2 Transformation of pairs of r.v.s

Given 2 random variables and their joint function, and given a function of them, find its distribution.

$f_{Y_1,Y_2}(y_1,y_2)=\sum_{i=1}^{k} f_{X_1,X_2}(h_1(y_1,y_2),h_2(y_1,y_2))|J_i|$

$J_{1,2}=\begin{vmatrix}\frac{\partial h_1}{\partial y_1} & \frac{\partial h_2}{\partial y_2}\\ \frac{\partial h_1}{\partial y_1} & \frac{\partial h_2}{\partial y_2}\end{vmatrix}=\frac{\partial h_1}{\partial y_1}\frac{\partial h_2}{\partial y_2}-\frac{\partial h_2}{\partial y_1}\frac{\partial h_1}{\partial y_2}$

$U\sim Geom(p=\frac{1}{2}), u=1,2..$ the number of trials needed to get the first head.

$V\sim NBin(p=\frac{1}{2},r=2), v=2,3..$ the number of trials needed to get two heads in repeated tosses of a fair coin.

the distribution of (U,V) is $\{(u,v):u=1,2,..;v=u+1,u+2,..\}$ is not a cross-product set. U and V are not independent.

3 Covariance and Correlation

Given a joint pdf, find the covariance, correlation, conditional expectation,conditional variance.

Conditional expectations and variances

$f(x_2|x_1)=\frac{f(x_1,x_2)}{f_1(x_1)}$;$p(x_2|x_1)=\frac{p(x_1,x_2)}{p_1(x_1)}$

4.2.1 $f(y|x)=P(Y=y|X=x)=\frac{f(x,y)}{f_X(x)}$

4.2.3 $E(g(x_2)|x_1)=\sum_{all\ x_2} g(x_2)p(x_2|x_1)$,$E(g(x_2)|x_1)=\int_{all\ x_2} g(x_2)f(x_2|x_1)dx_2$

4.2.4 $Var(y|x)=E(Y^2|x)-(E(Y|x))^2$, $V[X|Y]=E[(X-E[X|Y])^2|Y]$

4.2.7 $f(x,y)=g(x)h(y) \iff$ indep

4.2.10 $E(g(X)h(Y))=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x)h(y)f(x,y)dxdy=\int_{-\infty}^{\infty} h(y)f_Y(y)\left[\int_{-\infty}^{\infty} g(x)f_X(x)dx\right]dy=$ $E[g(x)]E[h(y)]=(Eg(X))(Eh(Y))$

$F_{XY}(x,y)=P(X\le x,Y\le y)$

Properties of $\bar{X}$ and S^2

5.2

$\bar{X}_{n+1}=\frac{\sum_{i=1}^{n+1}X_i}{n+1}=\frac{\sum_{i=1}^{n}X_i+X_{n+1}}{n+1}=\frac{n\bar{X}_n+X_{n+1}}{n+1}$

$nS_{n+1}^2=(n-1)S_n^2+(\frac{n}{n+1})(X_{n+1}-\bar{X}_n)^2$

when $X_1,..,X_n$ be iid $n(\mu,\sigma^2)$, $\frac{(n-1)S^2}{\sigma^2}\sim\chi^2_{n-1}$,$Var\chi^2_{n-1}=2(n-1)$

$Var[\frac{(n-1)S^2}{\sigma^2}]=\frac{(n-1)^2}{\sigma^4}Var[S^2]=2(n-1)\implies Var[S^2]=\frac{2\sigma^4}{n-1}$

1 Connection between N,χ^2,t,F

5.3

$X_1..X_n\sim iidN(\mu,\sigma^2)$. Given some function of these, find the distribution.

