4.1 4.2 Joint, Marginal, Conditional

$f_X(x) = \sum_{y \in \mathbf{R}} f_{X,Y}(x,y) = \int_{-\infty}^{\infty} f(x,y)dy$ 4.1.6/10 $f(x|y) = P(X=x|Y=y) = \frac{f(x,y)}{f_Y(y)}$ 4.2.1
Indep $f(x,y) = f_X(x)f_Y(y)$ 4.2.5 $= g(x)h(y)$ 4.2.7 X,Y indep r.v. $g(X)$,$h(Y)$ indep 4.3.5
$F(x,y) = P(X \le x, Y \le y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f(s,t)dsdt$ — Indep $F_{X,Y}(x,y) = F_X(x)F_Y(y)$
$M_Z(t) = M_X(t)M_Y(t)$ 4.2.12 — $M_Z(t) = (M_X(t))^n$ 4.6.7 — $M_{\bar{X}}(t) = [M_X(\frac{t}{n})]^n$ 5.2.7
$M_W(t) = M_X(t)M_Y(t) = e^{\mu_1(e^t-1)}e^{\mu_2(e^t-1)} = e^{(\mu_1+\mu_2)(e^t-1)}$ Use 4.2.12 proof 4.3.2
$U \sim Geom(\frac{1}{2}), u = 1,2..$ $V \sim NBin(2, \frac{1}{2}), v = 2,3..$ $\{(u,v) : u = 1,2,..; v = u+1, u+2,..\}$ not indep

Expectations

$E[X] = \int_{-\infty}^{\infty} xf(x)dx = E[E(X|Y)]$ 4.4.3 $E[g(x)] = \sum_{x \in D} h(x)p(x) = \int_{-\infty}^{\infty} g(x)f(x)dx$
$E[(X-\mu)^n] = \mu_n = \sum (x-\mu)^n p(x) = \int (x-\mu)^n f(x)dx$ — $E(aX+b) = aE(X)+b$
$E[g(\vec{X})] = \sum \cdots \sum_{all \vec{x}} g(\vec{x})p(\vec{x}) = \int \cdots \int_{\mathbf{R^n}} g(\vec{x})f(\vec{x})d\vec{x}$
$E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f(x,y)dxdy$ 4.1.10
$E[g(X)|y] = \sum_x g(x)f(x|y) = \int_{-\infty}^{\infty} g(x)f(x|y)dx$ r.v.(y) 4.2.3 $E[g(X)h(Y)|y] = h(Y)E[g(X)|y]$
$E[XY] = E[X]E[Y] = E[g(x)]E[h(y)]$ $P(X \le x, Y \le y) = P(X \le x)P(Y \le y)$ indep 4.2.10
$E[g_1(X_1)\cdots g_n(X_n)] = E[g_1(X_1)]\cdots E[g_n(X_n)]$ $X_1,..,X_n$ indep 4.6.6 $E[g(X)|y] = E[g(X)]$ indep

Variances

$V[X] = \sigma_x^2 = E(X^2) - [E(X)]^2 = E[V(X|Y)] + V[E(X|Y)]$ 4.4.7 — $V[aX+b] = a^2\sigma^2$
$V[X|Y] = E[(X - E[X|Y])^2|Y] = E[X^2|y] - (E[X|y])^2$ 4.2.4 — $\sigma_{ax+b} = |a|\sigma_x$
$V[aX \pm bY] = a^2VX + b^2VY \pm 2abCov(X,Y)$ — $V[X \pm Y] = VX + VY$ Indep 4.5.6
$E[\sum_{i=1}^n g(X_i)] = nE(g(X_1))$ — $V[\sum_{i=1}^n g(X_i)] = nVar(g(X_1))$ r.s. 5.25

