$U \sim Geom(p=\frac{1}{2}), u=1,2..$ the number of trials needed to get the first head.
$V \sim NBin(p=\frac{1}{2}, r=2), v=2,3..$ the number of trials needed to get two heads in repeated tosses of a fair coin.
the distribution of (U,V) is $\{(u,v): u=1,2,..; v=u+1,u+2,..\}$ is not a cross-product set. U and V are not independent.

Conditional expectations and variances

$f(x_2|x_1)=\frac{f(x_1,x_2)}{f_1(x_1)}$;$p(x_2|x_1)=\frac{p(x_1,x_2)}{p_1(x_1)}$

4.2.1 $f(y|x)=P(Y=y|X=x)=\frac{f(x,y)}{f_X(x)}$

4.2.3 $E(g(x_2)|x_1)=\sum_{all\ x_2} g(x_2)p(x_2|x_1)$,$E(g(x_2)|x_1)=\int_{all\ x_2} g(x_2)f(x_2|x_1)dx2$

4.2.4 $Var(y|x)=E(Y^2|x)-(E(Y|x))^2$, $V[X|Y]=E[(X-E[X|Y])^2|Y]$

4.2.7 $f(x,y)=g(x)h(y) \iff$ indep

4.2.10 $E(g(X)h(Y))=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x)h(y)f(x,y)dxdy = \int_{-\infty}^{\infty} h(y)f_Y(y)\left[\int_{-\infty}^{\infty} g(x)f_X(x)dx\right]dy = E[g(x)]E[h(y)] = (Eg(X))(Eh(Y))$

$F_{XY}(x,y)=P(X\le x, Y\le y)$

Covariance and Correlation

Transformation of pairs of r.v.s

$f_{Y_1,Y_2}(y_1,y_2)=\sum_{i=1}^k f_{X_1,X_2}(h_1(y_1,y_2),h_2(y_1,y_2))|J_i|$

$J_{1,2}=\begin{vmatrix}\frac{\partial h_1}{\partial y_1} & \frac{\partial h_2}{\partial y_2}\\ \frac{\partial h_1}{\partial y_1} & \frac{\partial h_2}{\partial y_2}\end{vmatrix} = \frac{\partial h_1}{\partial y_1}\frac{\partial h_2}{\partial y_2}-\frac{\partial h_2}{\partial y_1}\frac{\partial h_1}{\partial y_2}$

Properties of $\bar{X}$ and S^2

$\bar{X}_{n+1}=\frac{\sum_{i=1}^{n+1}X_i}{n+1}=\frac{\sum_{i=1}^n X_i+X_{n+1}}{n+1}=\frac{n\bar{X}_n+X_{n+1}}{n+1}$

$nS^2_{n+1}=(n-1)S_n^2+(\frac{n}{n+1})(X_{n+1}-\bar{X}_n)^2$

when $X_1,..,X_n$ be iid $n(\mu,\sigma^2)$,

$\frac{(n-1)S^2}{\sigma^2}\sim\chi^2_{n-1}$,$Var\chi^2_{n-1}=2(n-1)$

$Var[\frac{(n-1)S^2}{\sigma^2}]=\frac{(n-1)^2}{\sigma^4}Var[S^2]=2(n-1)\implies Var[S^2]=\frac{2\sigma^4}{n-1}$

Connection between N,χ^2,t,F

W Normal	$\bar{X}\sim N(\mu,\frac{\sigma^2}{n})$	$\bar{X}, S^2$ indep	$\frac{(n-1)S^2}{\sigma^2}\sim\chi^2_{n-1}$
W/o Normal	$E[\bar{X}]=\mu$	$V[\bar{X}]=\frac{\sigma^2}{n}$	$E[S^2]=\sigma^2$

5.2.8 Distribution of the mean: $\bar{X}\sim gamma(n\alpha,\beta/n)$ 5.2.9 Convolution formula $Z=X+Y$ $f_Z(z)=\int_{-\infty}^{\infty}f_X(w)f_Y(z-w)dw$

5.3.2 $Z\sim n(0,1)$,$Z^2\sim\chi_1^2$, $X_1..X_n$ indep $X_i\sim\chi^2_{p_i}$,$X_1=..=X_n\sim\chi^2_{p_1+..p_n}$

$\chi^2_p\sim gamma(p/2,2)$

5.3.4 T

5.3.5 $X_1..X_n,Y_1..Y_m$ indep.

