565 mid-term

model

$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$, for $\varepsilon_{ij} \sim iidN(0, \sigma^2), i = 1, 2, , a, j = 1, 2, , n$, $\sum_{i=1}^{a} \tau_i = 0$
Random effects: $\tau_i \sim iidN(0, \sigma_\tau^2)$,$\tau_i$ and ε_{ij} indep
$y_{ij} = (\mu + \tau_i + \beta_j) + \varepsilon_{ij}$, for $\varepsilon_{ij} \sim iidN(0, \sigma^2), i = 1, 2, , a, j = 1, 2, , b$
Fixed effects: $\sum_{i=1}^{a} \tau_i = 0, \sum_{j=1}^{b} \beta_j = 0$ Random effects: $\beta_j \sim iidN(0, \sigma_\beta^2)$

least square and normal equation

CRD:$SSE = \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \mu - \tau_i)^2$

$\frac{\partial SSE}{\partial \mu}\Big|_{\hat\mu,\hat\tau_i} = 2\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \hat\mu - \hat\tau_i)(-1) = 0$ $y_{..} = an\hat\mu + n\sum_{i=1}^{a}\hat\tau_i$

$\frac{\partial SSE}{\partial \tau_i}\Big|_{\hat\mu,\hat\tau_i} = 2\sum_{j=1}^{n}(y_{ij} - \hat\mu - \hat\tau_i)(-1) = 0$ $y_{i.} = n\hat\mu + n\hat\tau_i$

RCBD: $SSE = \sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij} - \mu - \tau_i - \beta_j)^2$

$\frac{\partial SSE}{\partial \mu}\Big|_{\hat\mu,\hat\tau_i,\hat\beta_j} = 2\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij} - \hat\mu - \hat\tau_i - \hat\beta_j)(-1) = 0$

$\frac{\partial SSE}{\partial \tau_i}\Big|_{\hat\mu,\hat\tau_i,\hat\beta_j} = 2\sum_{j=1}^{b}(y_{ij} - \hat\mu - \hat\tau_i - \hat\beta_j)(-1) = 0$

$\frac{\partial SSE}{\partial \beta_j}\Big|_{\hat\mu,\hat\tau_i,\hat\beta_j} = 2\sum_{i=1}^{a}(y_{ij} - \hat\mu - \hat\tau_i - \hat\beta_j)(-1) = 0$

$y_{..} = ab\hat\mu + b\sum_{i=1}^{a}\hat\tau_i + a\sum_{j=1}^{b}\hat\beta_j$
$y_{i.} = b\hat\mu + b\hat\tau_i + \sum_{j=1}^{b}\hat\beta_j$
$y_{.j} = a\hat\mu + \sum_{i=1}^{a}\hat\tau_i + a\hat\beta_j$

hypothesis test

model	H0	H1
FT-CRD	$\mu_1 = \mu_2 = .. = \mu_i$	at least two of μ_i are different
RT-CRD	$\sigma_\tau^2 = 0$	$\sigma_\tau^2 > 0$
FB-RCBD	$\mu_1 = \mu_2 = .. = \mu_i$	at least two of μ_i are different
-Block	$\beta_1 = \beta_2 = .. = \beta_j = 0$	at least one of $\beta_j \neq 0$
RB-RCBD	$\sigma_\beta^2 = 0$	$\sigma_\beta^2 > 0$
Contrast	$\Gamma = \sum_{i=1}^{a} c_i\mu_i = 0$	$\Gamma = \sum_{i=1}^{a} c_i\mu_i \neq 0$

ANOVA

	SS	df	MS	F
SS_{Trt}	$n\sum_{i=1}^{a}(\bar y_{i.} - \bar y_{..})^2$	$a-1$	$\frac{SS_{Trt}}{a-1}$	$\frac{MS_{Trt}}{MS_E}$
SS_E	$\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar y_{i.})^2$	$a(n-1)$	$\frac{SS_E}{N-a}$	
SS_T	$\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar y_{..})^2$	$an-1$		
SS_{Trt}	$b\sum_{i=1}^{a}(\bar y_{i.} - \bar y_{..})^2$	$a-1$	$\frac{SS_{Trt}}{a-1}$	$\frac{MS_{Trt}}{MS_E}$
SS_{Blk}	$a\sum_{j=1}^{b}(\bar y_{.j} - \bar y_{..})^2$	$b-1$	$\frac{SS_{Blk}}{b-1}$	$\frac{MS_{Blk}}{MS_E}$
SS_E	$SS_T - SS_{Trt} - SS_{Blk}$	$(a-1)(b-1)$	$\frac{SS_E}{df_E}$	
SS_T	$\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij} - \bar y_{..})^2$	$ab-1$		