4.5 Covariance and Correlation

$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = E(XY) - E(X)E(Y) = E[XY] - \mu_X\mu_Y = \sigma_{XY}$ 4.5.1/3
$Cov(aX, bY) = abCov(X,Y)$ $Cov(X, Y+Z) = Cov(X,Y) + Cov(X,Z)$ $Cov(X,c) = 0$
$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y}$ $Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$ 4.5.2

4.3 Transform

$f_{U,V}(u,v) = f_{X,Y}(h_1(u,v), h_2(u,v))|J|$ 4.3.2 $= \sum_{i=1}^k f_{X,Y}(h_{1i}(u,v),\ h_{2i}(u,v))|J_i|$ 4.3.5

$\begin{vmatrix} u = g_1(x,y) & x = h_1(u,v) \\ v = g_2(x,y) & y = h_2(u,v) \end{vmatrix}$ $J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial v} \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$

$f_Z(z) = \int_{-\infty}^{\infty} f_X(w)f_Y(z-w)dw$ $Z = X+Y$ 5.2.9 Convolution

Distribution

$X \sim n(\mu,\sigma^2)$, $Y \sim n(\gamma,\tau^2)$ — $X+Y \sim n(\mu+\gamma, \sigma^2+\tau^2)$indep 4.2.14
$X,Y \sim n(0,1)$ — $X+Y, X-Y \sim n(0,2)$, $X/Y \sim cauchy(0,1)$ indep 4.3.4/6
$X \sim Poisson(\theta)$, $Y \sim Poisson(\lambda)$ — $X+Y \sim Poisson(\theta+\lambda)$ indep 4.3.2
$X \sim Beta(\alpha,\beta)$, $Y \sim Beta(\alpha+\beta,\gamma)$ — $XY \sim n(\alpha, \beta+\gamma)$ indep 4.3.3
$X|Y \sim Bin(Y,p), Y|\Lambda \sim Pois(\Lambda), \Lambda \sim Expo(\beta)$ or $X|Y \sim Bin(Y,p), Y \sim NBin(1, \frac{1}{1+\beta})$ 4.4.5
$Y|\Lambda \sim Pois(\Lambda), \Lambda \sim Gamma(\alpha,\beta)$ then $Y \sim NBin(\alpha, \frac{1}{1+p\beta})$ Pois-Gamma
$X|P \sim Bin(n,P), P \sim Gamma(\alpha,\beta)$ $EX = E[E(X|P)] = E[nP] = n\frac{\alpha}{\alpha+\beta}$ Beta-Bin 4.4.6
$V[X] = V[E(X|P)] + E[V(X|P)] = \frac{n^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} + \frac{n\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)} = \frac{n\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^2(\alpha+\beta+1)}$ Beta-Bin 4.4.8
bivarialte normal$f_X(x) \sim n(\mu_X, \sigma_X^2)$ — $f_{X|Y}(x|y) \sim n\left(\mu_X + \rho\frac{\sigma_X}{\sigma_Y}(x-\mu_Y), \sigma_X^2(1-\rho^2)\right)$
4.5.10 $\rho_{XY} = \rho$ $f_Y(y) \sim n(\mu_Y, \sigma_Y^2)$ — $aX+bY \sim n(a\mu_X + b\mu_Y, a^2\mu_X^2 + b^2\mu_Y^2 + 2ab\rho\sigma_X\sigma_Y$
$X_1,..X_n \sim Gamma(\alpha,\beta)$, $X_1+..X_n \sim Gamma(\alpha_1+..\alpha_n, \beta)$ $\bar{X} \sim Gamma(n\alpha, \beta/n)$ indep 4.6.8
$X \sim Cauchy(0,\sigma), Y \sim Cauchy(0,\tau)$ — $X+Y \sim Cauchy(0, \sigma+\tau)$ 5.2.10
$X_1,..,X_n \sim Cauchy(0,\sigma)$ — $\bar{X} \sim Cauchy(0,\sigma), \sum_1^n X \sim Cauchy(0, n\sigma)$

