565 mid-term

model

$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$, for $\varepsilon_{ij} \sim iidN(0, \sigma^2), i = 1, 2, , a, j = 1, 2, , n,\ \sum_{i=1}^a \tau_i = 0$

Random effects: $\tau_i \sim iidN(0, \sigma_\tau^2)$,$\tau_i$ and ε_{ij} indep

$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$, for $\varepsilon_{ij} \sim iidN(0, \sigma^2), i = 1, 2, , a, j = 1, 2, , b$

Fixed effects: $\sum_{i=1}^a \tau_i = 0, \sum_{j=1}^b \beta_j = 0$ Random effects: $\beta_j \sim iidN(0, \sigma_\beta^2)$

$y_{..} = \sum_{i=1}^a (y_{i.} = \sum_{j=1}^n y_{ij}), \mu = \frac{\sum_{i=1}^a \mu_i}{a}$

$\bar{y}_{..} = \frac{1}{N}\left[y_{..} = \sum_{i=1}^a (y_{i.} = \sum_{j=1}^n y_{ij})\right] =$

$\frac{1}{a}\sum_{i=1}^a \left[\bar{y}_{i.} = \frac{1}{n}(y_{i.} = \sum_{j=1}^n y_{ij})\right]$

least square and normal equation

CRD:$SSE = \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \mu - \tau_i)^2$

$\frac{\partial SSE}{\partial \mu}\Big|_{\hat{\mu},\hat{\tau}_i} = 2\sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \hat{\mu} - \hat{\tau}_i)(-1) = 0\ y_{..} = an\hat{\mu} + n\sum_{i=1}^a \hat{\tau}_i$

$\frac{\partial SSE}{\partial \tau_i}\Big|_{\hat{\mu},\hat{\tau}_i} = 2\sum_{j=1}^n (y_{ij} - \hat{\mu} - \hat{\tau}_i)(-1) = 0\ y_{i.} = n\hat{\mu} + n\hat{\tau}_i$

RCBD: $SSE = \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \mu - \tau_i - \beta_j)^2$

$\frac{\partial SSE}{\partial \mu}\Big|_{\hat{\mu},\hat{\tau}_i,\hat{\beta}_j} = 2\sum_{i=1}^a \sum_{j=1}^b (y_{ij} - \hat{\mu} - \hat{\tau}_i - \hat{\beta}_j)(-1) = 0$

$\frac{\partial SSE}{\partial \tau_i}\Big|_{\hat{\mu},\hat{\tau}_i,\hat{\beta}_j} = 2\sum_{j=1}^b (y_{ij} - \hat{\mu} - \hat{\tau}_i - \hat{\beta}_j)(-1) = 0$

$\frac{\partial SSE}{\partial \beta_j}\Big|_{\hat{\mu},\hat{\tau}_i,\hat{\beta}_j} = 2\sum_{i=1}^a (y_{ij} - \hat{\mu} - \hat{\tau}_i - \hat{\beta}_j)(-1) = 0$

$y_{..} = ab\hat{\mu} + b\sum_{i=1}^a \hat{\tau}_i + a\sum_{j=1}^b \hat{\beta}_j$

$y_{i.} = b\hat{\mu} + b\hat{\tau}_i + \sum_{j=1}^b \hat{\beta}_j$

$y_{.j} = a\hat{\mu} + \sum_{i=1}^a \hat{\tau}_i + a\hat{\beta}_j$

hypothesis test

model	H0	H1
FT-CRD	$\mu_1 = \mu_2 = .. = \mu_i$	at least two of μ_i are different
RT-CRD	$\sigma_\tau^2 = 0$	$\sigma_\tau^2 > 0$
FB-RCBD	$\mu_1 = \mu_2 = .. = \mu_i$	at least two of μ_i are different
-Block	$\beta_1 = \beta_2 = .. = \beta_j = 0$	at least one of $\beta_j \neq 0$
RB-RCBD	$\sigma_\beta^2 = 0$	$\sigma_\beta^2 > 0$
Contrast	$\Gamma = \sum_{i=1}^a c_i\mu_i = 0$	$\Gamma = \sum_{i=1}^a c_i\mu_i \neq 0$