$X_i\sim n(\mu_X,\sigma_X^2)Y_j\sim n(\mu_Y,\sigma^2_Y)$,$\frac{S_X^2}{\sigma_X^2}\sim\chi^2$F: $\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2}$

Order statistics

5.4.6 $f_{X_{(i)},X_{(j)}}(u,v)=\frac{n!}{(i-1)!(j-1-i)!(n-j)!}f_X(u)f_X(v)[F_X(u)]^{i-1}[F_X(v)-F_X(u)]^{j-1-i}[1-F_X(v)]^{n-j}$

$f_{X_{(1)},...,X_{(n)}}(x_1,..,x_n)=\begin{cases} n!f_X(x_1)\cdot...\cdot f_X(x_n) & -\infty<x_1<..<x_n<\infty\\ 0 & \text{otherwise}\end{cases}$

Conditional/Baye's: $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(B|A)P(A)}{P(B)}$;disjoint $P(B)=P(B|A_1)P(A_1)+..P(B|A_k)P(A_k)=\sum_{i=1}^k P(B|A_i)P(A_i)$

Independence:

$P(A\cap B)=P(A)P(B)$	$P(X\le x,Y\le y)=P(X\le x)P(Y\le y)$
$E(XY)=E(X)E(Y)$	$F_{X,Y}(x,y)=F_X(x)F_Y(y)$
$V(X\pm Y)=VX+VY$	$f_{X,Y}(s,t)=f_X(s)f_Y(t)$

pdf/pmf

CDF: $F(x)=P(X\le x)=\sum_{y:y\le x}p(y)=\int_{-\infty}^x f(y)dy$

probabilities: $a\le b$:

PMF: $p(x)=P(X=x)=P(\forall w\in\mathcal{W}: X(w)=x)$

$P(a\le X\le b)=F(b)-F(a^-)$; $P(a<X\le b)=F(b)-F(a)$;
$P(a\le X\le a)=p(a)$; $P(a<X<b)=F(b^-)-F(a)$; (where a^- is the largest possible X value strictly less than a); Taking $a=b$ yields $P(X=a)=F(a)-F(a-1)$ as desired.

PDF: $P(\forall w\in\mathcal{W}: a\le X(w)\le b)=\int_a^b f(x)dx$
$P(a\le X\le b)=P(a<X\le b)=P(a<X<b)$;
$P(X>a)=1-F(a)$; $P(a\le X\le b)=F(b)-F(a)$

CDF

Condition:$f(x)\ge 0\forall x$ pmf $\sum_x f_X(x)=1, pdf\int_{-\infty}^{\infty}f(x)dx=1$

$c\sin x\ge 0, 0<x<\pi/2$	$\int_0^{\frac{\pi}{2}} c\sin xdx=1$	$c=1$
$ce^{-\lvert x\rvert}\ge 0,-\infty<x<\infty$	$c\int_{-\infty}^0 e^xdx+c\int_0^\infty e^{-x}dx=1$	$c=\frac{1}{2}$

mean/variance

$E[g(x)]=\mu=\sum_{x\in D}h(x)p(x)=\int_{-\infty}^{\infty}g(x)f(x)dx$

$E[(X-\mu)^n]=\mu_n=\sum(x-\mu)^np(x)=\int(x-\mu)^nf(x)dx$

$E(aX+b)=aE(X)+b$	$E(X)=\int_{-\infty}^0 F_X(t)dt+\int_0^\infty F_X(t)dt$
$V(aX+b)=a^2\sigma^2$	$V(X\pm Y)=VX+VY\pm 2Cov(X,Y)$
$\sigma_{ax+b}=\lvert a\rvert\cdot\sigma_x$	$Cov(x,y)=E(XY)-E(X)E(Y)$