5.3 Sampling from N

$\bar{X} = \frac{X_1+..+X_n}{n} = \frac{1}{n}\sum_{i=1}^n X_i$ 5.2.2 $\bar{X}_{n+1} = \frac{\sum_{i=1}^{n+1} X_i}{n+1} = \frac{\sum_{i=1}^n X_i + X_{n+1}}{n+1} = \frac{n\bar{X}_n + X_{n+1}}{n+1}$ 5.3.1
$S^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i^2 - n\bar{X}^2)$ 5.2.3 $nS_{n+1}^2 = (n-1)S_n^2 + (\frac{n}{n+1})(X_{n+1} - \bar{X}_n)^2$

5.2.6	$X_1,..,X_n$ iid	$E[\bar{X}] = \mu$	$E[S^2] = \sigma^2$	$V[\bar{X}] = \frac{\sigma^2}{n})$	W/o Normal
5.3.1	$\sim n(\mu,\sigma^2)$	$\bar{X}, S^2$ indep	$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$	$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$	W Normal

$X \sim n(\mu,\sigma^2)$, $\frac{x-\mu}{\sigma}, \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim n(0,1)$, $\frac{\bar{X}-\mu}{S/\sqrt{n}} = \frac{\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}}{\sqrt{S^2/\sigma^2}} = \frac{U}{\sqrt{\frac{\chi_{n-1}^2}{n-1}}} \sim t_{n-1}$ 5.3.4 — $t_1 = Cauchy(0,1)$
$x_i \sim n(0,1)$, $\sum_{i=1}^n x_i^2 \sim \chi_n^2$ — $x_i \sim n(0,\sigma^2)$, $\sum_{i=1}^n x_i^2 \sim \sigma^2\chi_n^2$, $\sum_{i=1}^n (x_i - \bar{x})^2 \sim \sigma^2\chi_{n-1}^2$
$\chi_2^2 \Leftrightarrow Expo(2)$ $\chi_p^2 \sim Gamma(\frac{p}{2}, 2)$ 4.6.8

$X_1,..X_n \sim \chi_{p_i}^2$, $X_1+..X_n \sim \chi_{p_1+..p_n}^2$ 5.3.2 $U \sim \chi_m^2, V \sim \chi_n^2$, $U+V \sim \chi_{m+n}^2$
$X_i \sim n(\mu_X, \sigma_X^2) Y_j \sim n(\mu_Y, \sigma_Y^2)$, $X_1..X_n, Y_1..Y_m$ indep,$\frac{S_X^2}{\sigma_X^2} \sim \chi^2$ — $\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \sim F$ 5.3.5
$X \sim F_{p,q}$, $\frac{1}{X} \sim F_{q,p}$ —$X \sim T_q$, $X^2 \sim F_{1,q}$ — $X \sim F_{p,q}$, $\frac{\frac{p}{q}X}{1+\frac{p}{q}X} \sim Beta(\frac{p}{2}, \frac{q}{2})$
$V\chi_{n-1}^2 = V[\frac{(n-1)S^2}{\sigma^2}] = \frac{(n-1)^2}{\sigma^4}Var[S^2] = 2(n-1) \implies Var[S^2] = \frac{2\sigma^4}{n-1}$

5.4 Order statistics

5.4.4 $f_K(x) = K\binom{n}{k}[F_X(x)]^{k-1}[1-F_X(x)]^{n-k}f(x)$ or $\frac{n!}{(k-1)!(n-k)!}$ 1-29p9

5.4.6 $f_{X_{(1)},..,X_{(n)}}(x_1,..,x_n) = \begin{cases} n!f_X(x_1)\cdot\ldots\cdot f_X(x_n) & -\infty < x_1 < .. < x_n < \infty \\ 0 & \text{otherwise} \end{cases}$

$f_{X_{(i)},X_{(j)}}(u,v) = \frac{n!}{(i-1)!(j-1-i)!(n-j)!}f_X(u)f_X(v)[F_X(u)]^{i-1}[F_X(v) - F_X(u)]^{j-1-i}[1-F_X(v)]^{n-j}$

1 from N to T to Chi to F

Given some function of these, find the distribution.