[f.]$SS_{Trt} = \frac{1}{n}\sum_{i=1}^{a} y_{i.}^2 - \frac{y_{..}^2}{N}$
[r.]$SS_{Trt} = \frac{1}{b}\sum_{i=1}^{a} y_{i.}^2 - \frac{y_{..}^2}{N}$ $SS_{Blk} = \frac{1}{a}\sum_{j=1}^{b} y_{.j}^2 - \frac{y_{..}^2}{N}$
$SSE = \sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij} - \bar y_{i.} - \bar y_{.j} + \bar y_{..})^2$
$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}^2 - \frac{y_{..}^2}{N}$, $S_i^2 = \frac{\sum_{j=1}^{n}(y_{ij} - \bar y_{i.})^2}{n-1}$

distribution

	y_{ij}	$\bar y_{i.}$	$\frac{y_{i.} - \mu_i}{\frac{\sigma}{\sqrt n}}$	$\bar y_{i.} - \bar y_{j.}$	$\frac{\bar y_{i.} - \bar y_{j.} - (\mu_i - \mu_j)}{\frac{2\sigma}{\sqrt n}}$
N	$(\mu + \tau_i, \sigma^2)$	$(\mu + \tau_i, \frac{\sigma^2}{n})$	$(0, 1)$	$(\mu_i - \mu_j, \frac{2\sigma^2}{n})$	$(0, 1)$

t_{df_E}	$\frac{\frac{y_{i.} - \mu_i}{\frac{\sigma}{\sqrt n}}}{\sqrt{\frac{SS_E}{\sigma^2(N-a)}}} = \frac{y_{i.} - \mu_i}{\sqrt{\frac{SS_E}{n(df_E)}}} = \frac{y_{i.} - \mu_i}{\sqrt{\frac{MS_E}{n}}}$	$\frac{\bar y_{i.} - \bar y_{j.} - (\mu_i - \mu_j)}{\sqrt{\frac{2MS_E}{n}}}$	$\frac{C}{\sqrt{\widehat{Var(C)}}}$

χ^2	$\frac{SS_E}{\sigma^2}$	$\frac{SS_{Trt}}{\sigma^2}$	$\frac{SS_{Blk}}{\sigma^2}$	SS_C
df	E	Trt	Blk	1

F	$\frac{MS_{Trt}}{MS_E}$	$\frac{(MS_{Blk}}{MS_E}$	$\frac{(\sum_{i=1}^{a} c_i\bar y_{i.})^2}{\frac{MS_E}{n}\sum_{i=1}^{a} c_i^2}$
df	(Trt, E)	(Blk, E)	$(C = 1, E)$

contrast

$\Gamma = \sum_{i=1}^{a} c_i\mu_i$,$C = \sum_{i=1}^{a} c_i\bar y_i$;$\sum_{i=1}^{a} c_i = 0$;Orthogonal $\sum_{i=1}^{a} c_i d_i = 0$
$SS_C = \frac{(\sum_{i=1}^{a} c_i\bar y_i)^2}{\frac{1}{n}\sum_{i=1}^{a} c_i^2}$,$\sum SS_C = SS_{Trt}$

CI

CI	balanced	unbalanced
μ_i	$\bar y_{i.} \pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n}}$	$\bar y_{i.} \pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n_i}}$
$\mu_i - \mu_j$	$\bar y_{i.} - \bar y_{j.} \pm t_{\frac{\alpha}{2}}\sqrt{\frac{2MS_E}{n}}$	$\bar y_{i.} - \bar y_{j.} \pm t_{\frac{\alpha}{2}}\sqrt{MS_E(\frac{1}{n_i} + \frac{1}{n_j})}$
Γ	$\sum_{i=1}^{a} c_i\bar y_i \pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n}\sum_{i=1}^{a} c_i^2}$	

