

2015F  
2015F1

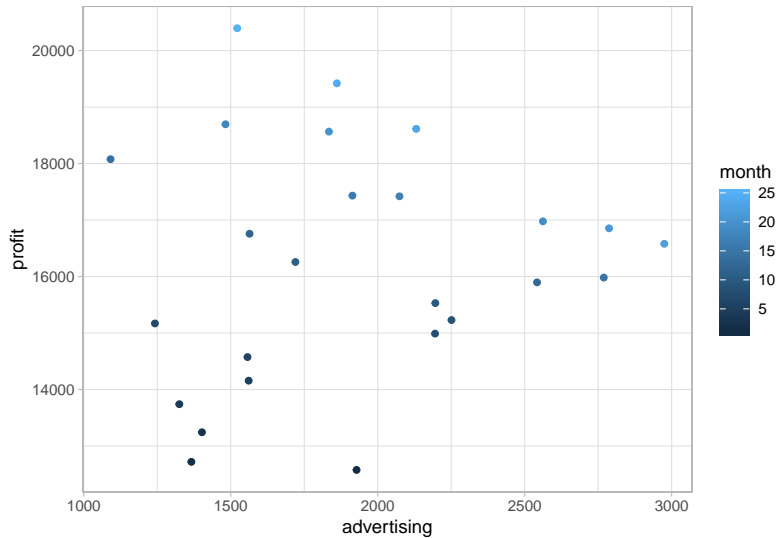
2017SR1 X1,X2 linear regression

Find the best model for predicting Y based on X1 and X2. Y is the amount of profit that a company makes in a month. X1 is the number of months that the company has been in business. X2 is the amount spent on advertising.

Consider as predictors all possible linear and quadratic terms ( $X1$ ,  $X1^2$ ,  $X2$ ,  $X2^2$ , and  $X1X2$ ). Consider possible transformations of Y. Include all appropriate diagnostics. When you have found your “best” model, predict a new Y when  $X1 = 20$  and  $X2 = \$1,500$ , giving a 95% prediction interval. The data set, shown below, appears in “Profits.xlsx”.

```
table_2015f1 <- readxl::read_xlsx("qe_lab/Profits_2015f.xlsx")
str(table_2015f1)
## Classes 'tbl_df', 'tbl' and 'data.frame':    25 obs. of  3 variables:
## $ X1: num  1 2 3 4 5 6 7 8 9 10 ...
## $ X2: num 1928 1366 1402 1325 1561 ...
## $ Y : num 12577 12720 13244 13741 14157 ...

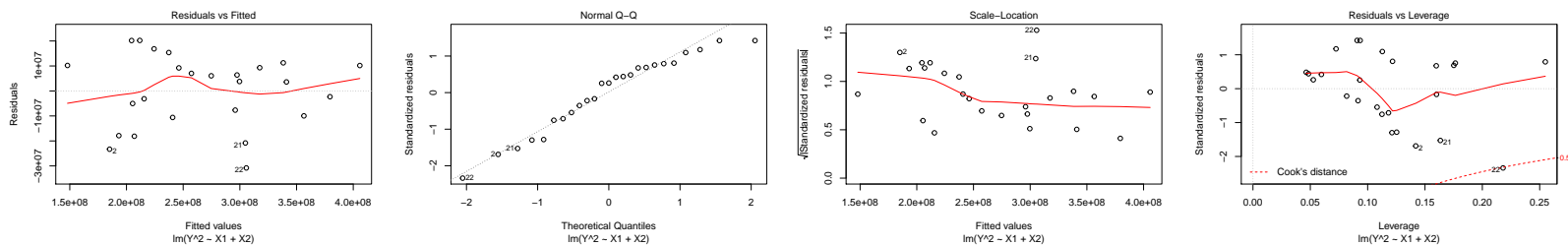
library(ggplot2)
ggplot(table_2015f1,aes(X2,Y, color=X1))+labs(x="advertising",y="profit",color="month")+geom_point()+theme_light()
```



```
model_2015f1_1 <- lm(Y^2~X1+X2,table_2015f1)
summary(model_2015f1_1)
##
## Call:
## lm(formula = Y^2 ~ X1 + X2, data = table_2015f1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -30805386 -9969025  3791394 10176772 20218197
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 231392970   11535085  20.060 1.25e-15 ***
## X1           9924495     455383    21.794 < 2e-16 ***
## X2          -48414         6402    -7.563 1.48e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14900000 on 22 degrees of freedom
## Multiple R-squared:  0.956, Adjusted R-squared:  0.952
## F-statistic: 239 on 2 and 22 DF, p-value: 1.194e-15

anova(model_2015f1_1)
## Analysis of Variance Table
##
## Response: Y^2
##      Df    Sum Sq   Mean Sq F value    Pr(>F)
## X1     1 9.3403e+16 9.3403e+16 420.892 7.822e-16 ***
## X2     1 1.2692e+16 1.2692e+16  57.192 1.482e-07 ***
## Residuals 22 4.8822e+15 2.2192e+14
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

plot(model_2015f1_1)
```

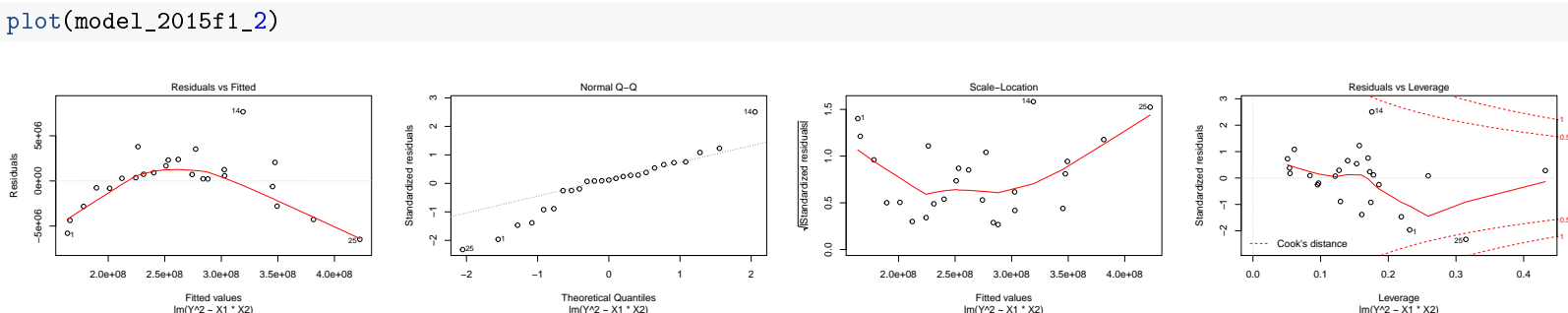


```
model_2015f1_2 <- lm(Y^2 ~ X1 * X2, table_2015f1)
summary(model_2015f1_2)

##
## Call:
## lm(formula = Y^2 ~ X1 * X2, data = table_2015f1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -6470217 -816187  356232  1674892  7659408
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.131e+08  6.398e+06  17.671 4.39e-14 ***
## X1           1.804e+07  4.136e+05  43.613 < 2e-16 ***
## X2           2.164e+04  3.749e+03   5.772 9.94e-06 ***
## X1:X2        -4.585e+03  2.264e+02 -20.252 2.92e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3365000 on 21 degrees of freedom
## Multiple R-squared:  0.9979, Adjusted R-squared:  0.9976
## F-statistic: 3260 on 3 and 21 DF, p-value: < 2.2e-16
```

```
anova(model_2015f1_2)

## Analysis of Variance Table
##
## Response: Y^2
##           Df      Sum Sq   Mean Sq F value    Pr(>F)
## X1          1 9.3403e+16 9.3403e+16 8248.18 < 2.2e-16 ***
## X2          1 1.2692e+16 1.2692e+16 1120.78 < 2.2e-16 ***
## X1:X2       1 4.6444e+15 4.6444e+15  410.13 2.92e-15 ***
## Residuals 21 2.3781e+14 1.1324e+13
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



```
sqrt(predict(model_2015f1_1, newdata=data.frame(X1 = 20 ,X2 =1500), interval="prediction", level=0.95 ))
##          fit          lwr          upr
## 1 18901.39 18003.89 19758.17
sqrt(predict(model_2015f1_2, newdata=data.frame(X1 = 20 ,X2 =1500), interval="prediction", level=0.95 ))
##          fit          lwr          upr
## 1 19201.9 19003.12 19398.65
```

## 2015F2

### 2018F2 5k1p Fractional Factorial Design

A replicated fractional factorial design is used to investigate the effect of five factors on the free height of leaf springs used in an automotive application. The factors are (A) furnace temperature, (B) heating time, (C) transfer time, (D) hold down time, and (E) quench oil temperature. There are 3 observations at each setting.

Write out the alias structure for this design. What is the resolution of this design? Analyze the data. What factors influence the mean free height? The data set appears in the file “Springs.xlsx”.

```
table_2015f2 <- readxl::read_xlsx("qe_lab/Springs_2015f.xlsx")
```

```
## New names:
## * -> ...7
## * -> ...8
```

```
library(tidyverse)
```

```
## -- Attaching packages ----- tidyverse 1.2.1 --
## v tibble 2.1.3          v purrr 0.3.2
## v tidyr 1.0.0.9000     v dplyr 0.8.3
## v readr 1.3.1          v stringr 1.4.0
## v tibble 2.1.3          v forcats 0.4.0
```

```
## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag() masks stats::lag()
table_2015f2 <- gather(table_2015f2, 'Height', '...7', '...8', key = "1", value = "height" )[, -6]
str(table_2015f2)
## Classes 'tbl_df', 'tbl' and 'data.frame': 48 obs. of 6 variables:
## $ A : num -1 1 -1 1 -1 1 -1 1 -1 1 ...
## $ B : num -1 -1 1 1 -1 -1 1 1 -1 -1 ...
## $ C : num -1 -1 -1 -1 1 1 1 1 -1 -1 ...
## $ D : num -1 1 1 -1 1 -1 -1 1 -1 1 ...
## $ E : num -1 -1 -1 -1 -1 -1 -1 -1 1 1 ...
## $ height: num 7.78 8.15 7.5 7.59 7.54 7.69 7.56 7.56 7.5 7.88 ...
```

```
kableExtra::kable(table_2015f2)
```

A	B	C	D	E	height
-1	-1	-1	-1	-1	7.78
-1	-1	-1	-1	-1	8.15
-1	-1	-1	-1	-1	7.50
-1	-1	-1	-1	-1	7.59
-1	-1	-1	-1	-1	7.54
-1	-1	-1	-1	-1	7.69
-1	-1	-1	-1	-1	7.56
-1	-1	-1	-1	-1	7.56
-1	-1	-1	-1	-1	7.50
-1	-1	-1	-1	-1	7.88
-1	-1	-1	-1	-1	7.50
-1	-1	-1	-1	-1	7.63
-1	-1	-1	-1	-1	7.32
-1	-1	-1	-1	-1	7.56
-1	-1	-1	-1	-1	7.18
-1	-1	-1	-1	-1	7.81
-1	-1	-1	-1	-1	7.78
-1	-1	-1	-1	-1	8.18
-1	-1	-1	-1	-1	7.56
-1	-1	-1	-1	-1	7.56
-1	-1	-1	-1	-1	8.00
-1	-1	-1	-1	-1	8.09
-1	-1	-1	-1	-1	7.52
-1	-1	-1	-1	-1	7.81
-1	-1	-1	-1	-1	7.25
-1	-1	-1	-1	-1	7.88
-1	-1	-1	-1	-1	7.56
-1	-1	-1	-1	-1	7.75
-1	-1	-1	-1	-1	7.44
-1	-1	-1	-1	-1	7.69
-1	-1	-1	-1	-1	7.18
-1	-1	-1	-1	-1	7.50
-1	-1	-1	-1	-1	7.81
-1	-1	-1	-1	-1	7.88
-1	-1	-1	-1	-1	7.50
-1	-1	-1	-1	-1	7.75
-1	-1	-1	-1	-1	7.88
-1	-1	-1	-1	-1	8.06
-1	-1	-1	-1	-1	7.44
-1	-1	-1	-1	-1	7.69
-1	-1	-1	-1	-1	7.12
-1	-1	-1	-1	-1	7.44
-1	-1	-1	-1	-1	7.50
-1	-1	-1	-1	-1	7.56
-1	-1	-1	-1	-1	7.44
-1	-1	-1	-1	-1	7.62
-1	-1	-1	-1	-1	7.25
1	1	1	1	1	7.59

```
library(devtools)
devtools::install_github("tidyverse/tidyr", force=T)
pivot_longer(table_2015f2, -A, -B, -C, -D, values_to = "Height")
I=ABCD;
A=BCD; B=ACD; C=ABD; D=ABC; E=ABCDE;
AB=CD; AC=BD; AD=BC; AE=BCDE; BE=ACDE; CE=ABDE; DE=ABCE;
ABE=CDE; BCE=ADE; BDE=ACE;
I=ABCD confounded
Resolution=IV
```

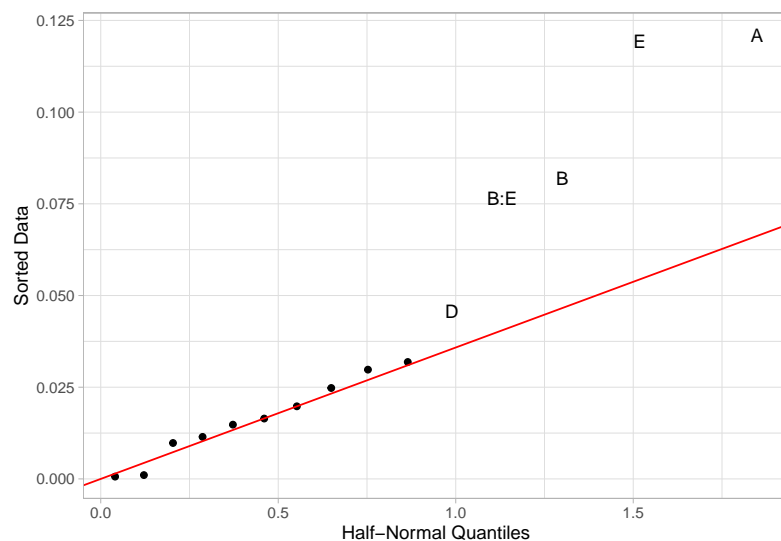
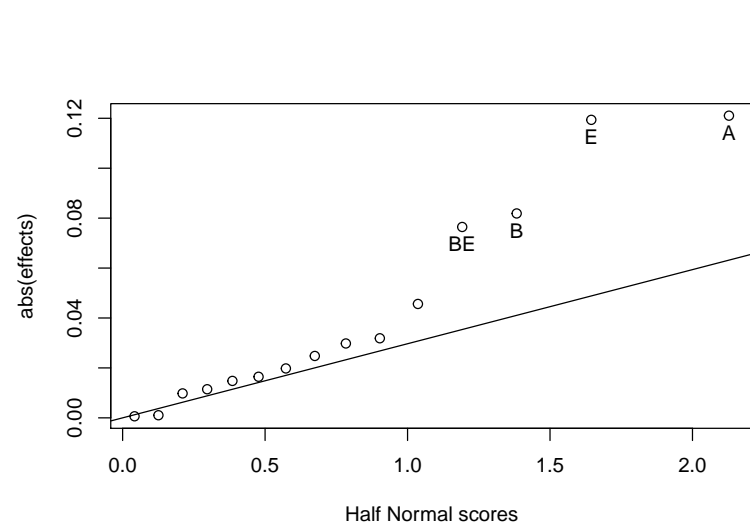
```
model_2015f2_1 <- aov(height~A*B*C*D*E, table_2015f2)
summary(model_2015f2_1)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
## A	1	0.7033	0.7033	35.888	1.12e-06 ***
## B	1	0.3218	0.3218	16.420	0.000302 ***
## C	1	0.0295	0.0295	1.506	0.228774
## D	1	0.0999	0.0999	5.099	0.030893 *
## E	1	0.6840	0.6840	34.906	1.42e-06 ***
## A:B	1	0.0105	0.0105	0.536	0.469451
## A:C	1	0.0000	0.0000	0.001	0.975515
## B:C	1	0.0063	0.0063	0.322	0.574603
## A:E	1	0.0488	0.0488	2.489	0.124500
## B:E	1	0.2806	0.2806	14.319	0.000640 ***
## C:E	1	0.0130	0.0130	0.664	0.421343
## D:E	1	0.0188	0.0188	0.959	0.334662
## A:B:E	1	0.0001	0.0001	0.003	0.959204
## A:C:E	1	0.0046	0.0046	0.235	0.631251
## B:C:E	1	0.0426	0.0426	2.174	0.150128
## Residuals	32	0.6271	0.0196		
## ---					
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1					

```
library(daewr)
halfnorm(coef(model_2015f2_1)[2:16],alpha=0.05)
## zscore= 0.0417893 0.1256613 0.2104284 0.2967378 0.3853205 0.4770404 0.5729675 0.6744898 0.7835004 0.9027348 1.03
library(gghalfnorm)
gghalfnorm(x =coef(model_2015f2_1)[2:16],labs = names(coef(model_2015f2_1)[2:16]) , nlab = 5)+ ggplot2::theme_light
model_2015f2_2 <- lm(height~A+B*E+D, table_2015f2)
summary(model_2015f2_2)
```

```
##
## Call:
## lm(formula = height ~ A + B * E + D, data = table_2015f2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.28875 -0.08687  0.03812  0.09094  0.20167
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.62563     0.01994 382.512 < 2e-16 ***
## A             0.12104     0.01994   6.072 3.13e-07 ***
## B            -0.08188     0.01994  -4.107 0.000181 ***
## E            -0.11938     0.01994  -5.988 4.13e-07 ***
## D             0.04562     0.01994   2.289 0.027199 *
## B:E           0.07646     0.01994   3.835 0.000414 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1381 on 42 degrees of freedom
## Multiple R-squared:  0.7228, Adjusted R-squared:  0.6898
## F-statistic: 21.91 on 5 and 42 DF, p-value: 9.877e-11
```

```
anova(model_2015f2_2)
## Analysis of Variance Table
##
## Response: height
##              Df Sum Sq Mean Sq F value    Pr(>F)
## A               1  0.70325  0.70325  36.8645 3.133e-07 ***
## B               1  0.32177  0.32177  16.8671 0.0001812 ***
## E               1  0.68402  0.68402  35.8563 4.133e-07 ***
## D               1  0.09992  0.09992   5.2377 0.0271986 *
## B:E             1  0.28060  0.28060  14.7092 0.0004145 ***
## Residuals     42  0.80122  0.01908
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



## 2016S

Fountain, Tableman\*

## 2016S1

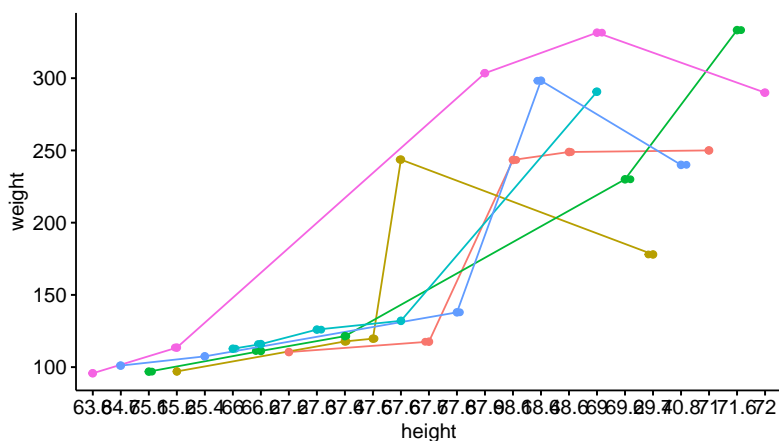
2017F1

Find the best model for predicting Y (weight) based on X1 (age), X2 (height), and X3 (indicator for male). Consider as predictors all possible linear and quadratic terms. Consider possible transformations of Y. Include all appropriate diagnostics. When you have found your “best” model, predict a new Y when X1 = 26, X2 = 70, and X3 = 1, giving a 95% prediction interval. The data set, shown below, appears in “RegressionSpr16.xlsx”.

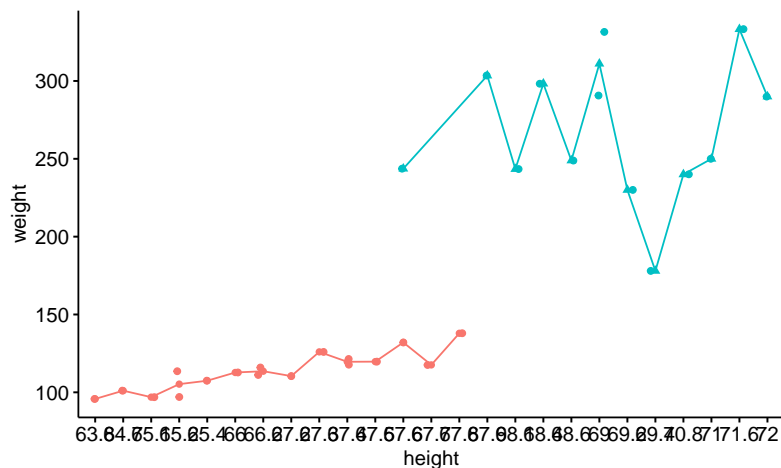
```
table_2016s1 <- readxl::read_xlsx("qe_lab/RegressionSpr16.xlsx")[-1,]
table_2016s1$weight <- round(as.numeric(table_2016s1$weight), 2)
table_2016s1$age <- as.factor(table_2016s1$age)
table_2016s1$height <- round(as.numeric(table_2016s1$height), 2)
table_2016s1$male <- factor(table_2016s1$male, labels=c("female","male"))
str(table_2016s1)
```



age  
 20 21 22 23 24 25



male female