2 Transformation of pairs of r.v.s

Given 2 r.v.s, f(x,y), and a function of them, find its distribution. 1-10p1

$f(x,y) = \frac{1}{4}e^{-\frac{x+y}{2}}, 0 < x < \infty, 0 < y < \infty$,$u = \frac{X-Y}{2}$

1. $\begin{vmatrix} U = \frac{x-y}{2} & X = 2u+v \\ V = y & Y = v \end{vmatrix}$ 2. $J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2$
3. $g(u,v) = f(x,y)|J| = \frac{1}{4}e^{-\frac{x+y}{2}}2 = \frac{1}{2}e^{-\frac{2u+v+v}{2}} = \frac{1}{2}e^{-(u+v)}$
4. $0 < x < \infty, 0 < y < \infty \implies 0 < 2u+v < \infty, 0 < v < \infty \implies v > -2u$
5. Double Exponential(Laplace)

$g_U(u) = \frac{1}{2}e^{|u|}$ $\left| \begin{array}{ll} \int_{-2u}^{\infty} \frac{1}{2}e^{-(u+v)}dv = \frac{1}{2}e^{-u}\int_{-2u}^{\infty} e^{-v}dv = \frac{1}{2}e^{-u}\left[-e^{-v}\right]_{-2u}^{\infty} = \frac{1}{2}e^{-u}\left[0 + e^{2u}\right] & u < 0 \\ \int_0^{\infty} \frac{1}{2}e^{-(u+v)}dv = \frac{1}{2}e^{-u}\int_0^{\infty} e^{-v}dv = \frac{1}{2}e^{-u}\left[-e^{-v}\right]_0^{\infty} = \frac{1}{2}e^{-u}\left[0+1\right] & u \ge 0 \end{array} \right.$

3

Given $f(x,y)$, find the
$f_X(x) =$ — $E[X] =$ — $V[X] =$
$f_Y(y) =$ — $E[Y] =$ — $V[Y] =$
$f(x,y) =$ — $E[XY] =$
$Cov(x,y) = E[XY] - EXEY =$
$f(X|Y) =$ — $E[X|Y] =$ — $V[X|Y] =$
$\rho = Cov(x,y)/\sqrt{V[X]V[Y]} =$
4.5.7 $E[Y|X] = a + bx$, $E[Y] = E[E[Y|X]] = E[a+bX] = a + bE[X]$ by 4.4.3,
$E[XE[Y|X]] = E[X(a+bX)] = aE[X] + bE[X^2]$, $E[XE[Y|X]] = \int_{-\infty}^{\infty} xE[Y|x]f_X(x)dx$ by 2.2.1,
$= \int_{-\infty}^{\infty} x\left[\int_{-\infty}^{\infty} yf_Y(y|x)dy\right]f_X(x)dx$ by 4.2.3,$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf(x,y)dydx = E[XY]$ by 4.1.10,
$\sigma_{XY} = E[XY] - \mu_X\mu_Y = a\mu_X + bE[X^2] - \mu_X\mu_Y = a\mu_X + b(\sigma_X^2 + \mu_X^2) - \mu_X(a + b\mu_X) = b\sigma_X^2$
$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y} = \frac{b\sigma_X^2}{\sigma_X\sigma_Y} = b\frac{\sigma_X}{\sigma_Y}$

