4.1 4.2 Joint, Marginal, Conditional

$f(x,y)=f_X(x)f_Y(y)$ 4.2.5 — Indep $f(x,y)=g(x)h(y)$ 4.2.7
$F(x,y)=P(X\le x,Y\le y)=\int_{-\infty}^{y}\int_{-\infty}^{x}f(s,t)dsdt$ — Indep $F_{X,Y}(x,y)=F_X(x)F_Y(y)$

$f(x|y)=P(X=x|Y=y)=\frac{f(x,y)}{f_Y(y)}$ 4.2.1
$f_X(x)=\sum_{y\in\mathbf{R}}f_{X,Y}(x,y)=\int_{-\infty}^{\infty}f(x,y)dy$
$M_Z(t)=M_X(t)M_Y(t)$ 4.2.12 — $M_Z(t)=(M_X(t))^n$ 4.6.7 — $M_{\bar X}(t)=[M_X(\frac{t}{n})]^n$ 5.2.7
$M_W(t)=M_X(t)M_Y(t)=e^{\mu_1(e^t-1)}e^{\mu_2(e^t-1)}=e^{(\mu_1+\mu_2)(e^t-1)}$ Use 4.2.12 proof 4.3.2

X,Y indep r.v. $g(X)$,$h(Y)$ indep 4.3.5
$U\sim Geom(p=\frac{1}{2}),u=1,2..$ $V\sim NBin(p=\frac{1}{2},r=2),v=2,3..$ (U,V)
$\{(u,v):u=1,2,..;v=u+1,u+2,..\}$ not indep

Expectations

$E[X]=\int_{-\infty}^{\infty}xf(x)dx=E[E(X|Y)]$ 4.4.3 $E[g(x)]=\sum_{x\in D}h(x)p(x)=\int_{-\infty}^{\infty}g(x)f(x)dx$
$E[(X-\mu)^n]=\mu_n=\sum(x-\mu)^np(x)=\int(x-\mu)^nf(x)dx$ — $E(aX+b)=aE(X)+b$
$E[g_1(X_1)\cdots g_n(X_n))=(E(g_1(X_1)]\cdots(E(g_n(X_n))$ $X_1,..,X_n$ indep 4.6.6
$E[g(\vec X)]=\sum\cdots\sum_{all\vec x}g(\vec x)p(\vec x)=\int\cdots\int_{\mathbf{R}^n}g(\vec x)f(\vec x)d\vec x$ $E[\sum_{i=1}^n g(X_i)]=nE(g(X_1))$ 5.2.5
$E[g(X,Y)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x,y)f(x,y)dxdy$ 4.1.10
$E[XY]=E[X]E[Y]$ $E[g(X)h(Y)]=E[g(x)]E[h(y)]$ $P(X\le x,Y\le y)=P(X\le x)P(Y\le y)$ 4.2.10
$E[g(X)|y]=\sum_x g(x)f(x|y)=\int_{-\infty}^{\infty}g(x)f(x|y)dx$ r.v.(y) 4.2.3

Variances

$V[X]=\sigma_x^2=E(X^2)-[E(X)]^2=E[V(X|Y)]+V[E(X|Y)]$ 4.4.7
$V[X|Y]=E[(X-E[X|Y])^2|Y]=E[X^2|y]-(E[X|y])^2$ 4.2.4
$V[aX\pm bY]=a^2VX+b^2VY\pm 2abCov(X,Y)$ 4.5.6 $V[aX+b]=a^2\sigma^2$ $\sigma_{ax+b}=|a|\sigma_x$
$V[X\pm Y]=VX+VY$ Indep 4.5.6 $V[\sum_{i=1}^n g(X_i)]=nVar(g(X_1))$ 5.25