```
library(GGally)
ggpairs(table_2016s1)
model_2016s1 <- lm(weight~height*male*age, table_2016s1)
olsrr::ols_step_both_aic(model_2016s1)

model_2016s1_1 <- lm((weight)~(height):male:age, table_2016s1)
summary(model_2016s1_1)
```

```
##
## Call:
## lm(formula = (weight) ~ (height):male:age, data = table_2016s1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -47.325  -7.574   0.820   7.236  45.739
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -16.955     304.536  -0.056   0.956
## height:malefemale:age20    1.942      4.524   0.429   0.673
## height:malemale:age20     3.818      4.403   0.867   0.398
## height:malefemale:age21    1.928      4.570   0.422   0.678
## height:malemale:age21     3.318      4.454   0.745   0.466
## height:malefemale:age22    1.916      4.603   0.416   0.682
## height:malemale:age22     4.253      4.333   0.981   0.340
## height:malefemale:age23    2.077      4.565   0.455   0.655
## height:malemale:age23     4.458      4.430   1.006   0.328
## height:malefemale:age24    2.012      4.621   0.435   0.669
## height:malemale:age24     4.102      4.383   0.936   0.362
## height:malefemale:age25    1.886      4.730   0.399   0.695
## height:malemale:age25     4.665      4.376   1.066   0.301
##
## Residual standard error: 26.62 on 17 degrees of freedom
## Multiple R-squared:  0.9399, Adjusted R-squared:  0.8976
## F-statistic: 22.17 on 12 and 17 DF, p-value: 5.073e-08
```

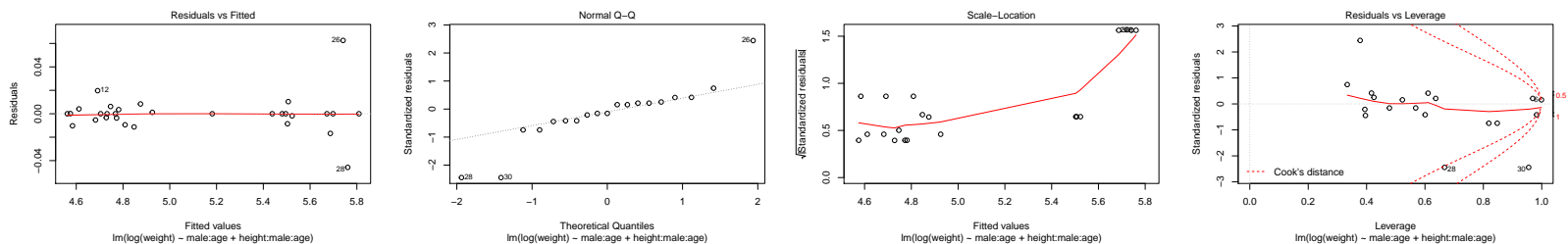
```
model_2016s1_2 <- lm(log(weight)~male:age+height:male:age, table_2016s1)
summary(model_2016s1_2)
```

```
##
## Call:
## lm(formula = log(weight) ~ male:age + height:male:age, data = table_2016s1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.045786 -0.003582  0.000000  0.000944  0.062574
##
## Coefficients: (2 not defined because of singularities)
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    6.992621   0.753588   9.279 3.50e-05 ***
## malefemale:age20  -10.754395   6.240000  -1.723 0.128468
## malemale:age20    -1.957457   1.272603  -1.538 0.167900
## malefemale:age21  -8.277268   1.398515  -5.919 0.000588 ***
## malemale:age21    10.295232   1.903129   5.410 0.000998 ***
## malefemale:age22  -8.773831   1.521712  -5.766 0.000687 ***
## malemale:age22   -12.248065   1.543569  -7.935 9.60e-05 ***
## malefemale:age23  -8.304076   1.748874  -4.748 0.002088 **
## malemale:age23   -1.320501   0.754287  -1.751 0.123474
## malefemale:age24  -8.932178   1.198331  -7.454 0.000143 ***
## malemale:age24     4.900144   1.530232   3.202 0.015019 *
## malefemale:age25 -10.180009   2.245823  -4.533 0.002690 **
## malemale:age25      NA         NA      NA      NA
## malefemale:age20:height  0.125984   0.091835   1.372 0.212458
## malemale:age20:height   0.006874   0.014810   0.464 0.656606
## malefemale:age21:height  0.089870   0.017661   5.089 0.001417 **
```

```
## malemale:age21:height -0.174439 0.025510 -6.838 0.000245 ***
## malefemale:age22:height 0.097775 0.019958 4.899 0.001755 **
## malemale:age22:height 0.154531 0.019132 8.077 8.57e-05 ***
## malefemale:age23:height 0.091513 0.023633 3.872 0.006114 **
## malemale:age23:height NA NA NA NA
## malefemale:age24:height 0.101254 0.014121 7.170 0.000182 ***
## malemale:age24:height -0.090567 0.019132 -4.734 0.002123 **
## malefemale:age25:height 0.121461 0.032798 3.703 0.007622 **
## malemale:age25:height -0.018138 0.010819 -1.677 0.137537
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.03247 on 7 degrees of freedom
## Multiple R-squared: 0.9987, Adjusted R-squared: 0.9948
## F-statistic: 251.1 on 22 and 7 DF, p-value: 3.905e-08
```

```
anova(model_2016s1_2)
## Analysis of Variance Table
##
## Response: log(weight)
##              Df Sum Sq Mean Sq F value    Pr(>F)
## male:age      11 5.5405  0.50368  477.776 5.658e-09 ***
## male:age:height 11 0.2839  0.02581   24.485 0.0001567 ***
## Residuals      7  0.0074  0.00105
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
plot(model_2016s1_2)
## Warning: not plotting observations with leverage one:
## 2, 4, 7, 10, 13, 15, 19, 23, 24, 27, 29
## Warning: not plotting observations with leverage one:
## 2, 4, 7, 10, 13, 15, 19, 23, 24, 27, 29
## Warning in sqrt(crit * p * (1 - hh)/hh): NaNs produced
## Warning in sqrt(crit * p * (1 - hh)/hh): NaNs produced
```



## 2016S2

### 2017F2

A process engineer is testing the yield of a product manufactured on three specific machines. Each machine can be operated at fixed high and low power settings, although the actual settings differ from one machine to the next. Furthermore, a machine has three stations on which the product is formed, and these are the same for each machine. An experiment is conducted in which each machine is tested at both power settings, and three observations on yield are taken from each station. The runs are made in random order. Analyze this experiment. The data set, shown below, appears in “DesignSpr16.xlsx”.

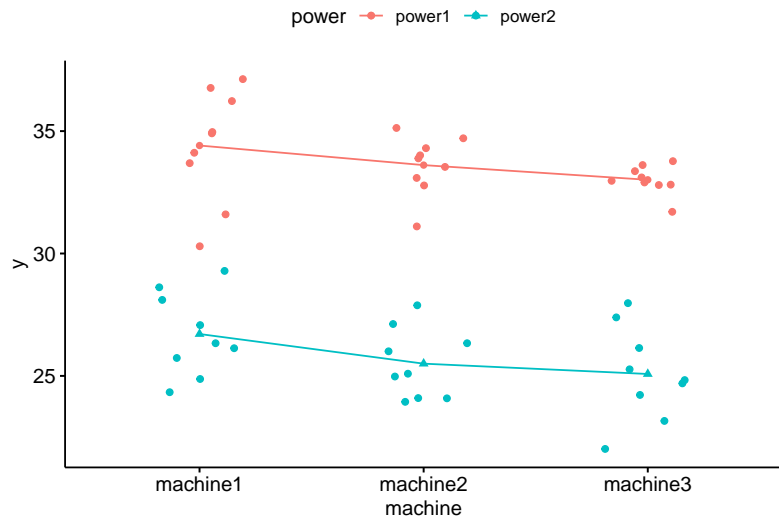
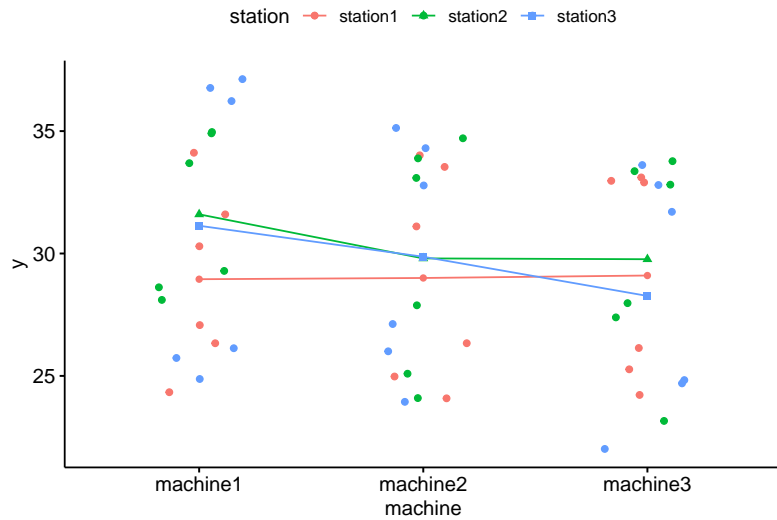
```
DesignSpr16 <- readxl::read_excel("qe_lab/DesignSpr16.xlsx")
```

```
## New names:
## * -> ...1
## * -> ...2
## * -> ...4
## * -> ...5
## * -> ...6
## * ... and 4 more problems
```

```
library(tidyverse)
table_2016s2 <- gather(DesignSpr16[c(2:4,6:8),c(2:4,6:8,10:12)])
names(table_2016s2) <- c("machine","y")
table_2016s2 <- table_2016s2[c("y","machine")]
table_2016s2$machine <- as.factor(c(rep("machine1",18),rep("machine2",18),rep("machine3",18)) )
table_2016s2$station <- as.factor(rep(c(rep("station1",6),rep("station2",6),rep("station3",6)),3) )
table_2016s2$power <- as.factor(rep(c(rep("power1",3),rep("power2",3)),9) )
str(table_2016s2)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame': 54 obs. of 4 variables:
## $ y : num 34.1 30.3 31.6 24.3 26.3 27.1 33.7 34.9 35 28.1 ...
## $ machine: Factor w/ 3 levels "machine1","machine2",...: 1 1 1 1 1 1 1 1 1 1 ...
## $ station: Factor w/ 3 levels "station1","station2",...: 1 1 1 1 1 1 2 2 2 2 ...
## $ power : Factor w/ 2 levels "power1","power2": 1 1 1 2 2 2 1 1 1 2 ...
```

```
library(ggpubr)
ggline(table_2016s2,"machine","y",add = c("mean","jitter"),color = "station",shape = "station")
ggline(table_2016s2,"machine","y",add = c("mean","jitter"),color = "power",shape = "power")
```



```
model_2016s2 <- aov(y~machine*power*station, table_2016s2)
summary(model_2016s2)

##               Df Sum Sq Mean Sq F value    Pr(>F)
## machine         2   21.4      10.7    6.248 0.00469 **
## power           1  845.7      845.7 492.959 < 2e-16 ***
## station         2   17.0       8.5    4.949 0.01262 *
## machine:power    2    0.4       0.2    0.112 0.89479
## machine:station  4   16.6       4.2    2.419 0.06625 .
## power:station    2   16.3       8.2    4.751 0.01475 *
## machine:power:station 4   12.9       3.2    1.881 0.13507
## Residuals      36   61.8       1.7
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(model_2016s2)

## Analysis of Variance Table
##
## Response: y
##               Df Sum Sq Mean Sq  F value    Pr(>F)
## machine         2  21.44   10.72    6.2475  0.004687 **
## power           1 845.70   845.70 492.9587 < 2.2e-16 ***
## station         2  16.98    8.49    4.9489  0.012623 *
## machine:power    2   0.38    0.19    0.1115  0.894793
## machine:station  4  16.60    4.15    2.4195  0.066255 .
## power:station    2  16.30    8.15    4.7514  0.014749 *
## machine:power:station 4  12.91    3.23    1.8806  0.135072
## Residuals      36  61.76    1.72
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
library(lme4)
model_2016s2_1 <- lmer(y~machine*station+(1|machine:power)+(1|machine:power:station),table_2016s2)
summary(model_2016s2_1)$varcor
```

```
## Groups          Name          Std.Dev.
## machine:power:station (Intercept) 1.0251
## machine:power      (Intercept) 5.5493
## Residual                    1.3098
```

```
anova(model_2016s2_1)

## Analysis of Variance Table
##               Df Sum Sq Mean Sq F value
## machine         2 0.1304  0.0652  0.0380
## station         2 5.9841  2.9921  1.7441
## machine:station  4 5.8511  1.4628  0.8527
pf(anova(model_2016s2_1)$'F value',df1=anova(model_2016s2_1)$'Df',df2=c(3,6,6), lower.tail = F)

## [1] 0.9631646 0.2528763 0.5409738
```

```
confint(model_2016s2_1)

##               2.5 %      97.5 %
## .sig01          0.0000000  1.2183770
## .sig02          2.4009309  7.9202750
## .sigma          1.0569419  1.6754545
## (Intercept)     22.3980316 35.5019668
## machinemachine2 -9.2158826  9.3158803
## machinemachine3 -9.1158826  9.4158803
## stationstation2  0.7334267  4.5665723
## stationstation3  0.2667600  4.0999057
## machinemachine2:stationstation2 -4.5604440  0.8604426
## machinemachine3:stationstation2 -4.6937773  0.7271093
## machinemachine2:stationstation3 -4.0271107  1.3937759
## machinemachine3:stationstation3 -5.7271107 -0.3062241
```

```
library(GAD)
table_2016s2$machine_f <- as.fixed(table_2016s2$machine)
table_2016s2$station_f <- as.fixed(table_2016s2$station)
table_2016s2$power_r <- as.random(table_2016s2$power)
```

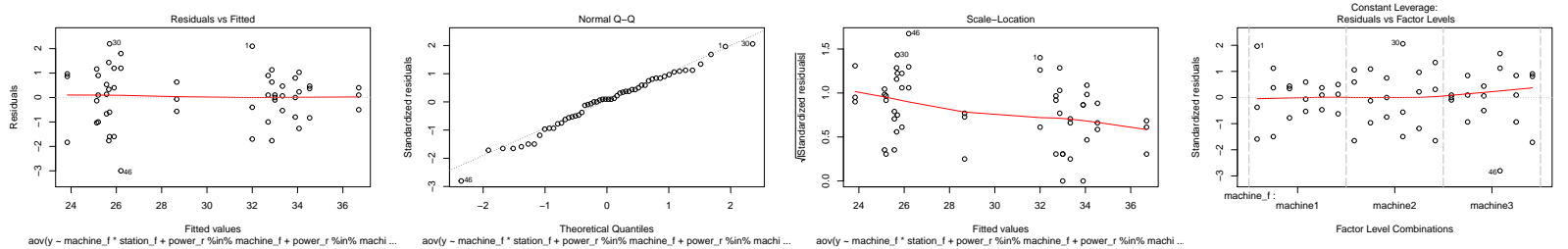


```

model_2016s2_2 <- aov(y~machine_f*station_f+power_r%in%machine_f+power_r%in%machine_f*station_f, table_2016s2)
gad(model_2016s2_2)
## Analysis of Variance Table
##
## Response: y
##
##           Df Sum Sq Mean Sq  F value    Pr(>F)
## machine_f    2  21.44   10.718    0.0380  0.96317
## station_f     2   16.98    8.490    1.7441  0.25287
## machine_f:station_f  4   16.60    4.151    0.8527  0.54097
## machine_f:power_r    3 846.08  282.027  164.3939 < 2e-16 ***
## machine_f:station_f:power_r  6  29.21    4.868    2.8375  0.02292 *
## Residual      36  61.76    1.716
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

plot(model_2016s2_2)

```



## 2016F

Jong Sung Kim\*, Brad Crain

### 2016F1

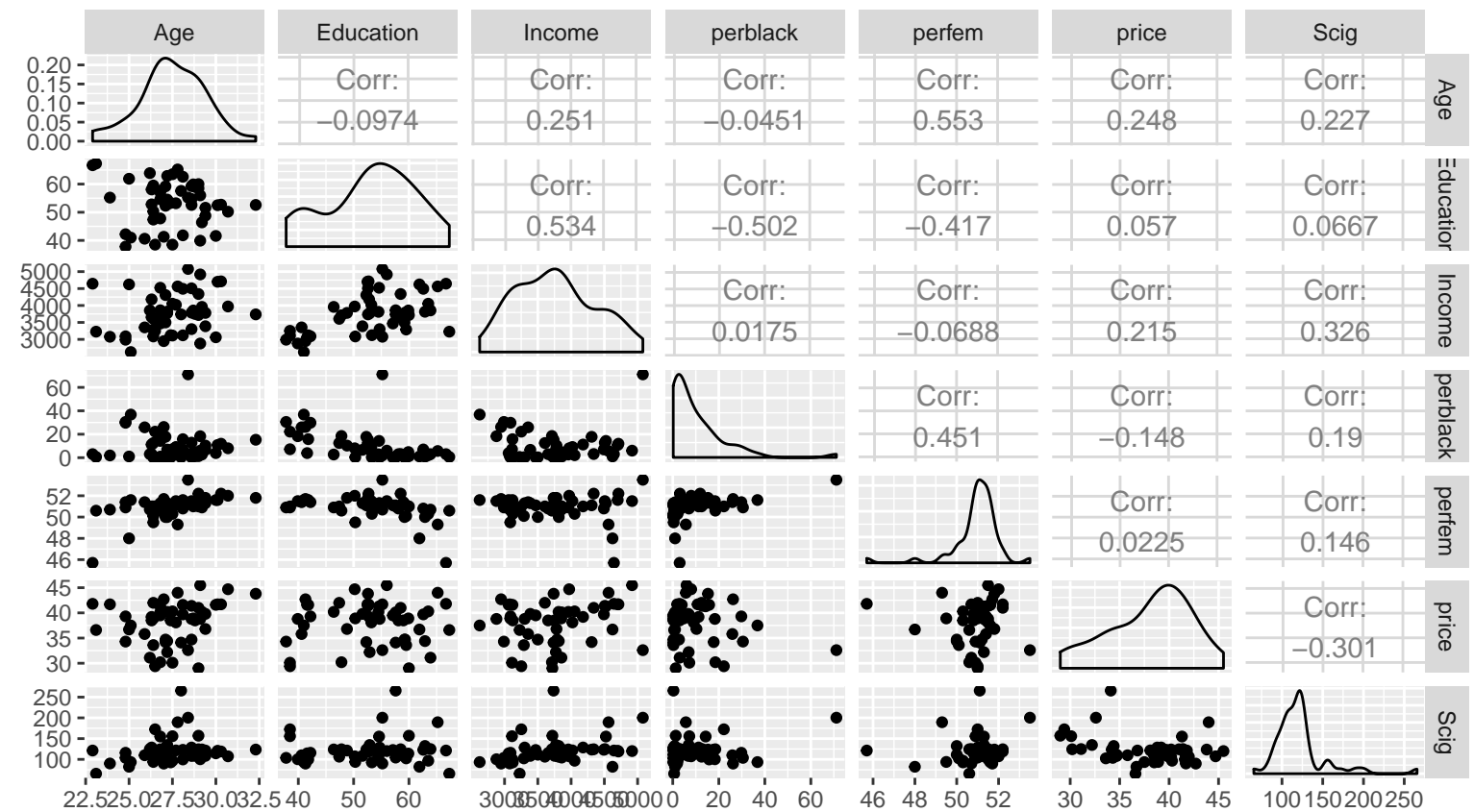
A national insurance organization wanted to study the consumption pattern of cigarettes in all 50 states and the District of Columbia. Data were collected for 1960, 1970, and 1980, but we will focus here on 1970. Using data from 1970, the organization wanted to construct a regression equation that relates statewide cigarette consumption (on a per capita basis) to various socioeconomic and demographic variables, and to determine whether these variables were useful in predicting the consumption of cigarettes. The variables chosen for study are given below. Age, x1: Median age of a person living in the state  
Education, x2: Percentage of people over 25 years of age in a state that had completed high school  
Income, x3: Per capita personal income for a state (in dollars)  
Perblack, x4: Percentage of blacks living in a state  
Perfem, x5: Percentage of females living in a state  
Price, x6: Average price of a pack of cigarettes in a state (in cents)  
Scig, y: Number of packs of cigarettes sold in a state on a per capita basis.  
The data on these variables are stored in 8 columns in the same order as listed above; a two-letter alphabetic code is given first, however. The data are saved as “cigcons.xlsx”  
Perform a complete regression analysis on these data; including checking of model assumptions and attempting appropriate remedies, if needed. The main objective of the analysis is to find the smallest number of variables that describes the state sale of cigarettes meaningfully and adequately. You might want to consider among others partial regression plot, interaction terms, outliers and influential cases analysis, Box-Cox transformation, and explanation of your final model.

