conversion of these dimensions is done by dividing the geopotential number by the value of gravity resulting to dimension of length. This process yields to another height systems which are dynamic, normal and orthometric (Idrissa, 2022).

Optimal Geopotential Vertical Datum ($W_0^{LVD,\ TZG13}$) for Tanzania was initially calculated in 2016 and found to be 62636863.32 m^2/s^2 (Masunga, 2016). This value was obtained from TZG13 Gravimetric Geoid Model and GPS & Ocean Levelling. But in 2018 there was an update of geoid for Tanzania to form TZG17 with a better accuracy (5cm) than the 10cm-TZG13 by Valerian, (2018). And in July 2015, the International Association of Geodesy (IAG) released a new conventional value of W_0 (62636853.4 m^2/s^2) to define the global equipotential reference surface. Thus, by using these updated values Kamugisha, (2019) come up with an update of Tanzania Local Geopotential Vertical Datum of 62636852.42 m^2/s^2 using $W_0^{LVD,TZG17}$ at Tanga Tide Gauge Benchmark using GPS-Levelling and TZG17 Geoid Model.

2.2.2 Orthometric height.

The distance of a surface point along the plumb line to the geoid, which is taken as the reference surface, is named as orthometric height. It can be obtained as follows;

$$H = \frac{C}{\bar{g}} \tag{2. 2}$$

where H is the orthometric height, C is the geopotential number and $\bar{g}$ is the mean gravity. For the computation of actual average gravity "g" along the plumb line, actual gravity is required between the geoid and earth surface. However, gravity inside the earth cannot be measured so a hypothesis regarding the mass distribution must be formed and g is computed on this basis. The orthometric height cannot be determined without a hypothesis. Orthometric heights are the natural "heights above sea level", that is, heights above the geoid. Therefore, they have an unequalled geometrical and physical significance (Yilmaz, 2008).

This is the length of the plumb line as measured from the geoid to the point on the surface of the earth. The orthometric height can also be given by the geopotential number divided by the integral mean value of gravity taken along the plumb line. Orthometric height has a clearer geometrical interpretation. The geoid reference surface is also unique, being the single equipotential surface of the Earth's gravity field that broadly corresponds with mean sea level in the open oceans (Featherstone & Kuhn, 2006). It's difficult to measure directly orthometric height due to the fact that it requires knowledge inside the earth's surface. Thus, different approaches have been made to realize the orthometric heights. Although there is other approach of using orthometric height as normal orthometric heights which involve use normal gravity along the levelling line as they are easy to be computed and they do not use surface gravity, but they are less likely to predict the fluid flow correctly.

2.2.3 Normal height

The ellipsoid is the reference surface best resembling the geoid, which is the earth's basic shape. An ellipsoid's gravity field is called a normal gravity field. The Earth's actual gravity field is slightly different from the normal gravity field. The difference is called the disturbing potential (Yilmaz, 2008).

We are going to assume that the actual gravity field is equal to the normal gravity field. In other words, we assume W =U, g = γ, T = 0 (Yilmaz, 2008). Hence, orthometric heights which correspond to this approximation are named normal heights and are shown by H^N they can be obtained as follows:

$$H^N = \frac{C}{\bar{\gamma}} \tag{2. 3}$$

where W is the actual gravity potential, U is the normal gravity potential, T is the disturbing potential and $\bar{\gamma}$ is the normal average gravity along a normal plumb line, H^N is normal height and C is the geopotential number.

A surface point P has W_p actual and U_p normal gravity potential and these potentials are not equal ($W_p \neq U_p$). But on the plumb line that crosses point P there is a Q point. In this Q point, the normal potential is equal to the actual potential ($W_p = U_Q$). The surface for which $W_p = U_Q$ hold for every point is called a telluroid. The normal height of a point P is equivalent to the height of the corresponding telluroid point Q above the ellipsoid. The physical and geometric meanings of normal heights are less obvious and they are dependent on the reference ellipsoid used (Yilmaz, 2008). Figure 2-1 show relationship between orthometric height and normal height.

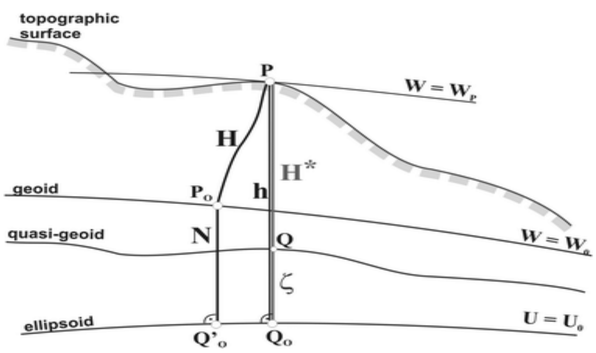

Figure 2-1: Orthometric height and normal height

Geodesist have developed different methods which can be employed to obtain normal heights. The normal heights can be obtained using following ways;

- **Determination of normal heights using quasigeoid**

Molodensky et al. (1962) formulated the geodetic boundary value problem on the earth surface and introduced two new surfaces called the telluroid and the quasi-geoid. The telluroid is the surface defined by plotting the points at a distance equal to height anomaly (ξ) below the earth surface (Idrissa, 2022).The quasi-geoid is not an equipotential surface of the earth's gravity field and thus has no physical meaning (Heiskanen and Moritz, 1967).

However, the quasi-geoid can be determined more directly from surface gravity data without prior knowledge of the topographic bulk density.

The separation between the reference ellipsoid and the quasi-geoid is called the height anomaly (ξ) and is defined as;

$$\xi = h - H^N \tag{2. 4}$$

Whereby ξ is the height anomaly, *h* is ellipsoidal height and H^N is normal height.

2.3 Geoid-Quasigeoid Separation

Different models have been developed by the geodesist to help in computation of normal height without depending on quasi geoid and geo potential number. The famous geoid-quasigeoid separation (GQGS) models are those of Prof. Lars Sjoberg of KTH Royal Institute of Technology, Sweden and Prof. Robert Tenzer of the Hong Kong polytechnic university.

The geoid-quasigeoid separation is typically referred to the difference between Molodensky's (1945, 1948) normal height and Helmert's (1884, 1890) orthometric height, because of their most common use for a practical vertical datum realization (Foroughi & Tenzer,2017).

$$N - \xi = H^N - H \tag{2. 5}$$

The basic rigorous formula for the geoid height (N) minus quasigeoid height (ξ) or normal height (H^N) minus orthometric height(H) is equation (2.6) (Heiskanen & Moritz,1967).

$$N - \xi = H^N - H = \frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}} H \tag{2. 6}$$

where $\bar{g}$ and $\bar{\gamma}$ are the mean gravity between the geoid and Earth's surface and mean normal gravity between the reference ellipsoid and telluroid, respectively. As their difference is not directly available, the traditional way to compute geoid-quasigeoid separation ($N-\xi$) is by approximating it by the (simple) Bouguer gravity anomaly (Δg_p^{bo}) at the computation point this formalism yields the formula for computing the geoid-quasigeoid separation as a function of the incomplete planar Bouguer gravity anomaly and the topographic height of the computation point referred as approximate formula for determining the separation (Sjoberg,2010).

$$N-\xi \approx \frac{\Delta g_p^{bo}}{\bar{\gamma}} H \tag{2. 7}$$

Where ($N-\xi$) is geoid-quasigeoid separation, (Δg_p^{bo}) is bouguer gravity anomaly, $\bar{\gamma}$ is mean normal gravity between the reference ellipsoid and telluroid and H is orthometric height. The approximate formula is not effective for the mountainous area but the it can still be used since the separation distance between geoid and quasi geoid (N-ξ) is mostly less than a meter, the error associated with the approximate formula is thus within tolerance. For example, for elevation of 3797m, (N−ξ) is around -35cm (Ulotu, 2009). Hence the geoid-quasigeoid separation has to be determined by using superior models which help to accurately determine the separation between the geoid and quasigeoid. The accurate model for determination of geoid and quasigeoid separation was developed by Tenzer et al. (2015) in the following form;

$$N-\xi = \frac{\overline{\partial g(\Omega)}}{\gamma(\varphi)} - 2\frac{\xi(\Omega)H(\Omega)}{R} \tag{2. 8}$$

Whereby $\overline{\partial g(\Omega)}$ is the mean gravity disturbance within topography is given as difference between the disturbing potential at the geoid $T_{(g,\Omega)}$ and the disturbing potential at the topographic surface $T_{(t,\Omega)}$. $\xi(\Omega)$ is the height anomaly, $H(\Omega)$ is the orthometric height, R is the radius of the mean earth sphere and $\gamma(\varphi)$ is the denotes normal gravity at the surface of the geocentric reference ellipsoid.

The geoid-quasigeoid separation (N-ξ) value can be used to obtain the normal height with ellipsoidal height from the reference ellipsoid. since the geoidal height (N) will be obtained from the geoid model hence the height anomaly (ξ) will be obtained. After obtaining height anomaly then it is subtracted to ellipsoidal height(h) to obtain normal height (H^N).

2.4 Overview Global Geopotential Models (GGMs)

The Global Gravitational Models are representations of the Earth's gravitational potential outside the masses of the Earth in terms of spherical harmonics i.e. external to Mean Earth Sphere (MES) as a bounding surface (Abdalla, 2009). A GGM is set of spherical harmonic coefficients of the earth gravity potential of external type i.e. external to the MES as the bounding surface (Ulotu, 2009). The spherical harmonics provide an efficient mathematical tool to compute an arbitrary 13 function of geopotential for any point on the earth's surface. Examples of these functions of geopotential include gravity anomaly, height anomaly, gravity disturbance, height anomaly and deflections of vertical (Bucha & Janak, 2013).

The need of GGM originated upon the launch of the artificial satellite as early as in 1957, specifically the launch of the first laser tracked satellite, Beacon-B, in 1964, has provided datasets which have allowed researchers to determine long to medium components of the gravitational field of the Earth. In particular, observational data recorded at satellite laser ranging tracking 4 stations have since been used to develop the GGMs that quantify the global long-wavelength and medium wavelength components of gravity field of the Earth (Lwehumbiza, 2017).

2.5 GGM sources

GGMs are products of the dedicated satellite gravity missions. But there are number of steps that are involved in turning the raw data gathered by the satellites into product suitable for the users worldwide. The part of the mission involves the 'ground segment', which is essentially made up of three main elements; the satellite control element, the processing element and the archiving element, as well as the services to provide users with the data (Lwehumbiza, 2017). Various research groups and institutions in the world use these data to develop different models. Examples of the groups and institutions include; European Improved Gravity Model of the Earth by New Techniques (EIGEN), Technical university of Denmark (DTU), Huazhong University of Science and Technology (HUST), Istanbul Technical University (ITU), Ohio State University (OSU), GeoForschungs Zentrum (GFZ) Potsdam, Goddard Earth Models (GEM), Joint Gravity Models (JGM), Texas Earth Gravity models (TEG), etc. Different development procedures are used to generate different GGMs. These procedures are accompanied with several corrections with respect to the physical properties of the earth.

Recently many organizations and research centers have developed multi–Global Geopotential Models (GGMs) depending on several types of available gravity and height datasets to estimate orthometric heights from GNSS measurements. These includes, XGM2016, XGM2019e which is published in three versions truncated to d/o 2160, 5540, and 760, EIGEN-6C4, GO_CONS_GCF_2_TIM_R6e, and EGM2008. Each of these GGMs has its own advantages, and their accuracy differs according to the region. For this research XGM2019e_5540 will be used for computation of disturbing potential at the geoid and at the topographic surface and for computation gravitational potential of all TAREF11 control points.

2.6 Overview of XGM2019e

XGM2019e is a combined GGM available in the ICGEM website http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html, complete to d/o 5540 corresponding to spatial resolution of 2′ (approximately 4km) released in the year 2019 that is presently the highest degree compared to existing models in the website. It includes the new improved combined satellite-only model named GOCO06s in the longer wavelength range up to d/o 300 combined with ground gravity grid that also covers the shorter wavelength. The ground data over land and oceans consist of gravity anomalies provided by the NGA (with 15′ spatial resolution) combined with the topography gravity field model (EARTH2014). Also, over the ocean part, gravity anomalies derived from satellite altimetry are used (DTU13 with 1' resolution) (Zingerle et al., 2020). The combination of the satellite-only model (GOCO06s) with the ground data up to d/o 719 in spheroidal harmonics coefficients was by weighted least squares adjustment (The XGM2019model with d/o 760 in spherical harmonics). The higher degree and order coefficients starting from 720 complete to 5480 were computed using the block diagonal technique. Then merging of these two sets of coefficients, whereby it was possible using the ellipsoidal harmonics coefficients since the reference surface of expansion (ellipsoid) coincides with the surface where ground data are located and thus the cut-off effects as experienced by spherical harmonics are avoided. The whole merged 18 spectrum of XGM2019e was transformed to spherical harmonic domain (The XGM2019e model complete to d/o 5540) (Zingerle et al., 2020). The functions of geopotential (gravitational potential) at any point on the surface of the earth are represented in spherical harmonics with an upper limit of summation, n_{max} that is related to the spatial resolution at the earth's surface

CHAPTER THREE

METHODOLOGY

This part describes the techniques, methods, practical applications of different mathematical models, all data and the software used in this study to achieve the main objective of this research. It explains the procedures that will be implied in the computation of normal height on the TAREF11 control points.