4.5 Covariance and Correlation

$Cov(X,Y)=E[(X-\mu_X)(Y-\mu_Y)]=E(XY)-E(X)E(Y)=E[XY]-\mu_X\mu_Y=\sigma_{XY}$ 4.5.1/3
$Cov(aX,bY)=abCov(X,Y)$ $Cov(X,Y+Z)=Cov(X,Y)+Cov(X,Z)$ $Cov(X,c)=0$
$\rho_{XY}=\frac{\sigma_{XY}}{\sigma_X\sigma_Y}$ $Corr(X,Y)=\frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$ 4.5.2

4.3 Transform

$f_{U,V}(u,v)=f_{X,Y}(h_1(u,v),h_2(u,v))|J|$ 4.3.2 $=\sum_{i=1}^k f_{X,Y}(h_{1i}(u,v),\ h_{2i}(u,v))|J_i|$ 4.3.5

$\begin{vmatrix}u=g_1(x,y) & x=h_1(u,v)\\ v=g_2(x,y) & y=h_2(u,v)\end{vmatrix}$ $J=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial y}{\partial v}\\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}=\frac{\partial x}{\partial u}\frac{\partial y}{\partial v}-\frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$

$f_Z(z)=\int_{-\infty}^{\infty}f_X(w)f_Y(z-w)dw$ $Z=X+Y$ 5.2.9 Convolution

Distribution

$X\sim n(\mu,\sigma^2)$, $Y\sim n(\gamma,\tau^2)$ — $X+Y\sim n(\mu+\gamma,\sigma^2+\tau^2)$indep 4.2.14
$X,Y\sim n(0,1)$ — $X+Y,X-Y\sim n(0,2)$, $X/Y\sim cauchy(0,1)$ indep 4.3.4/6
$X\sim Poisson(\theta)$, $Y\sim Poisson(\lambda)$ — $X+Y\sim Poisson(\theta+\lambda)$ indep 4.3.2
$X\sim Beta(\alpha,\beta)$, $Y\sim Beta(\alpha+\beta,\gamma)$ — $XY\sim n(\alpha,\beta+\gamma)$ indep 4.3.3
$X|Y\sim Bin(Y,p),Y|\Lambda\sim Pois(\Lambda),\Lambda\sim Expo(\beta)$ or $X|Y\sim Bin(Y,p),Y\sim NBin(1,\frac{1}{1+\beta})$ 4.4.5
$Y|\Lambda\sim Pois(\Lambda),\Lambda\sim Gamma(\alpha,\beta)$ then $Y\sim NBin(\alpha,\frac{1}{1+p\beta})$ Pois-Gamma
$X|P\sim Bin(n,P),P\sim Gamma(\alpha,\beta)$ $EX=E[E(X|P)]=E[nP]=n\frac{\alpha}{\alpha+\beta}$ Beta-Bin 4.4.6
$V[X]=V[E(X|P)]+E[V(X|P)]=\frac{n^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}+\frac{n\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)}=\frac{n\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^2(\alpha+\beta+1)}$ Beta-Bin 4.4.8
$f_X(x)\sim n(\mu_X,\sigma_X^2)$ $f_Y(y)\sim n(\mu_Y,\sigma_Y^2)$, $\rho_{XY}=\rho$ bivarialte normal 4.5.10
$f_{Y|X}(y|x)\sim n\left(\mu_Y+\rho\frac{\sigma_Y}{\sigma_X}(x-\mu_X),\sigma_Y^2(1-\rho^2)\right)$
$aX+bY\sim n(a\mu_X+b\mu_Y,a^2\mu_X^2+b^2\mu_Y^2+2ab\rho\sigma_X\sigma_Y$
$X_1,..X_n\sim Gamma(\alpha,\beta)$, $X_1+..X_n\sim Gamma(\alpha_1+..\alpha_n,\beta)$ $\bar X\sim Gamma(n\alpha,\beta/n)$ indep 4.6.8
$X\sim Cauchy(0,\sigma),Y\sim Cauchy(0,\tau)$ — $X+Y\sim Cauchy(0,\sigma+\tau)$ 5.2.10
$X_1,..,X_n\sim Cauchy(0,\sigma)$ — $\bar X\sim Cauchy(0,\sigma),\sum_1^n X\sim Cauchy(0,n\sigma)$