$V(X)=\sigma_x^2=E(X^2)-[E(X)]^2$

$f_X(x)$	EX	EX^2	$V(X)$
$\frac{1}{n}$	$\sum_1^n x\frac{1}{n}$ $\frac{1}{n}\frac{n(n+1)}{2}$	$\sum_1^n x^2\frac{1}{n}$ $\frac{1}{n}\frac{n(n+1)(2n+1)}{6}$	$\frac{n^2-1}{12}$
ax^{a-1}	$\int_0^1 xax^{a-1}dx$ $\frac{a}{a+1}x^{a+1}\big\vert_0^1$	$\int_0^1 x^2ax^{a-1}dx$ $\frac{a}{a+2}x^{a+2}\big\vert_0^1$	$\frac{a}{a+2}-(\frac{a}{a+1})^2$ $\frac{a}{(a+2)(a+1)^2}$
$\frac{3}{2}(x-1)^2$	$\int_0^2 x\frac{3}{2}(x-1)^2dx$ $\frac{3}{2}(\frac{x^4}{4}\big\vert_0^2..)$	$\int_0^2 x^2\frac{3}{2}(x-1)^2dx$ $\frac{3}{2}(\frac{x^5}{5}\big\vert_0^2..)$	$\frac{8}{5}-1^2$ $\frac{3}{5}$

transform

$g(x)\uparrow$	$F_Y(y)=F_X(g^{-1}(y))$
$g(x)\downarrow$	$F_Y(y)=1-F_X(g^{-1}(y))$

not monotone $\because X\le 0\ is\ \emptyset \therefore P(X\le -\sqrt{y})=0$

$F_Y(y)=P(Y\le y)=P(X^2\le y)=P(-\sqrt{y}\le X\le\sqrt{y})=P(X\le\sqrt{y})-P(X\le-\sqrt{y})=F_X(\sqrt{y})=\sqrt{y},\ 0<\sqrt{y}<1$

Y	X	$f_X(x)$	$F_Y(y)$	$f_Y(y)=\frac{dF_Y(y)}{dy}$
X^2	$\sqrt{Y}$	$\frac{e^{-x^2/2}}{\sqrt{2\pi}}$	$F_X(\sqrt{y})-F_X(-\sqrt{y})$	$\frac{e^{-y/2}}{2\sqrt{2\pi y}}, y>0$

monotone:$f_Y(y)=f_X(g^{-1}(y))|\frac{d(g^{-1}(y))}{dy}|$

	Y	X	$f_X(x)$
1	$-\log X$	e^{-Y}	$x^n(1-x)^m, 0<x<1$
2	e^X	$\ln Y$	$\frac{1}{\sigma^2}xe^{-(x/\sigma)^2/2}, 0<x<\infty$
3	$-2lnX$	$e^{-Y/2}$	$1, 0<x<1$
4	X^2	$\sqrt{Y}$	$1, 0<x<1$
5	$d\tan X$	$\arctan\frac{Y}{d}$	$\frac{1}{b-a}=\frac{2}{\pi}$

1 $(e^{-y})^n(1-e^{-y})^m|\frac{d(e^{-y})}{dy}|=e^{-(n+1)y}(1-e^{-y})^m,\ 0<y<\infty$

2 $\frac{1}{\sigma^2}\ln ye^{-(\ln y/\sigma)^2/2}|\frac{d(\ln y)}{dy}|=\frac{\ln(y)}{\sigma^2 y}e^-(\ln(y)/\sigma)^2/2, 1<y<\infty$