3.1 Computation of normal height

This research focus on determination of normal height from geoid-quasigeoid separation and validation of these normal heights will be performed using the normal height computed from geopotential number.

3.1.1 Determination of normal height using geoid-quasigeoid separation

Normal height will be determined by following various steps which are first by determination of geoid-quasigeoid separation, followed with determination of height anomaly which will be obtained by subtracting the geoid undulations from geoids model with the value of geoid-quasigeoid separation. Later on, the height anomaly obtained will be subtracted with ellipsoidal height and hence the normal height will be obtained.

3.1.1.1 Determination of geoid-quasigeoid separation (N − ξ)

For precise determination of geoid-quasigeoid separation, equation 2.8 will be employed whereby the mean gravity disturbance within topography is given as difference between the disturbing potential at the geoid and the disturbing potential at the topographic surface. This process of determining geoid-quasigeoid separation takes into account the topographic correction and gravimetric correction which make it to produce precise value of geoid-quasigeoid separation for all terrains.

$$N - \xi = \frac{T_{(g,\Omega)} - T_{(t,\Omega)}}{\gamma(\varphi)} - 2\frac{\xi(\Omega)H(\Omega)}{R} \qquad (3.\ 1)$$

N-ξ is the geoid-quasigeoid separation distance, $\xi(\Omega)$ is the quasigeoid height, $H(\Omega)$ is the orthometric height, R is the radius of the mean earth sphere and $\gamma(\varphi)$ is the denotes normal gravity at the surface of the geocentric reference ellipsoid, $T_{(g,\Omega)}$ is the disturbing potential at the geoid and $T_{(t,\Omega)}$ is the disturbing potential at the topography.

The disturbing potential values at the geoid and at the ellipsoid are computed as;

$$T_{(t,\Omega)} = \frac{GM}{R}\sum_{n=0}^{\bar{n}}\left(\frac{R}{R+H}\right)^{n+1}\sum_{m=-n}^{n} T_{n,m}Y_{n,m}(\Omega) \tag{3. 2}$$

$$T_{(g,\Omega)} = \frac{GM}{R}\sum_{n=0}^{\bar{n}}\sum_{m=-n}^{n} T_{n,m}Y_{n,m}(\Omega) \tag{3.3}$$

Equation (3.2) and (3.3) shows the disturbing potential at the topographic surface $T_{(t,\Omega)}$ and disturbing potential at the geoid $T_{(g,\Omega)}$ respectively. Where G is gravitational constant, M is the Mass of the earth sphere, R is radius of the Earth, H is the orthometric height of a point surface.

The data set of global geopotential model (GGM) are processed on matlab to give out the disturbing potential on topographic surface and disturbing potential at the geoid. The value of disturbing potential on the topography will be obtained using graflab and the value of disturbing potential on the geoid will be obtained using MATLAB coding. Figure 3-1 below shows a GUI of Graflab software that was used in computations of disturbing potential on the surface of the earth.

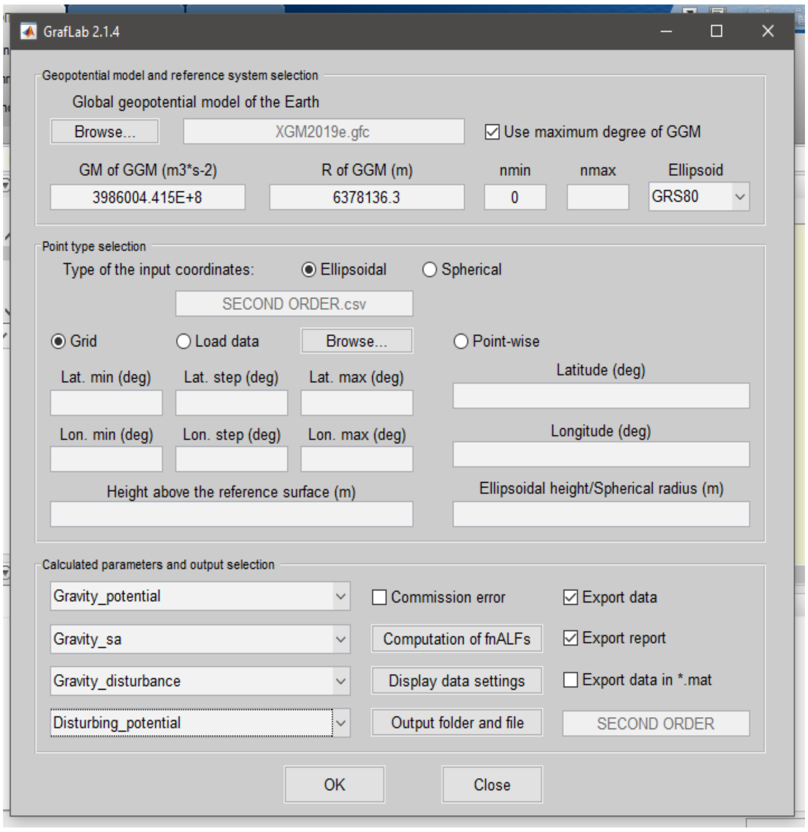

Figure 3-1: A GUI of Graf lab software that was used in computations of disturbing potential on the surface of the earth

3.1.1.2 Computation of normal height

The geoid-quasigeoid separation (N-ξ) value can be used to obtain the normal height with ellipsoidal height from the reference ellipsoid. x = separation between geoid and quasi geoid which in this study is obtained using Tenzer's explicit formula.

$$N - \xi = x \tag{3. 4}$$

since the geoidal height (N) will be obtained from the geoid model which in this study N is obtained from EGM08 and TZG17 hence the computed height anomaly (ζ) will be obtained.

$$\xi = N - x \tag{3. 5}$$

After obtaining height anomaly then it is subtracted to ellipsoidal height(h) to obtain normal height (H^N).

$$H^N = h - \xi \tag{3. 6}$$

3.1.2 Validation of normal heights using Geopotential number

The normal height computed using GQGS are validated using the normal computed using geopotential number. The difference between these heights systems will be evaluated in this dissertation in order to investigate if there are any differences between them.

Determination of normal height depends on computation of geopotential number and mean normal gravity. It is computed using the equation (2.3) which is called normal height by fundamental approach (Heiskanen and Moritz, 1967).

3.1.2.1 Determination of the Actual Gravity Potential of TAREF11 control points

Actual gravity potential is a resultant of gravitational potential and centrifugal potential. Due to the fact that TAREF11 control points are on the surface of the earth, geopotential numbers of all control points are required for computation of normal height. Gravitational potential can be expressed in terms of spherical harmonics as shown by Equation (3.7). Equation (3.7) was used in computing actual gravity potential on the surface of the earth.

$$W_P = W(r,\theta,\lambda) = V(r,\theta,\lambda) + Z(r,\theta) \tag{3. 7}$$

Where $W(r,\theta,\lambda)$ is the potential of a point on the surface of the Earth, $V(r,\theta,\lambda)$ is Gravitational potential and $Z(r,\theta)$ is centrifugal potential given by Equation (3.10).

$$V(r,\theta,\lambda) = \frac{KM}{R}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n+1} C_{nm} Y_{nm}(\theta,\lambda) \tag{3. 8}$$

where r is radial distance, θ is latitude, λ is longitude, kM is Product of mass of the earth and Newton's gravitational constant, R is Mean earth radius, C_{nm} is constant coefficient for degree n and order m, Y_{nm} is fully normalized equation of surface spherical harmonic function and $V(r,\theta,\lambda)$ is Gravitational potential.

$$Y_{nm} = \begin{cases} P_{nm}(\cos\theta)\cos|m|\lambda, m \geq 0 \\ P_{nm}(\cos\theta)\sin|m|\lambda, m < 0 \end{cases} \tag{3.9}$$

Where θ is latitude, λ is longitude and P_{nm} are fully normalized Legendre functions of the 1st kind and Y_{nm} is fully normalized equation of surface spherical harmonic function (Hofmann-Wellenhof & Helmut, 2005). The spherical harmonics coefficients are to be obtained from XGM2019e having maximum degree and order of 5540. Equation 3.10 shows the centrifugal potential.

$$Z(r,\theta) = \frac{W^2}{2} r^2 (\sin\theta)^2 \tag{3. 10}$$

where W is earth's rotation rate, θ is latitude, r is radial distance, $Z(r,\theta)$ is centrifugal potential. The centrifugal potential is the potential of a body undergoing circular motion due to rotation of the earth. Thus, by using the inputs from equation (3.8) and equation (3.10) and plug those in equation (3.7) actual gravity potential can be determined. All these mathematical equations have been programmed in the Graflab for simplification of computation of the actual gravity potential.

Graflab is MATLAB-based graphical user interface program for computing functionals of the geopotential including actual gravity potential (W_p^{GGM}) up to ultra-high degrees and orders, which allows: – evaluation of 38 functionals of the geopotential up to ultra-high degrees and orders. The computation of the geopotential functions is done by applying the technique so called lumped coefficients approach (Bucha & Janak, 2013). From Figure 3-1 the GUI has three main sections, the first section deals with selection of GGM. In case of this study XGM2019e having maximum degree of 5540 was used. In the same section for computation actual gravity potential minimum degree (n_{min}) was set to zero and maximum degree (n_{max}) was set as specified by selected GGM (5540). The second section is point type selection, where by ellipsoidal coordinates of the TAREF11 control points were imported. The last section is calculated parameters and output selection where by output folder and required parameter

(W_p^{GGM}) were set. Finally, actual gravity potential of the selected points on the surface of the earth was computed.

3.1.2.2 Determination of the Geopotential Number

By using actual gravity potential (W_P) on the surface of the earth computed using Combined Global Geopotential Model (XGM2019e) with maximum degree of 5540 and local geopotential vertical datum ($W_0^{LVD,\ TZG17}$) having the value of 62636852.42 m^2/s^2 computed by (Kamugisha,2019) was computed using GPS-levelling and TZG17 gravimetric geoid model. Mathematically; the geopotential number is obtained using Equation 3.11.

$$C = W_0^{LVD,TZG17} - W_P^{GGM} \tag{3. 11}$$

where, C is geopotential number, $W_0^{LVD,TZG17}$ is the actual potential on the reference surface (MSL) and W_p^{GGM} is the actual potential on the surface of the earth as obtained from XGM2019e of maximum degree and order of 5540.

3.1.2.3 Computation of Normal Gravity

The normal gravity on telluroid and reference ellipsoid can be computed using equation 3.12 and 3.13 respectively.

$$\gamma_Q = \left[\gamma_0 - (0.3087691 - 0.0004398\sin^2\varphi)h_Q + 7.2125\times 10^{-8} h_Q^2\right]\text{mga}l \tag{3. 12}$$

$$\gamma_0^{GRS80} = \left[978032.7(1 + 0.0053024\sin^2\varphi - 0.0000058\sin^2 2\varphi)\right]\text{mga}l \tag{3. 13}$$

Where γ_0^{GRS80} is the normal gravity on reference ellipsoid (GRS80), γ_Q is the normal gravity on telluroid, φ is latitude and h_Q Is the height of telluroid above a reference ellipsoid which further by

$$h_Q = h_p - \xi_p \tag{3. 14}$$

Where h_P is the ellipsoidal height of point on the topography is obtained through GNSS positioning and ξ_P is the height anomaly computed by using quasigeoid model.

3.2 Data availability

The data sets that will be used to execute this research are shown below on Table 3-1 with their respective sources.

Table 3-1: Data sets to be used and their sources.

	DATA	SOURCE
1	Geoid undulations of EGM08 and TZG17	ICGEM and DGST respectively
2	GGM data (XGM2019e) with d/o up to 5540 for computation of disturbing potential on the topographic surface and on the geoid and computation of actual gravity potential.	ICGEM (http://icgem.gfz-postdam.de/tom)
3	GPS positioning data of TAREF11 control points.	TAREF11 report
4	Height Anomalies of TZQ17	DGST

3.3 Software

In this research different software will be installed and used for performing different tasks. Table 3-2 below shows all important software that are used in this research;

Table 3-2:Software to be installed and its uses in this research.

Software Name	Use
Microsoft Excel	For data arrangement and in computation of different mathematical functions.
MATLAB	For computation of component of disturbing potential on the geoid.
Graflab	Act as program to be used in MATLAB for computation of different gravity parameters like disturbing potential on the topography and actual gravity potential on the earth surface.
Surfer software	Extraction of orthometric height, height anomaly and for statistical analysis.
Microsoft word	For report writing

CHAPTER FOUR

RESULT, ANALYSIS AND DISCUSSION

4.1 Results

This chapter present the results and discussion of the results obtained in this research. The outputs of this study are disturbing potential on the topography and on the geoid, normal gravity, extracted value of height anomaly from TZQ17 and GPS orthometric height, separation distance between geoid and quasigeoid, extracted geoid undulation from EGM08 and TZG17, computed height anomaly, normal height computed using GQQS, computed gravity potential, geopotential number and computed normal heights using geopotential numbers.