ANOVA

	SS	df	MS	F
SS_{Trt}	$n\sum_{i=1}^a (\bar{y}_{i.} - \bar{y}_{..})^2$	$a-1$	$\frac{SS_{Trt}}{a-1}$	$\frac{MS_{Trt}}{MS_E}$
SS_E	$\sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{i.})^2$	$a(n-1)$	$\frac{SS_E}{N-a}$	
SS_T	$\sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{..})^2$	$an-1$		
SS_{Trt}	$b\sum_{i=1}^a (\bar{y}_{i.} - \bar{y}_{..})^2$	$a-1$	$\frac{SS_{Trt}}{a-1}$	$\frac{MS_{Trt}}{MS_E}$
SS_{Blk}	$a\sum_{j=1}^b (\bar{y}_{.j} - \bar{y}_{..})^2$	$b-1$	$\frac{SS_{Trt}}{b-1}$	$\frac{MS_{Blk}}{MS_E}$
SS_E	$SS_T - SS_{Trt} - SS_{Blk}$	$(a-1)(b-1)$	$\frac{SS_E}{df_E}$	
SS_T	$\sum_{i=1}^a \sum_{j=1}^b (y_{ij} - \bar{y}_{..})^2$	$ab-1$		

distribution

	y_{ij}	$\bar{y}_{i.}$	$\frac{y_{i.}-\mu_i}{\frac{\sigma}{\sqrt{n}}}$	$\bar{y}_{i.} - \bar{y}_{j.}$	$\frac{\bar{y}_{i.}-\bar{y}_{j.}-(\mu_i-\mu_j)}{\frac{2\sigma}{\sqrt{n}}}$
N	$(\mu + \tau_i, \sigma^2)$	$(\mu + \tau_i, \frac{\sigma^2}{n})$	$(0, 1)$	$(\mu_i - \mu_j, \frac{2\sigma^2}{n})$	$(0, 1)$

t_{df_E}	$\frac{\frac{y_{i.}-\mu_i}{\frac{\sigma}{\sqrt{n}}}}{\sqrt{\frac{SS_E}{\sigma^2(N-a)}}} = \frac{y_{i.}-\mu_i}{\sqrt{\frac{SS_E}{n(df_E)}}} = \frac{y_{i.}-\mu_i}{\sqrt{\frac{MS_E}{n}}}$	$\frac{\bar{y}_{i.}-\bar{y}_{j.}-(\mu_i-\mu_j)}{\sqrt{\frac{2MS_E}{n}}}$	$\frac{C}{\sqrt{\widehat{Var(C)}}}$

χ^2	$\frac{SS_E}{\sigma^2}$	$\frac{SS_{Trt}}{\sigma^2}$	$\frac{SS_{Blk}}{\sigma^2}$
df	E	Trt	Blk

F	$\frac{MS_{Trt}}{MS_E}$	$\frac{(MS_{Blk}}{MS_E}$	$\frac{\sum_{i=1}^a c_i\bar{y}_{i.})^2}{\frac{MS_E}{n}\sum_{i=1}^a c_i^2}$
df	(Trt, E)	(Blk, E)	$(C = 1, E)$

$SS_{Trt} = \frac{1}{n}\sum_{i=1}^a y_{i.}^2 - \frac{y_{..}^2}{N}$

$SS_{Trt} = \frac{1}{b}\sum_{i=1}^a y_{i.}^2 - \frac{y_{..}^2}{N}$

$SS_{Blk} = \frac{1}{a}\sum_{j=1}^b y_{.j}^2 - \frac{y_{..}^2}{N}$

$SSE = \sum_{i=1}^a \sum_{j=1}^b (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2$

$SS_T = \sum_{i=1}^a \sum_{j=1}^b y_{ij}^2 - \frac{y_{..}^2}{N}$

$SS_C = \frac{(\sum_{i=1}^a c_i\bar{y}_i)^2}{\frac{1}{n}\sum_{i=1}^a c_i^2} = SS_{Trt}$

$Var[y_{ij} - \bar{y}_{i.}] =$

contrast

$C = \sum_{i=1}^a c_i\mu_i$;$\sum_{i=1}^a c_i = 0$;orthogonal $\sum_{i=1}^a c_i d_i = 0$