5.3 Sampling from N

$\bar X=\frac{X_1+..+X_n}{n}=\frac{1}{n}\sum_{i=1}^n X_i$ 5.2.2 $\bar X_{n+1}=\frac{\sum_{i=1}^{n+1}X_i}{n+1}=\frac{\sum_{i=1}^n X_i+X_{n+1}}{n+1}=\frac{n\bar X_n+X_{n+1}}{n+1}$ 5.3.1
$S^2=\frac{1}{n-1}\sum_{i=1}^n(X_i-\bar X)^2=\frac{1}{n-1}\sum_{i=1}^n(X_i^2-n\bar X^2)$ 5.2.3 $nS_{n+1}^2=(n-1)S_n^2+(\frac{n}{n+1})(X_{n+1}-\bar X_n)^2$

5.2.6 $X_1,..,X_n$ iid	$E[\bar X]=\mu$	$E[S^2]=\sigma^2$	$V[\bar X]=\frac{\sigma^2}{n})$	W/o Normal
5.3.1 $\sim n(\mu,\sigma^2)$	$\bar X,S^2$ indep	$\bar X\sim N(\mu,\frac{\sigma^2}{n})$	$\frac{(n-1)S^2}{\sigma^2}\sim\chi^2_{n-1}$	W Normal

$X\sim n(\mu,\sigma^2)$, $\frac{x-\mu}{\sigma}$, $\frac{\bar x-\mu}{\sigma/\sqrt n}\sim n(0,1)$, $\frac{\bar X-\mu}{S/\sqrt n}=\frac{\frac{\bar x-\mu}{\sigma/\sqrt n}}{\sqrt{S^2/\sigma^2}}=\frac{U}{\sqrt{\frac{\chi^2_{n-1}}{n-1}}}\sim t_{n-1}$ 5.3.4 — $t_1=Cauchy(0,1)$

$x_i\sim n(0,1)$, $\sum_{i=1}^n x_i^2\sim\chi_n^2$ — $x_i\sim n(0,\sigma^2)$, $\sum_{i=1}^n x_i^2\sim\sigma^2\chi_n^2$, $\sum_{i=1}^n(x_i-\bar x)^2\sim\sigma^2\chi^2_{n-1}$
$\chi_2^2\Leftrightarrow Expo(2)$ $\chi_p^2\sim Gamma(\frac{p}{2},2)$ 4.6.8
$X_1,..X_n\sim\chi^2_{p_i}$, $X_1+..X_n\sim\chi^2_{p_1+..p_n}$ 5.3.2 $U\sim\chi_m^2$,$V\sim\chi_n^2$, $U+V\sim\chi^2_{m+n}$
$X_i\sim n(\mu_X,\sigma_X^2)Y_j\sim n(\mu_Y,\sigma_Y^2)$, $X_1..X_n,Y_1..Y_m$ indep,$\frac{S_X^2}{\sigma_X^2}\sim\chi^2$ — $\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2}\sim F$ 5.3.5

$X\sim F_{p,q}$, $\frac{1}{X}\sim F_{q,p}$ —$X\sim T_q$, $X^2\sim F_{1,q}$ — $X\sim F_{p,q}$, $\frac{\frac{p}{q}X}{1+\frac{p}{q}X}\sim Beta(\frac{p}{2},\frac{q}{2})$

$Var\chi^2_{n-1}=Var[\frac{(n-1)S^2}{\sigma^2}]=\frac{(n-1)^2}{\sigma^4}Var[S^2]=2(n-1)\implies Var[S^2]=\frac{2\sigma^4}{n-1}$

5.4 Order statistics

5.4.4 $f_K(x)=\frac{n!}{(j-1)!(n-j)!}K[F_X(x)]^{k-1}[1-F_X(x)]^{n-k}f(x)$ 1-29p9

5.4.6 $f_{X_{(1)},..,X_{(n)}}(x_1,..,x_n)=\begin{cases}n!f_X(x_1)\cdot\ldots\cdot f_X(x_n) & -\infty<x_1<..<x_n<\infty\\ 0 & \text{otherwise}\end{cases}$