```

table_2016f1 <- readxl::read_xlsx("qe_lab/cigcons.xlsx")
table_2016f1$State <- as.factor(table_2016f1$State)
str(table_2016f1)
## Classes 'tbl_df', 'tbl' and 'data.frame':   51 obs. of  8 variables:
## $ State      : Factor w/ 51 levels "AK","AL","AR",...: 2 1 4 3 5 6 7 9 8 10 ...
## $ Age        : num  27 22.9 26.3 29.1 28.1 26.2 29.1 26.8 28.4 32.3 ...
## $ Education  : num  41.3 66.7 58.1 39.9 62.6 63.9 56 54.6 55.2 52.6 ...
## $ Income     : num  2948 4644 3655 2878 4493 ...
## $ perblack   : num  26.2 3 3 18.3 7 3 6 14.3 71.1 15.3 ...
## $ perfem     : num  51.7 45.7 50.8 51.5 50.8 50.7 51.5 51.3 53.5 51.8 ...
## $ price      : num  42.7 41.8 38.5 38.8 39.7 31.1 45.5 41.3 32.6 43.8 ...
## $ Scig       : num  89.8 121.3 115.2 100.3 123 ...

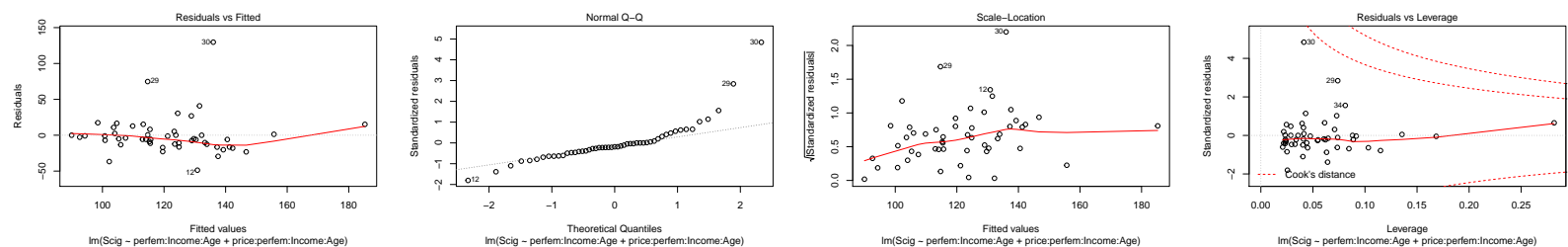
library(GGally)
## Registered S3 method overwritten by 'GGally':
##   method from
##   +.gg      ggplot2
##
## Attaching package: 'GGally'
## The following object is masked from 'package:dplyr':
##
##   nasa
ggpairs(table_2016f1[, -1])

```



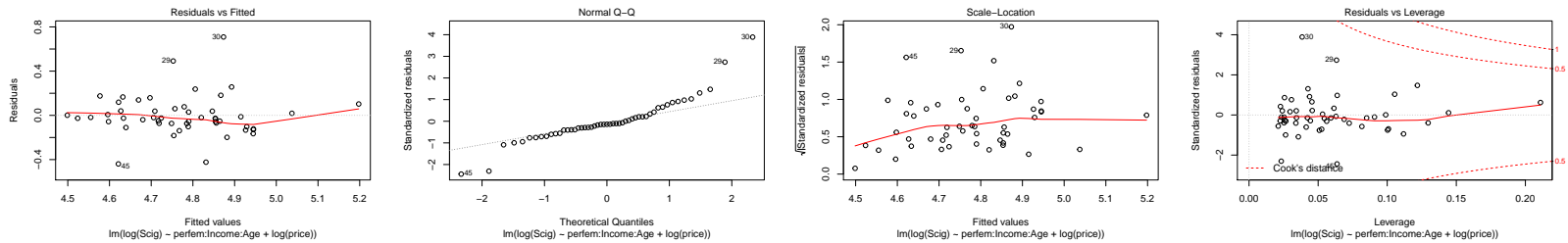
```
model_2016f1 <- lm(Scig~price*perfem*perblack*Income*Education*Age, table_2016f1)
ols_step_both_aic(model_2016f1)
ols_step_both_p(model_2016f1)
```

```
##          perfem:Income:Age perfem:Income:Age:price
##          5.380033          5.380033
##
## Call:
## lm(formula = Scig ~ perfem:Income:Age + price:perfem:Income:Age,
##     data = table_2016f1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -48.743 -12.457  -4.995   3.835  129.705
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    4.067e+01  2.127e+01  1.912  0.06187 .
## perfem:Income:Age  3.847e-05  8.784e-06  4.379  6.43e-05 ***
## perfem:Income:Age:price -6.053e-07  1.770e-07 -3.420  0.00129 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 27.36 on 48 degrees of freedom
## Multiple R-squared:  0.3013, Adjusted R-squared:  0.2721
## F-statistic: 10.35 on 2 and 48 DF, p-value: 0.0001835
## Analysis of Variance Table
##
## Response: Scig
##
##              Df Sum Sq Mean Sq F value    Pr(>F)
## perfem:Income:Age      1    6735   6734.6    8.9961 0.004279 **
## perfem:Income:Age:price 1    8758   8757.6   11.6985 0.001286 **
## Residuals            48   35933    748.6
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



```
##          log(price) perfem:Income:Age
##          1.066268      1.066268
##
## Call:
## lm(formula = log(Scig) ~ perfem:Income:Age + log(price), data = table_2016f1)
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.43922 -0.07364 -0.02540  0.05006  0.70893
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   7.407e+00  8.503e-01   8.711 1.89e-11 ***
## log(price)    -8.993e-01  2.405e-01  -3.739 0.000493 ***
## perfem:Income:Age 1.197e-07  2.657e-08   4.506 4.24e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1859 on 48 degrees of freedom
## Multiple R-squared:  0.3651, Adjusted R-squared:  0.3386
## F-statistic: 13.8 on 2 and 48 DF,  p-value: 1.843e-05
## Analysis of Variance Table
##
## Response: log(Scig)
##              Df Sum Sq Mean Sq F value    Pr(>F)
## log(price)      1  0.25199  0.25199    7.2931  0.009534 **
## perfem:Income:Age 1  0.70156  0.70156   20.3043  4.236e-05 ***
## Residuals      48  1.65850  0.03455
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



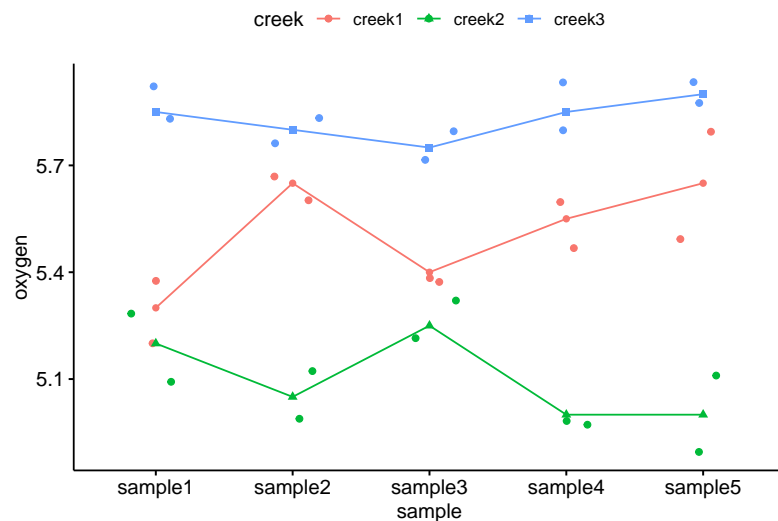
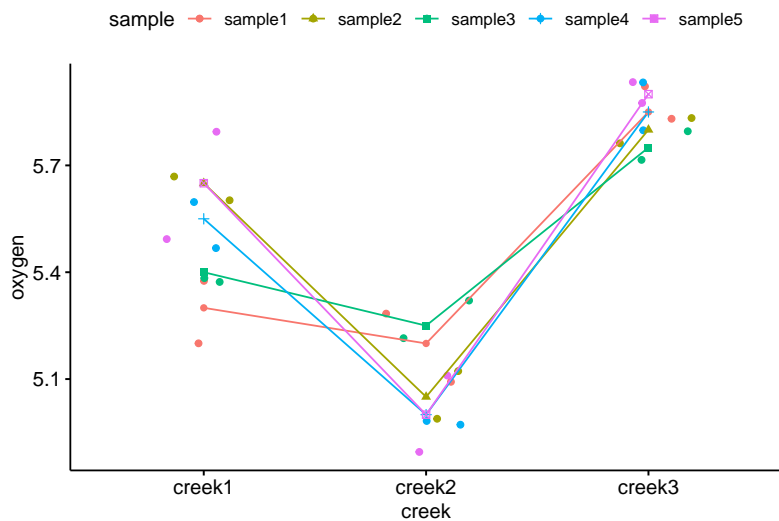
## 2016F2

An experiment is conducted to compare the water quality of three creeks in an area. Five water samples are selected from each creek. Each sample is divided into two parts, and the dissolved oxygen content is measured for each part. (Higher dissolved oxygen contents indicate higher water quality.) The results are given as follows:

Creek/Water Sample	1	2	3	4	5	
1	5.2,	5.4	5.6, 5.7	5.4, 5.4	5.6, 5.5	5.8, 5.5
2	5.1,	5.3	5.1, 5.0	5.3, 5.2	5.0, 5.0	4.9, 5.1
3	5.9,	5.8	5.8, 5.8	5.7, 5.8	5.8, 5.9	5.9, 5.9

- a. Write down an appropriate model with assumptions (including normality).
- One-stage nested design
- $$y = \mu + \tau_i + \beta_{j(i)} + \varepsilon_{k(ij)}, i = 1, 2, 3; j = 1, 2, 3, 4, 5; k = 1, 2$$
- b. Find the ANOVA table for the data.
- c. Perform the F-test comparing the creeks using a .05 level.
- d. Perform a Tukey multiple comparison on the creeks using a .05 level.

```
creek1 <- c(5.2, 5.4, 5.6, 5.7, 5.4, 5.4, 5.6, 5.5, 5.8, 5.5)
creek2 <- c(5.1, 5.3, 5.1, 5.0, 5.3, 5.2, 5.0, 5.0, 4.9, 5.1)
creek3 <- c(5.9, 5.8, 5.8, 5.8, 5.7, 5.8, 5.8, 5.9, 5.9, 5.9)
library(tidyverse)
table_2016f2 <- gather(data.frame(creek1, creek2, creek3), creek, oxygen)
table_2016f2$creek <- as.factor(table_2016f2$creek)
table_2016f2$sample <- as.factor(c(rep("sample1", 2), rep("sample2", 2), rep("sample3", 2), rep("sample4", 2), rep("sample5", 2)))
table_2016f2$rep <- as.factor(rep(c("rep1", "rep2"), 15))
str(table_2016f2)
## 'data.frame':   30 obs. of  4 variables:
## $ creek : Factor w/ 3 levels "creek1","creek2",...: 1 1 1 1 1 1 1 1 1 1 ...
## $ oxygen: num  5.2 5.4 5.6 5.7 5.4 5.4 5.6 5.5 5.8 5.5 ...
## $ sample: Factor w/ 5 levels "sample1","sample2",...: 1 1 2 2 3 3 4 4 5 5 ...
## $ rep   : Factor w/ 2 levels "rep1","rep2": 1 2 1 2 1 2 1 2 1 2 ...
library(ggpubr)
ggline(table_2016f2, "creek", "oxygen", add = c("mean", "jitter"), color = "sample", shape = "sample")
ggline(table_2016f2, "sample", "oxygen", add = c("mean", "jitter"), color = "creek", shape = "creek")
```

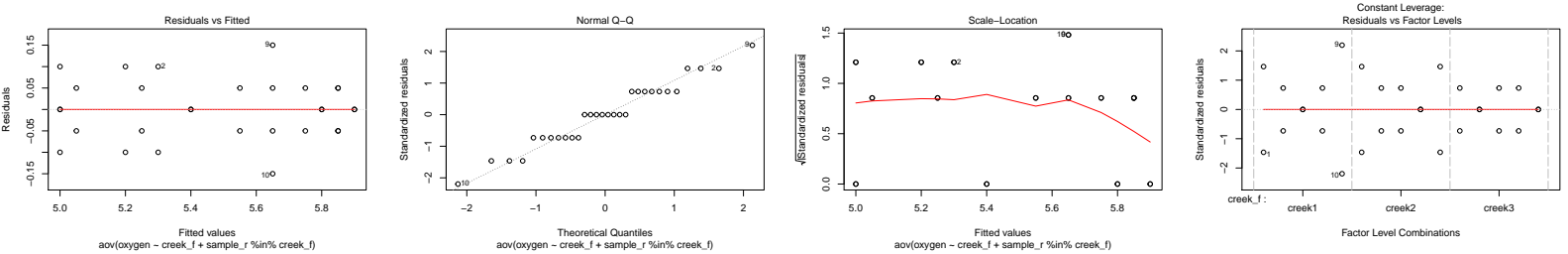


```
model_2016f2_1 <- lm(oxygen~creek/sample,table_2016f2)
anova(model_2016f2_1)
## Analysis of Variance Table
##
## Response: oxygen
##              Df Sum Sq Mean Sq  F value    Pr(>F)
## creek         2  2.678  1.33900  143.4643 1.665e-10 ***
## creek:sample  12  0.330  0.02750   2.9464  0.02559 *
## Residuals    15  0.140  0.00933
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

library(lme4)
model_2016f2_2 <- lmer(oxygen~creek+(1|creek:sample),table_2016f2)
summary(model_2016f2_2)
## Linear mixed model fit by REML ['lmerMod']
## Formula: oxygen ~ creek + (1 | creek:sample)
## Data: table_2016f2
##
## REML criterion at convergence: -29.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.77284 -0.43208 -0.06556  0.52633  2.04448
##
## Random effects:
## Groups Name Variance Std.Dev.
## creek:sample (Intercept) 0.009083 0.09531
## Residual 0.009333 0.09661
## Number of obs: 30, groups: creek:sample, 15
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  5.51000    0.05244 105.071
## creekcreek2 -0.41000    0.07416  -5.528
## creekcreek3  0.32000    0.07416   4.315
##
## Correlation of Fixed Effects:
##      (Intr) crkcr2
## creekcreek2 -0.707
## creekcreek3 -0.707  0.500
anova(model_2016f2_2)
## Analysis of Variance Table
##              Df Sum Sq Mean Sq F value
## creek         2  0.90889  0.45444  48.691
pf(anova(model_2016f2_2)$'F value',df1=anova(model_2016f2_2)$'Df',df2=12, lower.tail = F)
## [1] 1.743538e-06
confint(model_2016f2_2)
## Computing profile confidence intervals ...
##              2.5 %      97.5 %
## .sig01      0.00000000  0.1425879
## .sigma      0.07016639  0.1450729
## (Intercept) 5.41185731  5.6081427
## creekcreek2 -0.54879472 -0.2712053
## creekcreek3  0.18120528  0.4587947
library(GAD)
table_2016f2$creek_f <- as.fixed(table_2016f2$creek)
table_2016f2$sample_r <- as.random(table_2016f2$sample)
model_2016f2_3 <- aov(oxygen~creek_f+sample_r%in%creek_f, table_2016f2)
gad(model_2016f2_3)
## Analysis of Variance Table
##
## Response: oxygen
```

```
##               Df Sum Sq Mean Sq F value    Pr(>F)
## creek_f        2   2.678   1.33900   48.6909 1.743e-06 ***
## creek_f:sample_r 12  0.330   0.02750    2.9464 0.02559  *
## Residual       15   0.140   0.00933
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
plot(model_2016f2_3)
```



```
library(emmeans)
library(kableExtra)
kable(test(lsmmeans(model_2016f2_2,~creek,adjust=c("tukey"))))
```

creek	lsmean	SE	df	t.ratio	p.value
creek1	5.51	0.0524406	12	105.07124	0
creek2	5.10	0.0524406	12	97.25288	0
creek3	5.83	0.0524406	12	111.17339	0

```
kable(pairs(lsmmeans(model_2016f2_2,~creek,adjust=c("tukey"))))
```

contrast	estimate	SE	df	t.ratio	p.value
creek1 - creek2	0.41	0.0741622	12	5.528421	0.0003541
creek1 - creek3	-0.32	0.0741622	12	-4.314865	0.0026695
creek2 - creek3	-0.73	0.0741622	12	-9.843287	0.0000012

```
kable(TukeyHSD(model_2016f2_3,conf.level=0.95)$creek_f)
```

	diff	lwr	upr	p adj
creek2-creek1	-0.41	-0.5222235	-0.2977765	3.0e-07
creek3-creek1	0.32	0.2077765	0.4322235	6.2e-06
creek3-creek2	0.73	0.6177765	0.8422235	0.0e+00