3 $1|\frac{de^{-Y/2}}{dy}|=\frac{1}{2}e^{-Y/2}, 0<y<\infty$

4 $1|\frac{d\sqrt{y}}{dy}|=\frac{1}{2\sqrt{y}}, 0<y<1$

5 $\frac{2}{\pi}|\frac{d(\arctan\frac{y}{d})}{dy}|=\frac{2}{\pi}\frac{d(\frac{y}{d})}{[(\frac{y}{d})^2+1]}=\frac{2}{\pi d[(\frac{y}{d})^2+1]}0<y<\infty$

find mean/variance by mgf

$E(e^{tx})=\int e^{tx}f(x)dx=\sum e^{tx}f(x)$; $M_{aX+b}(t)=e^{tb}M_{aX}(t)$

$M_X(t)=E(e^{tx})$	$M_X(0)=1$
$M'_X(t)=E(xe^{tx})$	$M'_X(0)=E(X)$
$M''_X(t)=E(x^2e^{tx})$	$M''_X(0)=E(X^2)$
$M^n_X(t)=E(x^ne^{tx})$	$M^n_X(0)=E(X^n)$

Series

Finite	Binomial
$\sum_{k=1}^n k=\frac{n(n+1)}{2}$	$\sum_{k=0}^n\binom{n}{k}=2^n$
$\sum_{k=1}^n(2k-1)=n^2$	$\sum_{k=0}^n\binom{r+k}{k}=\binom{r+n+1}{n}$
$\sum_{k=1}^n k^2=\frac{n(n+1)(2n+1)}{6}$	$\sum_{k=0}^n\binom{k}{m}=\binom{n+1}{m+1}$
$\sum_{k=1}^n k^3=(\frac{n(n+1)}{2})^2$	$\sum_{k=0}^r\binom{m}{k}\binom{n}{r-k}=\binom{m+n}{r}$
$\sum_{k=0}^n c^k=\frac{c^{n+1}-1}{c-1}$	$\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k=(a+b)^n$

Infinite $\lvert p\rvert<1$	$\sum_{k=0}^\infty kp^{k-1}=\frac{d}{dp}\left(\sum_{k=0}^\infty p^k\right)=\frac{1}{(1-p)^2}$
$\sum_{k=0}^\infty p^k=\frac{1}{1-p}$	$\sum_{k=0}^\infty\binom{r+k-1}{k}x^k=(1-x)^{-r}r\in\mathbb{N}^+$
$\sum_{k=1}^\infty p^k=\frac{p}{1-p}$	$\sum_{k=0}^\infty\binom{\alpha}{k}p^k=(1+p)^\alpha\quad \lvert p\rvert<1\,,\,\alpha\in\mathbb{C}$

$\Gamma(n)=(n-1)!$	$\sum_{n=0}^\infty\frac{x^n}{n!}=e^x$
$\Gamma(a+1)=a\Gamma(a)$	$\sum_{n=0}^\infty ar^n=\frac{a}{1-r}, \lvert r\rvert<1$
$\Gamma(1/2)=\sqrt{\pi}$	$\sum_{k=0}^{n-1}ar^n=\frac{a(1-r^n)}{1-r}$
$\Gamma(0)=\Gamma(-1)=\infty$	$\int_0^1 x^{a-1}(1-x)^{b-1},dx=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$
$\Gamma(-1/2)=-2\Gamma(1/2)$	$\int_0^\infty x^{t-1}e^{-x}dx=\Gamma(t)$

Integrals+c

$\int kdx=kx$	$\int_a^b cdx=c(b-a)$	$\lvert\int_a^b f(x)dx\rvert\le\int_a^b\lvert f(x)\rvert dx$
$\int e^udu=e^u$	$\int lnudu=uln(u)-u$	$\int\frac{1}{a^2+u^2}du=\frac{1}{a}tan^{-1}(\frac{u}{a})$
$\int x^n=\frac{x^{n+1}}{n+1}$	$\int\frac{1}{ax+b}=\frac{1}{a}ln\lvert ax+b\rvert$	$\int\frac{1}{\sqrt{a^2-u^2}}du=\frac{1}{sin(\frac{u}{a})}$
$\int\ln aa^x=a^x$		

Substitution

$u=g(x)$	$du=g'(x)dx$	$\int_a^b f(g(x))g'(x)dx=\int_{g(a)}^{g(b)}f(u)du$
x^3	$du=3x^2dx$	$\int_1^2 5x^2cos(x^3)dx=\int_1^8\frac{5}{3}cos(u)du$