$f_{X_{(i)},X_{(j)}}(u,v)=\frac{n!}{(i-1)!(j-1-i)!(n-j)!}f_X(u)f_X(v)[F_X(u)]^{i-1}[F_X(v)-F_X(u)]^{j-1-i}[1-F_X(v)]^{n-j}$

1 from N to T to Chi to F

Given some function of these, find the distribution.

2 Transformation of pairs of r.v.s

Given 2 r.v.s, f(x,y), and a function of them, find its distribution. 1-10p1

$f(x,y)=\frac{1}{4}e^{-\frac{x+y}{2}},0<x<\infty,0<y<\infty,u=\frac{X-Y}{2}$

1\. $\begin{vmatrix}U=\frac{x-y}{2} & X=2u+v\\ V=y & Y=v\end{vmatrix}$ 2. $J=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}=\begin{vmatrix}2 & 1\\ 0 & 1\end{vmatrix}=2$

3\. $g(u,v)=f(x,y)|J|=\frac{1}{4}e^{-\frac{x+y}{2}}2=\frac{1}{2}e^{-\frac{2u+v+v}{2}}=\frac{1}{2}e^{-(u+v)}$
4\. $0<x<\infty,0<y<\infty\implies 0<2u+v<\infty,0<v<\infty\implies v>-2u$
5\. Double Exponential(Laplace)

$g_U(u)=\frac{1}{2}e^{|u|}$ $\begin{vmatrix}\int_{-2u}^{\infty}\frac{1}{2}e^{-(u+v)}dv=\frac{1}{2}e^{-u}\int_{-2u}^{\infty}e^{-v}dv=\frac{1}{2}e^{-u}\left[-e^{-v}\right]_{-2u}^{\infty}=\frac{1}{2}e^{-u}\left[0+e^{2u}\right] & u<0\\ \int_0^{\infty}\frac{1}{2}e^{-(u+v)}dv=\frac{1}{2}e^{-u}\int_0^{\infty}e^{-v}dv=\frac{1}{2}e^{-u}\left[-e^{-v}\right]_0^{\infty}=\frac{1}{2}e^{-u}\left[0+1\right] & u\ge 0\end{vmatrix}$

3

Given $f(x,y)$, find the
$f_X(x)=$ — $E[X]=$ — $V[X]=$
$f_Y(y)=$ — $E[Y]=$ — $V[Y]=$
$f(x,y)=$ — $E[XY]=$
$Cov(x,y)=E[XY]-EXEY=$
$f(X|Y)=$ — $E[X|Y]=$ — $V[X|Y]=$
$\rho=Cov(x,y)/\sqrt{V[X]V[Y]}=$
4.5.7 $E[Y|X]=a+bx$, $E[Y]=E[E[Y|X]]=E[a+bX]=a+bE[X]$ by 4.4.3,
$E[XE[Y|X]]=E[X(a+bX)]=aE[X]+bE[X^2]$, $E[XE[Y|X]]=\int_{-\infty}^{\infty}xE[Y|x]f_X(x)dx$ by 2.2.1,
$=\int_{-\infty}^{\infty}x\left[\int_{-\infty}^{\infty}yf_Y(y|x)dy\right]f_X(x)dx$ by 4.2.3,$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}xyf(x,y)dydx=E[XY]$ by 4.1.10,
$\sigma_{XY}=E[XY]-\mu_X\mu_Y=a\mu_X+bE[X^2]-\mu_X\mu_Y=a\mu_X+b(\sigma_X^2+\mu_X^2)-\mu_X(a+b\mu_X)=b\sigma_X^2$
$\rho_{XY}=\frac{\sigma_{XY}}{\sigma_X\sigma_Y}=\frac{b\sigma_X^2}{\sigma_X\sigma_Y}=b\frac{\sigma_X}{\sigma_Y}$

