Complex Operations:

- You can manipulate complex numbers arithmetically just like real numbers to carry out operations. You just have to be careful to keep all the *i*'s straight. You can't combine real parts with imaginary parts by using addition or subtraction, because they're not like terms, so you have to keep them separate. Also, when multiplying complex numbers, the product of two imaginary numbers is a real number; the product of a real and an imaginary number is still imaginary; and the product of two real numbers is real.

To add and subtract complex numbers: Simply combine like terms
To multiply two complex numbers: Simply follow the FOIL
To divide complex numbers: Multiply both the numerator and the denominator by the conjugate of the denominator, FOIL the numerator and denominator separately, and then combine like terms. This process is necessary because the imaginary part in the denominator is really a square root (of –1, remember?), and the denominator of the fraction must not contain an imaginary part.

Dirac Delta Functions:

At $t = a$ the Dirac Delta function is sometimes thought of has having an "infinite" value. So, the Dirac Delta function is a function that is zero everywhere except one point and at that point it can be thought of as either undefined or as having an "infinite" value.

Completing The Square:

$$y = ax^2 + bx + c$$

$$y = a\left(x^2 + \frac{b}{a}x + \quad\right) + c$$

$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$y = a(x - h)^2 + k$$

$$h = -\frac{b}{2a} \qquad k = c - \frac{b^2}{4a}$$

Quantum Mechanics:

Uncertainty:

- **the uncertainty principle states that the position and velocity cannot both be measured,exactly, at the same time (actually pairs of position, energy and time)**
- **uncertainty principle derives from the measurement problem, the intimate connection between the wave and particle nature of quantum objects**
- **the change in a velocity of a particle becomes more ill defined as the wave function is confined to a smaller region**

$$\triangle x \triangle p > \frac{h}{2} \qquad \triangle E \triangle t > \frac{h}{2}$$

Wave Functions and Probability

The wavefunction represents the probability amplitude for finding a particle at a given point in space at a given time. The actual probability of finding the particle is given by the product of the wavefunction with its complex conjugate (like the square of the amplitude for a complex function).

Wave function, in quantum mechanics, variable quantity that mathematically describes the wave characteristics of a particle. The value of the wave function of a particle at a given point of space and time is related to the likelihood of the particle's being there at the time. By analogy with waves such as those of sound, a wave function, designated by the Greek letter psi, Ψ, may be thought of as an expression for the amplitude of the particle wave (or de Broglie wave), although for such waves amplitude has no physical significance. The square of the wave function, Ψ2, however, does have physical significance: the probability of finding the particle described by a specific wave function Ψ at a given point and time is proportional to the value of Ψ2.

In classical mechanics, the configuration or state of a system is given by a point (x, p) in the space of coordinates and momenta. This specifies everything else in the system in a fully deterministic way, in that any observable Y that can be expressed as Y (x, p) can be found, and any that cannot is irrelevant. Yet, as we have seen with the diffraction of electrons, it is impossible to know both the position and momentum of the electron exactly at every point along the trajectory. This is mathematically expressed as the famous position-momentum uncertainty principle:

$$\triangle x \triangle p > \frac{h}{2}$$

Hence, specifying a state by (x, p) clearly will not work. So what specifies the state of a quantum system? **The configuration or state of a quantum object is completely specified by a wavefunction denoted as ψ(x).**