```
# for reference
cre_sam <- pairs(lsmmeans(model_2016f2_1,~creek|sample))
sam_cre <- pairs(lsmmeans(model_2016f2_1,~sample|creek))
kable(test(rbind(cre_sam,sam_cre),adjust="tukey"),format="latex")>%kable_styling("condensed",full_width=F,font_size=10)
cre_sam <- pairs(lsmmeans(model_2016f2_3,~creek_f|sample_r))
sam_cre <- pairs(lsmmeans(model_2016f2_3,~sample_r|creek_f))
kable(test(rbind(cre_sam,sam_cre),adjust="tukey"),format="latex")>%kable_styling("condensed",full_width=F,font_size=10)
```

## 2017S

Brad Crain, Jong Sung Kim\*

### 2017SR1

2015F1

Find the best model for predicting Y based on X1 and X2. Y is the amount of profit that a company makes in a month. X1 is the number of months that the company has been in business. X2 is the amount spent on advertising. Consider as predictors all possible linear and quadratic terms (X1, X1<sup>2</sup>, X2, X2<sup>2</sup>, and X1X2). Consider possible transformations of Y. Include all appropriate diagnostics. When you have found your “best” model, predict a new Y when X1 = 20 and X2 = /\$1,500, giving a 95% prediction interval. The data set, shown below, appears in “Profits.xlsx”.

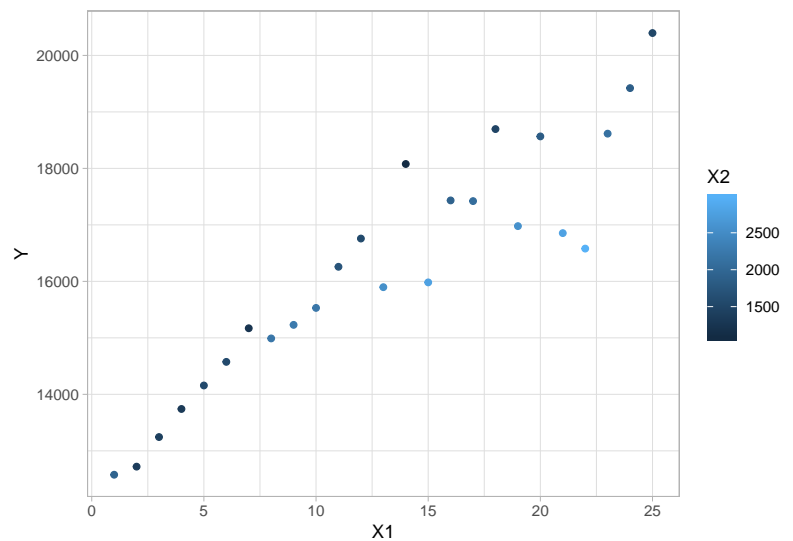
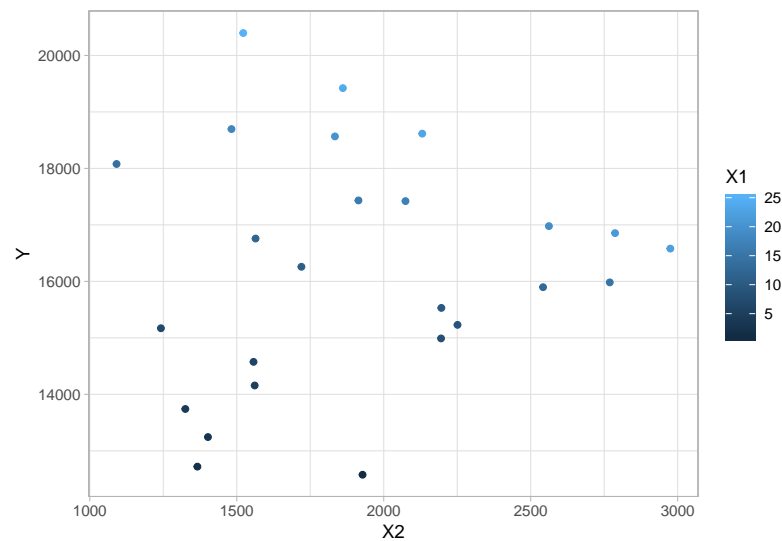
```
table_2017sr1 <- readxl::read_xlsx("qe_lab/Profits_2017s.xlsx")
# table_2017sr1$X1 <- as.factor(table_2017sr1$X1)
str(table_2017sr1)

## Classes 'tbl_df', 'tbl' and 'data.frame':   25 obs. of  3 variables:
## $ X1: num  1 2 3 4 5 6 7 8 9 10 ...
## $ X2: num 1928 1366 1402 1325 1561 ...
## $ Y : num 12577 12720 13244 13741 14157 ...

summary(table_2017sr1)

##      X1      X2      Y
## Min.   : 1   Min.   :1091   Min.   :12577
## 1st Qu.: 7   1st Qu.:1522   1st Qu.:14990
## Median :13   Median :1861   Median :16258
## Mean   :13   Mean   :1914   Mean   :16235
## 3rd Qu.:19   3rd Qu.:2196   3rd Qu.:17433
## Max.   :25   Max.   :2975   Max.   :20396
```

```
library(ggplot2)
ggplot(table_2017sr1, aes(X2,Y,color=X1))+geom_point()+theme_light()
ggplot(table_2017sr1, aes(X1,Y,color=X2))+geom_point()+theme_light()
```



```
model_2017sr1 <- lm(Y^2~X1+X2, table_2017sr1)
# car::vif(model_2017sr1)
summary(model_2017sr1)

##
## Call:
## lm(formula = Y^2 ~ X1 + X2, data = table_2017sr1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -30805386 -9969025  3791394 10176772 20218197
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 231392970  11535085  20.060 1.25e-15 ***
## X1           9924495    455383   21.794 < 2e-16 ***
## X2          -48414      6402   -7.563 1.48e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14900000 on 22 degrees of freedom
## Multiple R-squared:  0.956, Adjusted R-squared:  0.952
## F-statistic: 239 on 2 and 22 DF, p-value: 1.194e-15

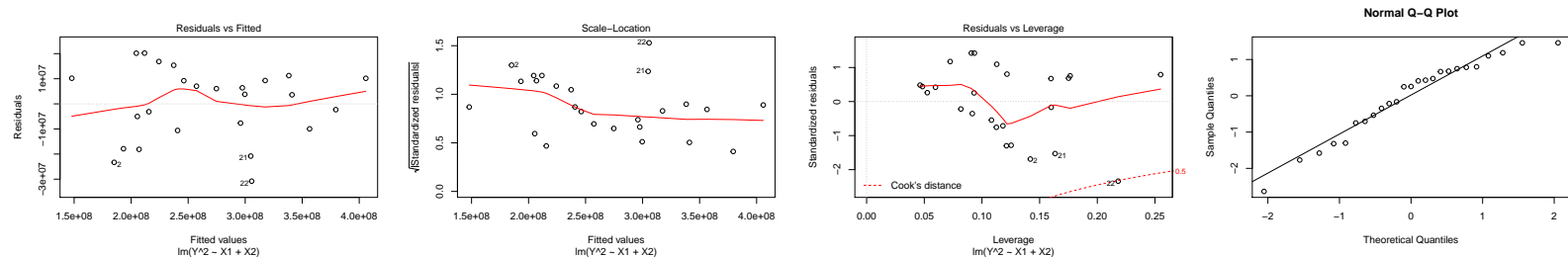
anova(model_2017sr1)

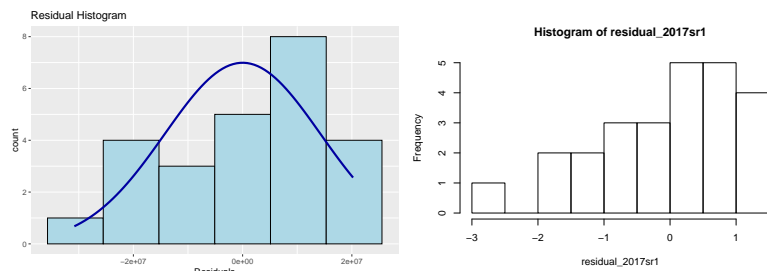
## Analysis of Variance Table
##
## Response: Y^2
##
##      Df Sum Sq Mean Sq F value    Pr(>F)
## X1     1 9.3403e+16 9.3403e+16 420.892 7.822e-16 ***
## X2     1 1.2692e+16 1.2692e+16  57.192 1.482e-07 ***
## Residuals 22 4.8822e+15 2.2192e+14
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

plot(model_2017sr1,c(1,3,5))
residual_2017sr1 <- rstudent(model_2017sr1)
qqnorm(residual_2017sr1)
qqline(residual_2017sr1)
olsrr::ols_plot_resid_hist(model_2017sr1)

## Registered S3 method overwritten by 'lava':
##   method      from
## print.equivalence partitions

hist(residual_2017sr1)
```





```
sqrt(predict(model_2017sr1,newdata = data.frame(X1=20,X2=1500),interval = "prediction", level = 0.95))
##          fit          lwr          upr
## 1 18901.39 18003.89 19758.17
```

## 2017SD1

Review the data provided in 'NBalance.xlsx'. Note, there were nine distinct treatments [Feed Rations] and three distinct animals. An experimental design was used to examine the means differences in the Nitrogen balance in ruminants. Provide the following in your answer

1. Which design was used, include the required parameters of the experimental design [ $t; b; k; r; \lambda$ ]

BIBD

$$y = \mu + \tau_i + \beta_j + \varepsilon_{ij} + \varepsilon$$

Treatment(Rations) $a = 9$ ,

Replication $r = 3$ ,

Block(animals) $b = 3$ ,

Block size $k = 9$

$\lambda = 3$

2. An appropriate ANOVA

3. A TukeyHSD analysis of the proper means differences

4. Conclusions on the impact of Feed Rations on Nitrogen Balance in Ruminants

Source: J.L. Gill (1978), Design and analysis of experiments in the animal and medical sciences, Vol2. Ames, Iowa: Iowa State University Press

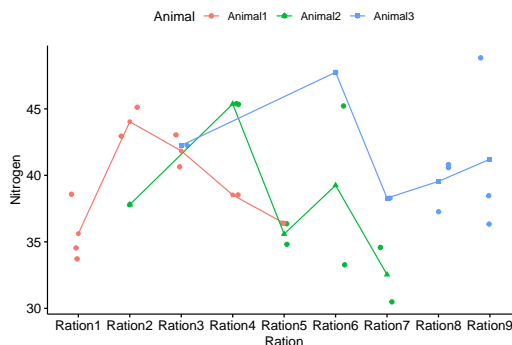
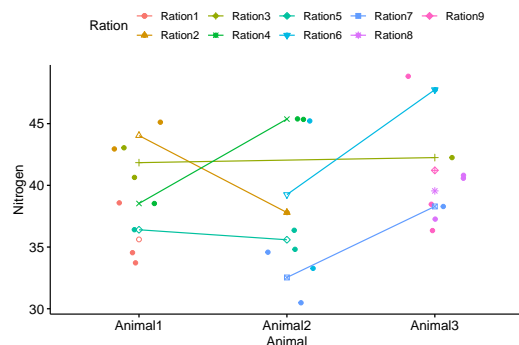
```
## Classes 'tbl_df', 'tbl' and 'data.frame': 27 obs. of 4 variables:
## $ Block : Factor w/ 9 levels "Blk1","Blk2",...: 1 1 1 2 2 2 3 3 3 4 ...
## $ Animal : Factor w/ 3 levels "Animal1","Animal2",...: 1 2 3 1 2 3 1 2 3 1 ...
## $ Ration : Factor w/ 9 levels "Ration1","Ration2",...: 1 2 3 1 4 6 1 5 7 2 ...
## $ Nitrogen: num 33.7 37.8 42.2 38.6 45.4 ...
```

```
library(ggpubr)
ggline(table_2017sd1, "Animal", "Nitrogen", add = c("mean", "jitter"), color = "Ration", shape = "Ration")
```

```
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
```

```
ggline(table_2017sd1, "Ration", "Nitrogen", add = c("mean", "jitter"), color = "Animal", shape = "Animal")
```

```
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
```



```
model_2017sd1 <- aov(Nitrogen~Animal+Ration, table_2017sd1)
summary(model_2017sd1)
```

```
##          Df Sum Sq Mean Sq F value Pr(>F)
## Animal    2  42.23   21.12    1.237  0.317
## Ration    8 274.92   34.36    2.013  0.111
## Residuals 16  273.10   17.07
```

```
anova(model_2017sd1)
```

```
## Analysis of Variance Table
##
## Response: Nitrogen
##           Df Sum Sq Mean Sq F value Pr(>F)
## Animal      2  42.233   21.117   1.2372 0.3165
## Ration       8 274.919   34.365   2.0133 0.1112
## Residuals   16 273.096   17.069
TukeyHSD(model_2017sd1,conf.level = 0.95)
##      Tukey multiple comparisons of means
##      95% family-wise confidence level
##
## Fit: aov(formula = Nitrogen ~ Animal + Ration, data = table_2017sd1)
##
## $Animal
##           diff           lwr           upr           p adj
## Animal2-Animal1 -1.137778 -6.163134 3.887579 0.8304041
## Animal3-Animal1  1.894444 -3.130912 6.919801 0.6039032
## Animal3-Animal2  3.032222 -1.993134 8.057579 0.2921780
##
## $Ration
##           diff           lwr           upr           p adj
## Ration2-Ration1  6.7192593 -5.281040 18.719558 0.5684881
## Ration3-Ration1  5.7285185 -6.271780 17.728817 0.7397514
## Ration4-Ration1  8.2318519 -3.768447 20.232151 0.3269033
## Ration5-Ration1  1.0018519 -10.998447 13.002151 0.9999968
## Ration6-Ration1  6.5937037 -5.406595 18.594003 0.5905654
## Ration7-Ration1 -1.0362963 -13.036595 10.964003 0.9999958
## Ration8-Ration1  2.0388889 -9.961410 14.039188 0.9993103
## Ration9-Ration1  3.7022222 -8.298077 15.702521 0.9663240
## Ration3-Ration2 -0.9907407 -12.991040 11.009558 0.9999970
## Ration4-Ration2  1.5125926 -10.487706 13.512892 0.9999235
## Ration5-Ration2 -5.7174074 -17.717706  6.282892 0.7415690
## Ration6-Ration2 -0.1255556 -12.125854 11.874743 1.0000000
## Ration7-Ration2 -7.7555556 -19.755854  4.244743 0.3959375
## Ration8-Ration2 -4.6803704 -16.680669  7.319929 0.8870802
## Ration9-Ration2 -3.0170370 -15.017336  8.983262 0.9901325
## Ration4-Ration3  2.5033333 -9.496966 14.503632 0.9971040
## Ration5-Ration3 -4.7266667 -16.726966  7.273632 0.8818327
## Ration6-Ration3  0.8651852 -11.135114 12.865484 0.9999990
## Ration7-Ration3 -6.7648148 -18.765114  5.235484 0.5605018
## Ration8-Ration3 -3.6896296 -15.689929  8.310669 0.9669660
## Ration9-Ration3 -2.0262963 -14.026595  9.974003 0.9993402
## Ration5-Ration4 -7.2300000 -19.230299  4.770299 0.4805005
## Ration6-Ration4 -1.6381481 -13.638447 10.362151 0.9998610
## Ration7-Ration4 -9.2681481 -21.268447  2.732151 0.2059952
## Ration8-Ration4 -6.1929630 -18.193262  5.807336 0.6609465
## Ration9-Ration4 -4.5296296 -16.529929  7.470669 0.9032344
## Ration6-Ration5  5.5918519 -6.408447 17.592151 0.7618186
## Ration7-Ration5 -2.0381481 -14.038447  9.962151 0.9993120
## Ration8-Ration5  1.0370370 -10.963262 13.037336 0.9999958
## Ration9-Ration5  2.7003704 -9.299929 14.700669 0.9951866
## Ration7-Ration6 -7.6300000 -19.630299  4.370299 0.4154158
## Ration8-Ration6 -4.5548148 -16.555114  7.445484 0.9006356
## Ration9-Ration6 -2.8914815 -14.891780  9.108817 0.9924771
## Ration8-Ration7  3.0751852 -8.925114 15.075484 0.9888726
## Ration9-Ration7  4.7385185 -7.261780 16.738817 0.8804680
## Ration9-Ration8  1.6633333 -10.336966 13.663632 0.9998443
```

## 2017F

Robert Fountain\*, Daniel Taylor-Rodriguez

### 2017F1

#### 2016S1

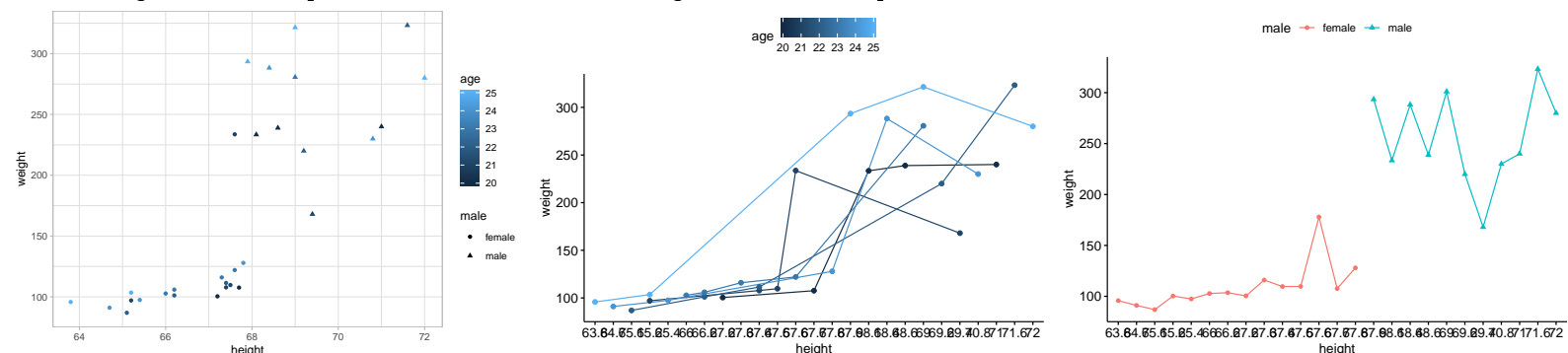
Find the best model for predicting Y (weight) based on X1 (age), X2 (height), and X3 (indicator for male). Consider as predictors all possible linear and quadratic terms. Consider possible transformations of Y. Include all appropriate diagnostics. When you have found your “best” model, predict a new Y when X1 = 26, X2 = 70, and X3 = 1, giving a 95% prediction interval. The data set, shown below, appears in “RegressionFall17.xlsx”.