$Var\left[\sum_{i=1}^a c_i y_{i.}\right] = \sigma^2\sum_{i=1}^a n_i c_i^2$

CI

CI	balanced	unbalanced
μ_i	$\bar{y}_i \pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n}}$	$\bar{y}_i \pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n_i}}$
$\mu_i - \mu_j$	$\bar{y}_{i.} - \bar{y}_{j.} \pm t_{\frac{\alpha}{2}}\sqrt{\frac{2MS_E}{n}}$	$\bar{y}_{i.} - \bar{y}_{j.} \pm t_{\frac{\alpha}{2}}\sqrt{MS_E(\frac{1}{n_i} + \frac{1}{n_j})}$

covariance

$Cov(y_{ij}, y_{kl}) = \{\ 0\ , i \neq k; \sigma_\tau^2, i = k, j \neq l; \sigma^2 + \sigma_\tau^2, i = k, j = l$

	ij	$\sum_{i=1}^a ij$	$\sum_{j=1}^b ij$	$\sum_{i=1}^a \sum_{j=1}^b ij$
ij	σ^2	σ^2	σ^2	σ^2
$\sum_{i=1}^a ij$		$a\sigma^2$	σ^2	$a\sigma^2$
$\sum_{j=1}^b ij$			$b\sigma^2$	$b\sigma^2$
$\sum_{i=1}^a \sum_{j=1}^b ij$				$ab\sigma^2$

$Cov[_{ij}, _{ij}] = Cov[\sum_{i1}^a {}_{ij}, \sum_{j=1}^b {}_{ij}] = \sigma^2$,$Cov[\tau_i, \tau_i] = {}^2_\tau$,$Cov[\beta_j, \beta_j] = {}^2_\beta$

$Cov[_{ij}, \tau_i] = Cov[_{ij}, \beta_j] = Cov[\tau_i, \beta_j] = Cov[_{ij}, _{ik}] = 0, k \neq j$

$Cov[_{ij}, \sum_{j=1}^n {}_{ij}] = Cov[_{ij}, (_{i1} + .. + _{ij} .. + _{in})] =$

$Cov[_{ij}, _{i1}] + .. + Cov[_{ij}, _{ij}] .. + Cov[_{ij}, _{in}] = \sigma^2$

$Cov[\sum_{j=1}^n {}_{ij}, \sum_{i=1}^a \sum_{j=1}^n {}_{ij}] = Cov[(_{i1} + _{i2} .. + _{in}), (_{i1} + _{i2} .. + _{in})] =$

$Cov[_{i1}, _{i1}] + Cov[_{i2}, _{i2}] .. + Cov[_{in}, _{in}] = n\sigma^2$

expectation and variance

	E[f.]	E[r.]
MS_{Trt}	$\sigma^2 + \frac{n}{a-1}\sum_{i=1}^a \tau_i^2$	$\sigma^2 + n\tau_i^2$
MS_{Trt}	$\sigma^2 + \frac{b}{a-1}\sum_{i=1}^a \tau_i^2$	
MS_{Blk}	$\sigma^2 + \frac{a}{b-1}\sum_{j=1}^b \beta_j^2$	$\sigma^2 + a\beta_j^2$
MS_E	σ^2	σ^2

	y_{ij}	$\bar{y}_{..}$	$\bar{y}_{i.}$	$\bar{y}_{.j}$
E[f.]	μ	μ	$\mu + \tau_i$	$\mu + \beta_j$
V[f.]	σ^2	$\frac{\sigma^2}{na}$	$\frac{\sigma^2}{n}$	
E[r.]	μ	μ	$\mu + \tau_i$	
V[r.]	$\sigma^2 + \sigma_\tau^2$	$\frac{\sigma_\tau^2}{a} + \frac{\sigma^2}{na}$	$\frac{\sigma^2}{n} + \sigma_\tau^2$	

$\hat{\mu}$	$\hat{\mu}_i$	$\hat{\tau}_i$	$\hat{\beta}_j$	$\hat{y}_{ij}$	ε_{ij}
$\bar{y}_{..}$	$\bar{y}_{i.}$	$\bar{y}_{i.} - \bar{y}_{..}$	$\bar{y}_{.j} - \bar{y}_{..}$	$\bar{y}_{i.} + \bar{y}_{.j} - \bar{y}_{..}$	$\bar{y}_{ij} - \bar{y}_{..}$
$\frac{1}{an}y_{..} - \frac{1}{a}\sum_{i=1}^a \hat{\tau}_i$		$\frac{1}{a}y_{i.} - \hat{\mu}$			