4 Order statistics

Find the distribution of X(k) or $X_{(j)}, X_{(k)}$
joint pmf 1-31p2-3
5.4.5 uniform order pdf 1-31p4-8

pdf/pmf

CDF: $F(x) = P(X \le x) = \sum_{y:y \le x} p(y) = \int_{-\infty}^{x} f(y)dy$
probabilities: $a \le b$:
PMF: $p(x) = P(X = x) = P(\forall w \in \mathcal{W} : X(w) = x)$
$P(a \le X \le b) = F(b) - F(a^-)$; $P(a < X \le b) = F(b) - F(a)$; $P(a \le X \le a) = p(a)$;
$P(a < X < b) = F(b^-) - F(a)$; (where a^- is the largest possible X value strictly less than a); Taking $a = b$ yields $P(X = a) = F(a) - F(a-1)$ as desired.
PDF: $P(\forall w \in \mathcal{W} : a \le X(w) \le b) = \int_a^b f(x)dx$
$P(a \le X \le b) = P(a < X \le b) = P(a < X < b)$; $P(X > a) = 1 - F(a)$; $P(a \le X \le b) = F(b) - F(a)$

CDF

Condition:$f(x) \ge 0 \forall x$ pmf $\sum_x f_X(x) = 1$, pdf $\int_{-\infty}^{\infty} f(x)dx = 1$

mgf

$E(e^{tx}) = \int e^{tx} f(x)dx = \sum e^{tx} f(x)$; $M_{aX+b}(t) = e^{tb} M_{aX}(t)$

$M_X(t) = E(e^{tx})$	$M_X(0) = 1$
$M'_X(t) = E(xe^{tx})$	$M'_X(0) = E(X)$
$M''_X(t) = E(x^2e^{tx})$	$M''_X(0) = E(X^2)$
$M^n_X(t) = E(x^ne^{tx})$	$M^n_X(0) = E(X^n)$

transform

$g(x) \uparrow$	$F_Y(y) = F_X(g^{-1}(y))$
$g(x) \downarrow$	$F_Y(y) = 1 - F_X(g^{-1}(y))$

not monotone $\because X \leq 0 \; is \; \emptyset \therefore P(X \leq -\sqrt{y}) = 0$

$F_Y(y) = P(Y \leq y) = P(X^2 \leq y) = P(-\sqrt{y} \leq X \leq \sqrt{y}) = P(X \leq \sqrt{y}) - P(X \leq -\sqrt{y}) = F_X(\sqrt{y}) = \sqrt{y},\ 0 < \sqrt{y} < 1$

monotone:$f_Y(y) = f_X(g^{-1}(y))|\frac{d(g^{-1}(y))}{dy}|$

Series

Finite	Binomial
$\sum_{k=1}^n k = \frac{n(n+1)}{2}$	$\sum_{k=0}^n \binom{n}{k} = 2^n$
$\sum_{k=1}^n (2k-1) = n^2$	$\sum_{k=0}^n \binom{r+k}{k} = \binom{r+n+1}{n}$
$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{k=0}^n \binom{k}{m} = \binom{n+1}{m+1}$
$\sum_{k=1}^n k^3 = (\frac{n(n+1)}{2})^2$	$\sum_{k=0}^r \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
$\sum_{k=0}^n c^k = \frac{c^{n+1}-1}{c-1}$	$\sum_{k=0}^n \binom{n}{k} a^{n-k} b^k = (a+b)^n$

Infinite $\|p\| < 1$	$\sum_{k=0}^\infty kp^{k-1} = \frac{d}{dp}\left(\sum_{k=0}^\infty p^k\right) = \frac{1}{(1-p)^2}$
$\sum_{k=0}^\infty p^k = \frac{1}{1-p}$	$\sum_{k=0}^\infty \binom{r+k-1}{k} x^k = (1-x)^{-r} r \in \mathbb{N}^+$
$\sum_{k=1}^\infty p^k = \frac{p}{1-p}$	$\sum_{k=0}^\infty \binom{\alpha}{k} p^k = (1+p)^\alpha \quad \|p\| < 1\,,\ \alpha \in \mathbb{C}$