W Normal	$\bar{X}\sim N(\mu,\frac{\sigma^2}{n})$	$\bar{X},S^2$ indep	$\frac{(n-1)S^2}{\sigma^2}\sim\chi^2_{n-1}$
W/o Normal	$E[\bar{X}]=\mu$	$V[\bar{X}]=\frac{\sigma^2}{n})$	$E[S^2]=\sigma^2$

5.2.8 Distribution of the mean: $\bar{X}\sim gamma(n\alpha,\beta/n)$ 5.2.9 Convolution formula $Z=X+Y$ $f_Z(z)=\int_{-\infty}^{\infty} f_X(w)f_Y(z-w)dw$

5.3.2 $Z\sim n(0,1)$,$Z^2\sim\chi^2_1$, $X_1..X_n$ indep $X_i\sim\chi^2_{p_i}$,$X_1=..=X_n\sim\chi^2_{p_1+..p_n}$

$\chi^2_p\sim gamma(p/2,2)$

5.3.4 T

5.3.5 $X_1..X_n$,$Y_1..Y_m$ indep. $X_i\sim n(\mu_X,\sigma_X^2)Y_j\sim n(\mu_Y,\sigma_Y^2)$,$\frac{S_X^2}{\sigma_X^2}\sim\chi^2$F: $\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2}$

4 Order statistics

Find the distribution of X(k) or find the joint distribution of $X_{(j)},X_{(k)}$

1-29p9

5.4.4 $f_K(x)=\frac{n!}{(j-1)!(n-j)!}K[F_X(x)]^{k-1}[1-F_X(x)]^{n-k}f(x)$

1-31p4-8

5.4.5 uniform order pdf

1-31p2-3

5.4.6

$f_{X_{(i)},X_{(j)}}(u,v)=\frac{n!}{(i-1)!(j-1-i)!(n-j)!}f_X(u)f_X(v)[F_X(u)]^{i-1}[F_X(v)-F_X(u)]^{j-1-i}[1-F_X(v)]^{n-j}$

$f_{X_{(1)},..,X_{(n)}}(x_1,..,x_n)=\begin{cases}n!f_X(x_1)\cdot...\cdot f_X(x_n) & -\infty<x_1<..<x_n<\infty\\ 0 & \text{otherwise}\end{cases}$

joint pmf

Conditional/Baye's: $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(B|A)P(A)}{P(B)}$;disjoint

$P(B)=P(B|A_1)P(A_1)+..P(B|A_k)P(A_k)=\sum_{i=1}^{k}P(B|A_i)P(A_i)$

	$P(A\cap B)=P(A)P(B)$	$P(X\le x,Y\le y)=P(X\le x)P(Y\le y)$
Independence:	$E(XY)=E(X)E(Y)$	$F_{X,Y}(x,y)=F_X(x)F_Y(y)$
	$V(X\pm Y)=VX+VY$	$f_{X,Y}(s,t)=f_X(s)f_Y(t)$

pdf/pmf

CDF: $F(x)=P(X\le x)=\sum_{y:y\le x}p(y)=\int_{-\infty}^{x}f(y)dy$

probabilities: $a\le b$:

PMF: $p(x)=P(X=x)=P(\forall w\in\mathcal{W}:X(w)=x)$

$P(a\le X\le b)=F(b)-F(a^-)$; $P(a<X\le b)=F(b)-F(a)$; $P(a\le X\le a)=p(a)$;

$P(a<X<b)=F(b^-)-F(a)$; (where a^- is the largest possible X value strictly less than a); Taking $a=b$ yields $P(X=a)=F(a)-F(a-1)$ as desired.

PDF: $P(\forall w\in\mathcal{W}:a\le X(w)\le b)=\int_a^b f(x)dx$

$P(a\le X\le b)=P(a<X\le b)=P(a<X<b)$; $P(X>a)=1-F(a)$; $P(a\le X\le b)=F(b)-F(a)$

CDF

Condition:$f(x)\ge 0\forall x$ pmf $\sum_x f_X(x)=1$,pdf $\int_{-\infty}^{\infty}f(x)dx=1$