	$\hat{\sigma}^2$	$\hat{\sigma}_{Trt}^2$	$\hat{\sigma}_{Blk}^2$	$\widehat{Var(\bar{y}_{i.})}$	$\hat{X}$
f.	MSE	$\frac{MS_{Trt}-MS_E}{n}$		$\frac{MS_E}{n}$	
r.	MSE	$\frac{MS_{Trt}-MS_E}{n}$	$\frac{MS_{Blk}-MS_E}{a}$	$\frac{MS_{Trt}}{n}$	$\frac{ay'_{i.}+by'_{.j}-y'_{..}}{(a-1)(b-1)}$

FT-	CRD	E	Var[]
$y_{ij} - \bar{y}_{i.}$	$\varepsilon_{ij} - \frac{\sum_{j=1}^n \varepsilon_{ij}}{n}$	0	$\frac{n-1}{n}\sigma^2$
$\bar{y}_{i.} - \bar{y}_{..}$	$\tau_i + \frac{\sum_{j=1}^n \varepsilon_{ij}}{n} - \frac{\sum_{i=1}^a \sum_{j=1}^n \varepsilon_{ij}}{an}$	τ_i	$\frac{(a-1)\sigma^2}{an}$
RT-	**CRD**	**E**	**Var[]**
$y_{ij} - \bar{y}_{i.}$	$\varepsilon_{ij} - \frac{\sum_{j=1}^n \varepsilon_{ij}}{n}$	0	$\frac{n-1}{n}\sigma^2$
$\bar{y}_{i.} - \bar{y}_{..}$	$\tau_i - \frac{\sum_{i=1}^a \tau_i}{a} + \frac{\sum_{j=1}^n \varepsilon_{ij}}{n} - \frac{\sum_{i=1}^a \sum_{j=1}^n \varepsilon_{ij}}{an}$	0	$\frac{(a-1)(n\sigma_\tau^2+\sigma^2}{an}$
F-	**RCBD**	**E**	**Var[]**
$y_{ij} - \bar{y}_{i.}$	$\beta_j + \varepsilon_{ij} - \frac{\sum_{j=1}^b \varepsilon_{ij}}{b}$	β_j	$\frac{(b-1)\sigma^2}{b}$
$\bar{y}_{i.} - \bar{y}_{..}$	$\tau_i + \frac{\sum_{j=1}^b \varepsilon_{ij}}{b} - \frac{\sum_{i=1}^a \sum_{j=1}^b \varepsilon_{ij}}{ab}$	τ_i	$\frac{(a-1)\sigma^2}{ab}$
$y_{ij} - \bar{y}_{.j}$	$\tau_i + \varepsilon_{ij} - \frac{\sum_{i=1}^a \varepsilon_{ij}}{a}$	τ_i	$\frac{(a-1)\sigma^2}{a}$
$\bar{y}_{.j} - \bar{y}_{..}$	$\beta_j + \frac{\sum_{i=1}^a \varepsilon_{ij}}{a} - \frac{\sum_{i=1}^a \sum_{j=1}^b \varepsilon_{ij}}{ab}$	β_j	$\frac{(b-1)\sigma^2}{ab}$
R-	**RCBD**	**E**	**Var[]**
$y_{ij} - \bar{y}_{i.}$	$\beta_j - \frac{\sum_{j=1}^b \beta_j}{b} + \varepsilon_{ij} - \frac{\sum_{j=1}^b \varepsilon_{ij}}{b}$	0	$\frac{(b-1)(\sigma_\beta^2+\sigma^2)}{b}$
$\bar{y}_{i.} - \bar{y}_{..}$	$\tau_i + \frac{\sum_{j=1}^b \varepsilon_{ij}}{b} - \frac{\sum_{i=1}^a \sum_{j=1}^b \varepsilon_{ij}}{ab}$	τ_i	$\frac{(a-1)\sigma^2}{ab}$
$y_{ij} - \bar{y}_{.j}$	$\tau_i + \varepsilon_{ij} - \frac{\sum_{i=1}^a \varepsilon_{ij}}{a}$	τ_i	$\frac{(a-1)\sigma^2}{a}$
$\bar{y}_{.j} - \bar{y}_{..}$	$\beta_j - \frac{\sum_{j=1}^b \beta_j}{b} + \frac{\sum_{i=1}^a \varepsilon_{ij}}{a} - \frac{\sum_{i=1}^a \sum_{j=1}^b \varepsilon_{ij}}{ab}$	0	$\frac{(b-1)(a\sigma_\beta^2+\sigma^2}{ab}$