4 Order statistics

Find the distribution of X(k) or $X_{(j)},X_{(k)}$
joint pmf 1-31p2-3
5.4.5 uniform order pdf 1-31p4-8

pdf/pmf

CDF: $F(x) = P(X \leq x) = \sum_{y:y\leq x} p(y) = \int_{-\infty}^{x} f(y)dy$

probabilities: $a \leq b$:

PMF: $p(x) = P(X = x) = P(\forall w \in \mathcal{W} : X(w) = x)$

$P(a \leq X \leq b) = F(b) - F(a^-)$; $P(a < X \leq b) = F(b) - F(a)$; $P(a \leq X \leq a) = p(a)$;

$P(a < X < b) = F(b^-) - F(a)$; (where a^- is the largest possible X value strictly less than a); Taking $a = b$ yields $P(X = a) = F(a) - F(a-1)$ as desired.

PDF: $P(\forall w \in \mathcal{W} : a \leq X(w) \leq b) = \int_a^b f(x)dx$

$P(a \leq X \leq b) = P(a < X \leq b) = P(a < X < b)$; $P(X > a) = 1 - F(a)$; $P(a \leq X \leq b) = F(b) - F(a)$

CDF

Condition:$f(x) \geq 0 \forall x$ pmf $\sum_x f_X(x) = 1, pdf \int_{-\infty}^{\infty} f(x)dx = 1$

mgf

$E(e^{tx}) = \int e^{tx} f(x)dx = \sum e^{tx} f(x)$; $M_{aX+b}(t) = e^{tb} M_{aX}(t)$

$M_X(t) = E(e^{tx})$	$M_X(0) = 1$
$M'_X(t) = E(xe^{tx})$	$M'_X(0) = E(X)$
$M''_X(t) = E(x^2e^{tx})$	$M''_X(0) = E(X^2)$
$M^n_X(t) = E(x^ne^{tx})$	$M^n_X(0) = E(X^n)$

transform

$g(x) \uparrow$	$F_Y(y) = F_X(g^{-1}(y))$
$g(x) \downarrow$	$F_Y(y) = 1 - F_X(g^{-1}(y))$

not monotone $\because X \leq 0\ is\ \emptyset \therefore P(X \leq -\sqrt{y}) = 0$

$F_Y(y) = P(Y \leq y) = P(X^2 \leq y) = P(-\sqrt{y} \leq X \leq \sqrt{y}) = P(X \leq \sqrt{y}) - P(X \leq -\sqrt{y}) = F_X(\sqrt{y}) = \sqrt{y},\ 0 < \sqrt{y} < 1$

monotone:$f_Y(y) = f_X(g^{-1}(y))|\frac{d(g^{-1}(y))}{dy}|$

Series

Finite	Binomial
$\sum_{k=1}^n k = \frac{n(n+1)}{2}$	$\sum_{k=0}^n \binom{n}{k} = 2^n$
$\sum_{k=1}^n (2k-1) = n^2$	$\sum_{k=0}^n \binom{r+k}{k} = \binom{r+n+1}{n}$
$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{k=0}^n \binom{k}{m} = \binom{n+1}{m+1}$
$\sum_{k=1}^n k^3 = (\frac{n(n+1)}{2})^2$	$\sum_{k=0}^r \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
$\sum_{k=0}^n c^k = \frac{c^{n+1}-1}{c-1}$	$\sum_{k=0}^n \binom{n}{k} a^{n-k}b^k = (a+b)^n$

Infinite $\lvert p\rvert < 1$	$\sum_{k=0}^{\infty} kp^{k-1} = \frac{d}{dp}\left(\sum_{k=0}^{\infty} p^k\right) = \frac{1}{(1-p)^2}$
$\sum_{k=0}^{\infty} p^k = \frac{1}{1-p}$	$\sum_{k=0}^{\infty} \binom{r+k-1}{k} x^k = (1-x)^{-r} r \in \mathbb{N}^+$
$\sum_{k=1}^{\infty} p^k = \frac{p}{1-p}$	$\sum_{k=0}^{\infty} \binom{\alpha}{k} p^k = (1+p)^{\alpha} \quad \lvert p\rvert < 1 ,\ \alpha \in \mathbb{C}$