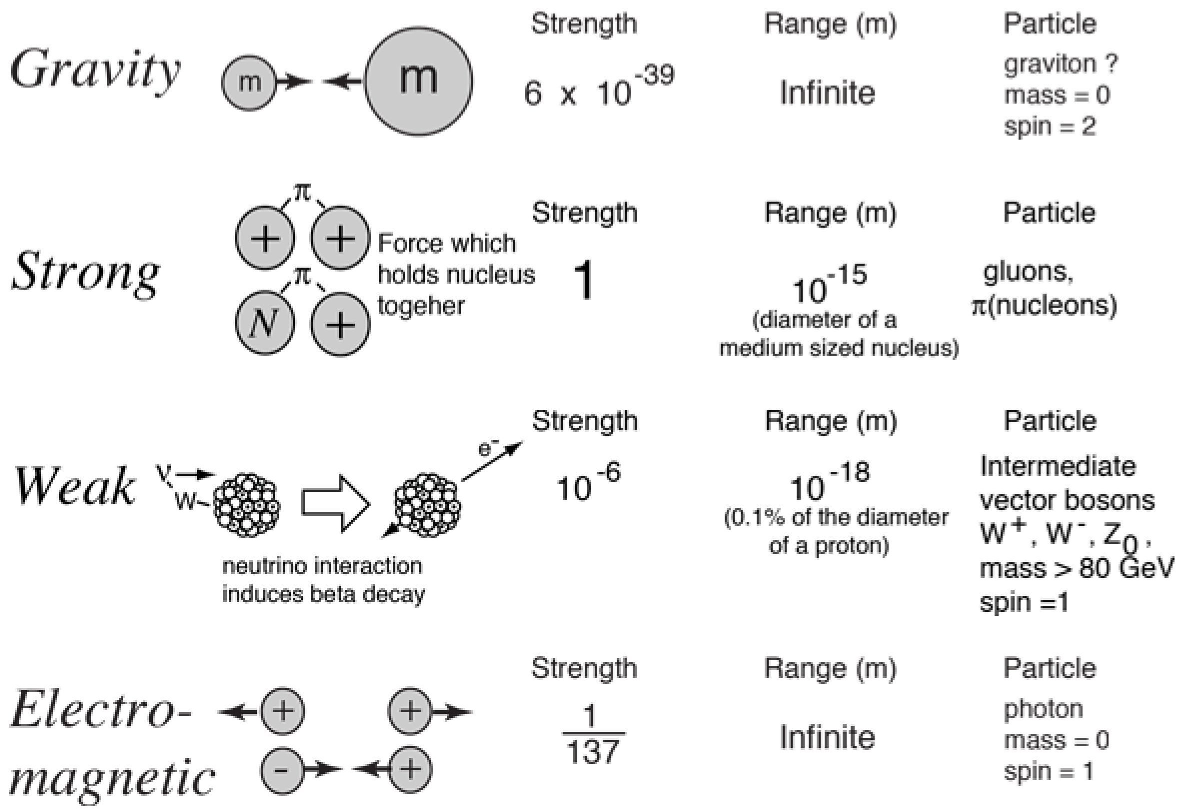
And what does ψ(x) mean?

$$p(x) = |\psi(x)|^2$$

determines the probability (density) that an object in the state ψ(x) will be found at position x.

Note that ψ ∈ C, meaning the wavefunction is complex! Here, the real part of ψ is being drawn for simplicity, as complex-plane paper is hard to find. Furthermore, ψ must be singly-valued and not “stupid”; the latter point will be elaborated later.

Four Fundamental Forces:



Particle Spin:

In quantum mechanics and particle physics, spin is an intrinsic form of angular momentum carried by elementary particles, composite particles (hadrons), and atomic nuclei.

Spin is one of two types of angular momentum in quantum mechanics, the other being orbital angular momentum. The orbital angular momentum operator is the quantum-mechanical counterpart to the classical angular momentum of orbital revolution: it arises when a particle executes a rotating or twisting trajectory (such as when an electron orbits a nucleus). The existence of spin angular momentum is inferred from experiments, such as the Stern–Gerlach experiment, in which particles are observed to possess angular momentum that cannot be accounted for by orbital angular momentum alone.