4.1.1 Disturbing potential on the topography and on the geoid

The disturbing potential on the geoid ($T_{(g,\Omega)}$) and on the topographic surface ($T_{(t,\Omega)}$) are required so as to be able to compute the mean gravity disturbance. The disturbing potential on the topography and that of the geoid was computed using equation (3.2) and (3.3) respectively. sample of the results are presented on Table 4-1 below.

Table 4-1:Sample of TAREF11 stations computed disturbing potential on the geoid and at the topographic surface.

S/No	St/Name	Location	Order	$T_{(g,\Omega)}$ (m^2/s^2)	$T_{(t,\Omega)}$ (m^2/s^2)
1	T01D	Karege	2	-274.236	-274.321
2	T02D	UGPS5 Mapinga	2	-274.673	-274.662
3	T03D	Wazo Block A	2	-275.969	-276.105
4	T04D	Wazo Block A	2	-275.992	-276.124
5	T05D	Ardhi University	2	-274.762	-274.861
6	T06D	Ardhi	2	-274.376	-274.458
7	T07D	Ubango	2	-273.486	-273.587
8	T08D	Beside Dar FBM	2	-277.737	-277.787
9	T09D	MabwePande	2	-271.923	-272.030
10	T10D	Pungu Mwakanga	2	-269.399	-269.543
11	T11D	Mzambarauni	2	-270.609	-270.797
12	T12D	NHC Quarters, Temeke	2	-274.154	-274.412
13	T13D	Kongowe	2	-273.442	-273.707

4.1.2 Normal gravity

The normal gravity information used in this study were computed using equation (3.12) as explained in sect (3.1.2.3) and the final results are in Table 4-2. Where, $\boldsymbol{\lambda}$ is geographical longitude φ is the geographical latitude and $\boldsymbol{\gamma}$ is the normal gravity.

Table 4-2:Sample of TAREF11 control points computed normal gravity.

St/Name	λ (d.ddd)	φ (d.ddd)	γ (m/s^2)
T01D	39.033594	-6.580906	9.781005
T02D	39.075094	-6.606430	9.781010
T03D	39.169451	-6.666885	9.781023
T04D	39.169226	-6.665853	9.781023
T05D	39.209671	-6.766281	9.781044
T06D	39.206800	-6.776309	9.781046
T07D	39.199759	-6.799314	9.781051
T08D	39.285448	-6.778390	9.781046
T09D	39.069018	-6.696115	9.781029
T10D	39.137358	-6.894070	9.781071
T11D	39.166149	-6.876714	9.781067
T12D	39.256386	-6.864195	9.781065
T13D	39.280354	-6.943123	9.781082
T14D	39.353300	-6.845500	9.781061
T15D	39.416499	-6.866626	9.781065
T16D	39.480220	-6.892843	9.781071
T17D	39.227936	-6.952074	9.781083
T18D	39.297818	-6.955226	9.781084
T19D	39.360392	-6.962594	9.781086
T20D	39.485141	-7.010491	9.781096
T21D	39.527237	-6.987391	9.781091

4.1.3 Extraction of height anomaly and orthometric height

The data set used for the topographic correction for equation (3.1) were GPS orthometric height from the TAREF11 reports, height anomaly value used was from TZQ17 quasi geoid model of Tanzania and the radius of the earth used was 6371Km. This data set was used due to their accuracy of their obtainment compared to the use of global topographical model i.e. Digital terrain model 2006.0 (DTM2006.0) which produces both orthometric height and height anomaly (Bucha & Janak , 2013). sample of the results are presented on Table 4-3 below;

Table 4-3: Sample data of height anomaly and GPS orthometric height that was used for topographic correction.

St/Name	λ (d.ddd)	φ (d.ddd)	H (m)	ζ (m)
T01D	39.033594	-6.580906	68.103	-26.884
T02D	39.075094	-6.606430	40.758	-26.938
T03D	39.169451	-6.666885	124.748	-27.109
T04D	39.169226	-6.665853	128.262	-27.110
T05D	39.209671	-6.766281	101.713	-27.059
T06D	39.206800	-6.776309	134.162	-27.019
T07D	39.199759	-6.799314	107.659	-26.926
T08D	39.285448	-6.778390	12.491	-27.482
T09D	39.069018	-6.696115	117.733	-26.721
T10D	39.137358	-6.894070	143.925	-26.391
T11D	39.166149	-6.876714	87.687	-26.600
T12D	39.256386	-6.864195	53.955	-27.031
T13D	39.280354	-6.943123	43.533	-26.949
T14D	39.353300	-6.845500	16.245	-27.661
T15D	39.416499	-6.866626	4.103	-28.006
T16D	39.480220	-6.892843	14.247	-28.264
T17D	39.227936	-6.952074	62.119	-26.647
T18D	39.297818	-6.955226	62.672	-27.016
T19D	39.360392	-6.962594	129.355	-27.359
T20D	39.485141	-7.010491	54.032	-27.990
T21D	39.527237	-6.987391	22.801	-28.258

4.1.4 Separation distance between geoid and quasi geoid (N-ζ)

The separation distance between geoid and quasigeoid were computed using equation (3.1) and the sample results are presented on Table 4-4 below. The maximum value obtained is 0.073m, minimum value is -0.538m and the mean value is -0.139m for all TAREF11 control points. Where ϕ is latitude, λ is longitude and N-ξ is separation distance between geoid and quasi geoid.

Table 4-4: Sample of TAREF11 stations computed separation distance between geoid and quasigeoid.

St/Name	Location	λ (d.ddd)	ϕ (d.ddd)	N-ξ(m)
T01D	Karege	39.033594	-6.580906	-0.008
T02D	UGPS5 Mapinga	39.075094	-6.606430	0.001
T03D	Wazo Block A	39.169451	-6.666885	-0.013
T04D	Wazo Block A	39.169226	-6.665853	-0.012
T05D	Ardhi University	39.209671	-6.766281	-0.009
T06D	Ardhi	39.206800	-6.776309	-0.007
T07D	Ubango	39.199759	-6.799314	-0.009
T08D	Beside Dar FBM	39.285448	-6.778390	-0.005
T09D	MabwePande	39.069018	-6.696115	-0.010
T10D	Pungu Mwakanga	39.137358	-6.894070	-0.014
T11D	Mzambarauni	39.166149	-6.876714	-0.018
T12D	NHC Quarters, Temeke	39.256386	-6.864195	-0.026
T13D	Kongowe	39.280354	-6.943123	-0.027
T14D	Mjimwema	39.353300	-6.845500	-0.027
T15D	Mbwamaji	39.416499	-6.866626	-0.020
T16D	Mbutu	39.480220	-6.892843	-0.024
T17D	Chamazi	39.227936	-6.952074	-0.029
T18D	Kipila	39.297818	-6.955226	-0.026
T19D	Lugwadu	39.360392	-6.962594	-0.051
T20D	Kijaka	39.485141	-7.010491	-0.032
T21D	Kimbiji	39.527237	-6.987391	-0.038

4.1.5 Extracted geoid undulation from EGM08 and TZG17

In this study, EGM08 and TZG17 geoid undulation were used so as be able to obtain the height anomaly as explained in equation (3.5) and sample results are represented on Table 4-5. These two-geoid models were used to observe the trend of the height anomaly obtained from global geoid model and regional geoid model. Where ϕ is latitude, λ is longitude and N is geoid undulations.

Table 4-5: Sample of EGM08 and TZG17 geoidal undulations for TAREF11 points

St/Name	λ(d.ddd)	ϕ(d.ddd)	N-EGM08(m)	N-TZG17(m)
T01D	39.033594	-6.580906	-27.466	-27.086
T02D	39.075094	-6.606430	-27.497	-27.140
T03D	39.169451	-6.666885	-27.579	-27.284
T04D	39.169226	-6.665853	-27.582	-27.285
T05D	39.209671	-6.766281	-27.459	-27.205
T06D	39.206800	-6.776309	-27.421	-27.158
T07D	39.199759	-6.799314	-27.334	-27.072
T08D	39.285448	-6.778390	-27.790	-27.474
T09D	39.069018	-6.696115	-27.212	-26.949
T10D	39.137358	-6.894070	-26.898	-26.619
T11D	39.166149	-6.876714	-27.032	-26.762
T12D	39.256386	-6.864195	-27.456	-27.156
T13D	39.280354	-6.943123	-27.400	-27.133
T14D	39.353300	-6.845500	-27.961	-27.661
T15D	39.416499	-6.866626	-28.200	-27.894
T16D	39.480220	-6.892843	-28.449	-28.178
T17D	39.227936	-6.952074	-27.171	-26.894
T18D	39.297818	-6.955226	-27.451	-27.188
T19D	39.360392	-6.962594	-27.698	-27.469
T20D	39.485141	-7.010491	-28.165	-27.959
T21D	39.527237	-6.987391	-28.436	-28.181

4.1.6 Computed height anomaly

Height anomalies were computed using equation (3.5) as explained in section 3.1.2 for EGM08 and TZG17 and the sample results are presented on Table 4-6 below. Whereby ϕ is latitude, λ is longitude and ξ is the height anomaly.

Table 4-6: Sample of EGM08 and TZG17 computed height anomalies

St/Name	λ(d.ddd)	ϕ(d.ddd)	ξ-EGM08(m)	ξ-TZG17(m)
T01D	39.033594	-6.580906	-27.457	-27.078
T02D	39.075094	-6.606430	-27.499	-27.142
T03D	39.169451	-6.666885	-27.566	-27.271
T04D	39.169226	-6.665853	-27.569	-27.273
T05D	39.209671	-6.766281	-27.450	-27.196
T06D	39.206800	-6.776309	-27.414	-27.151
T07D	39.199759	-6.799314	-27.324	-27.062
T08D	39.285448	-6.778390	-27.785	-27.469
T09D	39.069018	-6.696115	-27.203	-26.940
T10D	39.137358	-6.894070	-26.885	-26.606
T11D	39.166149	-6.876714	-27.013	-26.743
T12D	39.256386	-6.864195	-27.430	-27.130
T13D	39.280354	-6.943123	-27.374	-27.106
T14D	39.353300	-6.845500	-27.934	-27.634
T15D	39.416499	-6.866626	-28.180	-27.874
T16D	39.480220	-6.892843	-28.426	-28.155
T17D	39.227936	-6.952074	-27.142	-26.865
T18D	39.297818	-6.955226	-27.425	-27.162
T19D	39.360392	-6.962594	-27.647	-27.418
T20D	39.485141	-7.010491	-28.133	-27.927
T21D	39.527237	-6.987391	-28.398	-28.143

4.1.7 Normal height obtained using GQQS

Normal height on this research was computed using equation (3.6) as explained in section (3.1.1.2) and normal height were computed from the height anomalies obtained using EGM08 and TZG17 respectively and sample results are presented on Table 4-7 below. Whereby ϕ is latitude, λ is longitude h is ellipsoidal height and H^N is normal height.

Table 4-7: Sample of computed normal height of TAREF11 control points

St/Name	λ(d.ddd)	ϕ(d.ddd)	h(m)	H^N EGM08(m)	H^N TZG17(m)
T01D	39.033594	-6.580906	40.052	67.509	67.130
T02D	39.075094	-6.606430	12.689	40.187	39.830
T03D	39.169451	-6.666885	96.590	124.156	123.861
T04D	39.169226	-6.665853	100.102	127.671	127.375
T05D	39.209671	-6.766281	73.711	101.160	100.906
T06D	39.206800	-6.776309	106.201	133.615	133.351
T07D	39.199759	-6.799314	79.788	107.112	106.850
T08D	39.285448	-6.778390	-15.868	11.916	11.601
T09D	39.069018	-6.696115	89.964	117.166	116.903
T10D	39.137358	-6.894070	116.467	143.352	143.073
T11D	39.166149	-6.876714	60.096	87.109	86.839
T12D	39.256386	-6.864195	25.957	53.387	53.087
T13D	39.280354	-6.943123	15.583	42.956	42.689
T14D	39.353300	-6.845500	-12.272	15.662	15.362
T15D	39.416499	-6.866626	-24.663	3.517	3.211
T16D	39.480220	-6.892843	-14.771	13.654	13.383
T17D	39.227936	-6.952074	34.398	61.539	61.263
T18D	39.297818	-6.955226	34.674	62.099	61.836
T19D	39.360392	-6.962594	101.113	128.760	128.532
T20D	39.485141	-7.010491	25.310	53.443	53.237
T21D	39.527237	-6.987391	-6.189	22.209	21.955

4.1.8 computed actual gravity potential of TAREF11 control points

The actual gravity potential of TAREF11 points used in this study were computed using equation (3.7) and the results of the actual gravity potential on the surface of the earth were obtained and sample were presented on Table 4-8. where ϕ is latitude, λ is longitude and Wp is the actual gravity potential.