$y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..} = \varepsilon_{ij} - \frac{\sum_{i=1}^a \varepsilon_{ij}}{a} - \frac{\sum_{j=1}^b \varepsilon_{ij}}{b} + \frac{\sum_{i=1}^a \sum_{j=1}^b \varepsilon_{ij}}{ab}, E[] =$
$0, Var = \frac{(a-1)(b-1)\sigma^2}{ab}$

FT-CRD;RT-CRD

$Var[{}_{ij} - \frac{\sum_{j=1}^{n} {}_{ij}}{n}] =, Var[{}_{ij}] + \frac{\sum_{j=1}^{n} Var[{}_{ij}]}{n^2} - \frac{2Cov[{}_{ij},\sum_{j=1}^{n} {}_{ij}]}{n}, = \sigma^2 + \frac{\sigma^2}{n} - \frac{2}{n}\sigma^2 = \frac{n-1}{n}\sigma^2$

FB-RCBD

$Var[\beta_j + {}_{ij} - \frac{\sum_{j=1}^b {}_{ij}}{b}] =, Var[\beta_j] + Var[{}_{ij}] + \frac{\sum_{j=1}^b Var[{}_{ij}]}{b^2} - 2Cov[\beta_j, {}_{ij}] - \frac{2Cov[\beta_j,\sum_{j=1}^b {}_{ij}]}{b} - \frac{2Cov[{}_{ij},\sum_{j=1}^b {}_{ij}]}{b}, = 0 + \sigma^2 + \frac{\sigma^2}{b} - 0 - 0 - \frac{2\sigma^2}{b} = \frac{(b-1)\sigma^2}{b}$

RB-RCBD

$Var[\beta_j - \frac{\sum_{j=1}^b \beta_j}{b} + {}_{ij} - \frac{\sum_{j=1}^b {}_{ij}}{b}] =$

$Var[\beta_j] + \frac{\sum_{j=1}^b Var[\beta_j]}{b^2} + Var[{}_{ij}] + \frac{\sum_{j=1}^b Var[{}_{ij}]}{b^2} - \frac{2Cov[\beta_j,\sum_{j=1}^b \beta_j]}{b} + 2Cov[\beta_j, {}_{ij}] - \frac{2Cov[\beta_j,\sum_{j=1}^b {}_{ij}]}{b} - \frac{2Cov[\sum_{j=1}^b \beta_j, {}_{ij}]}{b}$

$+\frac{2Cov[\sum_{j=1}^b \beta_j,\sum_{j=1}^b {}_{ij}]}{b^2} - \frac{2Cov[{}_{ij},\sum_{j=1}^b {}_{ij}]}{b} = \sigma_\beta^2 + \frac{\sigma_\beta^2}{b} - \frac{2\sigma_\beta^2}{b} + \sigma^2 + \frac{\sigma^2}{b} - \frac{2\sigma^2}{b} = \frac{(\sigma_\beta^2+\sigma^2)(b-1)}{b}$

—

$Var[\bar{y}_{i.} - \bar{y}_{..}] =$

FT-CRD

$Var[\tau_i + \frac{\sum_{j=1}^n {}_{ij}}{n} - \frac{\sum_{i=1}^a\sum_{j=1}^n {}_{ij}}{an}] =$

$Var[\tau_i] + \frac{\sum_{j=1}^n Var[{}_{ij}]}{n^2} + \frac{\sum_{i=1}^a\sum_{j=1}^n Var[{}_{ij}]}{a^2n^2} + \frac{2Cov[\tau_i,\sum_{j=1}^n {}_{ij}]}{n} - \frac{2Cov[\tau_i,\sum_{i=1}^a\sum_{j=1}^n {}_{ij}]}{an} - \frac{2Cov[\sum_{j=1}^n {}_{ij},\sum_{i=1}^a\sum_{j=1}^n {}_{ij}]}{an^2}, = 0 + \frac{\sigma^2}{n} + \frac{\sigma^2}{an} - \frac{2\sigma^2}{an} = \frac{(a-1)\sigma^2}{an}$