```
table_2017f1 <- readxl::read_xlsx("qe_lab/RegressionFall17.xlsx")[-1,]
table_2017f1$weight <- round(as.numeric(table_2017f1$weight),2)
table_2017f1$age <- as.numeric(table_2017f1$age)
table_2017f1$height <- round(as.numeric(table_2017f1$height),2)
table_2017f1$male <- factor(table_2017f1$male, labels = c("female", "male"))
str(table_2017f1)
## Classes 'tbl_df', 'tbl' and 'data.frame':   30 obs. of  4 variables:
## $ weight: num  240 100 233 108 239 ...
## $ age : num  20 20 20 20 20 21 21 21 21 21 ...
## $ height: num  71 67.2 68.1 67.7 68.6 65.2 67.6 67.4 67.5 69.4 ...
## $ male : Factor w/ 2 levels "female","male": 2 1 2 1 2 1 1 1 1 2 ...
library(ggplot2)
ggplot(table_2017f1, aes(height,weight,color=age,shape=male))+geom_point()+theme_light()
library(ggpubr)
ggline(table_2017f1,"height","weight",add=c("mean","jitter"),color="age")
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
```



[illegible]

```
ggline(table_2017f1,"height","weight",add=c("mean","jitter"),color="male",shape = "male")
```

[illegible]

```
model_2017f1 <- lm(weight~height*age*male,table_2017f1)
```

```
library(olsrr)
```

```
ols_step_both_aic(model_2017f1)
```

## ## Stepwise Selection Method

## \_\_\_\_\_  
" "

```
##
## Candidate Terms:
```

```
##          canal_dose
##
## 1 . height
```

```
## 1 : sex
## 2 : age
```

## 3 . male

```
## 4 . height:age
```

```
## 5 . height:male
## 6 . age:male
## 7 . height:age:male
##
## Variables Entered/Removed:
## - height:age:male added
## - age:male added
## - height:age added
##
## No more variables to be added or removed.
##
##
## Stepwise Summary
## -----
## Variable          Method      AIC      RSS      Sum Sq      R-Sq      Adj. R-Sq
## height:age:male    addition    304.169    34051.024    163429.310    0.82757    0.81480
## age:male           addition    303.786    29423.052    168057.281    0.85101    0.82717
## height:age         addition    303.786    29423.052    168057.281    0.85101    0.82717
## -----
ols_step_both_p(model_2017f1)
## Stepwise Selection Method
## -----
## Candidate Terms:
## 1. height
## 2. age
## 3. male
## 4. height:age
## 5. height:male
## 6. age:male
## 7. height:age:male
##
## We are selecting variables based on p value...
##
## Variables Entered/Removed:
## - male added
## - age:male added
## - age added
## - height added
##
## No more variables to be added/removed.
##
## Final Model Output
## -----
##
## Model Summary
## -----
## R              0.921      RMSE              34.629
## R-Squared       0.848      Coef. Var      20.228
## Adj. R-Squared  0.824      MSE           1199.151
## Pred R-Squared  0.788      MAE           20.901
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
## ANOVA
## -----
## Sum of Squares      DF      Mean Square      F      Sig.
## Regression    167501.551      4      41875.388    34.921    0.0000
## Residual      29978.783     25      1199.151
## Total        197480.333     29
## -----
##
## Parameter Estimates
## -----
## model      Beta      Std. Error      Std. Beta      t      Sig      lower      upper
## (Intercept) -321.525      435.768      -0.738      -0.738    0.467    -1219.006    575.956
## malemale    -191.733      172.899      -1.158      -1.109    0.278    -547.826    164.360
## age         -1.245      5.847      -0.026      -0.213    0.833    -13.287    10.797
## height      6.951      5.487      0.172      1.267    0.217     -4.349    18.252
## malemale:age 14.058      7.892      1.929      1.781    0.087     -2.195    30.311
## -----
##
## Stepwise Selection Summary
## -----
## Step      Variable      Added/Removed      R-Square      Adj. R-Square      C(p)      AIC      RMSE
## 1          male      addition      0.795      0.788      6.9980      307.3529      38.0198
## 2      age:male      addition      0.838      0.820      2.0100      304.2144      35.0293
## 3          age      addition      0.838      0.820      4.0100      304.2144      35.0293
## 4      height      addition      0.848      0.824      4.4410      304.3477      34.6288
## -----
```

```
model_2017f1_1 <- lm(weight~height+age:male,table_2017f1)
model_2017f1_2 <- lm(log(weight)~height+age:male,table_2017f1)
car::vif(model_2017f1_2)

##          GVIF Df GVIF^(1/(2*Df))
## height    2.8472 1      1.687365
## age:male  2.8472 2      1.298986
```

```
summary(model_2017f1_2)

##
## Call:
## lm(formula = log(weight) ~ height + age:male, data = table_2017f1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35019 -0.06823 -0.03331  0.08138  0.70476
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.592786    2.103225   0.282   0.7803
## height        0.058601    0.028409   2.063   0.0493 *
## age:malefemale 0.009281    0.021315   0.435   0.6668
## age:malemale  0.038784    0.020107   1.929   0.0647 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1852 on 26 degrees of freedom
## Multiple R-squared:  0.8599, Adjusted R-squared:  0.8438
## F-statistic: 53.2 on 3 and 26 DF, p-value: 3.121e-11
```

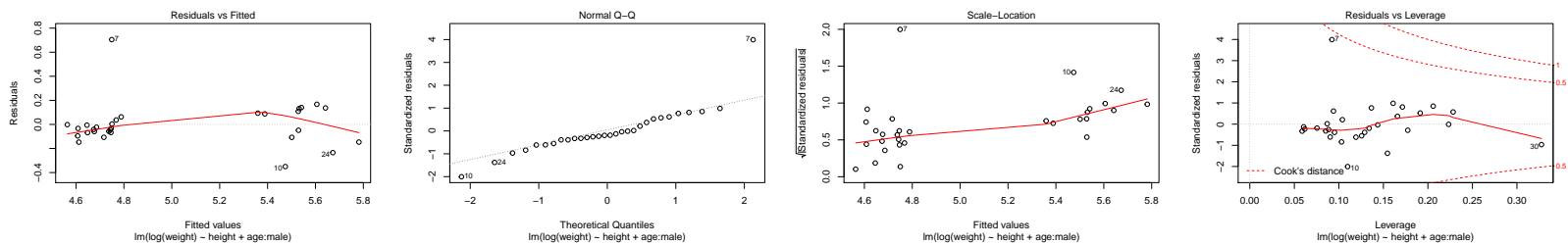
```
anova(model_2017f1_2)

## Analysis of Variance Table
##
## Response: log(weight)
##           Df Sum Sq Mean Sq F value    Pr(>F)
## height     1 4.1007  4.1007 119.540 3.203e-11 ***
## age:male    2 1.3744  0.6872  20.033 5.432e-06 ***
## Residuals  26 0.8919  0.0343
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
ols_regress(model_2017f1_2)

##
##                               Model Summary
## -----
## R                               0.927          RMSE                0.185
## R-Squared                       0.860          Coef. Var          3.679
## Adj. R-Squared                  0.844          MSE                0.034
## Pred R-Squared                  0.819          MAE                0.112
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##                               ANOVA
## -----
##              Sum of      DF      Mean Square      F      Sig.
## Regression    5.475         3         1.825    53.202  0.0000
## Residual      0.892        26         0.034
## Total         6.367        29
## -----
##
##                               Parameter Estimates
## -----
##      model      Beta      Std. Error      Std. Beta      t      Sig.      lower      upper
## (Intercept)   0.593         2.103         0.255      0.282   0.780      -3.730      4.916
## height        0.059         0.028         0.255      2.063   0.049       0.000      0.117
## age:malefemale 0.009         0.021         0.223      0.435   0.667      -0.035      0.053
## age:malemale  0.039         0.020         0.937      1.929   0.065      -0.003      0.080
## -----
```

```
plot(model_2017f1_2)
```



```
predict(model_2017f1_2, newdata=data.frame(age= 26, height= 70, male= "male"),interval = "prediction",level = 0.95)

##      fit      lwr      upr
## 1 5.703233 5.278615 6.127851
```

## 2017F2

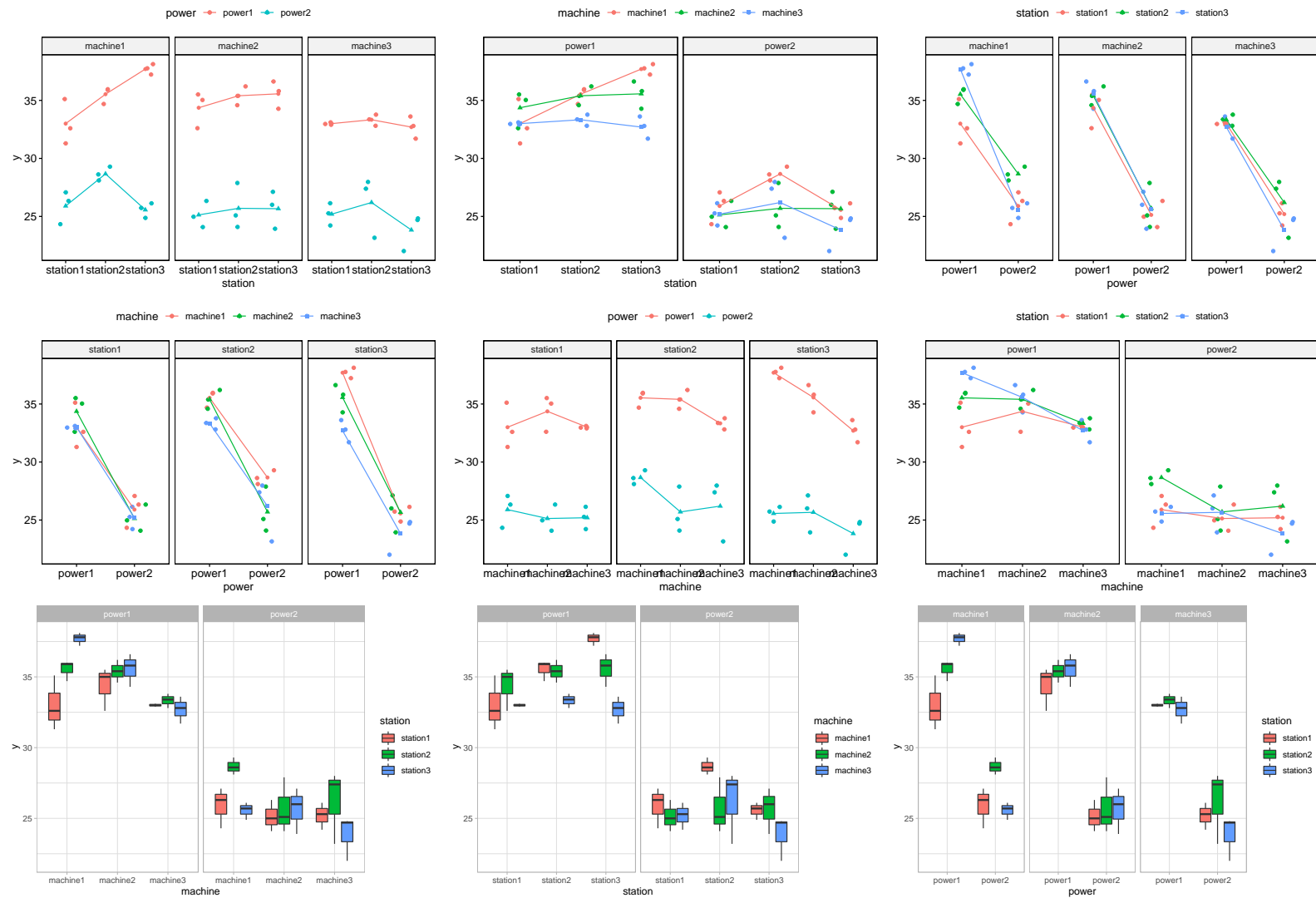
A process engineer is testing the yield of a product manufactured on three specific machines. Each machine can be operated at fixed high and low power settings, although the actual settings differ from one machine to the next. Furthermore, a machine has three stations on which the product is formed, and these are the same for each machine. An experiment is conducted in which each machine is tested at both power settings, and three observations on yield are taken from each station. The runs are made in random order. Analyze this experiment. The data set, shown below, appears in "DesignFall17.xlsx".

```
DesignFall17 <- readxl::read_excel("qe_lab/DesignFall17.xlsx")
```

```
## New_names:
## * --> ...1
## * --> ...2
## * --> ...4
## * --> ...5
## * --> ...6
## * ... and 4 more problems
```

```
library(tidyverse)
table_2017f2 <- gather(DesignFall17[c(2:4,6:8),c(2:4,6:8,10:12)])
names(table_2017f2) <- c("machine","y")
table_2017f2 <- table_2017f2[c("y","machine")]
table_2017f2$machine <- as.factor(c(rep("machine1",18),rep("machine2",18),rep("machine3",18)))
table_2017f2$station <- as.factor(rep(c(rep("station1",6),rep("station2",6),rep("station3",6)),3))
table_2017f2$power <- as.factor(rep(c(rep("power1",3),rep("power2",3)),9))
str(table_2017f2)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame': 54 obs. of 4 variables:
## $ y : num 35.1 31.3 32.6 24.3 26.3 27.1 34.7 35.9 36 28.1 ...
## $ machine: Factor w/ 3 levels "machine1","machine2",...: 1 1 1 1 1 1 1 1 1 1 ...
## $ station: Factor w/ 3 levels "station1","station2",...: 1 1 1 1 1 1 2 2 2 2 ...
## $ power : Factor w/ 2 levels "power1","power2": 1 1 1 2 2 2 1 1 1 2 ...
```



```
model_2017f2 <- lm(y~power*station*machine, table_2017f2)
summary(model_2017f2)
```

```
##
## Call:
## lm(formula = y ~ power * station * machine, data = table_2017f2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.0000 -0.6500  0.1000  0.7583  2.2000
##
```

```
## Coefficients:
##               Estimate Std. Error t value
## (Intercept)    3.300e+01  7.562e-01  43.639
## powerpower2   -7.100e+00  1.069e+00  -6.639
## stationstation2  2.533e+00  1.069e+00   2.369
## stationstation3  4.700e+00  1.069e+00   4.395
## machinemachine2  1.367e+00  1.069e+00   1.278
## machinemachine3 -2.722e-14  1.069e+00   0.000
## powerpower2:stationstation2  2.333e-01  1.512e+00   0.154
## powerpower2:stationstation3 -5.033e+00  1.512e+00  -3.328
## powerpower2:machinemachine2 -2.133e+00  1.512e+00  -1.411
## powerpower2:machinemachine3 -7.000e-01  1.512e+00  -0.463
## stationstation2:machinemachine2 -1.500e+00  1.512e+00  -0.992
## stationstation3:machinemachine2 -3.500e+00  1.512e+00  -2.314
## stationstation2:machinemachine3 -2.200e+00  1.512e+00  -1.455
## stationstation3:machinemachine3 -5.000e+00  1.512e+00  -3.306
## powerpower2:stationstation2:machinemachine2 -7.000e-01  2.139e+00  -0.327
## powerpower2:stationstation3:machinemachine2  4.367e+00  2.139e+00   2.042
## powerpower2:stationstation2:machinemachine3  4.333e-01  2.139e+00   0.203
## powerpower2:stationstation3:machinemachine3  3.967e+00  2.139e+00   1.855
##
##               Pr(>|t|)
## (Intercept)    < 2e-16 ***
## powerpower2    9.82e-08 ***
## stationstation2  0.02333 *
## stationstation3  9.38e-05 ***
## machinemachine2  0.20945
## machinemachine3  1.00000
## powerpower2:stationstation2  0.87825
## powerpower2:stationstation3  0.00203 **
## powerpower2:machinemachine2  0.16696
## powerpower2:machinemachine3  0.64627
## stationstation2:machinemachine2  0.32792
## stationstation3:machinemachine2  0.02648 *
## stationstation2:machinemachine3  0.15444
## stationstation3:machinemachine3  0.00215 **
## powerpower2:stationstation2:machinemachine2  0.74536
## powerpower2:stationstation3:machinemachine2  0.04858 *
## powerpower2:stationstation2:machinemachine3  0.84059
## powerpower2:stationstation3:machinemachine3  0.07187 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.31 on 36 degrees of freedom
## Multiple R-squared:  0.9486, Adjusted R-squared:  0.9243
## F-statistic: 39.08 on 17 and 36 DF,  p-value: < 2.2e-16
```

```
anova(model_2017f2)
## Analysis of Variance Table
##
## Response: y
##               Df Sum Sq Mean Sq F value Pr(>F)
## power           1 1033.16  1033.16  602.2284 < 2e-16 ***
## station          2   16.98    8.49   4.9489 0.01262 *
## machine           2   37.37   18.68  10.8913 0.00020 ***
## power:station    2   16.30    8.15   4.7514 0.01475 *
## power:machine     2    6.35    3.17   1.8505 0.17180
## station:machine   4   16.60    4.15   2.4195 0.06625 .
## power:station:machine  4   12.91    3.23   1.8806 0.13507
## Residuals       36   61.76    1.72
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
library(lme4)
model_2017f2_1 <- lmer(y~machine*station+(1|machine:power)+(1|machine:power:station),table_2017f2)
summary(model_2017f2_1)$varcor
```

```
## Groups          Name          Std.Dev.
## machine:power:station (Intercept) 1.0251
## machine:power         (Intercept) 6.1612
## Residual              1.3098
```

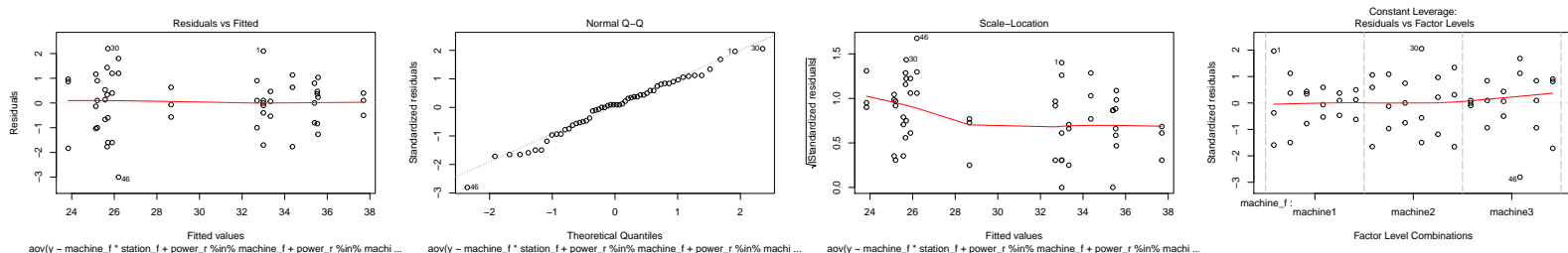
```
anova(model_2016s2_1)
## Analysis of Variance Table
##               Df Sum Sq Mean Sq F value
## machine         2  0.1304   0.0652   0.0380
## station          2  5.9841   2.9921   1.7441
## machine:station  4  5.8511   1.4628   0.8527
pf(anova(model_2017f2_1)$'F value',df1=anova(model_2017f2_1)$'Df',df2=c(3,6,6), lower.tail = F)
## [1] 0.9484030 0.2528741 0.5409705
pander::pander(confint(model_2017f2_1)[1:4,1:2])
```

Computing profile confidence intervals ...

	2.5 %	97.5 %
.sig01	0	1.218
.sig02	2.673	8.783
.sigma	1.057	1.675
(Intercept)	22.2	36.7

```
library(GAD)
table_2017f2$machine_f <- as.fixed(table_2017f2$machine)
table_2017f2$station_f <- as.fixed(table_2017f2$station)
table_2017f2$power_r <- as.random(table_2017f2$power)
model_2017f2_2 <- aov(y~machine_f*station_f+power_r%in%machine_f+power_r%in%machine_f*station_f, table_2017f2)
gad(model_2017f2_2)

## Analysis of Variance Table
##
## Response: y
##
##           Df Sum Sq Mean Sq  F value    Pr(>F)
## machine_f    2   37.37   18.68    0.0539  0.94840
## station_f    2   16.98    8.49    1.7441  0.25287
## machine_f:station_f  4   16.60    4.15    0.8527  0.54097
## machine_f:power_r    3 1039.51  346.50  201.9765 < 2e-16 ***
## machine_f:station_f:power_r  6   29.21    4.87    2.8375  0.02292 *
## Residual      36   61.76    1.72
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



## 2018S

Robert Fountain\*, Daniel Taylor-Rodriguez

## 2018S1

The data for this problem was obtained from research relating children smoking to pulmonary function. Today it is well established that smoking cigarettes is a very unhealthy habit, especially for children; however, this was not well-known in the past. This data corresponds to one of the first studies of the effects of smoking on pulmonary (i.e., lung) function, an observational study of 654 youths aged 3 to 19. The variables in the study are displayed in Table 1 below. The outcome variable is volume, which measures the liters of air exhaled by the child in the first second of a forced breath. Some evidence in the literature suggests that children under age 6 may not understand the instructions of the breath exhalation test, so that the quality of volume measurements for those children is suspect. We are interested in the relationship between smoking, gender and the volume of air exhaled. Smoking is expected to impair pulmonary function (i.e., decrease volume).