expectation and variance

$y_{..} = \sum_{i=1}^{a}(y_{i.} = \sum_{j=1}^{n} y_{ij})$,$\mu = \frac{\sum_{i=1}^{a}\mu_i}{a}$
$\bar y_{..} = \frac{1}{N}\left[y_{..} = \sum_{i=1}^{a}(y_{i.} = \sum_{j=1}^{n} y_{ij})\right] = \frac{1}{a}\sum_{i=1}^{a}\left[\bar y_{i.} = \frac{1}{n}(y_{i.} = \sum_{j=1}^{n} y_{ij})\right]$

	y_{ij}	$\bar y_{..}$ $\frac{\sum_{i=1}^{a}\sum_{j=1}^{n} y_{ij}}{N}$	$\bar y_{i.}$ $\frac{\sum_{j=1}^{n} y_{ij}}{n}$	$\bar y_{.j}$ $\frac{\sum_{i=1}^{a} y_{ij}}{a}$	C $\sum_{i=1}^{a} c_i\bar y_{i.}$
E[f.]	$\mu + \tau_i + \beta_j$	μ	$\mu + \tau_i$	$\mu + \beta_j$	$\sum_{i=1}^{a} c_i\mu_i$
V[f.]	σ^2	$\frac{\sigma^2}{na}$	$\frac{\sigma^2}{n}$	$\frac{\sigma^2}{a}$	$\frac{\sigma^2}{n}\sum_{i=1}^{a} c_i^2$
E[r.]	μ	μ	μ	μ	
V[r.]	$\sigma^2 + \sigma_\tau^2$	$\frac{\sigma_\tau^2}{a} + \frac{\sigma^2}{na}$	$\frac{\sigma^2}{n} + \sigma_\tau^2$	$\frac{\sigma^2}{a} + \sigma_\beta^2$	

	MS_E	MS_{Trt}	MS_{Trt}	MS_{Blk}
E[f.]	σ^2	$\sigma^2 + \frac{n}{a-1}\sum_{i=1}^{a}\tau_i^2$	$\sigma^2 + \frac{b}{a-1}\sum_{i=1}^{a}\tau_i^2$	$\sigma^2 + \frac{a}{b-1}\sum_{j=1}^{b}\beta_j^2$
E[r.]	σ^2	$\sigma^2 + n\tau_i^2$	$\sigma^2 + b\tau_i^2$	$\sigma^2 + a\beta_j^2$

	$\hat\sigma^2$	$\hat\sigma_{Trt}^2$	$\hat\sigma_{Blk}^2$	$\widehat{Var(\bar y_{i.})}$	$\widehat{Var(\bar y_{.j})}$
f.	MSE			$\frac{MS_E}{n}$	$\frac{MS_E}{a}$
r.	MSE	$\frac{MS_{Trt} - MS_E}{n}$	$\frac{MS_{Blk} - MS_E}{a}$	$\frac{MS_{Trt}}{n}$	$\frac{MS_{Trt}}{a}$

$\hat\mu$	$\hat\mu_i$	$\hat\tau_i$	$\hat\beta_j$	$\hat y_{ij}$	$\hat\varepsilon_{ij}$
$\bar y_{..}$	$\bar y_{i.}$	$\bar y_{i.} - \bar y_{..}$	$\bar y_{.j} - \bar y_{..}$	$\bar y_{i.} + \bar y_{.j} - \bar y_{..}$	$\bar y_{ij} - \bar y_{..}$
$\frac{1}{an}y_{..} - \frac{1}{a}\sum_{i=1}^{a}\hat\tau_i$		$\frac{1}{a}y_{i.} - \hat\mu$			