RT-CRD

$Var[\tau_i - \frac{\sum_{i=1}^a \tau_i}{a} + \frac{\sum_{j=1}^n {}_{ij}}{n} - \frac{\sum_{i=1}^a\sum_{j=1}^n {}_{ij}}{an}] =$

$Var[\tau_i] + \frac{\sum_{i=1}^a Var[\tau_i]}{a^2} + \frac{\sum_{j=1}^n Var[{}_{ij}]}{n^2} + \frac{\sum_{i=1}^a\sum_{j=1}^n Var[{}_{ij}]}{a^2n^2} - \frac{2Cov[\tau_i,\sum_{i=1}^a \tau_i]}{a} + \frac{2Cov[\tau_i,\sum_{j=1}^n {}_{ij}]}{n} - \frac{2Cov[\tau_i,\sum_{i=1}^a\sum_{j=1}^n {}_{ij}]}{an} - \frac{2Cov[\sum_{i=1}^a \tau_i,\sum_{j=1}^n {}_{ij}]}{an}$

$+\frac{2Cov[\sum_{i=1}^a \tau_i,\sum_{i=1}^a\sum_{j=1}^n {}_{ij}]}{a^2n} - \frac{2Cov[\sum_{j=1}^n {}_{ij},\sum_{i=1}^a\sum_{j=1}^n {}_{ij}]}{an^2} = \sigma_\tau^2 + \frac{\sigma_\tau^2}{a} - \frac{2\sigma_\tau^2}{a} + \frac{\sigma^2}{n} + \frac{\sigma^2}{an} - \frac{2\sigma^2}{an} = \frac{(n\sigma_\tau^2+\sigma^2)(a-1)}{an}$

FB-RCBD;RB-RCBD

$Var[\tau_i + \frac{\sum_{j=1}^b {}_{ij}}{b} - \frac{\sum_{i=1}^a\sum_{j=1}^b {}_{ij}}{ab}] =$

$Var[\tau_i] + \frac{\sum_{j=1}^b Var[{}_{ij}]}{b^2} + \frac{\sum_{i=1}^a\sum_{j=1}^b Var[{}_{ij}]}{a^2b^2} + \frac{2Cov[\tau_i,\sum_{j=1}^b {}_{ij}]}{b} - \frac{2Cov[\tau_i,\sum_{i=1}^a\sum_{j=1}^b {}_{ij}]}{ab} - \frac{2Cov[\sum_{j=1}^b {}_{ij},\sum_{i=1}^a\sum_{j=1}^b {}_{ij}]}{ab^2} = 0 + \frac{\sigma^2}{b} + \frac{\sigma^2}{ab} - \frac{2\sigma^2}{ab} = \frac{(a-1)\sigma^2}{ab}$

—

$Var[y_{ij} - \bar{y}_{.j}] =$

FB-RCBD; RB-RCBD

$Var[\tau_i + {}_{ij} - \frac{\sum_{i=1}^a {}_{ij}}{a}] = Var[\tau_i] + Var[{}_{ij}] + \frac{\sum_{i=1}^a Var[{}_{ij}]}{a^2} - \frac{2Cov[\tau_i, {}_{ij}]}{a} - \frac{2Cov[\tau_i,\sum_{i=1}^a {}_{ij}]}{a} - \frac{2Cov[{}_{ij},\sum_{i=1}^a {}_{ij}]}{a} = 0 + \sigma^2 + \frac{\sigma^2}{a} - \frac{2\sigma^2}{a} = \frac{(a-1)\sigma^2}{a}$

—

$Var[\bar{y}_{.j} - \bar{y}_{..}] =$

FB-RCBD

$Var[\beta_j + \frac{\sum_{i=1}^a {}_{ij}}{a} - \frac{\sum_{i=1}^a\sum_{j=1}^b {}_{ij}}{ab}] =$