Table 4-8: Sample of computed actual gravity potential

St/Name	**λ(d.ddd)**	**ϕ(d.ddd)**	**Wp (m^2/s^2)**
T01D	39.033594	-6.580906	62636194.87
T02D	39.075094	-6.606430	62636462.07
T03D	39.169451	-6.666885	62635640.15
T04D	39.169226	-6.665853	62635605.78
T05D	39.209671	-6.766281	62635865.13
T06D	39.206800	-6.776309	62635547.74
T07D	39.199759	-6.799314	62635806.97
T08D	39.285448	-6.778390	62636738.32
T09D	39.069018	-6.696115	62635709
T10D	39.137358	-6.894070	62635452.3
T11D	39.166149	-6.876714	62636002.44
T12D	39.256386	-6.864195	62636332.81
T13D	39.280354	-6.943123	62636434.99
T14D	39.353300	-6.845500	62636701.99
T15D	39.416499	-6.866626	62636338.23
T16D	39.480220	-6.892843	62636721.37
T17D	39.227936	-6.952074	62636252.98
T18D	39.297818	-6.955226	62636247.81
T19D	39.360392	-6.962594	62635595.58
T20D	39.485141	-7.010491	62636331.58
T21D	39.527237	-6.987391	62636637.07

4.1.9 Computed geopotential numbers of TAREF11 control points

The geopotential number of each TAREF11 control points used in this study were computed using equation (3.11) as explained in Sect. (3.1.2.2) and the following are the sample result on the Table 4-9 of the computed geopotential numbers. Where ϕ is latitude, λ is longitude, W_p is actual potential on the surface of the earth, W_0 is Local geopotential vertical datum (W_0^{LVD},^{TZG17}) and C_p is the geopotential number.

Table 4-9: Sample of computed geopotential numbers of TAREF11 control points

St/Name	λ(d.ddd)	ϕ(d.ddd)	$W_p(m^2/s^2)$	$W_o(m^2/s^2)$	$C_p(m^2/s^2)$
T01D	39.033594	-6.580906	62636194.87	62636852.42	657.550
T02D	39.075094	-6.606430	62636462.07	62636852.42	390.350
T03D	39.169451	-6.666885	62635640.15	62636852.42	1212.270
T04D	39.169226	-6.665853	62635605.78	62636852.42	1246.643
T05D	39.209671	-6.766281	62635865.13	62636852.42	987.290
T06D	39.206800	-6.776309	62635547.74	62636852.42	1304.680
T07D	39.199759	-6.799314	62635806.97	62636852.42	1045.453
T08D	39.285448	-6.778390	62636738.32	62636852.42	114.097
T09D	39.069018	-6.696115	62635709	62636852.42	1143.417
T10D	39.137358	-6.894070	62635452.3	62636852.42	1400.119
T11D	39.166149	-6.876714	62636002.44	62636852.42	849.976
T12D	39.256386	-6.864195	62636332.81	62636852.42	519.609
T13D	39.280354	-6.943123	62636434.99	62636852.42	417.428
T14D	39.353300	-6.845500	62636701.99	62636852.42	150.429
T15D	39.416499	-6.866626	62636338.23	62636852.42	31.737
T16D	39.480220	-6.892843	62636721.37	62636852.42	131.048
T17D	39.227936	-6.952074	62636252.98	62636852.42	599.441

4.1.10 Computed normal heights using geopotential numbers

The normal heights in this study were computed using equation (2.3) and the final results of the computed normal height using geopotential numbers are in Table 4-10. Where ϕ is latitude, λ is longitude, H^N is the normal height by fundamental approach, C_p is geopotential number and γ is normal gravity along the ellipsoidal normal.

Table 4-10: Sample of computed normal height of TAREF11 control points using geopotential number.

St/Name	λ(d.ddd)	ϕ(d.ddd)	γ(m/s^2)	C_p(m^2/s^2)	H^N(m)
T01D	39.033594	-6.580906	9.781005	657.550	67.229
T02D	39.075094	-6.606430	9.781010	390.350	39.910
T03D	39.169451	-6.666885	9.781023	1212.270	123.948
T04D	39.169226	-6.665853	9.781023	1246.643	127.463
T05D	39.209671	-6.766281	9.781044	987.290	100.944
T06D	39.206800	-6.776309	9.781046	1304.680	133.397
T07D	39.199759	-6.799314	9.781051	1045.453	106.891
T08D	39.285448	-6.778390	9.781046	114.097	11.665
T09D	39.069018	-6.696115	9.781029	1143.417	116.908
T10D	39.137358	-6.894070	9.781071	1400.119	143.155
T11D	39.166149	-6.876714	9.781067	849.976	86.904
T12D	39.256386	-6.864195	9.781065	519.609	53.125
T13D	39.280354	-6.943123	9.781082	417.428	42.678
T14D	39.353300	-6.845500	9.781061	150.429	15.380
T15D	39.416499	-6.866626	9.781065	31.737	3.245
T16D	39.480220	-6.892843	9.781071	131.048	13.398
T17D	39.227936	-6.952074	9.781083	599.441	61.288
T18D	39.297818	-6.955226	9.781084	604.614	61.816

4.1.11 Validation of the normal heights

These computed normal heights are the final outputs of this dissertation. The normal height computed using GQGS are validated using the normal computed using geopotential number. The difference between these heights systems were evaluated in this dissertation in order to investigate if there are any differences between them. The sample of the results are presented on Table 4-11. ***Δ*H1** is the difference of normal height computed using geoid to quasi-geoid separation model using EGM 08 and normal height computed using geopotential numbers, ***Δ*H2** is the difference of normal height computed using geoid to quasi-geoid separation model using TZG17 and normal height computed using geopotential numbers and ***Δ*H3** is the difference of normal height computed using geoid to quasi-geoid separation model using TZG17 and EGM 08.

Table 4-11: Sample of the normal height differences of TAREF 11 control points

St/Name	H^N EGM08(m)	H^N TZG17(m)	H^N(m)	ΔH1(m)	ΔH2(m)	ΔH3(m)
T01D	67.509	67.130	67.229	0.280	-0.100	0.379
T02D	40.187	39.830	39.910	0.278	-0.079	0.357
T03D	124.156	123.861	123.948	0.208	-0.088	0.295
T04D	127.671	127.375	127.463	0.208	-0.088	0.296
T05D	101.160	100.906	100.944	0.216	-0.038	0.254
T06D	133.615	133.351	133.397	0.218	-0.046	0.263
T07D	107.112	106.850	106.891	0.221	-0.041	0.262
T08D	11.916	11.601	11.665	0.251	-0.064	0.315
T09D	117.166	116.903	116.908	0.258	-0.005	0.263
T10D	143.352	143.073	143.155	0.196	-0.083	0.279
T11D	87.109	86.839	86.904	0.206	-0.065	0.270
T12D	53.387	53.087	53.125	0.262	-0.038	0.300

4.2 Analysis

4.2.1 Statistical analysis of Normal heights of TAREF11 controls

Golden surfer software version 16 grid math module was used to compare and assess statistically the fit between the normal heights pair. Comparison was made between the computed normal heights using geopotential number (H^N) and normal heights using geoid-quasigeoid separation from TZG17 and EGM08. Normal heights computed by geopotential number was used to validate the normal height produced using geoid-quasigeoid separation from TZG17 and EGM08. The statistic values obtained are shown in Table 4-12 below.

Table 4-12: Statistic of computed normal heights of TAREF11 controls

STATISTIC	H^N EGM08(m)	H^N TZG17(m)	H^N(m)
Number of values	577	577	577
Sum	574171.181	573923.129	574183.231
Minimum	3.326	3.076	3.011
Maximum	2743.766	2744.150	2745.857
Mean	995.097	994.667	995.118
Standard deviation	539.150	539.109	539.374
Root Mean Square:	1131.546	1131.149	1131.672

The statistics of the residuals was conducted by subtracting the normal heights obtained by separation model to those obtain using geopotential number so as to validate the heights. **ΔH1** is the difference of normal height computed using geoid-quasigeoid separation using EGM08 and normal height computed using geopotential numbers, **ΔH2** is the difference of normal height computed using geoid to quasi-geoid separation model using TZG17 and normal height computed using geopotential numbers and **ΔH3** is the difference of normal height computed using geoid-quasigeoid separation using TZG17 and EGM08. The statistic values of height differences obtained are shown in Table 4-13 below;

Table 4-13: Statistical analysis of TAREF 11 controls on normal heights differences

STATISTIC	ΔH1(m)	ΔH2(m)	ΔH3(m)
Number of values	577	577	577
Sum	-12.051	-260.102	248.052
Minimum	-15.032	-1.787	-14.790
Maximum	1.885	0.392	2.195
Mean	-0.021	-0.451	0.430
Standard deviation	0.786	0.328	0.743
95% confidence interval	0.064	0.027	0.061
Root Mean Square:	0.786	0.558	0.858

Later on, a test was conducted at 95% confidence interval to see how significantly consistency the results are at that confidence interval. Table 4-14 shows the statistical analysis of normal heights difference at 95% confidence level.

Table 4-14: Statistic analysis of normal heights differences at 95% confidence interval

STATISTIC AT 95% CI	ΔH1(m)	ΔH2(m)	ΔH3(m)
Number of values	80	48	110
Number of missing values	497	529	467
Sum	-1.472	-21.661	46.784
Minimum	-0.083	-0.477	0.370
Maximum	0.043	-0.424	0.490
Mean	-0.018	-0.451	0.425
Standard deviation	0.035	0.016	0.037
Root Mean Square:	0.040	0.452	0.427

4.3 Discussion of the results

Table 4-14 in sect 4.2 shows the statistical analysis for computed normal heights differences at 95% confidence interval of normal height computed using geopotential number and normal heights computed using geoid-quasigeoid separation from TZG17 and EGM 08.

Based on the statistics of normal height differences at 95% confidence interval as shown on Table4-14, Normal heights computed using geoid-quasigeoid separation from EGM08 agrees with the normal heights computed using geopotential number. It shows that there is a significance difference of 3.5 cm between them and with the root mean square of 4 cm. These better results were contributed by likeness of the explicit formula for geoid-quasigeoid separation and EGM08.The separation distance obtained behave best with the global geoid models.

Normal heights computed using geoid-quasigeoid separation from TZG17 and normal heights computed using geopotential number have the significance difference of 1.6 cm between them and with root mean square of 45.2 cm. They are not statistically fit and this is contributed by difference in datum between these heights system.

Normal height computed using geoid-quasigeoid separation from TZG17 and EGM08 have the significance difference of 3.7cm and with the root mean square of 42.7 cm. These results are contributed by the accuracy differences between the 2 geoidal model used in this study.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The normal heights were successful computed using geoid and quasi geoid separation model, analyzed and validation was performed using geopotential number.

Assessment of normal height was done based on statistical analysis in which a test was conducted at 95% and Normal heights computed using geoid-quasigeoid separation from EGM08 and normal heights computed using geopotential number showed better results with significance difference of 3.5 cm between them and root mean square of 4 cm. They are statistical fit and agree with each other.

Normal heights computed using geoid-quasigeoid separation from TZG17 and normal heights computed using geopotential number have the significance difference of 1.6 cm between them with root mean square of 45.2 cm. Normal height computed using geoid-quasigeoid separation from TZG17 and EGM08 have the significance difference of 3.7cm and with the root mean square of 42.7 cm.

5.2 Recommendations

Referring from the results and discussion in chapter four above and conclusion made in chapter five the following are recommended.

a) There should be more assessment on different GQGS model so as to know which one produces the best separation distance for the Tanzania.

b) There should be improvement on Local Geopotential Vertical Datum ($W_0^{LVD,\ TZG17}$) using the latest Gravimetric Geoid Models for Tanzania (TZG19) since geopotential heights depend much on local vertical datum.