Find the best model to predict volume considering as predictors all possible linear, quadratic and pairwise interaction terms. Additionally, consider possible transformations of the response (i.e., volume), and include all relevant diagnostic measures. Once you select the best model, write down and test the hypothesis to determine if the volume is influenced by the smoking status in terms of your best model's parameters. Using this same model, predict the volume for a 16-yearold male smoker who is 61 inches high, and provide a 95% prediction interval. A description of the variables is found in the table below, and the data is included in the file Problem1\_ChildSmoking.xlsx.

Variable Name and Description

age: age of child in years

volume: volume of air in exhaled breath in liters

height: height of child in inches

male=1 if child is male, and =0 otherwise

smoker=1 if child reports that he or she smokes cigarettes regularly, and =0 otherwise

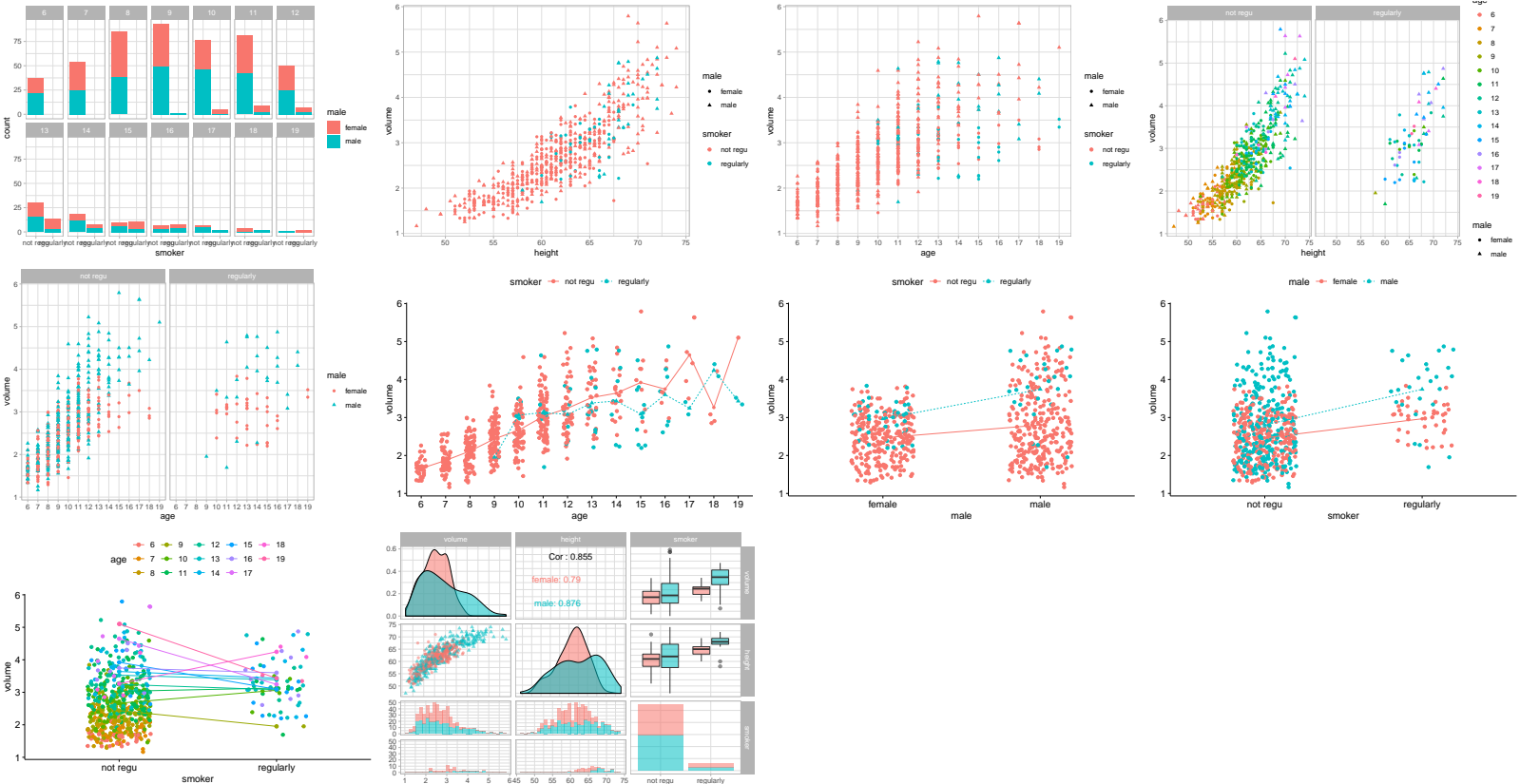
```
table_2018s1 <- readxl::read_xlsx("qe_lab/Problem1_ChildSmoking.xlsx")
table_2018s1_above6 <- table_2018s1[which(table_2018s1$age>5),]
table_2018s1_above6$age <- factor(table_2018s1_above6$age)
table_2018s1_above6$male <- factor(table_2018s1_above6$male, labels = c("female","male"))
table_2018s1_above6$smoker <- factor(table_2018s1_above6$smoker, labels = c("not regu","regularly"))
str(table_2018s1)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':   654 obs. of  5 variables:
## $ age : num  9 8 7 9 9 8 6 6 8 9 ...
## $ volume: num  1.71 1.72 1.72 1.56 1.9 ...
## $ height: num  57 67.5 54.5 53 57 61 58 56 58.5 60 ...
## $ male : num  0 0 0 1 1 0 0 0 0 0 ...
## $ smoker: num  0 0 0 0 0 0 0 0 0 0 ...
```

```
str(table_2018s1_above6)
## Classes 'tbl_df', 'tbl' and 'data.frame':   615 obs. of  5 variables:
## $ age : Factor w/ 14 levels "6","7","8","9",...: 4 3 2 4 4 3 1 1 3 4 ...
## $ volume: num  1.71 1.72 1.72 1.56 1.9 ...
## $ height: num  57 67.5 54.5 53 57 61 58 56 58.5 60 ...
## $ male : Factor w/ 2 levels "female","male": 1 1 1 2 2 1 1 1 1 1 ...
## $ smoker: Factor w/ 2 levels "not regu","regularly": 1 1 1 1 1 1 1 1 1 1 ...
```

```
summary(table_2018s1$height)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      46.00  57.00  61.50  61.14  65.50  74.00
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## Warning in stats::qt(ci/2 + 0.5, data_sum$length - 1): NaNs produced
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



```
model_2018s1 <- lm(volume~height*age*male*smoker,table_2018s1_above6)
ols_step_both_aic(model_2018s1)
ols_step_both_p(model_2018s1)

model_2018s1_2 <- lm(log(volume)~log(height):age:male+smoker,table_2018s1_above6)
summary(model_2018s1_2)
```

```
##
## Call:
## lm(formula = log(volume) ~ log(height):age:male + smoker, data = table_2018s1_above6)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.53620 -0.08805  0.01031  0.08931  0.32758
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -9.55944    0.50972  -18.754  <2e-16 ***
## smokerregularly -0.04161    0.02162   -1.925   0.0547 .
## log(height):age6:malefemale  2.52311    0.12828   19.669  <2e-16 ***
## log(height):age7:malefemale  2.53067    0.12722   19.892  <2e-16 ***
## log(height):age8:malefemale  2.52611    0.12503   20.205  <2e-16 ***
## log(height):age9:malefemale  2.53901    0.12444   20.403  <2e-16 ***
## log(height):age10:malefemale 2.55392    0.12371   20.644  <2e-16 ***
## log(height):age11:malefemale 2.55951    0.12321   20.773  <2e-16 ***
## log(height):age12:malefemale 2.56509    0.12310   20.838  <2e-16 ***
## log(height):age13:malefemale 2.56979    0.12291   20.907  <2e-16 ***
## log(height):age14:malefemale 2.54910    0.12263   20.788  <2e-16 ***
## log(height):age15:malefemale 2.54670    0.12327   20.659  <2e-16 ***
## log(height):age16:malefemale 2.56462    0.12322   20.813  <2e-16 ***
## log(height):age17:malefemale 2.61978    0.12811   20.450  <2e-16 ***
## log(height):age18:malefemale 2.56344    0.12435   20.615  <2e-16 ***
## log(height):age19:malefemale 2.58822    0.12449   20.791  <2e-16 ***
## log(height):age6:malemale    2.53430    0.12861   19.705  <2e-16 ***
## log(height):age7:malemale    2.54115    0.12729   19.964  <2e-16 ***
## log(height):age8:malemale    2.53915    0.12608   20.139  <2e-16 ***
## log(height):age9:malemale    2.54426    0.12419   20.487  <2e-16 ***
## log(height):age10:malemale   2.54385    0.12324   20.642  <2e-16 ***
## log(height):age11:malemale   2.55859    0.12183   21.002  <2e-16 ***
## log(height):age12:malemale   2.56512    0.12138   21.133  <2e-16 ***
```

```
## log(height):age13:malemale      2.58523      0.12074      21.412      <2e-16 ***
## log(height):age14:malemale      2.58262      0.12086      21.368      <2e-16 ***
## log(height):age15:malemale      2.60392      0.12106      21.509      <2e-16 ***
## log(height):age16:malemale      2.59467      0.12097      21.449      <2e-16 ***
## log(height):age17:malemale      2.59767      0.12076      21.511      <2e-16 ***
## log(height):age18:malemale      2.60975      0.12237      21.327      <2e-16 ***
## log(height):age19:malemale      2.61631      0.12363      21.163      <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1404 on 585 degrees of freedom
## Multiple R-squared:  0.7988, Adjusted R-squared:  0.7888
## F-statistic: 80.08 on 29 and 585 DF,  p-value: < 2.2e-16
```

```
anova(model_2018s1_2)
```

```
## Analysis of Variance Table
```

```
## Response: log(volume)
##              Df Sum Sq Mean Sq F value    Pr(>F)
## smoker          1  3.197   3.1974 162.111 < 2.2e-16 ***
## log(height):age:male 28 42.605   1.5216  77.148 < 2.2e-16 ***
## Residuals       585 11.538   0.0197
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
library(olsrr)
```

```
ols_regress(model_2018s1_2)
```

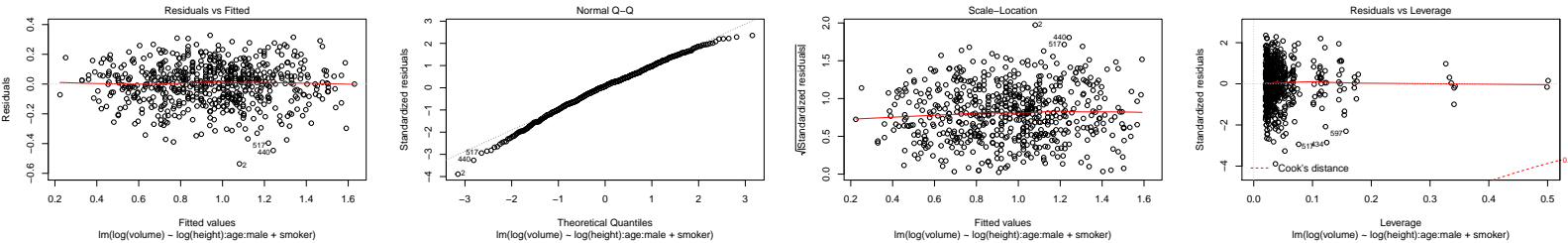
```
##              Model Summary
## -----
## R              0.894          RMSE              0.140
## R-Squared      0.799          Coef. Var         14.772
## Adj. R-Squared 0.789          MSE              0.020
## Pred R-Squared -Inf          MAE              0.108
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##              ANOVA
## -----
##              Sum of Squares      DF      Mean Square      F      Sig.
## -----
## Regression    45.803             29           1.579      80.078    0.0000
## Residual      11.538            585           0.020
## Total         57.341            614
## -----
##
##              Parameter Estimates
## -----
##              model      Beta      Std. Error      Std. Beta      t      Sig      lower      upper
## -----
##              (Intercept) -9.559          0.510              -18.754    0.000    -10.561    -8.558
##              smokerregularly -0.042          0.022              -0.042    -1.925    0.055    -0.084     0.001
##              log(height):age6:malefemale 2.523          0.128               5.078    19.669    0.000     2.271     2.775
##              log(height):age7:malefemale 2.531          0.127               7.048    19.892    0.000     2.281     2.781
##              log(height):age8:malefemale 2.526          0.125               8.879    20.205    0.000     2.281     2.772
##              log(height):age9:malefemale 2.539          0.124               8.785    20.403    0.000     2.295     2.783
##              log(height):age10:malefemale 2.554          0.124               7.886    20.644    0.000     2.311     2.797
##              log(height):age11:malefemale 2.560          0.123               9.041    20.773    0.000     2.318     2.802
##              log(height):age12:malefemale 2.565          0.123               7.386    20.838    0.000     2.323     2.807
##              log(height):age13:malefemale 2.570          0.123               6.778    20.907    0.000     2.328     2.811
##              log(height):age14:malefemale 2.549          0.123               4.189    20.788    0.000     2.308     2.790
##              log(height):age15:malefemale 2.547          0.123               4.388    20.659    0.000     2.305     2.789
##              log(height):age16:malefemale 2.565          0.123               3.442    20.813    0.000     2.323     2.807
##              log(height):age17:malefemale 2.620          0.128               1.427    20.450    0.000     2.368     2.871
##              log(height):age18:malefemale 2.563          0.124               2.428    20.615    0.000     2.319     2.808
##              log(height):age19:malefemale 2.588          0.124               2.020    20.791    0.000     2.344     2.833
##              log(height):age6:malemale 2.534          0.129               6.119    19.705    0.000     2.282     2.787
##              log(height):age7:malemale 2.541          0.127               6.591    19.964    0.000     2.291     2.791
##              log(height):age8:malemale 2.539          0.126               8.200    20.139    0.000     2.292     2.787
##              log(height):age9:malemale 2.544          0.124               9.353    20.487    0.000     2.300     2.788
##              log(height):age10:malemale 2.544          0.123               9.162    20.642    0.000     2.302     2.786
##              log(height):age11:malemale 2.559          0.122               9.139    21.002    0.000     2.319     2.798
##              log(height):age12:malemale 2.565          0.121               7.366    21.133    0.000     2.327     2.804
##              log(height):age13:malemale 2.585          0.121               6.200    21.412    0.000     2.348     2.822
##              log(height):age14:malemale 2.583          0.121               5.695    21.368    0.000     2.345     2.820
##              log(height):age15:malemale 2.604          0.121               4.334    21.509    0.000     2.366     2.842
##              log(height):age16:malemale 2.595          0.121               3.825    21.449    0.000     2.357     2.832
##              log(height):age17:malemale 2.598          0.121               3.833    21.511    0.000     2.361     2.835
##              log(height):age18:malemale 2.610          0.122               2.517    21.327    0.000     2.369     2.850
##              log(height):age19:malemale 2.616          0.124               1.476    21.163    0.000     2.373     2.859
## -----
```

```
plot(model_2018s1_2)
```

```
## Warning: not plotting observations with leverage one:
```



## 570, 591  
## Warning: not plotting observations with leverage one:  
## 570, 591



$$y = \mu + \beta_1 \ln(H) * Age * Male + \beta_2 Smoker + \varepsilon$$
$$H_0 : \beta_2 = 0, H_1 : \beta_2 \neq 0$$

```
predict(model_2018s1_2, newdata =data.frame(age="16",male="male",smoker="regularly",height=61), interval = "predict.
##          fit          lwr          upr
## 1 1.065319 0.7691739 1.361463
```

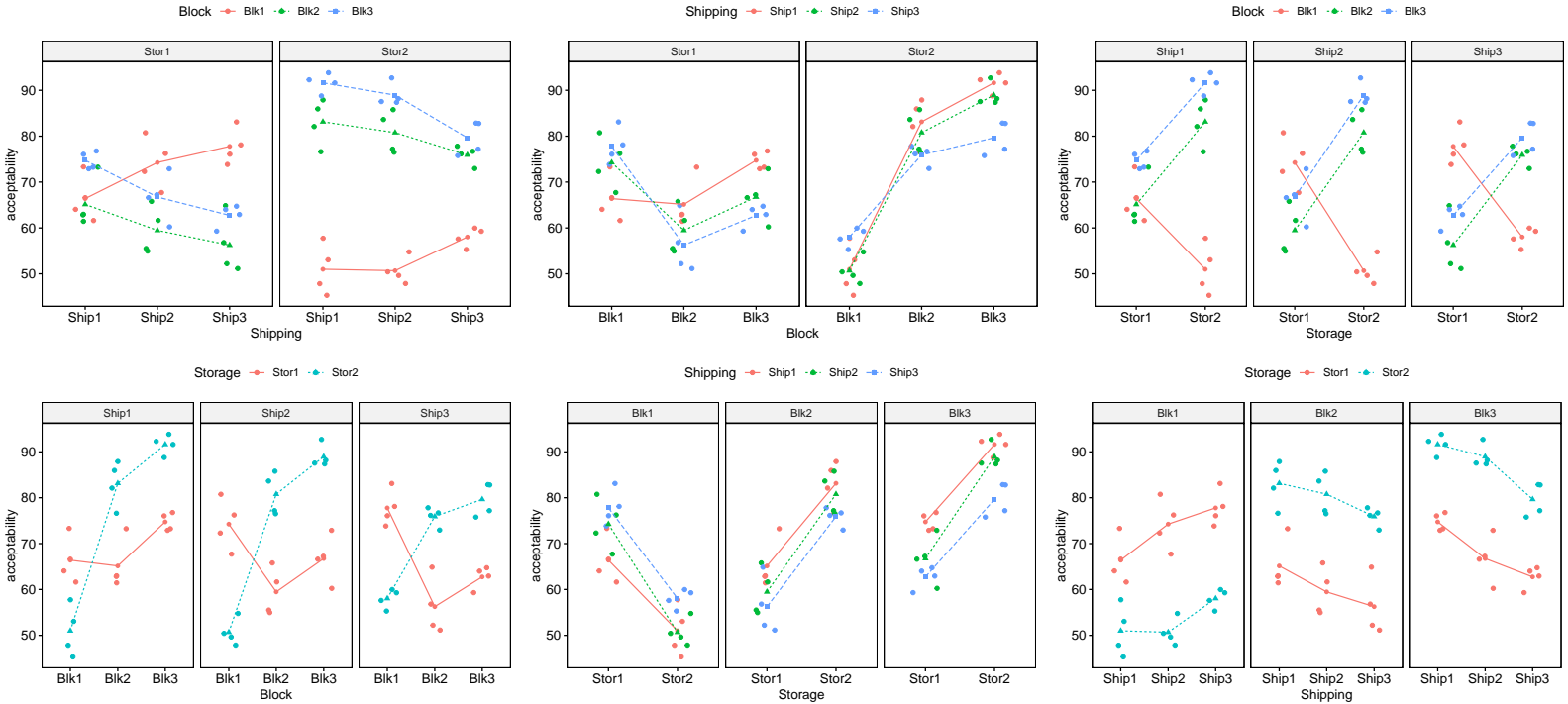
2018S2

[RCBD]

An experiment is conducted to assess the effect of shipping and storage on the acceptability of avocados. Three shipping methods (labeled 1, 2 and 3) and two storage methods (labeled 1 and 2) were considered. Each combination of shipping x storage was applied to a group of four crates. Additionally, three different shipments were made. The experiment's configuration is shown below. Analyze this experiment.

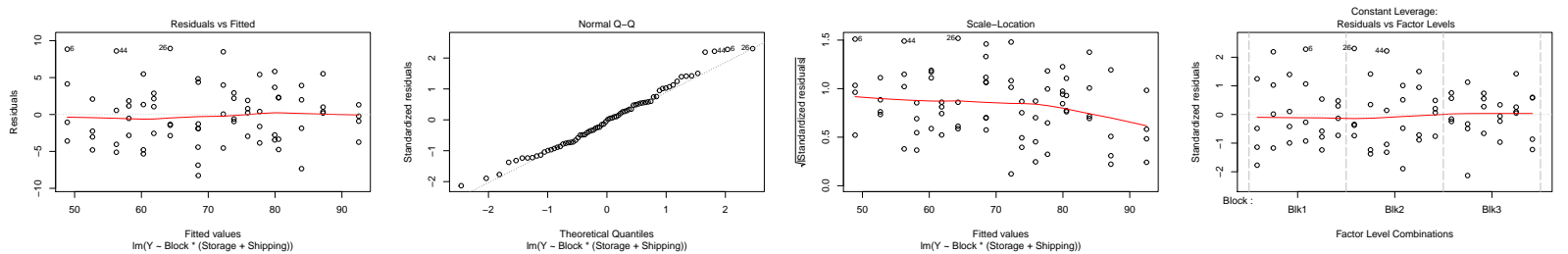
The data set can be found in the file Problem2\_Avocado.xlsx.