$c\sin x\ge 0, 0<x<\pi/2$	$\int_0^{\frac{\pi}{2}}c\sin xdx=1$	$c=1$
$ce^{-\|x\|}\ge 0,-\infty<x<\infty$	$c\int_{-\infty}^{0}e^x dx+c\int_0^{\infty}e^{-x}dx=1$	$c=\frac{1}{2}$

mean/variance

$E[g(x)]=\mu=\sum_{x\in D}h(x)p(x)=\int_{-\infty}^{\infty}g(x)f(x)dx$

$E[(X-\mu)^n]=\mu_n=\sum(x-\mu)^n p(x)=\int(x-\mu)^n f(x)dx$

$E(aX+b)=aE(X)+b$	$E(X)=\int_{-\infty}^{0}F_X(t)dt+\int_0^{\infty}F_X(t)dt$	
$V(aX+b)=a^2\sigma^2$	$V(X\pm Y)=VX+VY\pm 2Cov(X,Y)$	$V(X)=\sigma_x^2=E(X^2)-[E(X)]^2$
$\sigma_{ax+b}=\|a\|\cdot\sigma_x$	$Cov(x,y)=E(XY)-E(X)E(Y)$	

$f_X(x)$	EX	EX^2	$V(X)$
$\frac{1}{n}$	$\sum_1^n x\frac{1}{n}$ $\frac{1}{n}\frac{n(n+1)}{2}$	$\sum_1^n x^2\frac{1}{n}$ $\frac{1}{n}\frac{n(n+1)(2n+1)}{6}$	$\frac{n^2-1}{12}$
ax^{a-1}	$\int_0^1 xax^{a-1}dx$ $\frac{a}{a+1}x^{a+1}\|_0^1$	$\int_0^1 x^2ax^{a-1}dx$ $\frac{a}{a+2}x^{a+2}\|_0^1$	$\frac{a}{a+2}-(\frac{a}{a+1})^2$ $\frac{a}{(a+2)(a+1)^2}$
$\frac{3}{2}(x-1)^2$	$\int_0^2 x\frac{3}{2}(x-1)^2dx$ $\frac{3}{2}(\frac{x^4}{4}\|_0^2..)$	$\int_0^2 x^2\frac{3}{2}(x-1)^2dx$ $\frac{3}{2}(\frac{x^5}{5}\|_0^2..)$	$\frac{8}{5}-1^2$ $\frac{3}{5}$

transform

$g(x)\uparrow$	$F_Y(y)=F_X(g^{-1}(y))$
$g(x)\downarrow$	$F_Y(y)=1-F_X(g^{-1}(y))$

not monotone $\because X\le 0\ is\ \emptyset\therefore P(X\le-\sqrt{y})=0$

$F_Y(y)=P(Y\le y)=P(X^2\le y)=P(-\sqrt{y}\le X\le\sqrt{y})=P(X\le\sqrt{y})-P(X\le-\sqrt{y})=F_X(\sqrt{y})=$ $\sqrt{y},\ 0<\sqrt{y}<1$

Y	X	$f_X(x)$	$F_Y(y)$	$f_Y(y)=\frac{dF_Y(y)}{dy}$
X^2	$\sqrt{Y}$	$\frac{e^{-x^2/2}}{\sqrt{2\pi}}$	$F_X(\sqrt{y})-F_X(-\sqrt{y})$	$\frac{e^{-y/2}}{2\sqrt{2\pi y}},y>0$

monotone:$f_Y(y)=f_X(g^{-1}(y))|\frac{d(g^{-1}(y))}{dy}|$

	Y	X	$f_X(x)$
1	$-\log X$	e^{-Y}	$x^n(1-x)^m,0<x<1$
2	e^X	$\ln Y$	$\frac{1}{\sigma^2}xe^{-(x/\sigma)^2/2},0<x<\infty$
3	$-2lnX$	$e^{-Y/2}$	$1,0<x<1$
4	X^2	$\sqrt{Y}$	$1,0<x<1$
5	$d\tan X$	$\arctan\frac{Y}{d}$	$\frac{1}{b-a}=\frac{2}{\pi}$

1 $(e^{-y})^n(1-e^{-y})^m|\frac{d(e^{-y})}{dy}|=e^{-(n+1)y}(1-e^{-y})^m,\ 0<y<\infty$

2 $\frac{1}{\sigma^2}\ln ye^{-(\ln y/\sigma)^2/2}|\frac{d(\ln y)}{dy}|=\frac{\ln(y)}{\sigma^2 y}e^-(\ln(y)/\sigma)^2/2,1<y<\infty$