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APPENDICES

APPENDIX 1: Computed normal heights for CORS and zero order TAREF11 stations

S/n	Station	$\lambda(d.ddd)$	θ(d.ddd)	N-ζ(m)	Cp(m^2/s^2)	H^N EGM08(m)	H^N TZG17(m)	H^N(m)
1	ARIT	32.825455	-5.040146	-0.194	11956.782	1222.683	1222.701	1223.192
2	MOSH	37.337103	-3.349947	-0.011	8396.226	858.836	858.344	858.214
3	TANZ	39.207926	-6.765585	-0.009	955.448	97.905	97.652	97.688
4	DARA	39.297181	-6.817814	-0.013	487.735	50.138	49.825	49.866
5	ZNZB	39.210716	-6.218534	-0.011	232.587	24.135	23.709	23.780
6	DODC	35.748304	-6.169644	-0.160	10947.968	1119.956	1119.515	1119.912
7	T01Z	34.800277	-11.27367	-0.050	5295.829	541.343	541.049	541.507
8	T02Z	37.340529	-11.06259	-0.083	6798.339	695.305	694.915	695.197
9	T03Z	40.184944	-10.33472	-0.015	1091.676	112.018	111.515	111.606
10	T05Z	35.753157	-7.675926	-0.210	13968.260	1429.248	1428.423	1428.031
11	T07Z	31.605715	-7.951187	-0.302	17619.133	1802.439	1802.305	1802.843
12	T09Z	39.116630	-5.087755	0.012	113.192	11.741	11.609	11.573
13	T11Z	32.825463	-5.040230	-0.194	11924.027	1219.333	1219.351	1219.839
14	T12Z	29.668363	-4.886718	-0.221	8053.181	823.368	823.248	823.696
15	T13Z	31.799225	-1.325547	-0.148	12244.008	1252.590	1252.211	1252.641
16	T14Z	32.922518	-2.446997	-0.097	11194.750	1145.395	1144.838	1145.229
17	T15Z	33.837949	-1.537530	-0.139	11589.690	1185.673	1185.299	1185.661
18	T16Z	37.804929	-6.748471	-0.018	4919.717	503.323	502.862	503.105

APPENDIX 2: Computed normal heights for Dar es salaam second order TAREF11 control points

S/No	Station	$\lambda(d.ddd)$	θ(d.ddd)	N-ζ(m)	Cp(m^2/s^2)	H^N EGM08(m)	H^N TZG17(m)	H^N(m)
1	T01D	39.033594	-6.580906	-0.008	657.550	67.509	67.130	67.229
2	T02D	39.075094	-6.606430	0.001	390.350	40.187	39.830	39.910
3	T03D	39.169451	-6.666885	-0.013	1212.270	124.156	123.861	123.948
4	T04D	39.169226	-6.665853	-0.012	1246.643	127.671	127.375	127.463
5	T05D	39.209671	-6.766281	-0.009	987.290	101.160	100.906	100.944
6	T06D	39.206800	-6.776309	-0.007	1304.680	133.615	133.351	133.397
7	T07D	39.199759	-6.799314	-0.009	1045.453	107.112	106.850	106.891
8	T08D	39.285448	-6.778390	-0.005	114.097	11.916	11.601	11.665
9	T09D	39.069018	-6.696115	-0.010	1143.417	117.166	116.903	116.908
10	T10D	39.137358	-6.894070	-0.014	1400.119	143.352	143.073	143.155
11	T11D	39.166149	-6.876714	-0.018	849.976	87.109	86.839	86.904
12	T12D	39.256386	-6.864195	-0.026	519.609	53.387	53.087	53.125
13	T13D	39.280354	-6.943123	-0.027	417.428	42.956	42.689	42.678
14	T14D	39.353300	-6.845500	-0.027	150.429	15.662	15.362	15.380
15	T15D	39.416499	-6.866626	-0.020	31.737	3.517	3.211	3.245
16	T16D	39.480220	-6.892843	-0.024	131.048	13.654	13.383	13.398
17	T17D	39.227936	-6.952074	-0.029	599.441	61.539	61.263	61.288
18	T18D	39.297818	-6.955226	-0.026	604.614	62.099	61.836	61.816
19	T19D	39.360392	-6.962594	-0.051	1256.840	128.760	128.532	128.505
20	T20D	39.485141	-7.010491	-0.032	520.844	53.443	53.237	53.251
21	T21D	39.527237	-6.987391	-0.038	215.347	22.209	21.955	22.017

APPENDIX 3: Computed normal heights for first order TAREF11 control points

S/No	Station	$\lambda(d.ddd)$	θ(d.ddd)	N-ζ(m)	Cp(m^2/s^2)	H^N EGM08(m)	H^N TZG17(m)	H^N(m)
1	T01F	31.150543	-1.531898	-0.255	16115.975	1648.743	1648.097	1648.075
2	T02F	34.681635	-1.834966	-0.297	15803.445	1616.609	1616.290	1617.069
3	T03F	30.669423	-2.506662	-0.274	17468.558	1786.942	1786.806	1787.585
4	T04F	31.365769	-2.664347	-0.226	14056.707	1437.999	1437.652	1438.206
5	T05F	34.819758	-2.436597	-0.236	14693.495	1503.101	1502.730	1503.407
6	T06F	35.545128	-2.067269	-0.371	19780.442	2023.347	2023.202	2024.397
7	T07F	36.704096	-2.742595	-0.203	12991.343	1328.865	1328.485	1329.133
8	T08F	30.708531	-3.593106	-0.242	14467.467	1480.033	1479.777	1480.248
9	T09F	31.524587	-3.335702	-0.178	11994.623	1227.064	1226.688	1227.094
10	T10F	32.407390	-3.405327	-0.199	12322.112	1260.662	1260.156	1260.616
11	T11F	33.406123	-3.370276	-0.197	11957.589	1223.268	1222.806	1223.302
12	T12F	34.305680	-3.517024	-0.190	11686.961	1195.512	1195.175	1195.599
13	T13F	35.685937	-3.336089	-0.212	14972.644	1531.596	1531.258	1531.977
14	T14F	36.706703	-3.437632	-0.176	12245.708	1252.553	1252.248	1252.794
15	T15F	30.274507	-4.480611	-0.198	13332.248	1364.161	1363.456	1364.007
16	T16F	31.337040	-5.046428	-0.128	10613.109	1086.122	1085.175	1085.662
17	T17F	32.861783	-4.277974	-0.179	11795.501	1206.485	1206.259	1206.697
18	T18F	33.449134	-4.531640	-0.214	12410.920	1269.215	1269.170	1269.689
19	T19F	34.740144	-4.813875	-0.254	14664.147	1500.160	1499.573	1500.360
20	T20F	35.771961	-4.896166	-0.201	13671.453	1398.550	1398.219	1398.725
21	T21F	36.935376	-4.205005	-0.205	14096.236	1441.832	1441.521	1442.228
22	T22F	37.914662	-4.514897	-0.020	6312.612	645.789	645.393	645.616
23	T23F	38.294000	-4.803467	-0.255	13460.858	1376.936	1377.101	1377.167
24	T24F	30.477565	-5.614652	-0.158	12717.174	1302.017	1300.562	1301.016
25	T25F	31.652573	-5.328049	-0.125	10700.453	1094.819	1094.182	1094.596
26	T26F	32.743926	-5.612446	-0.179	11507.686	1176.859	1176.718	1177.212
27	T27F	33.155147	-5.323667	-0.203	11862.651	1212.983	1213.003	1213.551
28	T28F	34.490344	-5.700680	-0.209	12755.526	1304.983	1304.347	1304.940
29	T29F	35.970240	-5.967471	-0.121	10438.003	1067.817	1067.436	1067.724
30	T30F	36.577800	-5.300954	-0.243	15054.839	1540.050	1539.779	1540.351
31	T31F	37.289189	-5.567020	-0.117	12366.194	1265.069	1264.370	1265.089

32	T32F	38.485105	-5.500057	-0.037	3880.011	397.057	396.788	396.771
33	T33F	30.940304	-6.367420	-0.104	10148.728	1038.970	1037.540	1038.110
34	T34F	31.237071	-6.294712	-0.161	11853.440	1213.244	1212.295	1212.587
35	T35F	32.062679	-6.724554	-0.156	11560.065	1182.860	1181.793	1182.547
36	T36F	33.206459	-6.806987	-0.154	11696.635	1196.608	1195.969	1196.523
37	T37F	34.969521	-6.353896	-0.187	11222.774	1147.685	1147.781	1148.034
38	T38F	35.334424	-6.431252	-0.119	8698.353	889.818	889.169	889.688
39	T39F	36.405720	-6.434562	-0.081	7678.678	785.590	784.697	785.354
40	T40F	37.568723	-6.501059	0.008	3731.740	381.772	381.302	381.599
41	T41F	38.386437	-6.333831	-0.052	2742.510	280.642	280.563	280.430
42	T42F	31.021036	-7.494289	-0.247	15290.725	1563.909	1563.565	1564.434
43	T43F	32.485201	-7.858072	-0.205	11464.163	1172.486	1172.439	1172.702
44	T44F	33.352548	-7.835440	-0.219	14330.666	1466.248	1464.928	1466.127
45	T45F	35.082538	-7.832083	-0.120	9428.549	963.460	963.978	964.378
46	T46F	36.420554	-7.534399	-0.003	5553.031	568.509	566.943	567.076
47	T47F	38.609581	-7.484352	-0.026	1733.666	179.145	176.950	177.259
48	T48F	39.193472	-7.650947	-0.009	4440.623	455.968	454.039	454.095
49	T49F	31.492911	-8.668020	-0.318	1407.687	128.895	143.684	143.927
50	T50F	31.493589	-8.682976	-0.288	15895.454	1625.699	1625.282	1625.303
51	T51F	32.771788	-8.874605	-0.193	15213.705	1556.330	1555.932	1556.493
52	T52F	33.440158	-8.559334	-0.276	15110.031	1545.627	1545.347	1545.892
53	T53F	34.380764	-8.744546	-0.137	10732.609	1097.713	1097.323	1097.803
54	T54F	35.349751	-8.525336	-0.354	20143.544	2060.902	2059.913	2060.369
55	T55F	36.426015	-8.667373	-0.016	3423.324	350.129	350.089	350.038
56	T56F	37.036299	-7.800338	0.003	2643.968	270.782	270.074	270.344
57	T57F	38.882518	-8.801428	0.005	800.087	82.228	81.739	81.799
58	T58F	39.522126	-8.917558	-0.029	110.464	11.659	11.372	11.293
59	T59F	33.873317	-9.609341	-0.038	4824.636	493.433	492.786	493.343
60	T60F	34.585379	-9.594812	-0.314	21474.640	2196.489	2196.350	2197.660
61	T61F	35.103256	-9.549573	-0.239	16904.284	1729.353	1728.606	1728.560
62	T62F	35.893576	-10.08224	-0.134	10079.707	1031.314	1030.523	1030.947
63	T63F	37.596380	-9.668195	-0.047	6738.889	689.506	688.869	689.147
64	T64F	38.362250	-9.417253	-0.032	3770.611	385.983	385.573	385.546
65	T65F	39.544328	-9.666848	-0.005	1430.959	146.624	146.393	146.298

APPENDIX 4: Computed normal heights for second order TAREF11 control points

Station	$\lambda(d.ddd)$	θ(d.ddd)	N-ζ(m)	Cp(m^2/s^2)	HN EGM08(m)	H^N TZG17(m)	H^N(m)
T001	30.673411	-1.205802	-0.281	16235.266	1661.006	1660.657	1661.294
T002	30.939904	-1.037301	-0.172	12198.893	1248.247	1247.707	1248.024
T003	31.405357	-1.204689	-0.175	11309.855	1157.108	1156.276	1157.019
T004	31.812719	-1.221719	-0.173	11992.510	1226.790	1226.433	1226.896
T005	30.946814	-1.529348	-0.212	14204.521	1453.342	1452.646	1453.350
T006	31.725840	-1.607506	-0.163	12988.913	1328.925	1328.493	1328.895
T007	31.112566	-1.751691	-0.273	15287.741	1563.901	1563.375	1564.261
T008	31.654815	-1.851175	-0.139	12611.539	1290.369	1290.006	1290.261
T011	33.089901	-2.092851	-0.121	11598.188	1186.567	1186.102	1186.527
T012	33.449786	-2.149593	-0.102	11281.092	1154.213	1153.685	1154.069
T013	33.860276	-2.000975	-0.217	12152.949	1243.287	1242.769	1243.315
T014	33.924368	-1.682146	-0.220	13342.426	1364.984	1364.642	1365.087
T020	34.038905	-1.178224	-0.171	11695.883	1196.479	1196.138	1196.533
T021	34.371790	-1.351443	-0.248	13915.701	1423.432	1422.914	1423.781
T022	34.125766	-1.455278	-0.196	12156.795	1243.621	1243.182	1243.712
T023	34.544233	-1.438293	-0.208	12986.604	1328.479	1327.979	1328.660
T024	34.234147	-1.735294	-0.241	12630.058	1292.030	1291.461	1292.158
T025	34.031519	-1.903265	-0.253	13912.083	1423.168	1422.725	1423.406
T026	34.348268	-1.445562	-0.190	11684.072	1195.181	1194.547	1195.322
T027	34.402429	-1.792076	-0.247	13794.669	1411.239	1410.563	1411.386
T030	30.857752	-2.490313	-0.257	15343.460	1569.473	1569.225	1569.959
T031	30.514951	-2.600305	-0.231	16240.186	1661.208	1661.048	1661.784
T032	30.723995	-2.799635	-0.231	14936.751	1527.980	1527.480	1528.310
T033	30.932243	-2.638830	-0.210	13104.639	1340.463	1340.013	1340.733
T035	31.445215	-2.138457	-0.174	11996.940	1227.360	1227.058	1227.344
T036	31.578764	-2.114792	-0.143	11261.338	1152.188	1151.722	1152.047
T037	31.683730	-2.388912	-0.158	11504.182	1177.070	1176.320	1176.902
T038	31.734609	-2.636356	-0.138	11782.888	1205.651	1204.855	1205.428
T039	31.007979	-2.783247	-0.176	12810.617	1310.444	1309.882	1310.632
T040	31.200497	-2.907148	-0.198	13210.964	1351.457	1351.036	1351.615
T041	31.479374	-2.804722	-0.189	13204.239	1350.909	1350.394	1350.928