Integreation by parts

u	du	dv	v	$\int_a^b udv$
x	dx	e^{-x}	$-e^{-x}$	$\int xe^{-x}dx$
lnx	$\frac{1}{x}dx$	dx	x	$\int_3^5 xlnxdx$

$=uv|_a^b-\int_a^b vdu$
$=-xe^{-x}+\int xe^{-x}dx=-xe^{-x}-e^{-x}+c$
$=xlnx|_3^5-\int_3^5 dx=(xlnx-x)|_3^5=5ln5-3ln3-2$
$\int_a^b f(x)dx=F(b)-F(a)=-\int_b^a f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx;$
$\frac{d}{dx}\int_{v(x)}^{u(x)}f(t)dt=u'(x)f[u(x)]-v'(x)f[v(x)]$

Derivatives

$(cf)'=cf'(x)$	$(fg)'=f'g+fg'$	$(f\pm g)'=f'(x)\pm g'(x)$
$\frac{dx}{dx}=1$	$(\frac{f}{g})'=\frac{(f'g-fg')}{g^2}$	$(f(g(x)))'=f'(g(x))g'(x)$
$\frac{de^x}{dx}=e^x$	$\frac{d(x^n)}{dx}=nx^{n-1}$	$\frac{d\ln(x)}{dx}=\frac{1}{x}, x>0$
$\frac{da^x}{dx}=a^xln(a)$	$\frac{d\tan^{-1}x}{dx}=\frac{1}{1+x^2}$	$\frac{d\log_a(x)}{dx}=\frac{1}{xlna}, x>0$
$\frac{d\cos x}{dx}=-\sin x$	$\frac{d\tan x}{dx}=\sec^2 x$	$\frac{dx(\ln x-1)}{dx}=\ln x$