$Var[\beta_j] + \frac{\sum_{i=1}^a Var[{}_{ij}]}{a^2} + \frac{\sum_{i=1}^a\sum_{j=1}^b Var[{}_{ij}]}{a^2b^2} + \frac{2Cov[\beta_j,\sum_{i=1}^a {}_{ij}]}{a} - \frac{2Cov[\beta_j,\sum_{i=1}^a\sum_{j=1}^b {}_{ij}]}{ab} - \frac{2Cov[\sum_{i=1}^a {}_{ij},\sum_{i=1}^a\sum_{j=1}^b {}_{ij}]}{a^2b} = 0 + \frac{\sigma^2}{a} + \frac{\sigma^2}{ab} - \frac{2\sigma^2}{ab} = \frac{(b-1)\sigma^2}{ab}$

RB-RCBD

$Var[\beta_j - \frac{\sum_{j=1}^b \beta_j}{b} + \frac{\sum_{i=1}^a {}_{ij}}{a} - \frac{\sum_{i=1}^a\sum_{j=1}^b {}_{ij}}{ab}] =,$

$Var[\beta_j] + \frac{\sum_{j=1}^b Var[\beta_j]}{b^2} + \frac{\sum_{i=1}^a Var[{}_{ij}]}{a^2} + \frac{\sum_{i=1}^a\sum_{j=1}^b Var[{}_{ij}]}{a^2b^2} - \frac{2Cov[\beta_j,\sum_{j=1}^b \beta_j]}{b} + \frac{2Cov[\beta_j,\sum_{i=1}^a {}_{ij}]}{a} - \frac{2Cov[\beta_j,\sum_{i=1}^a\sum_{j=1}^b {}_{ij}]}{ab} - \frac{2Cov[\sum_{j=1}^b \beta_j,\sum_{i=1}^a {}_{ij}]}{ab}$

$+\frac{2Cov[\sum_{j=1}^b \beta_j,\sum_{i=1}^a\sum_{j=1}^b {}_{ij}]}{ab^2} - \frac{2Cov[\sum_{i=1}^a {}_{ij},\sum_{i=1}^a\sum_{j=1}^b {}_{ij}]}{a^2b} = \sigma_\beta^2 + \frac{\sigma_\beta^2}{b} - \frac{2\sigma_\beta^2}{b} + \frac{\sigma^2}{a} + \frac{\sigma^2}{ab} - \frac{2\sigma^2}{ab} = \frac{(a\sigma_\beta^2+\sigma^2)(b-1)}{ab}$

—

$Var[y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..}] =$ FB-RCBD

$Var[{}_{ij} - \frac{1}{a}\sum_{i=1}^a {}_{ij} - \frac{1}{b}\sum_{j=1}^b {}_{ij} + \frac{1}{ab}\sum_{i=1}^a\sum_{j=1}^b {}_{ij}] =$

$Var[{}_{ij}] + \frac{\sum_{i=1}^a Var[{}_{ij}]}{a^2} + \frac{\sum_{j=1}^b Var[{}_{ij}]}{b^2} + \frac{\sum_{i=1}^a\sum_{j=1}^b Var[{}_{ij}]}{a^2b^2} - \frac{2Cov[{}_{ij},\sum_{i=1}^a {}_{ij}]}{a} - \frac{2Cov[{}_{ij},\sum_{j=1}^b {}_{ij}]}{b} + \frac{2Cov[{}_{ij},\sum_{i=1}^a\sum_{j=1}^b {}_{ij}]}{ab} + \frac{2Cov[\sum_{i=1}^a {}_{ij},\sum_{j=1}^b {}_{ij}]}{ab}$

$-\frac{2Cov[\sum_{i=1}^a {}_{ij},\sum_{i=1}^a\sum_{j=1}^b {}_{ij}]}{a^2b} - \frac{2Cov[\sum_{j=1}^b {}_{ij},\sum_{i=1}^a\sum_{j=1}^b {}_{ij}]}{ab^2} = \sigma^2 + \frac{\sigma^2}{a} + \frac{\sigma^2}{b} + \frac{\sigma^2}{ab} - \frac{2\sigma^2}{a} - \frac{2\sigma^2}{b} + \frac{2\sigma^2}{ab} + \frac{2\sigma^2}{ab} - \frac{2\sigma^2}{ab} - \frac{2\sigma^2}{ab} = \frac{(ab-a-b+1)\sigma^2}{ab}$