T042	31.914058	-3.014300	-0.202	12515.415	1280.462	1279.855	1280.409
T045	32.410541	-2.451055	-0.118	11355.954	1161.760	1161.389	1161.729
T046	32.093443	-2.561619	-0.143	11570.698	1183.770	1183.182	1183.709
T047	32.653438	-2.649659	-0.181	12264.366	1254.679	1254.139	1254.714
T048	32.737879	-2.539729	-0.134	11352.587	1161.497	1160.922	1161.384
T049	32.989913	-2.686796	-0.183	12020.465	1229.727	1229.106	1229.746
T050	32.262529	-2.889815	-0.203	12313.414	1259.457	1259.337	1259.732
T051	32.429776	-2.843666	-0.178	11710.727	1197.903	1197.571	1198.040
T052	32.849484	-2.921510	-0.144	11889.901	1216.370	1215.716	1216.379
T056	33.096451	-2.851159	-0.169	11492.140	1175.647	1174.985	1175.664
T057	33.458898	-2.602168	-0.156	11241.566	1150.047	1149.602	1150.019
T058	33.339380	-2.970714	-0.179	11957.724	1223.258	1222.781	1223.321
T059	33.997188	-2.787275	-0.196	12564.095	1285.433	1284.836	1285.395
T060	33.862311	-2.219449	-0.143	11141.410	1139.868	1139.183	1139.771
T065	34.239369	-2.782216	-0.244	14138.530	1446.453	1445.803	1446.581
T066	34.144412	-2.170610	-0.186	11664.118	1193.380	1192.670	1193.275
T067	34.330607	-1.971381	-0.189	12255.753	1253.834	1253.104	1253.839
T068	34.640164	-2.084682	-0.223	13254.039	1355.952	1355.462	1356.034
T069	34.907608	-2.656215	-0.294	16314.242	1668.951	1668.411	1669.366
T070	34.780002	-2.758445	-0.279	15923.593	1629.039	1628.434	1629.360
T071	34.458489	-2.286134	-0.212	12614.268	1290.485	1290.072	1290.537
T072	34.174006	-2.403463	-0.205	12541.957	1283.143	1282.626	1283.133
T080	30.969487	-3.289493	-0.173	12159.077	1243.885	1243.163	1243.928
T081	30.602296	-3.501096	-0.196	12877.395	1317.281	1317.053	1317.458
T082	30.904200	-3.639432	-0.201	12727.896	1302.260	1301.597	1302.152
T083	30.609444	-3.903085	-0.226	13200.728	1350.545	1350.286	1350.553
T085	31.216241	-3.042647	-0.164	12150.195	1242.981	1242.449	1243.022
T086	31.777406	-3.033451	-0.187	12201.451	1248.448	1247.760	1248.269
T087	31.900714	-3.467964	-0.199	12117.249	1239.474	1239.027	1239.644
T088	31.420275	-3.294572	-0.169	11904.912	1217.908	1217.459	1217.911
T089	31.351063	-3.188737	-0.187	12329.301	1261.341	1260.844	1261.355
T091	32.239296	-3.118771	-0.213	12894.921	1318.881	1318.648	1319.258
T092	32.606136	-3.187429	-0.185	11708.858	1197.771	1197.369	1197.844
T093	33.018352	-3.069331	-0.197	12185.608	1246.564	1246.003	1246.647

T094	32.866721	-3.311899	-0.190	11836.621	1210.892	1210.496	1210.920
T095	32.964400	-3.905928	-0.187	11524.750	1178.917	1178.399	1178.989
T096	32.590755	-3.823846	-0.193	12511.631	1279.866	1279.486	1280.010
T097	32.436172	-3.769972	-0.184	12082.638	1236.102	1235.544	1236.097
T098	32.501480	-3.910396	-0.177	11803.848	1207.470	1207.053	1207.557
T101	33.254307	-3.172351	-0.185	11518.936	1178.474	1177.912	1178.404
T102	33.519776	-3.140455	-0.188	12127.247	1240.559	1240.169	1240.672
T103	33.790751	-3.187279	-0.231	13018.600	1331.685	1331.287	1331.918
T104	33.825337	-3.626612	-0.190	11231.065	1148.571	1148.511	1148.933
T105	33.510485	-3.612760	-0.196	11471.340	1173.378	1172.993	1173.527
T106	33.194303	-3.872771	-0.203	11390.790	1165.112	1164.518	1165.278
T107	33.703830	-3.872488	-0.190	10953.442	1120.083	1120.063	1120.514
T108	33.974020	-3.917522	-0.174	10601.651	1083.700	1084.011	1084.507
T111	34.116780	-3.420971	-0.195	11789.265	1205.843	1205.375	1206.072
T112	34.447661	-3.541948	-0.201	11932.377	1220.641	1220.298	1220.719
T113	34.813704	-3.383942	-0.288	15842.979	1620.760	1620.595	1621.096
T114	34.200741	-3.039648	-0.211	13390.171	1369.888	1369.124	1369.960
T115	34.749269	-3.017684	-0.304	16345.967	1672.148	1671.668	1672.610
T116	34.210545	-3.851204	-0.182	10543.437	1078.023	1078.086	1078.550
T117	34.307675	-3.953540	-0.153	10222.854	1045.292	1045.198	1045.738
T118	34.519588	-3.904154	-0.159	10203.211	1043.754	1043.307	1043.728
T120	35.245436	-1.858035	-0.344	17044.606	1743.569	1742.972	1744.173
T121	35.155159	-2.013181	-0.321	16962.202	1735.155	1734.466	1735.732
T122	35.502447	-2.584384	-0.319	15523.724	1587.836	1587.695	1588.417
T123	35.137727	-2.912641	-0.299	15880.143	1624.518	1623.856	1624.909
T124	36.004893	-3.002355	0.007	8137.980	832.559	831.740	832.392
T126	35.878507	-2.614311	-0.036	6368.677	651.338	650.781	651.366
T127	35.697107	-2.171839	-0.216	12986.822	1328.369	1328.200	1328.677
T128	35.884848	-2.136975	-0.217	11901.801	1217.356	1217.108	1217.606
T129	35.508071	-2.380822	-0.445	18367.888	1878.764	1878.917	1879.699
T131	35.270699	-3.223118	-0.387	17804.872	1821.344	1820.880	1822.018
T132	35.896900	-3.390227	-0.138	9581.157	980.136	979.396	980.072
T133	35.341913	-3.520379	-0.172	10271.198	1050.923	1049.891	1050.691
T134	35.691086	-3.034183	-0.498	25738.303	2632.928	2633.391	2634.886

T135	35.457922	-3.211354	-0.538	24292.723	2485.147	2484.932	2486.719
T136	35.673278	-3.446048	-0.195	13706.048	1402.119	1401.673	1402.293
T137	35.550953	-3.863624	-0.297	17144.690	1754.122	1754.210	1754.392
T138	35.533361	-3.710293	-0.349	18955.571	1939.319	1939.434	1939.870
T139	35.746098	-3.670148	-0.093	9438.012	965.768	965.178	965.420
T140	35.929162	-3.706405	-0.151	9975.056	1020.559	1019.849	1020.380
T141	37.125362	-2.867334	-0.220	16538.686	1691.904	1691.493	1692.347
T142	37.466986	-2.934159	-0.324	19437.781	1988.392	1988.557	1989.281
T145	36.779403	-2.561278	-0.142	12887.898	1318.304	1317.860	1318.545
T146	36.482302	-2.529237	-0.334	16565.666	1694.380	1694.592	1695.117
T147	36.695657	-3.040999	-0.197	13723.095	1403.708	1403.402	1404.044
T148	36.300303	-2.824402	-0.213	12385.084	1266.834	1266.459	1267.070
T149	36.114245	-2.730315	-0.166	11621.514	1188.603	1188.287	1188.909
T151	32.235856	-4.205382	-0.187	11649.078	1191.341	1191.137	1191.711
T152	32.521049	-4.002499	-0.192	12548.072	1283.567	1283.282	1283.738
T153	32.514466	-4.400174	-0.178	11643.339	1190.728	1190.754	1191.120
T154	32.687443	-4.537083	-0.191	11931.412	1220.097	1220.150	1220.605
T155	32.090519	-4.635129	-0.160	11014.120	1126.357	1126.111	1126.712
T156	32.837587	-4.739923	-0.177	11340.753	1159.606	1159.590	1160.142
T157	32.590536	-4.921784	-0.173	11261.715	1151.680	1151.622	1152.049
T158	32.308419	-4.952096	-0.165	11046.434	1129.839	1129.496	1130.014
T161	33.131501	-4.036534	-0.181	11068.561	1132.243	1131.577	1132.294
T162	33.205495	-4.225444	-0.189	12043.148	1231.829	1231.388	1232.047
T163	33.707561	-4.001504	-0.183	10631.896	1087.141	1087.283	1087.602
T164	33.883493	-4.303304	-0.179	10524.915	1076.106	1076.114	1076.648
T165	33.859484	-4.567941	-0.176	10658.037	1089.820	1089.747	1090.268
T166	33.236034	-4.770170	-0.200	12188.456	1246.383	1246.219	1246.912
T167	33.102201	-4.866148	-0.209	12287.042	1256.389	1256.318	1257.001
T168	33.169829	-4.452769	-0.196	12067.480	1234.121	1233.799	1234.534
T171	34.384440	-4.267931	-0.315	15312.839	1566.145	1566.106	1566.794
T172	34.097481	-4.388658	-0.178	10587.394	1082.734	1082.412	1083.041
T173	34.529587	-4.419272	-0.279	14934.059	1527.691	1527.417	1528.006
T174	34.748342	-4.136518	-0.299	15516.701	1587.417	1587.014	1587.671
T175	34.841259	-4.354201	-0.267	15103.038	1545.094	1544.626	1545.309

T176	34.870905	-4.672387	-0.283	15803.030	1616.640	1616.162	1616.978
T177	34.634673	-4.555874	-0.215	13477.132	1378.654	1378.236	1378.837
T178	34.258960	-4.684231	-0.212	12007.604	1228.215	1227.656	1228.401
T181	35.022183	-4.197213	-0.325	16987.906	1738.003	1737.344	1738.327
T182	35.361999	-4.067246	-0.386	18935.701	1937.356	1936.999	1937.826
T183	35.790692	-3.997563	-0.133	9776.584	1000.358	999.563	1000.065
T184	35.717269	-4.220693	-0.217	14041.219	1436.541	1435.741	1436.595
T185	35.833802	-4.729294	-0.257	14617.857	1495.404	1495.035	1495.623
T186	35.379503	-4.528306	-0.322	16915.804	1730.495	1729.760	1730.935
T187	35.315996	-4.963541	-0.203	12251.152	1253.263	1252.729	1253.326
T188	35.180924	-4.665769	-0.258	16649.961	1703.281	1702.597	1703.707
T191	32.072636	-5.059803	-0.142	10906.491	1115.709	1115.250	1115.689
T192	32.319725	-5.104675	-0.167	11188.465	1144.415	1144.011	1144.548
T193	32.629636	-5.101882	-0.182	11311.068	1156.719	1156.619	1157.097
T194	32.280923	-5.266976	-0.159	11138.874	1139.496	1138.891	1139.470
T195	32.695287	-5.370907	-0.181	11464.780	1172.441	1172.313	1172.825
T196	32.505421	-5.730781	-0.161	10770.968	1102.019	1101.312	1101.806
T197	32.611712	-5.919240	-0.161	10717.625	1096.536	1095.885	1096.342
T198	32.888621	-5.802676	-0.192	11227.961	1148.109	1148.096	1148.577
T201	33.130528	-5.131427	-0.214	12459.105	1274.060	1274.021	1274.609
T202	33.475891	-5.314486	-0.181	11445.251	1170.411	1170.136	1170.828
T203	33.837002	-5.045328	-0.160	10251.526	1048.414	1048.348	1048.656
T204	33.497252	-5.147052	-0.200	12067.077	1234.020	1233.867	1234.480
T205	32.983950	-5.552035	-0.225	12448.080	1272.804	1273.107	1273.472
T206	33.262914	-5.935529	-0.218	12009.927	1227.923	1228.130	1228.613
T207	33.900943	-5.977141	-0.260	14830.560	1516.915	1516.650	1517.369
T208	33.647850	-5.434325	-0.193	11735.478	1200.210	1199.840	1200.532
T211	34.044311	-5.152873	-0.153	10365.726	1060.289	1060.030	1060.342
T212	34.754855	-5.006150	-0.281	15251.898	1560.257	1559.677	1560.536
T213	34.933736	-5.264183	-0.223	13523.304	1383.424	1382.780	1383.549
T214	34.664678	-5.554911	-0.232	13559.630	1387.172	1386.619	1387.260
T215	34.825895	-5.768677	-0.200	12262.370	1254.445	1253.951	1254.457
T216	34.366306	-5.642944	-0.211	12772.653	1306.767	1306.125	1306.695
T217	34.395378	-5.304725	-0.196	12042.336	1232.102	1231.469	1231.944