Distribution	**CDF**	**P(X=x),f(x)**	μ	EX^2	**Var**	**MGF**	**M'(t)**	**M"(t)**	$M^n(t)$
Bern(p)		$p^x q^{1-x}, x \in \{1,0\}$	p	p	pq	$pe^t + q$			
Bin(n,p)	$I_{1-p}(n-x, x+1)$	$\binom{n}{x} p^x q^{n-x}; x \in \{0,1..n\}$	np	$\mu(\mu+q)$	$q\mu$	$(pe^t+q)^n$			
Geom(p)	$1-q^x$	$pq^{x-1}, x \in 1,2,..$	$\frac{1}{p}$	$\frac{p+2q}{p^2}$	$\frac{q}{p^2}$	$\frac{pe^t}{1-qe^t}, t < -\ln q$			
	$1-q^{x+1}$	$pq^x, x \in 0,1,..$	$\frac{q}{p}$	$\frac{q^2+q}{p^2}$	$\frac{q}{p^2}$	$\frac{p}{1-qe^t}, qe^t < 1$	$\frac{pqe^t}{(1-qe^t)^2}$	$\frac{2pqe^t}{(1-qe^t)^3} - M'(t)$	
NBin(r,p)		$\binom{x-1}{r-1} p^r q^{x-r}, x \in r, r+1..$	$\frac{r}{p}$		$\frac{rq}{p^2}$	$(\frac{pe^t}{1-qe^t})^r$			
		$\binom{x+r-1}{r-1} p^r q^x, x \in 0,1..$	$\frac{rq}{p}$		$\frac{rq}{p^2}$	$(\frac{p}{1-qe^t})^r, qe^t < 1$			
HGeom(N,m,k)		$\frac{\binom{m}{x}\binom{N-m}{k-x}}{\binom{N}{k}}$	$\frac{km}{N}$		$\mu\frac{(N-m)(N-k)}{N(N-1)}$				
HGeom(w,b,k)		$\frac{\binom{w}{x}\binom{b}{k-x}}{\binom{w+b}{k}}$	$\frac{kw}{w+b}$		$\mu\frac{b(w+b-k)}{(w+b)(w+b-1)}$				
Pois(μ)	$e^{-\mu}\sum_{i=0}^{x} \frac{\mu^i}{i!}$	$\frac{\mu^x}{x!}e^{-\mu}, x \in 0,1..$	μ	$\mu^2+\mu$	μ	$e^{\mu(e^t-1)}$	$\mu e^t M(t)$	$\mu e^t(1+\mu e^t)M(t)$	
Unif(n)		$\frac{1}{n}, x \in 1,2..n$	$\frac{n+1}{2}$	$\frac{(n+1)(2n+1)}{6}$	$\frac{(n^2-1)}{12}$	$\frac{\sum_{i=1}^{n} e^{ti}}{n}$			
Unif(a,b)	$\frac{x-a}{b-a}$	$\frac{1}{b-a}, x \in (a,b)$	$\frac{a+b}{2}$		$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$			
$\mathcal{N}(\mu,\sigma^2)$		$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	$\mu^2+\sigma^2$	σ^2	$e^{\mu t+\frac{\sigma^2 t^2}{2}}$	$(\mu+\sigma^2 t)M(t)$	$[(\mu+\sigma^2 t)^2+\sigma^2]M(t)$	
$\mathcal{N}(0,1)$		$\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$	0	1	1	$e^{\frac{t^2}{2}}$			
$\mathcal{LN}(\mu,\sigma^2)$		$\frac{1}{x\sigma\sqrt{2\pi}}e^{\frac{-(\ln x-\mu)^2}{2\sigma^2}}$	$e^{\mu+\frac{\sigma^2}{2}}$	$e^{2\mu+2\sigma^2}$	$\theta^2(e^{\sigma^2}-1)$	$\times$			
$Cauchy(\theta,\sigma^2)$		$\frac{1}{\pi\sigma}\frac{1}{1+(\frac{x-\theta}{\sigma})^2}$	$\times$	$\times$	$\times$				
DExpo(μ,σ^2)		$\frac{1}{2\pi\sigma}e^{-\lvert\frac{x-\mu}{\sigma}\rvert}$	μ	$\mu^2+2\sigma^2$	$2\sigma^2$	$\frac{e^{\mu t}}{1-\sigma^2 t^2}$			
Expo(λ)	$1-e^{-\lambda x}$	$\lambda e^{-\lambda x}, x \in (0,\infty)$	$\frac{1}{\lambda}$		$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}, t < \lambda$			
Expo(β)		$\frac{1}{\beta}e^{-\frac{x}{\beta}}$	β		β^2	$\frac{1}{1-\beta t}$	$\beta(1-\beta t)^{-2}$	$2\beta^2(1-\beta t)^{-3}$	
Gamma(a,λ)		$\frac{\lambda^a}{\Gamma(a)}x^{a-1}e^{-\lambda x}, x \in (0,\infty)$	$\frac{a}{\lambda}$	$\frac{a}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-t}\right)^a, t < \lambda$				
Gamma(α,β)		$\frac{1}{\Gamma(a)\beta^\alpha}x^{a-1}e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$\left(\frac{1}{1-\beta t}\right)^a, t < \frac{1}{\beta}$				
Beta(a,b)		$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$	$\frac{a}{a+b}$		$\frac{\mu(1-\mu)}{(a+b+1)}$				$\frac{\Gamma(\alpha+n)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+n)\Gamma(\alpha)}$
$B(\alpha,\beta) =$		$\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, x \in (0,1)$		$\frac{a(a+1)}{(a+b)(a+b+1)}$	$\frac{ab}{(a+b)^2(a+b+1)}$				
χ_p^2		$\frac{1}{2^{p/2}\Gamma(p/2)}x^{p/2-1}e^{-x/2}$	p	$2p+p^2$	$2p$	$(1-2t)^{-p/2}, t < \frac{1}{2}$			
t_n		$\frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}(1+\frac{x^2}{n})^{-\frac{n+1}{2}}$	$0, n > 1$		$\frac{n}{n-2}, n > 2$	$\times$			