$\Gamma(n) = (n-1)!$	$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$
$\Gamma(a+1) = a\Gamma(a)$	$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \lvert r\rvert < 1$
$\Gamma(1/2) = \sqrt{\pi}$	$\sum_{k=0}^{n-1} ar^n = \frac{a(1-r^n)}{1-r}$
$\Gamma(0) = \Gamma(-1) = \infty$	$\int_0^1 x^{a-1}(1-x)^{b-1}, dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$
$\Gamma(-1/2) = -2\Gamma(1/2)$	$\int_0^{\infty} x^{t-1}e^{-x}dx = \Gamma(t)$

Integrals+c

$\int kdx = kx$	$\int_a^b cdx = c(b-a)$	$\lvert\int_a^b f(x)dx\rvert \leq \int_a^b \lvert f(x)\rvert dx$
$\int e^u du = e^u$	$\int lnudu = uln(u) - u$	$\int \frac{1}{a^2+u^2}du = \frac{1}{a}tan^{-1}(\frac{u}{a})$
$\int x^n = \frac{x^{n+1}}{n+1}$	$\int \frac{1}{ax+b} = \frac{1}{a}ln\lvert ax+b\rvert$	$\int \frac{1}{\sqrt{a^2-u^2}}du = \frac{1}{sin(\frac{u}{a})}$
$\int \ln a a^x = a^x$		

Substitution	$u = g(x)$	$du = g'(x)dx$	$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$
	x^3	$du = 3x^2dx$	$\int_1^2 5x^2cos(x^3)dx = \int_1^8 \frac{5}{3}cos(u)du$

	u	du	dv	v	$\int_a^b udv$
Integreation by parts	x	dx	e^{-x}	$-e^{-x}$	$\int xe^{-x}dx$
	lnx	$\frac{1}{x}dx$	dx	x	$\int_3^5 xlnxdx$

$= uv|_a^b - \int_a^b vdu$

$= -xe^{-x} + \int xe^{-x}dx = -xe^{-x} - e^{-x} + c$

$= xlnx|_3^5 - \int_3^5 dx = (xlnx - x)|_3^5 = 5ln5 - 3ln3 - 2$

$\int_a^b f(x)dx = F(b) - F(a) = -\int_b^a f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx;$

$\frac{d}{dx}\int_{v(x)}^{u(x)} f(t)dt = u'(x)f[u(x)] - v'(x)f[v(x)]$

Derivatives

$(cf)' = cf'(x)$	$(fg)' = f'g + fg'$	$(f \pm g)' = f'(x) \pm g'(x)$
$\frac{dx}{dx} = 1$	$(\frac{f}{g})' = \frac{(f'g-fg')}{g^2}$	$(f(g(x)))' = f'(g(x))g'(x)$
$\frac{de^x}{dx} = e^x$	$\frac{d(x^n)}{dx} = nx^{n-1}$	$\frac{d\ln(x)}{dx} = \frac{1}{x}, x > 0$
$\frac{da^x}{dx} = a^xln(a)$	$\frac{d\tan^{-1}x}{dx} = \frac{1}{1+x^2}$	$\frac{d\log_a(x)}{dx} = \frac{1}{xlna}, x > 0$
$\frac{d\cos x}{dx} = -\sin x$	$\frac{d\tan x}{dx} = \sec^2 x$	$\frac{dx(\ln x - 1)}{dx} = \ln x$