```
## Classes 'tbl_df', 'tbl' and 'data.frame': 72 obs. of 4 variables:
## $ Block : Factor w/ 3 levels "Blk1","Blk2",...: 1 1 1 1 1 1 1 1 1 1 ...
## $ Shipping: Factor w/ 3 levels "Ship1","Ship2",...: 1 1 1 1 1 1 1 1 2 2 ...
## $ Storage : Factor w/ 2 levels "Stor1","Stor2": 1 1 1 1 2 2 2 2 1 1 ...
## $ Y : num 73.3 66.6 61.6 64 53 ...
```



```
##
## Call:
## lm(formula = Y ~ Block * (Storage + Shipping), data = table_2018s2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -8.2704 -2.8865 -0.0842  2.2082  8.9433
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      68.476      1.735   39.478 < 2e-16 ***
## BlockBlk2        -4.170      2.453   -1.700  0.09436 .
## BlockBlk3         5.370      2.453    2.189  0.03248 *
## StorageStor2     -19.573      1.735  -11.284 < 2e-16 ***
## ShippingShip2      3.776      2.124    1.778  0.08054 .
## ShippingShip3      9.217      2.124    4.339 5.58e-05 ***
## BlockBlk2:StorageStor2 39.227      2.453   15.991 < 2e-16 ***
## BlockBlk3:StorageStor2 38.242      2.453   15.589 < 2e-16 ***
## BlockBlk2:ShippingShip2 -7.786      3.004   -2.592  0.01198 *
## BlockBlk3:ShippingShip2 -9.112      3.004   -3.033  0.00357 **
## BlockBlk2:ShippingShip3 -17.272      3.004   -5.749 3.21e-07 ***
```

```
## BlockBlk3:ShippingShip3 -21.206      3.004 -7.059 1.99e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.249 on 60 degrees of freedom
## Multiple R-squared:  0.9054, Adjusted R-squared:  0.8881
## F-statistic: 52.23 on 11 and 60 DF, p-value: < 2.2e-16
## Analysis of Variance Table
##
## Response: Y
##
##      Df Sum Sq Mean Sq  F value    Pr(>F)
## Block      2 2483.3  1241.65   68.7810 2.975e-16 ***
## Storage     1  703.2   703.19   38.9529 4.841e-08 ***
## Shipping     2  156.3    78.16    4.3297 0.01752 *
## Block:Storage  2 6004.3 3002.13 166.3021 < 2.2e-16 ***
## Block:Shipping  4 1024.0  256.00   14.1809 3.329e-08 ***
## Residuals    60 1083.1   18.05
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



```
library(emmeans)
Blk_Stor <- pairs(lsmeans(model_2018s2,~Block|Storage))
Stor_Blz <- pairs(lsmeans(model_2018s2,~Storage|Block))
Blk_Ship <- pairs(lsmeans(model_2018s2,~Block|Shipping))
Ship_Blz <- pairs(lsmeans(model_2018s2,~Shipping|Block))
Stor_Ship <- pairs(lsmeans(model_2018s2,~Storage|Shipping))
Ship_Stor <- pairs(lsmeans(model_2018s2,~Shipping|Storage))
library(kableExtra)
kable(test(rbind(Blk_Stor,Stor_Blz),adjust="tukey"),format="latex")%>%kable_styling("condensed",full_width=F,font_s
```

	Storage	Block	contrast	estimate	SE	df	t.ratio	p.value
	Stor1	.	Blk1 - Blk2	12.522500	1.734563	60	7.219399	0.0000000
	Stor1	.	Blk1 - Blk3	4.735833	1.734563	60	2.730275	0.0561879
	Stor1	.	Blk2 - Blk3	-7.786667	1.734563	60	-4.489124	0.0002805
	Stor2	.	Blk1 - Blk2	-26.704167	1.734563	60	-15.395331	0.0000000
	Stor2	.	Blk1 - Blk3	-33.505833	1.734563	60	-19.316588	0.0000000
	Stor2	.	Blk2 - Blk3	-6.801667	1.734563	60	-3.921257	0.0018712
	.	Blk1	Stor1 - Stor2	19.572500	1.734563	60	11.283824	0.0000000
	.	Blk2	Stor1 - Stor2	-19.654167	1.734563	60	-11.330906	0.0000000
	.	Blk3	Stor1 - Stor2	-18.669167	1.734563	60	-10.763039	0.0000000

```
kable(test(rbind(Blk_Ship,Ship_Blz),adjust="tukey"),format="latex")%>%kable_styling("condensed",full_width=F,font_s
```

Shipping	Block	contrast	estimate	SE	df	t.ratio	p.value
Ship1	.	Blk1 - Blk2	-15.44375	2.124397	60	-7.2697107	0.0000000
Ship1	.	Blk1 - Blk3	-24.49125	2.124397	60	-11.5285668	0.0000000
Ship1	.	Blk2 - Blk3	-9.04750	2.124397	60	-4.2588560	0.0011835
Ship2	.	Blk1 - Blk2	-7.65750	2.124397	60	-3.6045526	0.0094034
Ship2	.	Blk1 - Blk3	-15.37875	2.124397	60	-7.2391138	0.0000000
Ship2	.	Blk2 - Blk3	-7.72125	2.124397	60	-3.6345612	0.0085960
Ship3	.	Blk1 - Blk2	1.82875	2.124397	60	0.8608326	0.9679465
Ship3	.	Blk1 - Blk3	-3.28500	2.124397	60	-1.5463213	0.6800338
Ship3	.	Blk2 - Blk3	-5.11375	2.124397	60	-2.4071539	0.1928931
.	Blk1	Ship1 - Ship2	-3.77625	2.124397	60	-1.7775634	0.5309345
.	Blk1	Ship1 - Ship3	-9.21750	2.124397	60	-4.3388788	0.0009051
.	Blk1	Ship2 - Ship3	-5.44125	2.124397	60	-2.5613153	0.1405152
.	Blk2	Ship1 - Ship2	4.01000	2.124397	60	1.8875947	0.4606427
.	Blk2	Ship1 - Ship3	8.05500	2.124397	60	3.7916646	0.0053248
.	Blk2	Ship2 - Ship3	4.04500	2.124397	60	1.9040699	0.4503529
.	Blk3	Ship1 - Ship2	5.33625	2.124397	60	2.5118895	0.1559587
.	Blk3	Ship1 - Ship3	11.98875	2.124397	60	5.6433667	0.0000083
.	Blk3	Ship2 - Ship3	6.65250	2.124397	60	3.1314772	0.0357208

```
kable(test(rbind(Stor_Ship,Ship_Stor),adjust="tukey"),format="latex")%>%kable_styling("condensed",full_width=F,font_s
```

Shipping	Storage	contrast	estimate	SE	df	t.ratio	p.value
Ship1	.	Stor1 - Stor2	-6.250278	1.001450	60	-6.241226	0.0000004
Ship2	.	Stor1 - Stor2	-6.250278	1.001450	60	-6.241226	0.0000004
Ship3	.	Stor1 - Stor2	-6.250278	1.001450	60	-6.241226	0.0000004
.	Stor1	Ship1 - Ship2	1.856667	1.226521	60	1.513767	0.5320163
.	Stor1	Ship1 - Ship3	3.608750	1.226521	60	2.942265	0.0329359
.	Stor1	Ship2 - Ship3	1.752083	1.226521	60	1.428498	0.5860213
.	Stor2	Ship1 - Ship2	1.856667	1.226521	60	1.513767	0.5320163
.	Stor2	Ship1 - Ship3	3.608750	1.226521	60	2.942265	0.0329359
.	Stor2	Ship2 - Ship3	1.752083	1.226521	60	1.428498	0.5860213

# 2018F

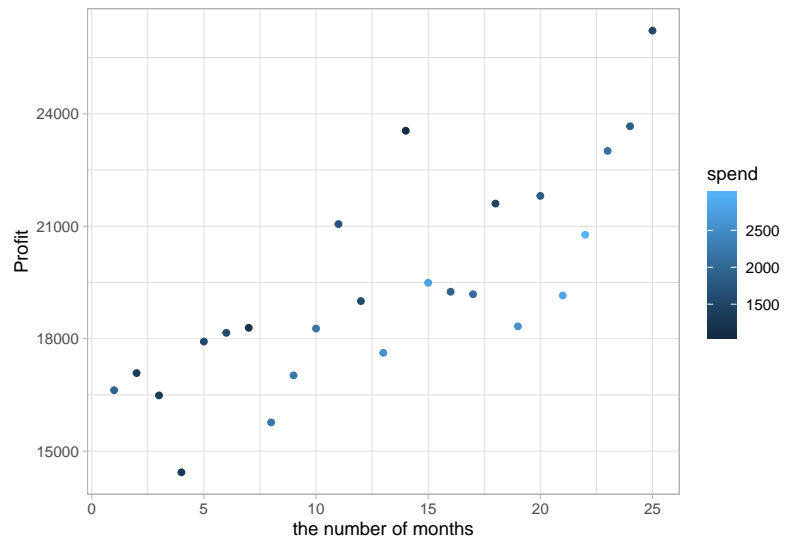
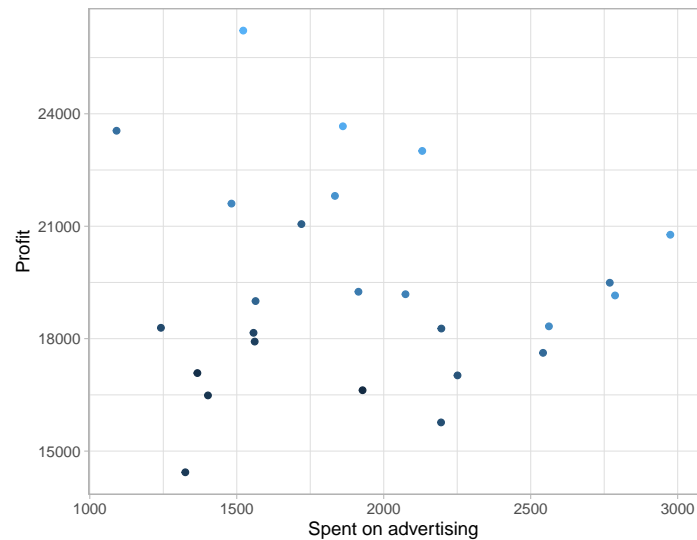
Robert Fountain\*, Daniel Taylor-Rodriguez

## 2018F1

2015F1 [2017S1]

Find the best model for predicting Y based on X1 and X2. Y is the amount of profit that a company makes in a month. X1 is the number of months that the company has been in business. X2 is the amount spent on advertising. Consider as predictors all possible linear and quadratic terms (X1, X1<sup>2</sup>, X2, X2<sup>2</sup>, and X1X2). Consider possible transformations of Y. Include all appropriate diagnostics. When you have found your “best” model, predict a new Y when X1 = 20 and X2 = \$1,900, giving a 95% prediction interval. The data set, shown below, appears in “Profits.xlsx”.

```
## Classes 'tbl_df', 'tbl' and 'data.frame':   25 obs. of  3 variables:
## $ X1: num  1 2 3 4 5 6 7 8 9 10 ...
## $ X2: num 1928 1366 1402 1325 1561 ...
## $ Y : num 16624 17082 16486 14435 17922 ...
```



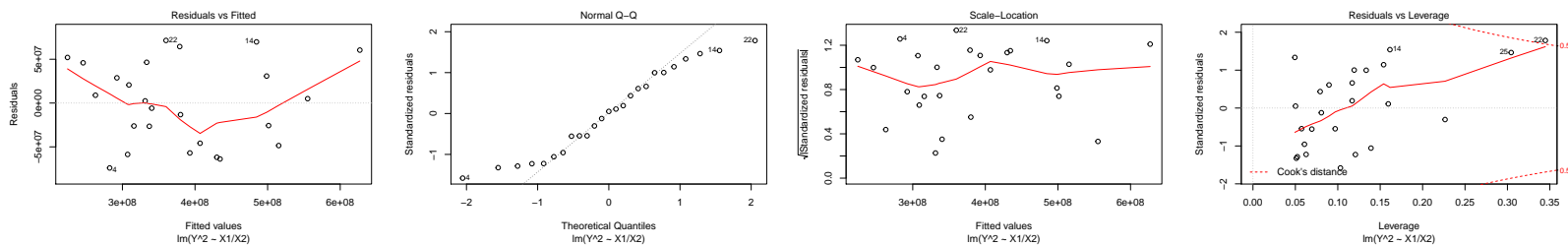
```
model_2018f1_1 <- lm(Y~X1/X2, table_2018f1)
summary(model_2018f1_1)

##
## Call:
## lm(formula = Y~X1/X2, data = table_2018f1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -74055527 -45860111  2480827  45931196  71468204
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 210415592  20657349  10.186 8.62e-10 ***
## X1           26996837   3171564   8.512 2.08e-08 ***
## X1:X2         -6788     1284   -5.286 2.64e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 49490000 on 22 degrees of freedom
## Multiple R-squared:  0.8238, Adjusted R-squared:  0.8078
## F-statistic: 51.44 on 2 and 22 DF, p-value: 5.069e-09

# library(olsrr)
# ols_regress(model_2018f1_1)
# car::Anova(model_2018f1_1)
# car::vif(model_2018f1_1)
anova(model_2018f1_1)

## Analysis of Variance Table
##
## Response: Y^2
##           Df      Sum Sq    Mean Sq F value    Pr(>F)
## X1           1 1.8355e+17 1.8355e+17  74.943 1.554e-08 ***
## X1:X2         1 6.8443e+16 6.8443e+16  27.945 2.641e-05 ***
## Residuals    22 5.3882e+16 2.4492e+15
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

plot(model_2018f1_1)
```



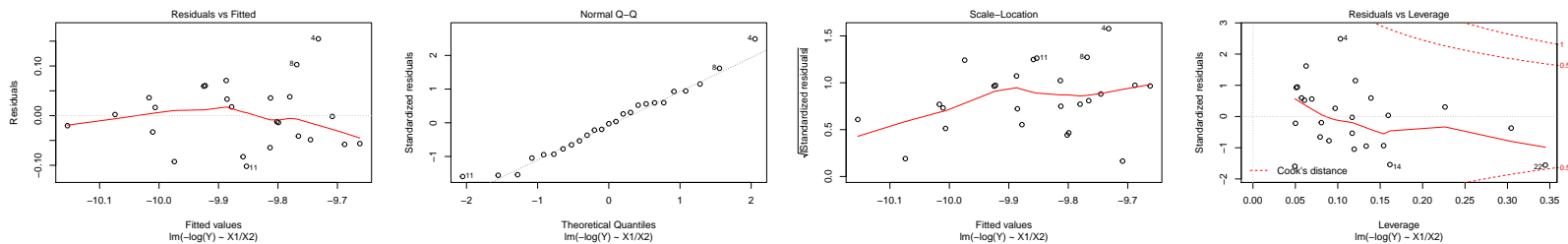
```
model_2018f1_2 <- lm(-log(Y)~X1/X2, table_2018f1)
summary(model_2018f1_2)
```

```
##
## Call:
## lm(formula = -log(Y) ~ X1/X2, data = table_2018f1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.102059 -0.048517 -0.001664  0.036092  0.154694
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -9.645e+00  2.740e-02 -352.012 < 2e-16 ***
## X1           -3.152e-02  4.207e-03  -7.493 1.72e-07 ***
## X1:X2         7.328e-06  1.703e-06   4.303 0.000288 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06564 on 22 degrees of freedom
## Multiple R-squared:  0.8004, Adjusted R-squared:  0.7823
## F-statistic: 44.12 on 2 and 22 DF,  p-value: 1.999e-08
```

```
anova(model_2018f1_2)
```

```
## Analysis of Variance Table
##
## Response: -log(Y)
##              Df Sum Sq Mean Sq F value    Pr(>F)
## X1              1  0.300460  0.300460   69.733 2.875e-08 ***
## X1:X2            1  0.079762  0.079762   18.512 0.0002882 ***
## Residuals      22  0.094793  0.004309
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
plot(model_2018f1_2)
```



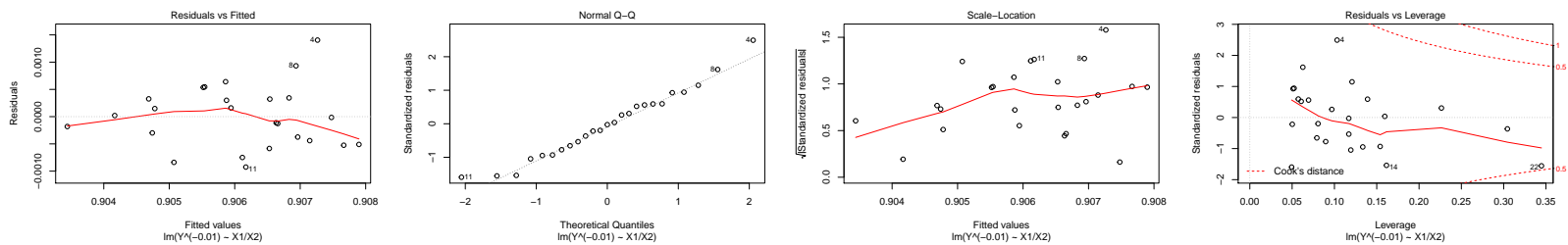
```
model_2018f1_3 <- lm(Y^(-0.01)~X1/X2, table_2018f1)
summary(model_2018f1_3)
```

```
##
## Call:
## lm(formula = Y^(-0.01) ~ X1/X2, data = table_2018f1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -9.254e-04 -4.402e-04 -1.469e-05  3.259e-04  1.405e-03
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  9.081e-01  2.484e-04 3656.193 < 2e-16 ***
## X1          -2.854e-04  3.813e-05  -7.485 1.75e-07 ***
## X1:X2         6.633e-08  1.544e-08   4.296 0.000293 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.000595 on 22 degrees of freedom
## Multiple R-squared:  0.8002, Adjusted R-squared:  0.782
## F-statistic: 44.06 on 2 and 22 DF,  p-value: 2.025e-08
```

```
anova(model_2018f1_3)
```

```
## Analysis of Variance Table
##
## Response: Y^(-0.01)
##              Df Sum Sq Mean Sq F value    Pr(>F)
## X1              1 2.4660e-05 2.4660e-05   69.656 2.902e-08 ***
## X1:X2            1  6.5346e-06  6.5346e-06   18.458 0.0002926 ***
## Residuals      22 7.7887e-06 3.5400e-07
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
plot(model_2018f1_3)
```



```
predict(model_2018f1_1, newdata = data.frame(X1=20,X2=1900), interval = "prediction", level=0.95)
```

```
##          fit          lwr          upr
## 1 492395512 385311070 599479954
```

## 2018F2

2015F2 [7.4] [8.E.10]

A replicated fractional factorial design is used to investigate the effect of four factors on the free height of leaf springs used in an automotive application. The factors are (A) furnace temperature, (B) heating time, (C) transfer time, and (D) hold down time. There are 3 observations at each setting.