3 $1|\frac{de^{-Y/2}}{dy}|=\frac{1}{2}e^{-Y/2},0<y<\infty$

4 $1|\frac{d\sqrt{y}}{dy}|=\frac{1}{2\sqrt{y}},0<y<1$

5 $\frac{2}{\pi}|\frac{d(\arctan\frac{y}{d})}{dy}|=\frac{2}{\pi}\frac{d(\frac{y}{d})}{[(\frac{y}{d})^2+1]}=\frac{2}{\pi d[(\frac{y}{d})^2+1]}0<y<\infty$

find mean/variance by mgf

$E(e^{tx}) = \int e^{tx} f(x)dx = \sum e^{tx} f(x)$; $M_{aX+b}(t) = e^{tb} M_{aX}(t)$

$M_X(t) = E(e^{tx})$	$M_X(0) = 1$
$M'_X(t) = E(xe^{tx})$	$M'_X(0) = E(X)$
$M''_X(t) = E(x^2 e^{tx})$	$M''_X(0) = E(X^2)$
$M^n_X(t) = E(x^n e^{tx})$	$M^n_X(0) = E(X^n)$

Series

Finite	Binomial
$\sum_{k=1}^n k = \frac{n(n+1)}{2}$	$\sum_{k=0}^n \binom{n}{k} = 2^n$
$\sum_{k=1}^n (2k-1) = n^2$	$\sum_{k=0}^n \binom{r+k}{k} = \binom{r+n+1}{n}$
$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{k=0}^n \binom{k}{m} = \binom{n+1}{m+1}$
$\sum_{k=1}^n k^3 = (\frac{n(n+1)}{2})^2$	$\sum_{k=0}^r \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
$\sum_{k=0}^n c^k = \frac{c^{n+1}-1}{c-1}$	$\sum_{k=0}^n \binom{n}{k} a^{n-k} b^k = (a+b)^n$

Infinite $\lvert p\rvert < 1$	$\sum_{k=0}^\infty kp^{k-1} = \frac{d}{dp}\left(\sum_{k=0}^\infty p^k\right) = \frac{1}{(1-p)^2}$
$\sum_{k=0}^\infty p^k = \frac{1}{1-p}$	$\sum_{k=0}^\infty \binom{r+k-1}{k} x^k = (1-x)^{-r} r \in \mathbb{N}^+$
$\sum_{k=1}^\infty p^k = \frac{p}{1-p}$	$\sum_{k=0}^\infty \binom{\alpha}{k} p^k = (1+p)^\alpha \quad \lvert p\rvert < 1 ,\ \alpha \in \mathbb{C}$

$\Gamma(n) = (n-1)!$	$\sum_{n=0}^\infty \frac{x^n}{n!} = e^x$
$\Gamma(a+1) = a\Gamma(a)$	$\sum_{n=0}^\infty ar^n = \frac{a}{1-r}, \lvert r\rvert < 1$
$\Gamma(1/2) = \sqrt{\pi}$	$\sum_{k=0}^{n-1} ar^n = \frac{a(1-r^n)}{1-r}$
$\Gamma(0) = \Gamma(-1) = \infty$	$\int_0^1 x^{a-1}(1-x)^{b-1}, dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$
$\Gamma(-1/2) = -2\Gamma(1/2)$	$\int_0^\infty x^{t-1} e^{-x} dx = \Gamma(t)$

Integrals+c

$\int kdx = kx$	$\int_a^b cdx = c(b-a)$	$\lvert\int_a^b f(x)dx\rvert \leq \int_a^b \lvert f(x)\rvert dx$
$\int e^u du = e^u$	$\int lnudu = uln(u) - u$	$\int \frac{1}{a^2+u^2} du = \frac{1}{a} tan^{-1}(\frac{u}{a})$
$\int x^n = \frac{x^{n+1}}{n+1}$	$\int \frac{1}{ax+b} = \frac{1}{a} ln\lvert ax+b\rvert$	$\int \frac{1}{\sqrt{a^2-u^2}} du = \frac{1}{sin(\frac{u}{a})}$
$\int \ln a a^x = a^x$		