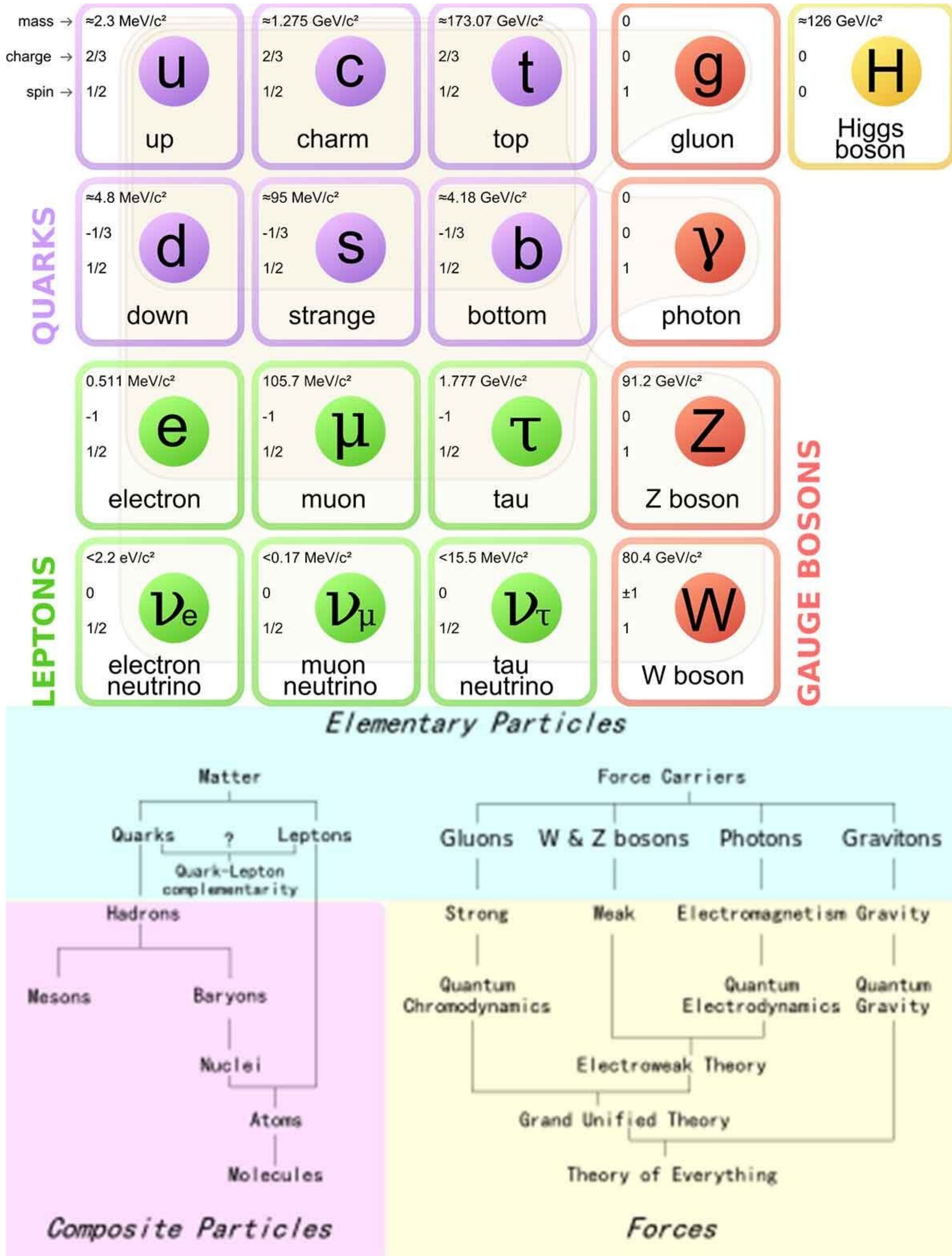
In some ways, spin is like a vector quantity; it has a definite magnitude, and it has a "direction" (but quantization makes this "direction" different from the direction of an ordinary vector). All elementary particles of a given kind have the same magnitude of spin angular momentum, which is indicated by assigning the particle a spin quantum number.

Particle “Charge”:

Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field. There are two types of electric charges: positive and negative (commonly carried by protons and electrons respectively). Like charges repel and unlike attract. An absence of net charge is referred to as neutral. An object is negatively charged if it has an excess of electrons, and is otherwise positively charged or uncharged.

The electric charge is a fundamental conserved property of some subatomic particles, which determines their electromagnetic interaction. Electrically charged matter is influenced by, and produces, electromagnetic fields. The interaction between a moving charge and an electromagnetic field is the source of the electromagnetic force, which is one of the four fundamental forces.

The total electric charge of an isolated system remains constant regardless of changes within the system itself.