Distribution	**CDF**	**P(X=x),f(x)**	μ	EX^2	**Var**	**MGF**	**M'(t)**	**M"(t)**	$M^n(t)$
Bern(p)		$p^x q^{1-x}, x \in \{1,0\}$	p	p	pq	$pe^t + q$			
Bin(n,p)	$I_{1-p}(n-x, x+1)$	$\binom{n}{x} p^x q^{n-x}; x \in \{0, 1..n\}$	np	$\mu(\mu+q)$	$q\mu$	$(pe^t+q)^n$			
Geom(p)	$1-q^x$	$pq^{x-1}, x \in 1, 2, ..$	$\frac{1}{p}$	$\frac{p+2q}{p^2}$	$\frac{q}{p^2}$	$\frac{pe^t}{1-qe^t}, t < -\ln q$			
	$1-q^{x+1}$	$pq^x, x \in 0, 1, ..$	$\frac{q}{p}$	$\frac{q^2+q}{p^2}$	$\frac{q}{p^2}$	$\frac{p}{1-qe^t}, qe^t < 1$	$\frac{pqe^t}{(1-qe^t)^2}$	$\frac{2pqe^t}{(1-qe^t)^3} - M'(t)$	
NBin(r,p)		$\binom{x-1}{r-1} p^r q^{x-r}, x \in r, r+1..$	$\frac{r}{p}$		$\frac{rq}{p^2}$	$(\frac{pe^t}{1-qe^t})^r$			
		$\binom{x+r-1}{r-1} p^r q^x, x \in 0, 1..$	$\frac{rq}{p}$		$\frac{rq}{p^2}$	$(\frac{p}{1-qe^t})^r, qe^t < 1$			
HGeom(N,m,k)		$\frac{\binom{m}{x}\binom{N-m}{k-x}}{\binom{N}{k}}$	$\frac{km}{N}$		$\mu \frac{(N-m)(N-k)}{N(N-1)}$				
HGeom(w,b,k)		$\frac{\binom{w}{x}\binom{b}{k-x}}{\binom{w+b}{k}}$	$\frac{kw}{w+b}$		$\mu \frac{b(w+b-k)}{(w+b)(w+b-1)}$				
Pois(μ)	$e^{-\mu} \sum_{i=0}^{x} \frac{\mu^i}{i!}$	$\frac{\mu^x}{x!} e^{-\mu}, x \in 0, 1..$	μ	$\mu^2 + \mu$	μ	$e^{\mu(e^t-1)}$	$\mu e^t M(t)$	$\mu e^t (1 + \mu e^t) M(t)$	
Unif(n)		$\frac{1}{n}, x \in 1, 2..n$	$\frac{n+1}{2}$	$\frac{(n+1)(2n+1)}{6}$	$\frac{(n^2-1)}{12}$	$\frac{\sum_{i=1}^{n} e^{ti}}{n}$			
Unif(a,b)	$\frac{x-a}{b-a}$	$\frac{1}{b-a}, x \in (a,b)$	$\frac{a+b}{2}$		$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$			
$\mathcal{N}(\mu, \sigma^2)$		$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	$\mu^2 + \sigma^2$	σ^2	$e^{\mu t + \frac{\sigma^2 t^2}{2}}$	$(\mu + \sigma^2 t) M(t)$	$[(\mu + \sigma^2 t)^2 + \sigma^2] M(t)$	
$\mathcal{N}(0,1)$		$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$	0	1	1	$e^{\frac{t^2}{2}}$			
$\mathcal{LN}(\mu, \sigma^2)$		$\frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(\ln x - \mu)^2}{2\sigma^2}}$	$e^{\mu + \frac{\sigma^2}{2}}$	$e^{2\mu + 2\sigma^2}$	$\theta^2 (e^{\sigma^2} - 1)$	$\times$			
$Cauchy(\theta, \sigma^2)$		$\frac{1}{\pi\sigma} \frac{1}{1+(\frac{x-\theta}{\sigma})^2}$	$\times$	$\times$	$\times$				
DExpo(μ, σ^2)		$\frac{1}{2\pi\sigma} e^{-\lvert \frac{x-\mu}{\sigma} \rvert}$	μ	$\mu^2 + 2\sigma^2$	$2\sigma^2$	$\frac{e^{\mu t}}{1-\sigma^2 t^2}$			