Substitution	$u = g(x)$	$du = g'(x)dx$	$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$
	x^3	$du = 3x^2 dx$	$\int_1^2 5x^2 cos(x^3)dx = \int_1^8 \frac{5}{3} cos(u)du$

	u	du	dv	v	$\int_a^b udv$
Integreation by parts	x	dx	e^{-x}	$-e^{-x}$	$\int xe^{-x}dx$
	lnx	$\frac{1}{x}dx$	dx	x	$\int_3^5 xlnxdx$

$= uv|_a^b - \int_a^b vdu$

$= -xe^{-x} + \int xe^{-x}dx = -xe^{-x} - e^{-x} + c$

$= xlnx|_3^5 - \int_3^5 dx = (xlnx - x)|_3^5 = 5ln5 - 3ln3 - 2$

$\int_a^b f(x)dx = F(b) - F(a) = -\int_b^a f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx;$

$\frac{d}{dx}\int_{v(x)}^{u(x)} f(t)dt = u'(x)f[u(x)] - v'(x)f[v(x)]$

Derivatives

$(cf)' = cf'(x)$	$(fg)' = f'g + fg'$	$(f \pm g)' = f'(x) \pm g'(x)$
$\frac{dx}{dx} = 1$	$(\frac{f}{g})' = \frac{(f'g - fg')}{g^2}$	$(f(g(x)))' = f'(g(x))g'(x)$
$\frac{de^x}{dx} = e^x$	$\frac{d(x^n)}{dx} = nx^{n-1}$	$\frac{d\ln(x)}{dx} = \frac{1}{x}, x > 0$
$\frac{da^x}{dx} = a^x ln(a)$	$\frac{d\tan^{-1} x}{dx} = \frac{1}{1+x^2}$	$\frac{d\log_a(x)}{dx} = \frac{1}{xlna}, x > 0$
$\frac{d\cos x}{dx} = -\sin x$	$\frac{d\tan x}{dx} = \sec^2 x$	$\frac{dx(\ln x - 1)}{dx} = \ln x$