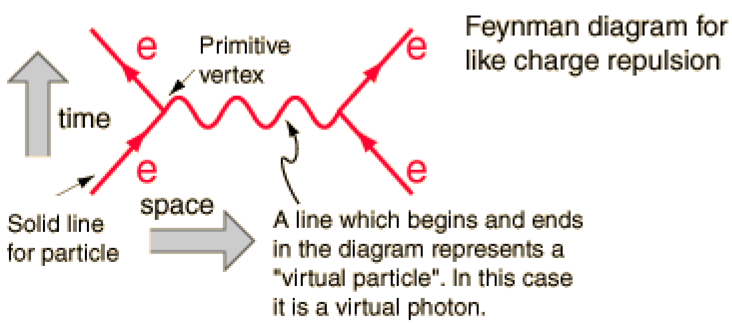


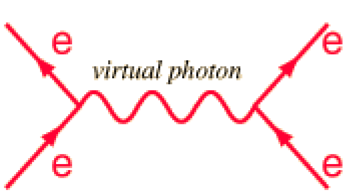
mass →
charge →
spin →
≈2.3 MeV/c²
2/3
1/2
u
up
≈1.275 GeV/c²
2/3
1/2
c
charm
≈173.07 GeV/c²
2/3
1/2
t
top
0
0
1
g
gluon
≈126 GeV/c²
0
0
H
Higgs boson
≈4.8 MeV/c²
-1/3
1/2
d
down
≈95 MeV/c²
-1/3
1/2
s
strange
≈4.18 GeV/c²
-1/3
1/2
b
bottom
0
0
1
γ
photon
QUARKS
0.511 MeV/c²
-1
1/2
e
electron
105.7 MeV/c²
-1
1/2
μ
muon
1.777 GeV/c²
-1
1/2
τ
tau
91.2 GeV/c²
0
1
Z
Z boson
<2.2 eV/c²
0
1/2
νe
electron neutrino
<0.17 MeV/c²
0
1/2
νμ
muon neutrino
<15.5 MeV/c²
0
1/2
ντ
tau neutrino
80.4 GeV/c²
±1
1
W
W boson
LEPTONS
GAUGE BOSONS
Elementary Particles
Matter
Quarks
?
Leptons
Quark-Lepton complementarity
Force Carriers
Gluons
W & Z bosons
Photons
Gravitons
Hadrons
Mesons
Baryons
Nuclei
Atoms
Molecules
Composite Particles
Strong
Weak
Electromagnetism
Gravity
Quantum Chromodynamics
Quantum Electrodynamics
Quantum Gravity
Electroweak Theory
Grand Unified Theory
Theory of Everything
Forces

Feynman Diagrams:

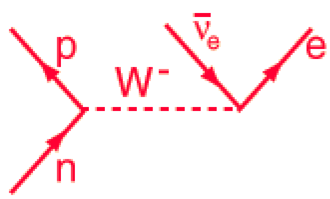
Feynman diagrams are graphical ways to represent exchange forces. Each point at which lines come together is called a vertex, and at each vertex one may examine the conservation laws which govern particle interactions. Each vertex must conserve charge, baryon number and lepton number.

Developed by Feynman to decribe the interactions in quantum electrodynamics (QED), the diagrams have found use in describing a variety of particle interactions. They are spacetime diagrams, ct vs x. The time axis points upward and the space axis to the right. (Particle physicists often reverse that orientation.) Particles are represented by lines with arrows to denote the direction of their travel, with antiparticles having their arrows reversed. Virtual particles are represented by wavy or broken lines and have no arrows. All electromagnetic interactions can be described with combinations of primitive diagrams like this one.

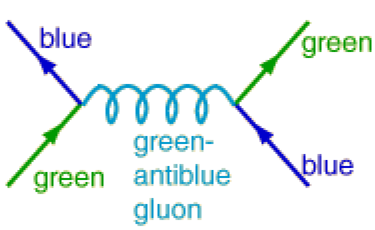




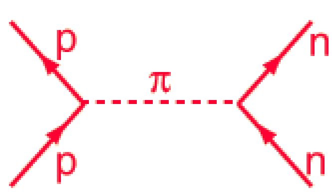
Electromagnetic



Weak



between quarks



between nucleons

Strong Interaction

Gluons:

Gluons are the exchange particles for the color force between quarks, analogous to the exchange of photons in the electromagnetic force between two charged particles. The gluon can be considered to be the fundamental exchange particle underlying the strong interaction between protons and neutrons in a nucleus. That short-range nucleon-nucleon interaction can be considered to be a residual color force extending outside the boundary of the proton or neutron. That strong interaction was modeled by Yukawa as involving an exchange of pions, and indeed the pion range calculation was helpful in developing our understanding of the strong force.

Special Relativity:

Relativistic Energy:

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$