FT-	CRD	E	Var[]
$y_{ij} - \bar y_{i.}$	$\varepsilon_{ij} - \frac{\sum_{j=1}^{n}\varepsilon_{ij}}{n}$	0	$\frac{n-1}{n}\sigma^2$
$\bar y_{i.} - \bar y_{..}$	$\tau_i + \frac{\sum_{j=1}^{n}\varepsilon_{ij}}{n} - \frac{\sum_{i=1}^{a}\sum_{j=1}^{n}\varepsilon_{ij}}{an}$	τ_i	$\frac{(a-1)\sigma^2}{an}$
RT-	**CRD**	**E**	**Var[]**
$y_{ij} - \bar y_{i.}$	$\varepsilon_{ij} - \frac{\sum_{j=1}^{n}\varepsilon_{ij}}{n}$	0	$\frac{n-1}{n}\sigma^2$
$\bar y_{i.} - \bar y_{..}$	$\tau_i - \frac{\sum_{i=1}^{a}\tau_i}{a} + \frac{\sum_{j=1}^{n}\varepsilon_{ij}}{n} - \frac{\sum_{i=1}^{a}\sum_{j=1}^{n}\varepsilon_{ij}}{an}$	0	$\frac{(a-1)(n\sigma_\tau^2+\sigma^2}{an}$
F-	**RCBD**	**E**	**Var[]**
$y_{ij} - \bar y_{i.}$	$\beta_j + \varepsilon_{ij} - \frac{\sum_{j=1}^{b}\varepsilon_{ij}}{b}$	β_j	$\frac{(b-1)\sigma^2}{b}$
$\bar y_{i.} - \bar y_{..}$	$\tau_i + \frac{\sum_{j=1}^{b}\varepsilon_{ij}}{b} - \frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\varepsilon_{ij}}{ab}$	τ_i	$\frac{(a-1)\sigma^2}{ab}$
$y_{ij} - \bar y_{.j}$	$\tau_i + \varepsilon_{ij} - \frac{\sum_{i=1}^{a}\varepsilon_{ij}}{a}$	τ_i	$\frac{(a-1)\sigma^2}{a}$
$\bar y_{.j} - \bar y_{..}$	$\beta_j + \frac{\sum_{i=1}^{a}\varepsilon_{ij}}{a} - \frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\varepsilon_{ij}}{ab}$	β_j	$\frac{(b-1)\sigma^2}{ab}$
R-	**RCBD**	**E**	**Var[]**
$y_{ij} - \bar y_{i.}$	$\beta_j - \frac{\sum_{j=1}^{b}\beta_j}{b} + \varepsilon_{ij} - \frac{\sum_{j=1}^{b}\varepsilon_{ij}}{b}$	0	$\frac{(b-1)(\sigma_\beta^2+\sigma^2)}{b}$
$\bar y_{i.} - \bar y_{..}$	$\tau_i + \frac{\sum_{j=1}^{b}\varepsilon_{ij}}{b} - \frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\varepsilon_{ij}}{ab}$	τ_i	$\frac{(a-1)\sigma^2}{ab}$
$y_{ij} - \bar y_{.j}$	$\tau_i + \varepsilon_{ij} - \frac{\sum_{i=1}^{a}\varepsilon_{ij}}{a}$	τ_i	$\frac{(a-1)\sigma^2}{a}$
$\bar y_{.j} - \bar y_{..}$	$\beta_j - \frac{\sum_{j=1}^{b}\beta_j}{b} + \frac{\sum_{i=1}^{a}\varepsilon_{ij}}{a} - \frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\varepsilon_{ij}}{ab}$	0	$\frac{(b-1)(a\sigma_\beta^2+\sigma^2}{ab}$

$y_{ij} - \bar y_{i.} - \bar y_{.j} + \bar y_{..} = \varepsilon_{ij} - \frac{\sum_{i=1}^{a}\varepsilon_{ij}}{a} - \frac{\sum_{j=1}^{b}\varepsilon_{ij}}{b} + \frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\varepsilon_{ij}}{ab}$
$E[] = 0, Var = \frac{(a-1)(b-1)\sigma^2}{ab}$

covariance

R-CRD: $Cov(y_{ij}, y_{kl}) = \{\, 0\ , i \neq k; \sigma_\tau^2, i = k, j \neq l; \sigma^2 + \sigma_\tau^2, i = k, j = l$

	ε_{ij}	$\sum_{i=1}^{a}\varepsilon_{ij}$	$\sum_{j=1}^{b}\varepsilon_{ij}$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\varepsilon_{ij}$
ε_{ij}	σ^2	σ^2	σ^2	σ^2
$\sum_{i=1}^{a}\varepsilon_{ij}$		$a\sigma^2$	σ^2	$a\sigma^2$
$\sum_{j=1}^{b}\varepsilon_{ij}$			$b\sigma^2$	$b\sigma^2$
$\sum_{i=1}^{a}\sum_{j=1}^{b}\varepsilon_{ij}$				$ab\sigma^2$