$\Gamma(n) = (n-1)!$	$\sum_{n=0}^\infty \frac{x^n}{n!} = e^x$
$\Gamma(a+1) = a\Gamma(a)$	$\sum_{n=0}^\infty ar^n = \frac{a}{1-r}, \|r\| < 1$
$\Gamma(1/2) = \sqrt{\pi}$	$\sum_{k=0}^{n-1} ar^n = \frac{a(1-r^n)}{1-r}$
$\Gamma(0) = \Gamma(-1) = \infty$	$\int_0^1 x^{a-1}(1-x)^{b-1}, dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$
$\Gamma(-1/2) = -2\Gamma(1/2)$	$\int_0^\infty x^{t-1} e^{-x} dx = \Gamma(t)$

Integrals+c

$\int kdx = kx$	$\int_a^b cdx = c(b-a)$	$\|\int_a^b f(x)dx\| \leq \int_a^b \|f(x)\|dx$
$\int e^u du = e^u$	$\int lnudu = uln(u) - u$	$\int \frac{1}{a^2+u^2} du = \frac{1}{a} tan^{-1}(\frac{u}{a})$
$\int x^n = \frac{x^{n+1}}{n+1}$	$\int \frac{1}{ax+b} = \frac{1}{a} ln\|ax+b\|$	$\int \frac{1}{\sqrt{a^2-u^2}} du = \frac{1}{sin(\frac{u}{a})}$
$\int \ln a a^x = a^x$		

Substitution	$u = g(x)$	$du = g'(x)dx$	$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$
	x^3	$du = 3x^2dx$	$\int_1^2 5x^2cos(x^3)dx = \int_1^8 \frac{5}{3} cos(u)du$

Integreation by parts	u	du	dv	v	$\int_a^b udv$
	x	dx	e^{-x}	$-e^{-x}$	$\int xe^{-x}dx$
	lnx	$\frac{1}{x}dx$	dx	x	$\int_3^5 xlnxdx$

$= uv|_a^b - \int_a^b vdu$

$= -xe^{-x} + \int xe^{-x}dx = -xe^{-x} - e^{-x} + c$

$= xlnx|_3^5 - \int_3^5 dx = (xlnx - x)|_3^5 = 5ln5 - 3ln3 - 2$

$\int_a^b f(x)dx = F(b) - F(a) = -\int_b^a f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx;$

$\frac{d}{dx}\int_{v(x)}^{u(x)} f(t)dt = u'(x)f[u(x)] - v'(x)f[v(x)]$

Derivatives

$(cf)' = cf'(x)$	$(fg)' = f'g + fg'$	$(f \pm g)' = f'(x) \pm g'(x)$
$\frac{dx}{dx} = 1$	$(\frac{f}{g})' = \frac{(f'g - fg')}{g^2}$	$(f(g(x)))' = f'(g(x))g'(x)$
$\frac{de^x}{dx} = e^x$	$\frac{d(x^n)}{dx} = nx^{n-1}$	$\frac{d\ln(x)}{dx} = \frac{1}{x}, x > 0$
$\frac{da^x}{dx} = a^x ln(a)$	$\frac{d\tan^{-1}x}{dx} = \frac{1}{1+x^2}$	$\frac{d\log_a(x)}{dx} = \frac{1}{xlna}, x > 0$
$\frac{d\cos x}{dx} = -\sin x$	$\frac{d\tan x}{dx} = \sec^2 x$	$\frac{dx(\ln x - 1)}{dx} = \ln x$

	Replace	No Replace
Fixed # trials (n)	Binomial (Bern if $n = 1$)	HGeom
Draw until r success	NBin (Geom if $r = 1$)	NHGeom

$U \sim Geom(\frac{1}{2}), u = 1, 2..$ # of trials needed to get the first head.

$V \sim NBin(2, \frac{1}{2}r), v = 2, 3..$ # of trials needed to get two heads in repeated tosses of a fair coin.