Distribution	**CDF**	**P(X=x),f(x)**	μ	EX^2	**Var**	**MGF**	**M'(t)**	**M"(t)**	$M^n(t)$
Bern(p)		$p^x q^{1-x}, x \in \{1,0\}$	p	p	pq	$pe^t + q$			
Bin(n,p)	$I_{1-p}(n-x, x+1)$	$\binom{n}{x} p^x q^{n-x}; x \in \{0,1..n\}$	np	$\mu(\mu+q)$	$q\mu$	$(pe^t+q)^n$			
Geom(p)	$1-q^x$	$pq^{x-1}, x \in 1,2,..$	$\frac{1}{p}$	$\frac{p+2q}{p^2}$	$\frac{q}{p^2}$	$\frac{pe^t}{1-qe^t}, t < -\ln q$			
	$1-q^{x+1}$	$pq^x, x \in 0,1,..$	$\frac{q}{p}$	$\frac{q^2+q}{p^2}$	$\frac{q}{p^2}$	$\frac{p}{1-qe^t}, qe^t < 1$	$\frac{pqe^t}{(1-qe^t)^2}$	$\frac{2pqe^t}{(1-qe^t)^3} - M'(t)$	
NBin(r,p)		$\binom{x-1}{r-1} p^r q^{x-r}, x \in r, r+1..$	$\frac{r}{p}$		$\frac{rq}{p^2}$	$(\frac{pe^t}{1-qe^t})^r$			
		$\binom{x+r-1}{r-1} p^r q^x, x \in 0,1..$	$\frac{rq}{p}$		$\frac{rq}{p^2}$	$(\frac{p}{1-qe^t})^r, qe^t < 1$			
HGeom(N,m,k)		$\frac{\binom{m}{x}\binom{N-m}{k-x}}{\binom{N}{k}}$	$\frac{km}{N}$		$\mu\frac{(N-m)(N-k)}{N(N-1)}$				
HGeom(w,b,k)		$\frac{\binom{w}{x}\binom{b}{k-x}}{\binom{w+b}{k}}$	$\frac{kw}{w+b}$		$\mu\frac{b(w+b-k)}{(w+b)(w+b-1)}$				
Pois(μ)	$e^{-\mu}\sum_{i=0}^{x}\frac{\mu^i}{i!}$	$\frac{\mu^x}{x!}e^{-\mu}, x \in 0,1..$	μ	$\mu^2+\mu$	μ	$e^{\mu(e^t-1)}$	$\mu e^t M(t)$	$\mu e^t(1+\mu e^t)M(t)$	
Unif(n)		$\frac{1}{n}, x \in 1,2..n$	$\frac{n+1}{2}$	$\frac{(n+1)(2n+1)}{6}$	$\frac{(n^2-1)}{12}$	$\frac{\sum_{i=1}^{n} e^{ti}}{n}$			
Unif(a,b)	$\frac{x-a}{b-a}$	$\frac{1}{b-a}, x \in (a,b)$	$\frac{a+b}{2}$		$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$			
$\mathcal{N}(\mu,\sigma^2)$		$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	$\mu^2+\sigma^2$	σ^2	$e^{\mu t+\frac{\sigma^2 t^2}{2}}$	$(\mu+\sigma^2 t)M(t)$	$[(\mu+\sigma^2 t)^2+\sigma^2]M(t)$	
$\mathcal{N}(0,1)$		$\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$	0	1	1	$e^{\frac{t^2}{2}}$			
$\mathcal{LN}(\mu,\sigma^2)$		$\frac{1}{x\sigma\sqrt{2\pi}}e^{\frac{-(\ln x-\mu)^2}{2\sigma^2}}$	$e^{\mu+\frac{\sigma^2}{2}}$	$e^{2\mu+2\sigma^2}$	$\theta^2(e^{\sigma^2}-1)$	$\times$			
$Cauchy(\theta,\sigma^2)$		$\frac{1}{\pi\sigma}\frac{1}{1+(\frac{x-\theta}{\sigma})^2}$	$\times$	$\times$	$\times$				
DExpo(μ,σ^2)		$\frac{1}{2\pi\sigma}e^{-\lvert\frac{x-\mu}{\sigma}\rvert}$	μ	$\mu^2+2\sigma^2$	$2\sigma^2$	$\frac{e^{\mu t}}{1-\sigma^2 t^2}$			
Expo(λ)	$1-e^{-\lambda x}$	$\lambda e^{-\lambda x}, x \in (0,\infty)$	$\frac{1}{\lambda}$		$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}, t < \lambda$			
Expo(β)		$\frac{1}{\beta}e^{-\frac{x}{\beta}}$	β		β^2	$\frac{1}{1-\beta t}$	$\beta(1-\beta t)^{-2}$	$2\beta^2(1-\beta t)^{-3}$	
Gamma(a,λ)		$\frac{\lambda^a}{\Gamma(a)}x^{a-1}e^{-\lambda x}, x \in (0,\infty)$	$\frac{a}{\lambda}$	$\frac{a}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-t}\right)^a, t < \lambda$				
Gamma(α,β)		$\frac{1}{\Gamma(a)\beta^\alpha}x^{a-1}e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$\left(\frac{1}{1-\beta t}\right)^a, t < \frac{1}{\beta}$				
Beta(a,b)		$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$	$\frac{a}{a+b}$		$\frac{\mu(1-\mu)}{(a+b+1)}$				$\frac{\Gamma(\alpha+n)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+n)\Gamma(\alpha)}$
$B(\alpha,\beta) =$		$\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, x \in (0,1)$		$\frac{a(a+1)}{(a+b)(a+b+1)}$	$\frac{ab}{(a+b)^2(a+b+1)}$				
χ^2_p		$\frac{1}{2^{p/2}\Gamma(p/2)}x^{p/2-1}e^{-x/2}$	p	$2p+p^2$	$2p$	$(1-2t)^{-p/2}, t < \frac{1}{2}$			
t_n		$\frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}(1+\frac{x^2}{n})^{-\frac{n+1}{2}}$	$0, n > 1$		$\frac{n}{n-2}, n > 2$	$\times$			