$Cov[\varepsilon_{ij}, \varepsilon_{ij}] = Cov[\sum_{i1}^{a}\varepsilon_{ij}, \sum_{j=1}^{b}\varepsilon_{ij}] = \sigma^2$,$Cov[\tau_i, \tau_i] = {}^2_\tau$,$Cov[\beta_j, \beta_j] = {}^2_\beta$
$Cov[\varepsilon_{ij}, \tau_i] = Cov[\varepsilon_{ij}, \beta_j] = Cov[\tau_i, \beta_j] = Cov[\varepsilon_{ij}, \varepsilon_{ik}] = 0, k \neq j$
$Cov[\varepsilon_{ij}, \sum_{j=1}^{n}\varepsilon_{ij}] = Cov[\varepsilon_{ij}, (\varepsilon_{i1} + .. + \varepsilon_{ij} .. + \varepsilon_{in})] = Cov[\varepsilon_{ij}, \varepsilon_{i1}] + .. + Cov[\varepsilon_{ij}, \varepsilon_{ij}] .. + Cov[\varepsilon_{ij}, \varepsilon_{in}] = \sigma^2$
$Cov[\sum_{j=1}^{n}\varepsilon_{ij}, \sum_{i=1}^{a}\sum_{j=1}^{n}\varepsilon_{ij}] = Cov[(\varepsilon_{i1} + \varepsilon_{i2} .. + \varepsilon_{in}), (\varepsilon_{i1} + \varepsilon_{i2} .. + \varepsilon_{in})] = Cov[\varepsilon_{i1}, \varepsilon_{i1}] + Cov[\varepsilon_{i2}, \varepsilon_{i2}] .. + Cov[\varepsilon_{in}, \varepsilon_{in}] = n\sigma^2$
$Var[y_{ij} - \bar y_{i.}] =$
FT-CRD;RT-CRD

$Var[\varepsilon_{ij} - \frac{\sum_{j=1}^n \varepsilon_{ij}}{n}] =, Var[\varepsilon_{ij}] + \frac{\sum_{j=1}^n Var[\varepsilon_{ij}]}{n^2} - \frac{2Cov[\varepsilon_{ij},\sum_{j=1}^n \varepsilon_{ij}]}{n}, = \sigma^2 + \frac{\sigma^2}{n} - \frac{2}{n}\sigma^2 = \frac{n-1}{n}\sigma^2$

FB-RCBD

$Var[\beta_j+\varepsilon_{ij}-\frac{\sum_{j=1}^b \varepsilon_{ij}}{b}] =, Var[\beta_j]+Var[\varepsilon_{ij}]+\frac{\sum_{j=1}^b Var[\varepsilon_{ij}]}{b^2} - 2Cov[\beta_j,\varepsilon_{ij}] - \frac{2Cov[\beta_j,\sum_{j=1}^b \varepsilon_{ij}]}{b} - \frac{2Cov[\varepsilon_{ij},\sum_{j=1}^b \varepsilon_{ij}]}{b}, = 0+\sigma^2+\frac{\sigma^2}{b}-0-0-\frac{2\sigma^2}{b} = \frac{(b-1)\sigma^2}{b}$

RB-RCBD

$Var[\beta_j - \frac{\sum_{j=1}^b \beta_j}{b} + \varepsilon_{ij} - \frac{\sum_{j=1}^b \varepsilon_{ij}}{b}] =$

$Var[\beta_j] + \frac{\sum_{j=1}^b Var[\beta_j]}{b^2} + Var[\varepsilon_{ij}] + \frac{\sum_{j=1}^b Var[\varepsilon_{ij}]}{b^2} - \frac{2Cov[\beta_j,\sum_{j=1}^b \beta_j]}{b} + 2Cov[\beta_j,\varepsilon_{ij}] - \frac{2Cov[\beta_j,\sum_{j=1}^b \varepsilon_{ij}]}{b} - \frac{2Cov[\sum_{j=1}^b \beta_j,\varepsilon_{ij}]}{b}$

$+\frac{2Cov[\sum_{j=1}^b \beta_j,\sum_{j=1}^b \varepsilon_{ij}]}{b^2} - \frac{2Cov[\varepsilon_{ij},\sum_{j=1}^b \varepsilon_{ij}]}{b} = \sigma_\beta^2 + \frac{\sigma_\beta^2}{b} - \frac{2\sigma_\beta^2}{b} + \sigma^2 + \frac{\sigma^2}{b} - \frac{2\sigma^2}{b} = \frac{(\sigma_\beta^2+\sigma^2)(b-1)}{b}$

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$Var[\bar{y}_{i.} - \bar{y}_{..}] =$

FT-CRD

$Var[\tau_i + \frac{\sum_{j=1}^n \varepsilon_{ij}}{n} - \frac{\sum_{i=1}^a\sum_{j=1}^n \varepsilon_{ij}}{an}] =$