Distribution	CDF	P(X=x),f(x)	μ	EX^2	Var	MGF	M'(t)	M''(t)	$M^n(t)$
Bern(p)		$p^x q^{1-x}, x \in \{1,0\}$	p	p	pq	$pe^t + q$			
Bin(n,p)	$I_{1-p}(n-x, x+1)$	$\binom{n}{x} p^x q^{n-x}; x \in \{0,1..n\}$	np	$\mu(\mu+q)$	μq	$(pe^t+q)^n$			
Geom(p)	$1-q^x$	$pq^{x-1}, x \in 1,2,..$	$\frac{1}{p}$	$\frac{p+2q}{p^2}$	$\frac{q}{p^2}$	$\frac{pe^t}{1-qe^t}, t < -\ln q$			
	$1-q^{x+1}$	$pq^x, x \in 0,1,..$	$\frac{q}{p}$	$\frac{q^2+q}{p^2}$	$\frac{q}{p^2}$	$\frac{p}{1-qe^t}, qe^t < 1$	$\frac{pqe^t}{(1-qe^t)^2}$	$\frac{2pqe^t}{(1-qe^t)^3} - M'(t)$	
NBin(r,p)		$\binom{x-1}{r-1} p^r q^{x-r}, x \in r, r+1..$	$\frac{r}{p}$		$\frac{rq}{p^2}$	$(\frac{pe^t}{1-qe^t})^r$			
		$\binom{x+r-1}{r-1} p^r q^x, x \in 0,1..$	$\frac{rq}{p}$		$\frac{rq}{p^2}$	$(\frac{p}{1-qe^t})^r, qe^t < 1$			
HGeom(N,m,k)		$\frac{\binom{m}{x}\binom{N-m}{k-x}}{\binom{N}{k}}$	$\frac{km}{N}$		$\mu\frac{(N-m)(N-k)}{N(N-1)}$				
HGeom(w,b,k)		$\frac{\binom{w}{x}\binom{b}{k-x}}{\binom{w+b}{k}}$	$\frac{kw}{w+b}$		$\mu\frac{b(w+b-k)}{(w+b)(w+b-1)}$				
Pois(μ)	$e^{-\mu}\sum_{i=0}^{x}\frac{\mu^i}{i!}$	$\frac{\mu^x}{x!}e^{-\mu}, x \in 0,1..$	μ	$\mu^2+\mu$	μ	$e^{\mu(e^t-1)}$	$\mu e^t M(t)$	$\mu e^t(1+\mu e^t)M(t)$	
Unif(n)		$\frac{1}{n}, x \in 1,2..n$	$\frac{n+1}{2}$	$\frac{(n+1)(2n+1)}{6}$	$\frac{(n^2-1)}{12}$	$\frac{\sum_{i=1}^{n} e^{ti}}{n}$			
Unif(a,b)	$\frac{x-a}{b-a}$	$\frac{1}{b-a}, x \in (a,b)$	$\frac{a+b}{2}$		$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$			
$\mathcal{N}(\mu,\sigma^2)$		$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	$\mu^2+\sigma^2$	σ^2	$e^{\mu t+\frac{\sigma^2 t^2}{2}}$	$(\mu+\sigma^2 t)M(t)$	$[(\mu+\sigma^2 t)^2+\sigma^2]M(t)$	
$\mathcal{N}(0,1)$		$\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$	0	1	1	$e^{\frac{t^2}{2}}$			
$\mathcal{LN}(\mu,\sigma^2)$		$\frac{1}{x\sigma\sqrt{2\pi}}e^{\frac{-(\ln x-\mu)^2}{2\sigma^2}}$	$e^{\mu+\frac{\sigma^2}{2}}$	$e^{2\mu+2\sigma^2}$	$\theta^2(e^{\sigma^2}-1)$	$\times$			
$Cauchy(\theta,\sigma^2)$		$\frac{1}{\pi\sigma}\frac{1}{1+(\frac{x-\theta}{\sigma})^2}$	$\times$	$\times$	$\times$				
