565 mid-term

model

response variable=parameters(coefficients).predictor+random error; 5 Assumption: linear relationship between x,y; $E(\varepsilon)=0, Var(\varepsilon)=\sigma^2$;homoscedasticity;$Cov(\varepsilon_i,\varepsilon_j)=0 for i\neq j$ $y_i=\beta_0+\beta_1x_i+\varepsilon_i$; $\varepsilon_i \sim^{iid} N(0,\sigma^2)$ $i=1,2..n$;

least square

$y_{ij}=\mu+\tau_i+\varepsilon_{ij}$, for $_{ij}\sim iidN(0,\sigma^2), i=1,2,,a, j=1,2,,n$, $\sum_{i=1}^a \tau_i=0$ $SSE=\sum_{i=1}^a\sum_{j=1}^n(y_{ij}-\mu-\tau_i)^2$,or $\sum_{i=1}^a\sum_{j=1}^n(y_{ij}-\mu-\tau_i-\beta_j)^2$,

$\frac{\partial SSE}{\partial\mu}\Big|_{\hat{\mu},\hat{\tau}_i}=2\sum_{i=1}^a\sum_{j=1}^n(y_{ij}-\hat{\mu}-\hat{\tau}_i)(-1)=0$,$y_{..}=an\hat{\mu}+n\sum_{i=1}^a\hat{\tau}_i$ $\frac{\partial SSE}{\partial\tau_i}\Big|_{\hat{\mu},\hat{\tau}_i}=2\sum_{j=1}^n(y_{ij}-\hat{\mu}-\hat{\tau}_i)(-1)=0$,$y_{i.}=n\hat{\mu}+n\hat{\tau}_i$

$\frac{\partial SSE}{\partial\mu}\Big|_{\hat{\mu},\hat{\tau}_i,\hat{\beta}_j}=2\sum_{i=1}^a\sum_{j=1}^b(y_{ij}-\hat{\mu}-\hat{\tau}_i-\hat{\beta}_j)(-1)=0$,$y_{..}=ab\hat{\mu}+b\sum_{i=1}^a\hat{\tau}_i+a\sum_{j=1}^b\hat{\beta}_j$

$\frac{\partial SSE}{\partial\tau_i}\Big|_{\hat{\mu},\hat{\tau}_i,\hat{\beta}_j}=2\sum_{j=1}^b(y_{ij}-\hat{\mu}-\hat{\tau}_i-\hat{\beta}_j)(-1)=0$,$y_{i.}=b\hat{\mu}+b\hat{\tau}_i+\sum_{j=1}^b\hat{\beta}_j$ $\frac{\partial SSE}{\partial\beta_j}\Big|_{\hat{\mu},\hat{\tau}_i,\hat{\beta}_j}=2\sum_{i=1}^a(y_{ij}-\hat{\mu}-\hat{\tau}_i-\hat{\beta}_j)(-1)=0$,$y_{.j}=a\hat{\mu}+\sum_{i=1}^a\hat{\tau}_i+a\hat{\beta}_j$

normal equation

$$\hat{\mu}=\frac{1}{an}y_{..}-\frac{1}{a}\sum_{i=1}^a\hat{\tau}_i=\bar{y}_{..}$$

$$\hat{\tau}_i=\frac{1}{a}y_{i.}-\hat{\mu}=\bar{y}_{i.}-\bar{y}_{..}$$

hypothesis test

model	H0	H1
FT-CRD	$\mu_1=\mu_2=..=\mu_i$	at least two of μ_i are different
RT-CRD	$\sigma_\tau^2=0$	$\sigma_\tau^2>0$
FB-RCBD	$\mu_1=\mu_2=..=\mu_i$	at least two of μ_i are different
-Block	$\beta_1=\beta_2=..=\beta_j=0$	at least one of $\beta_j\neq 0$
RB-RCBD	$\sigma_\beta^2=0$	$\sigma_\beta^2>0$