$Var[\tau_i] + \frac{\sum_{j=1}^n Var[\varepsilon_{ij}]}{n^2} + \frac{\sum_{i=1}^a\sum_{j=1}^n Var[\varepsilon_{ij}]}{a^2n^2} + \frac{2Cov[\tau_i,\sum_{j=1}^n \varepsilon_{ij}]}{n} - \frac{2Cov[\tau_i,\sum_{i=1}^a\sum_{j=1}^n \varepsilon_{ij}]}{an} - \frac{2Cov[\sum_{j=1}^n \varepsilon_{ij},\sum_{i=1}^a\sum_{j=1}^n \varepsilon_{ij}]}{an^2},$

$= 0 + \frac{\sigma^2}{n} + \frac{\sigma^2}{an} - \frac{2\sigma^2}{an} = \frac{(a-1)\sigma^2}{an}$

RT-CRD

$Var[\tau_i - \frac{\sum_{i=1}^a \tau_i}{a} + \frac{\sum_{j=1}^n \varepsilon_{ij}}{n} - \frac{\sum_{i=1}^a\sum_{j=1}^n \varepsilon_{ij}}{an}] =$

$Var[\tau_i] + \frac{\sum_{i=1}^a Var[\tau_i]}{a^2} + \frac{\sum_{j=1}^n Var[\varepsilon_{ij}]}{n^2} + \frac{\sum_{i=1}^a\sum_{j=1}^n Var[\varepsilon_{ij}]}{a^2n^2} - \frac{2Cov[\tau_i,\sum_{i=1}^a \tau_i]}{a} + \frac{2Cov[\tau_i,\sum_{j=1}^n \varepsilon_{ij}]}{n} - \frac{2Cov[\tau_i,\sum_{i=1}^a\sum_{j=1}^n \varepsilon_{ij}]}{an} - \frac{2Cov[\sum_{i=1}^a \tau_i,\sum_{j=1}^n \varepsilon_{ij}]}{an}$

$+\frac{2Cov[\sum_{i=1}^a \tau_i,\sum_{i=1}^a\sum_{j=1}^n \varepsilon_{ij}]}{a^2n} - \frac{2Cov[\sum_{j=1}^n \varepsilon_{ij},\sum_{i=1}^a\sum_{j=1}^n \varepsilon_{ij}]}{an^2} = \sigma_\tau^2 + \frac{\sigma_\tau^2}{a} - \frac{2\sigma_\tau^2}{a} + \frac{\sigma^2}{n} + \frac{\sigma^2}{an} - \frac{2\sigma^2}{an} = \frac{(n\sigma_\tau^2+\sigma^2)(a-1)}{an}$

FB-RCBD;RB-RCBD

$Var[\tau_i + \frac{\sum_{j=1}^b \varepsilon_{ij}}{b} - \frac{\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}}{ab}] =$

$Var[\tau_i] + \frac{\sum_{j=1}^b Var[\varepsilon_{ij}]}{b^2} + \frac{\sum_{i=1}^a\sum_{j=1}^b Var[\varepsilon_{ij}]}{a^2b^2} + \frac{2Cov[\tau_i,\sum_{j=1}^b \varepsilon_{ij}]}{b} - \frac{2Cov[\tau_i,\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}]}{ab} - \frac{2Cov[\sum_{j=1}^b \varepsilon_{ij},\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}]}{ab^2} = 0 + \frac{\sigma^2}{b} + \frac{\sigma^2}{ab} - \frac{2\sigma^2}{ab} = \frac{(a-1)\sigma^2}{ab}$

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$Var[y_{ij} - \bar{y}_{.j}] =$

FB-RCBD; RB-RCBD

$Var[\tau_i + \varepsilon_{ij} - \frac{\sum_{i=1}^a \varepsilon_{ij}}{a}] = Var[\tau_i] + Var[\varepsilon_{ij}] + \frac{\sum_{i=1}^a Var[\varepsilon_{ij}]}{a^2} - \frac{2Cov[\tau_i,\varepsilon_{ij}]}{a} - \frac{2Cov[\tau_i,\sum_{i=1}^a \varepsilon_{ij}]}{a} - \frac{2Cov[\varepsilon_{ij},\sum_{i=1}^a \varepsilon_{ij}]}{a} = 0 + \sigma^2 + \frac{\sigma^2}{a} - \frac{2\sigma^2}{a} = \frac{(a-1)\sigma^2}{a}$