DExpo(μ,σ^2)		$\frac{1}{2\pi\sigma}e^{-\lvert\frac{x-\mu}{\sigma}\rvert}$	μ	$\mu^2+2\sigma^2$	$2\sigma^2$	$\frac{e^{\mu t}}{1-\sigma^2 t^2}$			
Expo(λ)	$1-e^{-\lambda x}$	$\lambda e^{-\lambda x}, x \in (0,\infty)$	$\frac{1}{\lambda}$		$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}, t < \lambda$			
Expo(β)		$\frac{1}{\beta}e^{-\frac{x}{\beta}}$	β		β^2	$\frac{1}{1-\beta t}$	$\beta(1-\beta t)^{-2}$	$2\beta^2(1-\beta t)^{-3}$	
Gamma(a,λ)		$\frac{\lambda^a}{\Gamma(a)}x^{a-1}e^{-\lambda x}, x \in (0,\infty)$	$\frac{a}{\lambda}$	$\frac{a}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-t}\right)^a, t < \lambda$				
Gamma(α,β)		$\frac{1}{\Gamma(a)\beta^\alpha}x^{a-1}e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$\left(\frac{1}{1-\beta t}\right)^a, t < \frac{1}{\beta}$				
Beta(a,b)		$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$	$\frac{a}{a+b}$		$\frac{\mu(1-\mu)}{(a+b+1)}$				$\frac{\Gamma(\alpha+n)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+n)\Gamma(\alpha)}$
$B(\alpha,\beta) =$		$\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, x \in (0,1)$		$\frac{a(a+1)}{(a+b)(a+b+1)}$	$\frac{ab}{(a+b)^2(a+b+1)}$				
χ^2_p		$\frac{x^{\frac{p}{2}-1}}{\Gamma\frac{p}{2}2^{\frac{p}{2}}}e^{-\frac{x}{2}}$	p	$2p+p^2$	$2p$	$(1-2t)^{-p/2}, t < \frac{1}{2}$			
t_p		$\frac{\Gamma(\frac{p+1}{2})}{\Gamma(\frac{p}{2})\sqrt{p\pi}}\left(1+\frac{x^2}{p}\right)^{-\frac{p+1}{2}}$	$0, p > 1$		$\frac{p}{p-2}, p > 2$	$\times$			
F	$x > 0$	$\frac{\Gamma(\frac{p+q}{2})}{\Gamma(\frac{p}{2})\Gamma(\frac{q}{2})}(\frac{p}{q})^{\frac{p}{2}}\frac{x^{\frac{p}{2}-1}}{(1+\frac{p}{q}x)^{\frac{p+q}{2}}}$	$\frac{q}{q-2}$	$q > 2$	$2(\frac{q}{q-2})^2\frac{p+q-2}{p(q-4)}$	$q > 4$			
Arcsine	$\frac{1}{\pi\arcsin\sqrt{x}}$	$\frac{1}{\pi\sqrt{x(1-x)}}, x \in [0,1]$	$\frac{1}{2}$		$\frac{1}{8}$				Beta($\frac{1}{2},\frac{1}{2}$)
Dirichlet	$B(a) = \frac{\Pi_{i=1}^{k}\Gamma(a_i)}{\Gamma(\sum_{i=1}^{k}a_i)}$	$\frac{1}{B(a)}\Pi_{i=1}^{k}x_i^{a_i-1}, x \in (0,1)$	$\frac{a_i}{\sum_k a_k}$	$\sum_{i=1}^{k}x_i = 1$	$\frac{a_i(a_0-a_i)}{a_0^2(a_0+1)}$	$Cov(X_i,X_j) =$	$\frac{-a_i a_j}{a_0^2(a_0+1)}$	$a_0 = \sum_{i=1}^{k}a_i$	