Expo(λ)	$1 - e^{-\lambda x}$	$\lambda e^{-\lambda x}, x \in (0, \infty)$	$\frac{1}{\lambda}$		$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}, t < \lambda$			
Expo(β)		$\frac{1}{\beta} e^{-\frac{x}{\beta}}$	β		β^2	$\frac{1}{1-\beta t}$	$\beta(1-\beta t)^{-2}$	$2\beta^2 (1-\beta t)^{-3}$	
Gamma(a, λ)		$\frac{\lambda^a}{\Gamma(a)} x^{a-1} e^{-\lambda x}, x \in (0, \infty)$	$\frac{a}{\lambda}$	$\frac{a}{\lambda^2}$	$\left(\frac{\lambda}{\lambda - t}\right)^a, t < \lambda$				
Gamma(α, β)		$\frac{1}{\Gamma(a)\beta^\alpha} x^{a-1} e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$\left(\frac{1}{1-\beta t}\right)^a, t < \frac{1}{\beta}$				
Beta(a,b)		$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}$	$\frac{a}{a+b}$		$\frac{\mu(1-\mu)}{(a+b+1)}$				$\frac{\Gamma(\alpha+n)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+n)\Gamma(\alpha)}$
$B(\alpha, \beta) =$		$\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, x \in (0,1)$		$\frac{a(a+1)}{(a+b)(a+b+1)}$	$\frac{ab}{(a+b)^2(a+b+1)}$				
χ_p^2		$\frac{x^{\frac{p}{2}-1}}{\Gamma \frac{p}{2} 2^{\frac{p}{2}}} e^{-\frac{x}{2}}$	p	$2p + p^2$	$2p$	$(1-2t)^{-p/2}, t < \frac{1}{2}$			
t_p		$\frac{\Gamma(\frac{p+1}{2})}{\Gamma(\frac{p}{2})\sqrt{p\pi}} \left(1 + \frac{x^2}{p}\right)^{-\frac{p+1}{2}}$	$0, p > 1$		$\frac{p}{p-2}, p > 2$	$\times$			
F	$x > 0$	$\frac{\Gamma(\frac{p+q}{2})}{\Gamma(\frac{p}{2})\Gamma(\frac{q}{2})} (\frac{p}{q})^{\frac{p}{2}} \frac{x^{\frac{p}{2}-1}}{(1+\frac{p}{q}x)^{\frac{p+q}{2}}}$	$\frac{q}{q-2}$	$q > 2$	$2(\frac{q}{q-2})^2 \frac{p+q-2}{p(q-4)}$	$q > 4$			
Arcsine	$\frac{1}{\pi \arcsin \sqrt{x}}$	$\frac{1}{\pi\sqrt{x(1-x)}}, x \in [0,1]$	$\frac{1}{2}$		$\frac{1}{8}$				Beta$(\frac{1}{2}, \frac{1}{2})$
Dirichlet	$B(a) = \frac{\Pi_{i=1}^{k} \Gamma(a_i)}{\Gamma(\sum_{i=1}^{k} a_i)}$	$\frac{1}{B(a)} \Pi_{i=1}^{k} x_i^{a_i - 1}, x \in (0,1)$	$\frac{a_i}{\sum_k a_k}$	$\sum_{i=1}^{k} x_i = 1$	$\frac{a_i(a_0 - a_i)}{a_0^2(a_0+1)}$	$Cov(X_i, X_j) =$	$\frac{-a_i a_j}{a_0^2(a_0+1)}$	$a_0 = \sum_{i=1}^{k} a_i$	

$U \sim Geom(p = \frac{1}{2}), u = 1, 2..$ the number of trials needed to get the first head.

$V \sim NBin(p = \frac{1}{2}, r = 2), v = 2, 3..$ the number of trials needed to get two heads in repeated tosses of a fair coin.