T218	34.055667	-6.021394	-0.270	15154.480	1550.131	1549.675	1550.533
T221	35.025209	-5.055804	-0.174	12277.217	1255.974	1255.308	1255.992
T222	35.435929	-5.206455	-0.195	11587.716	1185.363	1184.967	1185.412
T223	35.809386	-5.091616	-0.199	12465.658	1275.247	1274.715	1275.281
T224	35.847509	-5.485278	-0.154	11273.746	1153.363	1152.676	1153.270
T225	35.077939	-5.321924	-0.186	11980.847	1225.647	1225.053	1225.649
T226	35.125233	-5.882914	-0.096	8308.403	850.009	849.366	849.796
T227	35.324854	-5.962900	-0.119	8250.666	844.064	843.473	843.887
T228	35.704396	-5.642978	-0.161	11064.981	1131.945	1131.552	1131.900
T231	35.061504	-6.145109	-0.081	8104.749	829.124	828.660	828.954
T232	35.491433	-6.100261	-0.110	9174.347	938.510	937.942	938.402
T233	35.012923	-6.648822	-0.158	10349.632	1058.152	1058.330	1058.665
T234	35.392597	-6.905029	-0.126	8985.580	919.191	918.671	919.070
T235	35.621538	-6.602822	-0.149	9716.044	994.010	993.475	993.826
T236	35.976386	-6.693324	-0.054	7212.818	738.026	737.431	737.687
T237	35.990626	-6.223820	-0.145	10161.173	1039.461	1038.999	1039.387
T238	35.749429	-6.324825	-0.167	11028.458	1128.149	1127.724	1128.147
T301	36.099309	-3.556225	-0.141	10486.448	1072.790	1072.258	1072.721
T302	36.450969	-3.318731	-0.223	14296.090	1462.326	1462.207	1462.706
T303	36.678837	-3.156098	-0.368	17270.690	1766.636	1766.535	1767.310
T304	36.817193	-3.171935	-0.296	15561.229	1591.791	1591.591	1592.248
T305	36.936134	-3.552964	-0.100	9040.014	924.806	924.145	924.692
T306	36.721324	-3.835939	-0.124	10876.506	1112.747	1112.016	1112.639
T307	36.389568	-3.821258	-0.231	15415.978	1577.021	1576.590	1577.363
T308	36.604983	-3.890796	-0.193	13808.196	1412.530	1411.986	1412.744
T311	37.142381	-3.334915	-0.049	9416.552	963.196	962.551	963.227
T312	37.012849	-3.151444	-0.191	13404.512	1371.121	1370.659	1371.426
T313	37.526254	-3.419020	-0.036	7865.101	804.446	803.918	804.467
T314	37.587960	-3.657526	-0.075	8207.843	839.552	839.266	839.535
T315	37.134058	-4.086256	-0.152	11525.836	1178.968	1178.859	1179.098
T316	37.486698	-3.926185	-0.053	6477.160	662.597	662.321	662.456
T321	36.077512	-4.803407	-0.166	11986.801	1226.216	1225.867	1226.269
T322	36.724648	-4.653055	-0.107	10112.699	1034.341	1033.897	1034.454
T323	36.376483	-4.274129	-0.169	12801.302	1309.464	1309.233	1309.656

T326	37.727748	-4.096268	-0.062	8450.439	864.374	864.113	864.355
T327	37.199242	-4.465259	-0.191	13361.027	1366.531	1366.366	1366.953
T328	37.157822	-4.811800	-0.096	9735.410	995.657	995.411	995.840
T329	37.685363	-4.560538	-0.024	6071.753	621.068	620.546	620.975
T340	38.012903	-4.164976	0.032	5999.137	613.923	613.353	613.549
T341	38.078644	-4.649228	0.002	4550.452	465.778	465.206	465.352
T342	38.434065	-4.407591	0.029	4073.282	417.705	416.205	416.546
T343	39.113375	-4.835970	-0.003	605.547	62.271	62.003	61.914
T344	38.909645	-4.539006	-0.012	1418.116	145.525	144.892	145.002
T345	38.723148	-4.772862	-0.014	3542.922	362.810	361.951	362.299
T346	38.573838	-4.904498	0.054	3319.930	340.001	339.222	339.491
T347	38.308709	-4.979307	0.073	3950.242	404.353	403.645	403.958
T348	38.297963	-4.917746	0.043	4433.678	453.787	453.267	453.406
T351	36.165256	-5.154564	-0.238	14382.517	1471.265	1470.578	1471.517
T352	36.421724	-5.624742	-0.218	14055.029	1437.763	1437.462	1437.977
T353	36.652349	-5.319715	-0.156	12917.432	1321.450	1321.070	1321.523
T354	36.160661	-5.574007	-0.156	12590.316	1288.004	1287.314	1288.031
T355	36.451263	-5.895305	-0.162	11689.136	1195.767	1195.399	1195.778
T356	36.723603	-5.934101	-0.201	14206.335	1453.228	1452.856	1453.459
T361	37.185009	-5.400812	-0.196	13370.146	1367.594	1367.237	1367.866
T362	37.513761	-5.394164	-0.102	9925.709	1015.327	1015.024	1015.305
T363	37.936180	-5.394392	-0.051	7066.433	722.892	722.654	722.729
T364	37.780277	-5.902039	0.021	4339.466	444.066	443.684	443.762
T365	37.323342	-5.789525	-0.029	8187.288	837.705	836.870	837.405
T366	37.018374	-5.926850	-0.107	10870.818	1112.053	1111.303	1112.020
T371	38.446731	-5.164366	0.033	3099.734	317.294	316.588	316.970
T372	38.805799	-5.173700	-0.030	2129.958	217.929	218.015	217.793
T373	38.988261	-5.409980	0.026	698.487	71.486	71.522	71.417
T374	38.854083	-5.784032	0.054	51.080	5.538	4.987	5.222
T375	38.030109	-5.433395	-0.060	6886.502	704.513	704.172	704.319
T376	38.297075	-5.776447	-0.011	3899.406	399.074	398.684	398.753
T377	38.581211	-5.864851	-0.021	2071.281	212.070	211.956	211.790
T378	38.659754	-5.583003	-0.021	1145.679	117.415	117.263	117.142
T381	36.411672	-6.201665	-0.141	9914.395	1014.243	1013.588	1014.132

T382	36.872705	-6.139286	-0.142	12222.089	1250.246	1249.853	1250.324
T383	36.980678	-6.853118	0.001	4764.753	487.528	486.936	487.253
T384	36.702449	-6.655504	-0.046	6624.392	677.751	677.089	677.487
T385	36.256747	-6.960179	-0.029	7316.439	748.473	747.924	748.284
T386	36.096314	-6.614945	-0.085	8561.947	875.926	875.410	875.727
T387	36.472430	-6.342246	-0.112	10020.675	1025.032	1024.402	1025.006
T388	36.137784	-6.363964	-0.089	8584.597	878.264	877.646	878.049
T391	37.163331	-6.251110	-0.055	7445.604	761.695	761.275	761.510
T392	37.625267	-6.139424	-0.016	3663.494	374.965	374.774	374.622
T393	37.917687	-6.731806	-0.023	3895.037	398.499	398.048	398.298
T394	37.134726	-6.783573	-0.039	4274.623	437.444	436.779	437.122
T395	37.403427	-6.935859	-0.033	5061.600	517.963	517.489	517.616
T396	37.808620	-6.950075	0.050	3255.146	333.320	332.836	332.853
T397	37.120402	-6.477934	-0.027	5417.442	554.213	553.804	554.020
T398	37.842259	-6.070648	-0.014	3571.117	365.316	364.993	365.174
T401	38.241898	-6.089664	-0.014	3765.456	385.229	384.884	385.050
T402	38.910428	-6.447138	0.012	58.039	6.223	5.707	5.934
T403	38.993665	-6.750733	-0.017	1352.195	138.524	138.253	138.256
T404	38.325306	-6.639125	-0.009	2078.245	212.704	212.616	212.499
T405	38.789249	-6.635298	0.009	802.741	82.346	82.045	82.074
T406	38.172133	-6.842595	0.012	1578.139	161.683	161.282	161.359
T407	39.062404	-6.922130	-0.015	2558.169	261.782	261.470	261.575
T408	38.774323	-6.047456	0.040	55.434	6.070	5.435	5.668
T411	38.934123	-7.730389	-0.029	1669.587	170.934	170.726	170.706
T412	38.766215	-7.904083	0.011	351.421	36.274	35.896	35.928
T413	38.322332	-7.305557	0.013	706.033	72.662	72.018	72.185
T414	38.244001	-7.777380	0.009	535.341	55.205	54.660	54.733
T415	38.689301	-7.437217	-0.016	1383.473	141.737	141.430	141.452
T416	38.505312	-7.138510	0.012	848.467	87.164	86.676	86.749
T417	38.124783	-7.670739	0.028	584.123	60.278	59.576	59.720
T421	35.590786	-7.848504	-0.217	15095.867	1544.522	1543.520	1544.469
T422	35.863378	-7.754925	-0.198	15394.886	1575.183	1574.394	1575.088
T423	35.773114	-7.592378	-0.196	13277.709	1358.576	1357.876	1358.341
T424	35.797947	-7.421071	-0.173	12173.859	1245.586	1245.118	1245.353

T425	35.492893	-7.359502	-0.051	7152.180	731.963	731.079	731.472
T426	35.422197	-7.674028	-0.097	9656.264	987.968	987.212	987.683
T427	35.981083	-7.109738	-0.033	7098.457	726.290	725.666	725.980
T428	35.792395	-7.961060	-0.271	18116.586	1853.569	1852.697	1853.787
T429	35.320398	-7.218387	-0.116	8624.307	882.283	881.715	882.096
T431	36.490440	-7.114392	-0.165	10915.839	1116.657	1116.392	1116.601
T432	36.126511	-7.474729	-0.323	14636.505	1497.300	1496.916	1497.452
T433	36.908577	-7.072275	-0.028	5382.455	550.773	550.094	550.434
T434	36.983523	-7.408192	-0.043	4985.418	510.310	509.541	509.818
T435	36.977720	-7.665996	-0.003	2812.616	288.155	287.460	287.591
T436	36.654276	-7.523634	-0.006	5058.057	517.871	516.968	517.246
T437	36.741894	-7.339948	-0.035	7508.036	768.286	767.564	767.880
T438	36.120345	-7.660860	-0.228	12610.436	1290.201	1289.329	1290.033
T440	37.816223	-7.367909	0.003	1339.853	137.667	136.826	136.992
T441	37.537193	-7.134623	-0.160	13280.024	1358.607	1358.635	1358.594
T442	37.782748	-7.107856	0.033	3858.825	395.273	394.773	394.592
T443	37.768047	-7.533754	-0.001	1800.715	184.766	183.928	184.115
T444	37.180851	-7.347881	-0.052	6169.424	631.312	630.625	630.933
T449	37.268554	-7.104925	-0.044	5280.525	540.378	539.796	540.007
T452	39.178042	-7.161251	0.001	1058.625	108.454	108.108	108.237
T453	39.068513	-7.359379	-0.003	896.749	91.928	91.526	91.685
T454	39.336423	-7.405451	0.002	99.869	10.429	10.146	10.210
T455	39.147870	-7.495744	-0.010	676.771	69.474	69.157	69.193
T456	39.531080	-7.086054	-0.025	189.800	19.586	19.364	19.405
T465	38.798752	-8.559804	-0.001	2011.328	206.049	205.686	205.646
T466	38.827210	-8.937246	0.005	1426.186	146.323	145.731	145.813
T467	38.799609	-8.368697	-0.026	4255.492	435.507	435.178	435.148
T468	38.759943	-8.006063	-0.006	588.547	60.521	60.111	60.172
T471	39.182390	-8.142726	0.022	46.853	5.130	4.608	4.790
T472	39.282703	-8.399408	-0.002	78.807	8.442	7.959	8.057
T473	39.273788	-8.611135	-0.006	253.554	26.302	25.916	25.922
T474	39.339151	-8.818141	-0.039	986.010	101.140	100.781	100.808
T476	39.026314	-8.787968	0.018	519.549	53.560	53.061	53.117
T501	31.078898	-6.361802	-0.125	10406.954	1065.369	1064.210	1064.537

T502	31.140204	-6.613606	-0.113	9858.848	1009.113	1008.037	1008.439
T503	31.283495	-6.726435	-0.126	9557.239	978.213	977.170	977.571
T504	31.352220	-6.451903	-0.118	10169.795	1040.961	1039.970	1040.264
T505	31.724057	-6.502807	-0.156	11730.937	1200.511	1199.350	1200.042
T506	32.203611	-6.306038	-0.131	10724.364	1097.427	1096.597	1097.024
T511	32.489162	-6.047568	-0.149	10655.073	1090.320	1089.561	1089.938
T512	31.813951	-6.703480	-0.167	12113.903	1239.615	1238.676	1239.236
T513	32.713336	-6.351088	-0.147	10823.251	1107.235	1106.731	1107.144
T514	33.033077	-6.300467	-0.175	11133.232	1138.643	1138.246	1138.871
T515	32.164686	-6.836766	-0.143	10990.819	1124.621	1123.491	1124.282
T516	32.488081	-6.748021	-0.150	11355.834	1161.792	1161.136	1161.643
T518	30.570216	-7.055338	-0.078	7981.344	815.004	815.970	816.312
T519	30.657817	-7.433698	-0.087	7745.581	791.142	791.608	792.182
T520	31.139827	-7.119171	-0.077	9468.727	969.053	968.190	968.505
T521	31.089698	-7.346604	-0.208	15167.071	1551.335	1551.003	1551.779
T522	31.228641	-7.537695	-0.258	16562.307	1694.075	1693.587	1694.635
T523	31.354551	-7.750705	-0.312	18779.270	1920.841	1920.459	1921.668
T524	31.391587	-7.254428	-0.061	9045.065	925.669	924.798	925.149
T525	31.477232	-7.581587	-0.079	8136.208	832.800	831.851	832.146
T526	31.209583	-7.853280	-0.272	16695.258	1707.787	1707.062	1708.236
T527	31.674854	-7.335288	-0.077	7973.174	815.911	815.426	815.470
T531	33.564215	-7.927913	-0.209	13409.955	1372.645	1370.851	1371.868
T532	33.402657	-7.625418	-0.221	14072.421	1439.995	1438.825	1439.696
T533	33.436755	-7.432705	-0.159	13025.188	1332.881	1331.889	1332.496
T534	33.755395	-7.315909	-0.174	13683.852	1400.685	1399.367	1399.927
T535	33.484734	-7.149314	-0.152	12588.103	1288.040	1287.244	1287.763
T536	33.526289	-6.938200	-0.150	12097.907	1237.591	1236.839	1237.592
T537	33.363487	-6.822284	-0.159	11771.621	1204.148	1203.457	1204.198
T540	31.049233	-8.125732	-0.273	13548.145	1385.563	1385.149	1386.007
T541	31.205664	-8.472826	-0.188	11739.276	1200.561	1199.930	1200.839
T542	31.381701	-8.257427	-0.275	16768.882	1715.174	1714.470	1715.757
T543	31.672715	-8.139438	-0.254	18641.191	1907.012	1907.021	1907.508
T544	31.668812	-8.660947	-0.318	17829.910	1823.644	1823.120	1824.394
T545	31.770226	-8.865894	-0.236	15288.360	1563.610	1562.957	1564.135