ANOVA

	SS	df	MS	F
SS_{Trt}	$n\sum_{i=1}^a(\bar{y}_{i.}\bar{y}_{..})^2=\frac{1}{n}\sum_{i=1}^a y_{i.}^2-\frac{y_{..}^2}{N}$	$a-1$	$\frac{SS_{Trt}}{a-1}$	$\frac{MS_{Trt}}{MS_E}$
SS_E	$\sum_{i=1}^a\sum_{j=1}^n(y_{ij}\bar{y}_{i.})^2$	$N-a=a(n-1)$	$\frac{SS_E}{N-a}$	
SS_{Trt}	$b\sum_{i=1}^a(\bar{y}_{i.}\bar{y}_{..})^2=\frac{1}{b}\sum_{i=1}^a y_{i.}^2-\frac{y_{..}^2}{N}$	$a-1$	$\frac{SS_{Trt}}{a-1}$	$\frac{MS_{Trt}}{MS_E}$
SS_{Blk}	$a\sum_{j=1}^b(\bar{y}_{.j}\bar{y}_{..})^2=\frac{1}{b}\sum_{j=1}^b y_{.j}^2-\frac{y_{..}^2}{N}$	$b-1$	$\frac{SS_{Trt}}{b-1}$	$\frac{MS_{Blk}}{MS_E}$
SS_E	$\sum_{i=1}^a\sum_{j=1}^b(y_{ij}\bar{y}_{i.})^2$	$(a-1)(b-1)$	$\frac{SS_E}{df_E}$	
SS_T	$\sum_{i=1}^a\sum_{j=1}^b(y_{ij}\bar{y}_{..})^2=\sum_{i=1}^a\sum_{j=1}^b y_{ij}^2-\frac{y_{..}^2}{N}$	$N-1=ab-1$		

CI

CI	balanced	unbalanced
μ_i	$\bar{y}_i\pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n}}$	$\bar{y}_i\pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n_i}}$
$\mu_i-\mu_j$	$\bar{y}_{i.}-\bar{y}_{j.}\pm t_{\frac{\alpha}{2}}\sqrt{\frac{2MS_E}{n}}$	$\bar{y}_{i.}-\bar{y}_{j.}\pm t_{\frac{\alpha}{2}}\sqrt{MS_E(\frac{1}{n_i}+\frac{1}{n_j})}$

expected value

	y_{ij}	$\bar{y}_{..}$	$\bar{y}_{i.}$	$\bar{y}_{.j}$	MS_{Trt}	MS_{Blk}	MS_E
E[f.]	μ	μ	$\mu+\tau_i$	$\mu+\beta_j$	$\sigma^2+\frac{n}{a-1}\sum_{i=1}^a\tau_i^2$		σ^2
V[f.]	σ^2	$\frac{\sigma^2}{na}$	$\frac{\sigma^2}{n}$				
$E[f.^2]$	$\mu^2+\sigma^2$	$\mu^2+\frac{\sigma^2}{na}$	$\mu_i^2+\frac{\sigma^2}{n}$				
E[r.]	μ	μ	$\mu+\tau_i$		$\sigma^2+\frac{b}{a-1}\sum_{i=1}^a\tau_i^2$	$\sigma^2+\frac{a}{b-1}\sum_{j=1}^b\beta_j^2$	
V[r.]	$\sigma^2+\sigma_\tau^2$	$\frac{\sigma_\tau^2}{a}+\frac{\sigma^2}{na}$	$\frac{\sigma^2}{n}+\sigma_\tau^2$				
$E[r.^2]$	$\mu^2+\sigma^2+\sigma_\tau^2$	$\mu^2+\frac{\sigma_\tau^2}{a}+\frac{\sigma^2}{na}$	$\mu_i^2+\frac{\sigma^2}{n}+\sigma_\tau^2$				

$\hat{\mu}_i$	$\hat{\tau}_i$	$\hat{\beta}_j$	$\hat{y}_{ij}$	$_{ij}$
$\bar{y}_{..}$	$\bar{y}_{i.}-\bar{y}_{..}$	$\bar{y}_{.j}-\bar{y}_{..}$	$\bar{y}_{i.}+\bar{y}_{.j}-\bar{y}_{..}$	$\bar{y}_{ij}-\bar{y}_{..}$

$\hat{\sigma}^2$	$\hat{\sigma}^2_{Trt}$	$\hat{\sigma}^2_{Blk}$	$\widehat{Var(\bar{y}_{i.})}$	$\hat{X}$
MSE	$\frac{MS_{Trt}-MS_E}{n}$	$\frac{MS_{Blk}-MS_E}{a}$	$\frac{MS_{Trt}}{n}$	$\frac{ay'_{i.}+by'_{.j}-y'_{..}}{(a-1)(b-1)}$

covariance

distribution, assumption

contrast