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$Var[\bar{y}_{.j} - \bar{y}_{..}] =$

FB-RCBD

$Var[\beta_j + \frac{\sum_{i=1}^a \varepsilon_{ij}}{a} - \frac{\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}}{ab}] =$

$Var[\beta_j] + \frac{\sum_{i=1}^a Var[\varepsilon_{ij}]}{a^2} + \frac{\sum_{i=1}^a\sum_{j=1}^b Var[\varepsilon_{ij}]}{a^2b^2} + \frac{2Cov[\beta_j,\sum_{i=1}^a \varepsilon_{ij}]}{a} - \frac{2Cov[\beta_j,\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}]}{ab} - \frac{2Cov[\sum_{i=1}^a \varepsilon_{ij},\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}]}{a^2b}$

$= 0 + \frac{\sigma^2}{a} + \frac{\sigma^2}{ab} - \frac{2\sigma^2}{ab} = \frac{(b-1)\sigma^2}{ab}$

RB-RCBD

$Var[\beta_j - \frac{\sum_{j=1}^b \beta_j}{b} + \frac{\sum_{i=1}^a \varepsilon_{ij}}{a} - \frac{\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}}{ab}] =,$

$Var[\beta_j] + \frac{\sum_{j=1}^b Var[\beta_j]}{b^2} + \frac{\sum_{i=1}^a Var[\varepsilon_{ij}]}{a^2} + \frac{\sum_{i=1}^a\sum_{j=1}^b Var[\varepsilon_{ij}]}{a^2b^2} - \frac{2Cov[\beta_j,\sum_{j=1}^b \beta_j]}{b} + \frac{2Cov[\beta_j,\sum_{i=1}^a \varepsilon_{ij}]}{a} - \frac{2Cov[\beta_j,\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}]}{ab} - \frac{2Cov[\sum_{j=1}^b \beta_j,\sum_{i=1}^a \varepsilon_{ij}]}{ab}$

$+\frac{2Cov[\sum_{j=1}^b \beta_j,\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}]}{ab^2} - \frac{2Cov[\sum_{i=1}^a \varepsilon_{ij},\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}]}{a^2b} = \sigma_\beta^2 + \frac{\sigma_\beta^2}{b} - \frac{2\sigma_\beta^2}{b} + \frac{\sigma^2}{a} + \frac{\sigma^2}{ab} - \frac{2\sigma^2}{ab} = \frac{(a\sigma_\beta^2+\sigma^2)(b-1)}{ab}$

—

$Var[y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..}] =$ FB-RCBD

$Var[\varepsilon_{ij} - \frac{1}{a}\sum_{i=1}^a \varepsilon_{ij} - \frac{1}{b}\sum_{j=1}^b \varepsilon_{ij} + \frac{1}{ab}\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}] =$

$Var[\varepsilon_{ij}] + \frac{\sum_{i=1}^a Var[\varepsilon_{ij}]}{a^2} + \frac{\sum_{j=1}^b Var[\varepsilon_{ij}]}{b^2} + \frac{\sum_{i=1}^a\sum_{j=1}^b Var[\varepsilon_{ij}]}{a^2b^2} - \frac{2Cov[\varepsilon_{ij},\sum_{i=1}^a \varepsilon_{ij}]}{a} - \frac{2Cov[\varepsilon_{ij},\sum_{j=1}^b \varepsilon_{ij}]}{b} + \frac{2Cov[\varepsilon_{ij},\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}]}{ab} + \frac{2Cov[\sum_{i=1}^a \varepsilon_{ij},\sum_{j=1}^b \varepsilon_{ij}]}{ab}$

$-\frac{2Cov[\sum_{i=1}^a \varepsilon_{ij},\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}]}{a^2b} - \frac{2Cov[\sum_{j=1}^b \varepsilon_{ij},\sum_{i=1}^a\sum_{j=1}^b \varepsilon_{ij}]}{ab^2} = \sigma^2 + \frac{\sigma^2}{a} + \frac{\sigma^2}{b} + \frac{\sigma^2}{ab} - \frac{2\sigma^2}{a} - \frac{2\sigma^2}{b} + \frac{2\sigma^2}{ab} + \frac{2\sigma^2}{ab} - \frac{2\sigma^2}{ab} - \frac{2\sigma^2}{ab} = \frac{(ab-a-b+1)\sigma^2}{ab}$