T546	31.945592	-8.769825	-0.213	14665.744	1499.954	1499.369	1500.395
T547	31.836764	-8.356204	-0.222	15780.245	1614.235	1613.883	1614.521
T548	31.979659	-8.266834	-0.056	8467.921	866.435	865.961	866.072
T551	32.062172	-8.582026	-0.227	15384.895	1573.563	1573.403	1574.032
T552	32.132293	-8.935387	-0.213	14268.117	1459.313	1458.665	1459.681
T553	32.276735	-8.844166	-0.183	13636.995	1394.883	1394.213	1395.076
T554	32.329680	-8.307105	-0.076	7998.870	818.286	817.965	818.080
T555	32.479423	-8.453859	-0.104	8507.912	870.152	869.765	870.160
T556	32.606984	-8.744224	-0.186	12784.818	1307.899	1307.580	1307.848
T557	32.752713	-8.535031	-0.055	8097.131	828.586	828.043	828.128
T558	33.000764	-8.395219	-0.134	10140.686	1037.118	1036.879	1037.238
T559	32.790500	-7.819040	-0.161	11632.928	1190.025	1189.335	1189.976
T560	32.684381	-8.071713	-0.175	10539.250	1077.750	1077.768	1078.035
T562	32.883636	-9.155182	-0.209	14951.279	1529.228	1528.535	1529.612
T563	32.790101	-9.298247	-0.226	15274.125	1562.399	1561.532	1562.658
T564	32.500415	-9.087447	-0.252	15102.930	1544.835	1544.077	1545.141
T800	33.258170	-8.035726	-0.202	12983.931	1328.214	1327.270	1328.254
T801	33.657823	-8.434429	-0.243	14242.351	1456.803	1456.587	1457.064
T802	33.161918	-8.418335	-0.179	11758.953	1202.760	1202.291	1202.856
T803	33.274363	-8.720094	-0.112	9992.027	1022.330	1021.454	1022.015
T804	33.033289	-8.620126	-0.108	8687.496	889.274	888.462	888.530
T805	33.451411	-8.902492	-0.281	16513.896	1689.101	1688.643	1689.618
T806	33.837821	-8.818554	-0.126	11669.220	1193.686	1193.144	1193.658
T807	33.131387	-8.968452	-0.196	16359.382	1673.560	1672.785	1673.793
T810	34.217418	-8.810215	-0.120	10883.650	1113.255	1112.763	1113.259
T811	34.610781	-8.919872	-0.186	14324.031	1465.427	1464.451	1465.405
T812	34.804783	-8.840581	-0.224	16093.857	1646.394	1645.735	1646.610
T813	34.956336	-8.617920	-0.237	16794.558	1717.963	1717.519	1718.370
T814	34.499776	-8.559094	-0.120	10220.284	1044.929	1045.074	1045.379
T815	34.843432	-8.219711	-0.145	10484.373	1071.354	1072.062	1072.414
T816	35.035198	-8.249321	-0.249	15266.776	1560.957	1561.085	1561.952
T817	35.288686	-8.743106	-0.167	12493.452	1278.339	1277.186	1278.023
T818	35.311457	-8.270499	-0.285	18454.805	1887.898	1886.948	1888.411
T819	35.880259	-8.237091	-0.308	19274.025	1971.622	1971.767	1972.320

T820	35.580741	-8.503336	-0.258	17156.593	1755.311	1754.509	1755.449
T821	35.028652	-8.838768	-0.208	14621.392	1495.872	1495.047	1495.852
T822	35.475255	-8.058409	-0.280	17565.557	1796.977	1795.964	1797.350
T825	32.955251	-9.135357	-0.225	15875.928	1623.987	1623.330	1624.283
T826	33.418354	-9.071462	-0.328	19916.394	2037.318	2036.830	2038.070
T827	33.177774	-9.373029	-0.203	15655.975	1601.442	1600.855	1601.751
T828	33.772811	-9.336348	-0.053	7526.565	769.871	769.161	769.735
T829	33.900777	-9.095905	-0.508	26824.051	2743.766	2744.150	2745.857
T830	33.555441	-9.288634	-0.072	11887.746	1216.173	1215.565	1216.007
T831	34.216745	-9.455489	-0.387	20723.108	2119.294	2119.371	2120.682
T832	34.099239	-9.262686	-0.448	22264.319	2276.886	2277.443	2278.583
T833	34.412766	-9.310747	-0.363	22427.687	2294.061	2293.860	2295.317
T834	34.784669	-9.367785	-0.275	19121.684	1956.162	1955.144	1956.653
T835	34.804843	-9.111124	-0.256	18048.538	1846.443	1845.507	1846.760
T836	34.536610	-9.767682	-0.307	17493.628	1788.857	1788.974	1789.899
T837	34.799707	-9.857563	-0.153	13486.600	1379.564	1378.880	1379.639
T841	35.242021	-9.283395	-0.248	16077.422	1644.819	1644.254	1644.907
T842	35.259158	-9.639952	-0.192	14992.684	1533.950	1533.238	1533.829
T843	35.373823	-9.870431	-0.159	12540.661	1283.181	1282.487	1282.813
T844	35.616953	-9.963547	-0.101	9324.188	954.327	953.250	953.641
T850	37.758105	-9.152070	-0.028	3685.168	377.592	376.662	376.811
T852	37.918007	-9.807228	-0.032	4998.299	511.800	510.948	511.102
T855	35.314968	-10.045851	-0.080	10554.505	1080.143	1079.160	1079.534
T856	34.677487	-10.106315	-0.249	13537.537	1384.153	1384.193	1384.843
T857	34.936703	-10.132333	-0.229	13075.209	1337.538	1337.107	1337.517
T858	35.651368	-10.118305	-0.087	9036.062	924.926	923.802	924.155
T859	35.670381	-10.036883	-0.129	10172.065	1041.017	1039.946	1040.399
T860	35.656328	-10.077815	-0.109	11064.715	1132.346	1131.244	1131.746
T861	35.879450	-10.055754	-0.133	9890.944	1012.027	1011.187	1011.631
T862	35.488730	-10.090982	-0.062	9103.107	931.756	930.687	931.016
T863	34.992162	-10.104562	-0.174	13115.914	1341.634	1341.670	1341.647
T864	35.190142	-10.089635	-0.065	9243.289	946.099	944.977	945.340
T865	34.682418	-10.071006	-0.260	4732.615	481.485	483.449	483.912
T866	34.906663	-10.056221	-0.246	9272.638	946.959	947.902	948.349

T867	34.596127	-10.047200	-0.164	4912.729	499.645	501.685	502.339
T871	36.136960	-10.027510	-0.090	7613.487	778.773	778.186	778.611
T872	36.323823	-10.036573	-0.088	8167.782	835.329	835.079	835.320
T873	36.523897	-10.027039	-0.052	8494.653	868.650	868.698	868.763
T874	36.733923	-10.022197	-0.055	6956.636	711.358	711.315	711.410
T875	36.952560	-10.074308	-0.052	8036.392	821.831	821.489	821.876
T876	36.521894	-10.084815	-0.059	9435.449	964.629	964.684	965.001
T878	36.096379	-10.095381	-0.060	7892.849	807.522	806.716	807.190
T879	36.685428	-10.086438	-0.068	8590.449	878.161	878.387	878.542
T881	37.798083	-10.085935	-0.033	5844.736	598.498	597.573	597.674
T884	37.451107	-10.086120	-0.104	5052.071	516.373	516.197	516.584
T891	34.647821	-11.104891	0.026	5008.379	511.562	511.709	512.111
T892	34.912588	-11.101065	-0.296	15757.943	1611.416	1611.959	1612.103
T893	34.959697	-11.059030	-0.301	4821.771	491.136	492.580	493.019
T894	35.231006	-11.036310	-0.053	10395.557	1064.053	1062.638	1063.234
T895	35.173945	-11.128347	-0.099	9511.094	973.345	972.322	972.728
T896	35.450990	-11.031900	-0.081	7400.041	757.741	756.433	756.750
T897	35.630493	-11.045479	-0.066	6074.856	621.824	620.968	621.192
T898	35.402312	-11.039772	-0.125	6226.477	637.780	636.398	636.701
T901	36.165692	-11.035430	-0.059	7003.745	716.676	716.014	716.210
T902	36.039795	-11.050894	-0.053	6218.065	636.178	635.679	635.841
T905	36.659128	-11.050251	-0.059	5828.510	596.461	595.679	595.995
T907	36.860903	-11.022012	-0.080	7074.320	723.911	723.270	723.430
T908	36.484092	-11.142614	-0.041	5924.034	606.291	605.411	605.763
T912	37.579240	-11.029367	-0.021	4375.886	447.363	447.111	447.425
T913	37.800821	-10.104949	-0.028	3946.780	404.424	403.498	403.554
T918	37.085376	-11.056911	-0.086	4504.013	461.085	460.502	460.529
T921	38.479467	-10.125112	-0.004	2639.486	269.977	269.683	269.868
T922	38.129682	-10.068518	-0.026	4807.419	491.999	491.441	491.575
T923	38.937474	-10.036460	-0.014	4392.755	449.420	449.074	449.165
T924	38.757973	-10.048782	-0.010	4344.139	444.361	444.057	444.193
T925	38.240548	-10.035465	-0.015	3668.434	375.354	374.919	375.089
T926	38.238565	-10.102072	-0.018	3272.925	334.887	334.463	334.642
T927	38.517928	-10.103580	-0.007	2981.092	304.918	304.632	304.799

T928	38.454962	-11.038682	-0.020	2947.627	301.140	301.261	301.368
T929	38.946999	-11.039191	-0.015	2364.811	241.794	241.730	241.774
T933	38.214761	-9.137180	0.004	2641.099	270.617	269.867	270.041
T934	38.702771	-9.336535	-0.005	2719.774	278.606	278.128	278.084
T935	38.685367	-9.176455	-0.009	2711.385	277.783	277.328	277.228
T936	38.015340	-9.701268	-0.028	4173.416	427.232	426.549	426.738
T937	38.310657	-9.941380	-0.004	3421.392	350.028	349.689	349.827
T938	38.509670	-9.790627	-0.022	3158.512	323.132	322.947	322.946
T939	39.716485	-10.005346	0.011	33.625	3.792	3.403	3.437
T940	39.266286	-10.096106	-0.106	7172.942	733.601	733.466	733.541
T941	39.026139	-10.049191	-0.022	2929.325	299.845	299.635	299.506
T942	39.337253	-10.071542	-0.037	4561.003	466.572	466.302	466.372
T943	39.433589	-10.093223	-0.047	4850.806	496.196	495.839	496.012
T944	39.811275	-10.070226	0.006	2416.204	247.416	246.975	247.036
T945	39.668131	-10.021009	0.017	550.866	56.677	56.272	56.316
T946	39.778810	-10.039891	0.004	911.650	93.591	93.146	93.202
T947	40.054287	-10.059370	-0.006	1605.769	164.543	164.123	164.170
T948	40.346378	-10.055392	0.024	108.915	11.613	10.912	11.134
T949	39.273391	-10.152005	-0.068	6701.413	685.364	685.218	685.303
T950	39.108059	-9.667169	0.035	998.787	102.586	101.763	102.112
T951	39.125328	-9.911704	-0.010	3221.232	329.722	329.352	329.358
T952	39.407847	-9.295504	-0.017	1573.810	161.249	160.830	160.906
T953	39.563032	-9.120455	-0.030	169.019	17.671	17.284	17.279
T954	39.315660	-9.063846	0.004	375.344	38.720	38.315	38.373
T955	39.707030	-9.742948	-0.039	29.453	3.326	3.076	3.011
T958	39.052693	-9.116938	0.016	1601.200	164.196	163.480	163.707
T959	38.994243	-9.436270	0.007	1961.918	201.047	200.288	200.589