Write out the alias structure for this design. What is the resolution of this design?

I=ABCD, AB=CD, AC=BD, BC=AD; A=BCD, B=ACD, C=ABD, D=ABC; III

Analyze the data. What factors influence the mean free height? The data set appears in the file "Springs.xlsx".

A, B

```
table_2018f2 <- readxl::read_xlsx("qe_lab/Springs_2018f.xlsx")
table_2018f2 <- table_2018f2[order(table_2018f2$D, table_2018f2$C, table_2018f2$B, table_2018f2$A),]
str(table_2018f2)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':  48 obs. of  5 variables:
## $ A      : num -1 -1 -1 -1 -1 -1 -1 1 1 1 1 ...
## $ B      : num -1 -1 -1 -1 -1 -1 1 1 1 1 1 ...
## $ C      : num -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 ...
## $ D      : num -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 ...
## $ Heights: num 8.56 8 8.56 7.5 8.62 7.24 8.18 8.26 8.12 8.5 ...
```

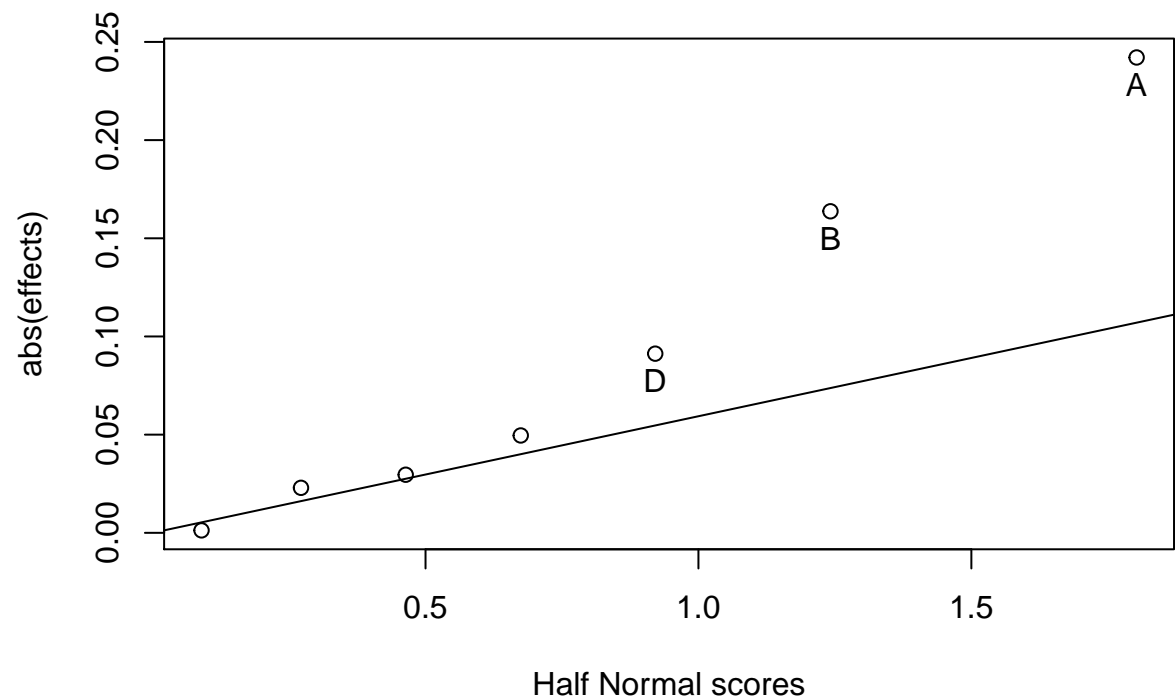
```
kableExtra::kable(table_2018f2)
```

A	B	C	D	Heights
-1	-1	-1	-1	8.56
-1	-1	-1	-1	8.00
-1	-1	-1	-1	8.56
-1	-1	-1	-1	7.50
-1	-1	-1	-1	8.62
-1	-1	-1	-1	7.24
-1	-1	-1	-1	8.18
-1	-1	-1	-1	8.26
-1	-1	-1	-1	8.12
-1	-1	-1	-1	8.50
-1	-1	-1	-1	8.50
-1	-1	-1	-1	8.12
-1	-1	-1	-1	8.38
-1	-1	-1	-1	8.12
-1	-1	-1	-1	9.18
-1	-1	-1	-1	8.38
-1	-1	-1	-1	9.12
-1	-1	-1	-1	8.24
-1	-1	-1	-1	8.12
-1	-1	-1	-1	7.36
-1	-1	-1	-1	8.04
-1	-1	-1	-1	7.36
-1	-1	-1	-1	7.88
-1	-1	-1	-1	7.50
-1	-1	-1	-1	9.30
-1	-1	-1	-1	8.76
-1	-1	-1	-1	9.36
-1	-1	-1	-1	8.76
-1	-1	-1	-1	8.76
-1	-1	-1	-1	7.88
-1	-1	-1	-1	8.00
-1	-1	-1	-1	8.00
-1	-1	-1	-1	8.12
-1	-1	-1	-1	8.12
-1	-1	-1	-1	8.00
-1	-1	-1	-1	8.00
-1	-1	-1	-1	8.08
-1	-1	-1	-1	7.64
-1	-1	-1	-1	9.00
-1	-1	-1	-1	7.88
-1	-1	-1	-1	8.76
-1	-1	-1	-1	7.88
-1	-1	-1	-1	7.88
-1	-1	-1	-1	8.12
-1	-1	-1	-1	8.62
-1	-1	-1	-1	8.62
-1	-1	-1	-1	8.00
-1	-1	-1	-1	8.38
-1	-1	-1	-1	8.18

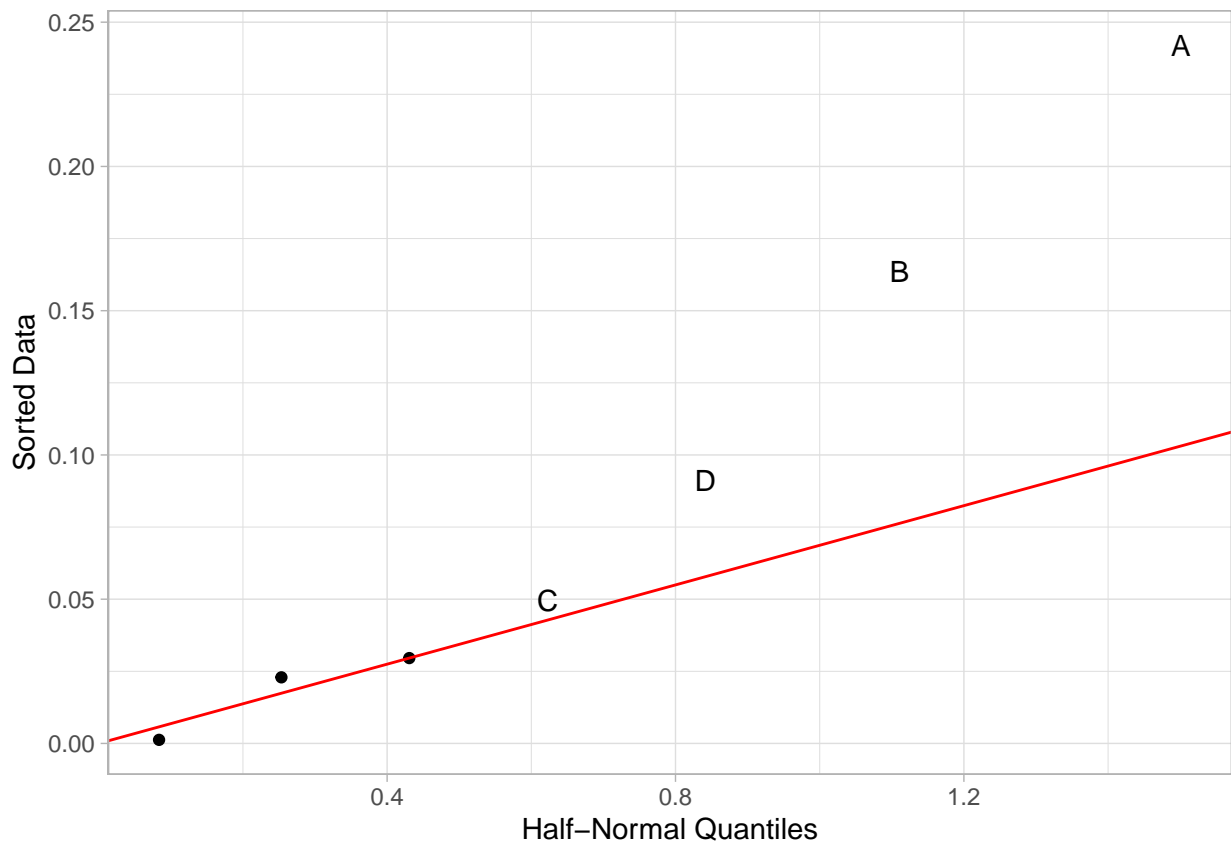
```
model_2018f2_1 <- aov(Heights~A*B*C*D, table_2018f2)
summary(model_2018f2_1)
```

```
##           Df Sum Sq Mean Sq F value    Pr(>F)
## A             1   2.813    2.8130   16.359 0.000232 ***
## B             1   1.287    1.2871    7.485 0.009232 **
## C             1   0.118    0.1180    0.686 0.412346
## D             1   0.400    0.3997    2.324 0.135232
## A:B           1   0.042    0.0420    0.244 0.623819
## A:C           1   0.000    0.0001    0.000 0.983441
## B:C           1   0.025    0.0252    0.147 0.703832
## Residuals    40   6.878    0.1720
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
library(daewr)
halfnorm(coef(model_2018f2_1)[2:8],alpha=0.4)
```



```
## zscore= 0.08964235 0.27188 0.4637078 0.6744898 0.920823 1.241867 1.802743effp= 0.00125 0.02291667 0.02958333 0.04
library(gghalfnorm)
gghalfnorm(x =coef(model_2018f2_1)[2:8],labs = names(coef(model_2018f2_1)[2:8]) , nlab = 4)+ ggplot2::theme_light()
```



```
model_2018f2_2 <- lm(Heights~A+B, table_2018f2)
summary(model_2018f2_2)

##
## Call:
## lm(formula = Heights ~ A + B, data = table_2018f2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.93292 -0.28104  0.07667  0.27458  0.82708
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   8.25125    0.05878  140.375 < 2e-16 ***
## A             0.24208    0.05878   4.118 0.000161 ***
## B            -0.16375    0.05878  -2.786 0.007788 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4072 on 45 degrees of freedom
## Multiple R-squared:  0.3546, Adjusted R-squared:  0.3259
## F-statistic: 12.36 on 2 and 45 DF,  p-value: 5.265e-05
```

**2019S**  
 Robert Fountain\*, Daniel Taylor-Rodriguez

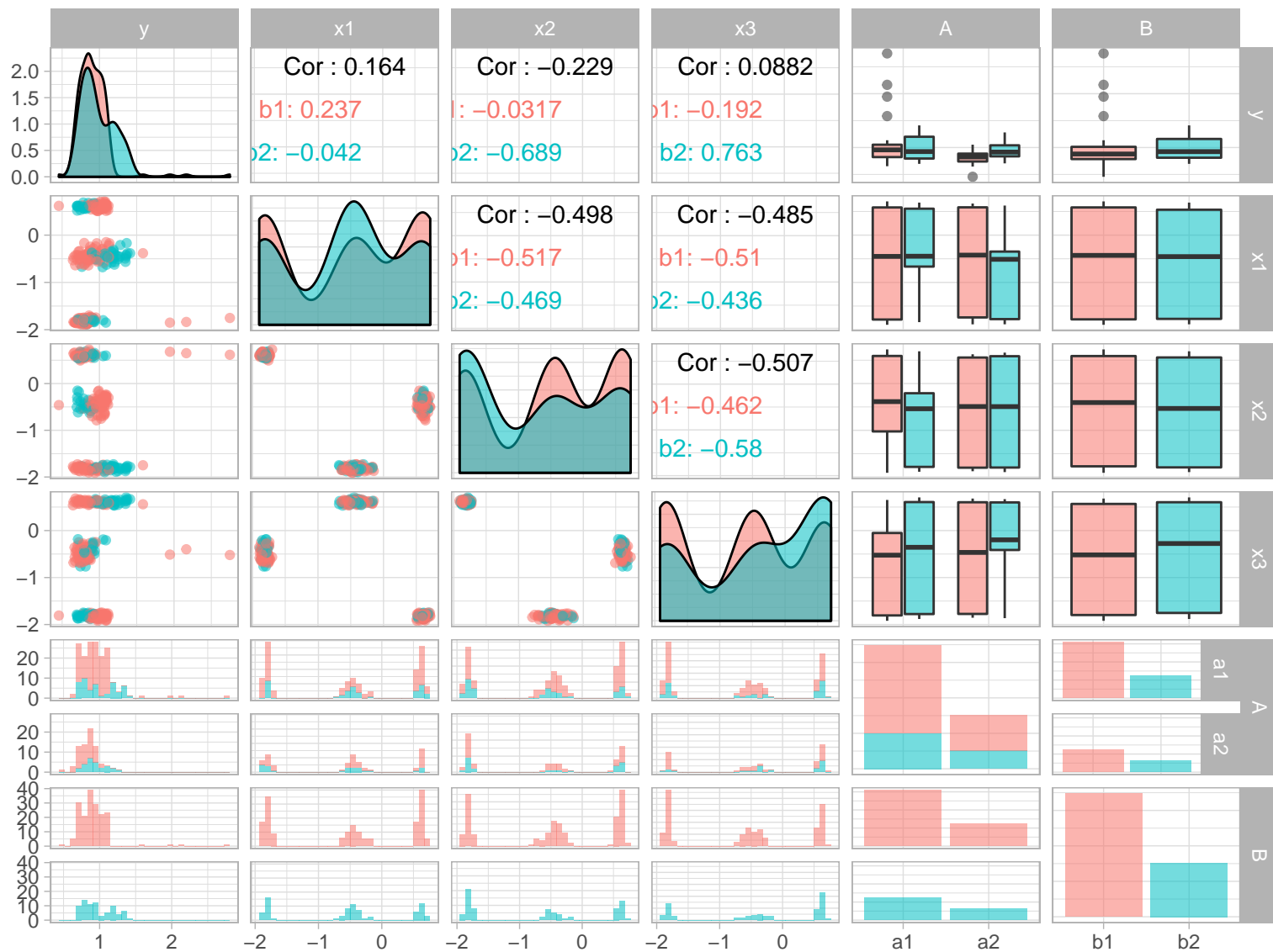
- Instructions:
1. Two 8.5" x 11" pages of notes (front and back) are allowed.
  2. Perform the statistical analysis in your software of preference for the two problems below. The data sets for each problem are on the flash drive provided. Create a word or pdf document with your findings. Save the document to the flash drive provided with your name as the file name. You may use scratch paper during the exam, but everything you want considered for grading must be included in your document. Additionally, you must copy and paste the code used for the analysis at the end of the word/pdf document you submit.
  3. For each question discuss all relevant aspects of your analysis (exploratory and modeling) supporting them with graphical and numerical summaries that are important for communicating results. It should also include a discussion of diagnostics and model adequacy, and rationale for any transformations or other key modeling decisions. The report should include interpretations of the output, written so that a statistically literate person can understand and apply the findings in each case.

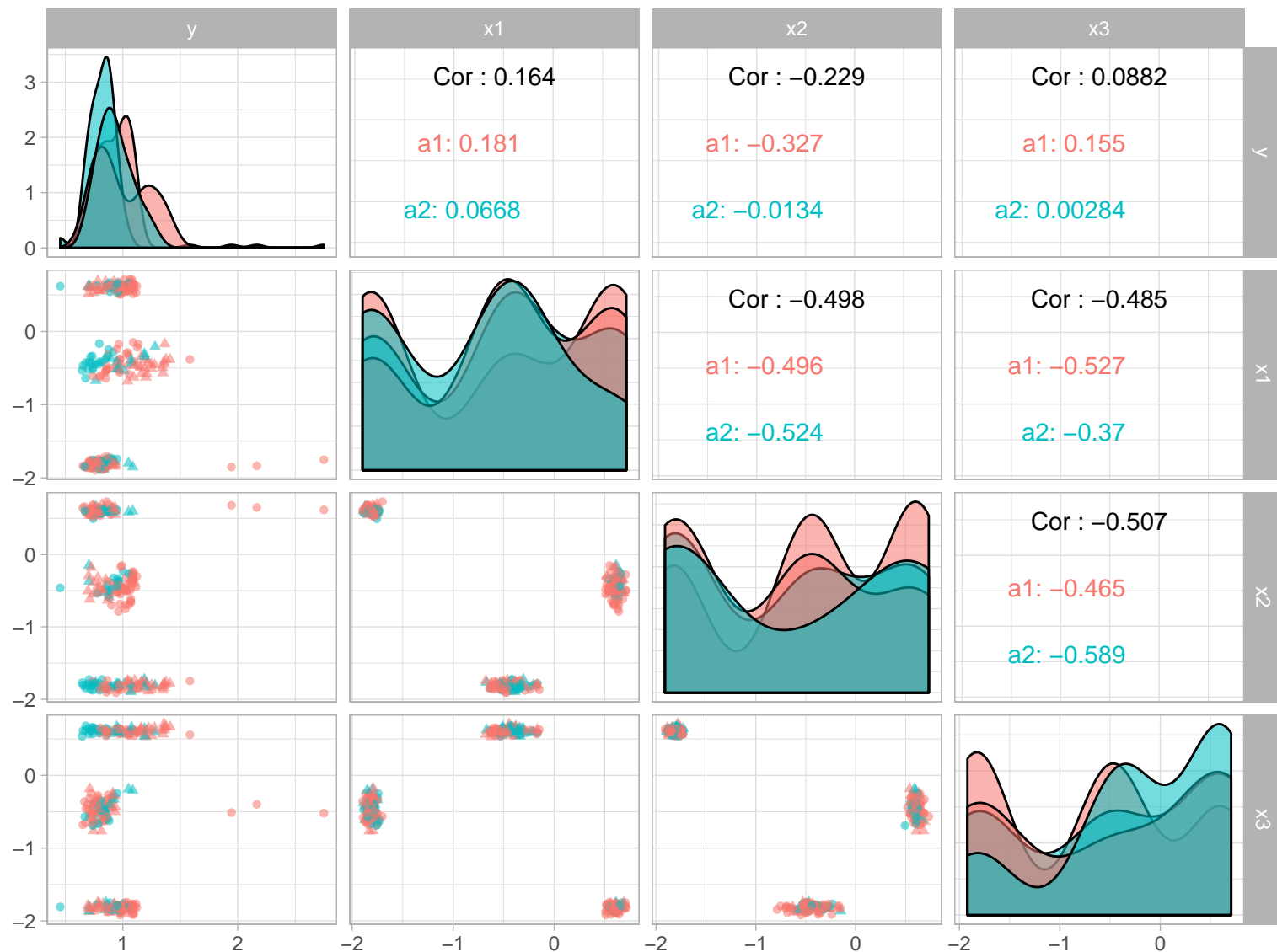
**2019S1**  
 [4.2.1 PRESS residuals]  
 The goal of this exercise is to find the best model for predicting (out-of-sample) Y based on the continuous variables x1, x2, x3, and on the binary variables A and B. The data set is in the dataset "ModelBuildingData.xlsx". Consider possible transformations of Y, and for the linear predictor consider 2-way interactions and quadratic terms. Include all appropriate diagnostics, and make any necessary adjustments to the data so model assumptions are met.

Use only the first 250 observations for model training model (i.e., selection, fitting and diagnostics). With your top model, obtain predictions for all 250 remaining observations (the hold-out samples), and their corresponding 95% predictive intervals. Finally, calculate and interpret (in term of the model predictive ability) the Prediction Root Mean Square Error (PRMSE), as follows:

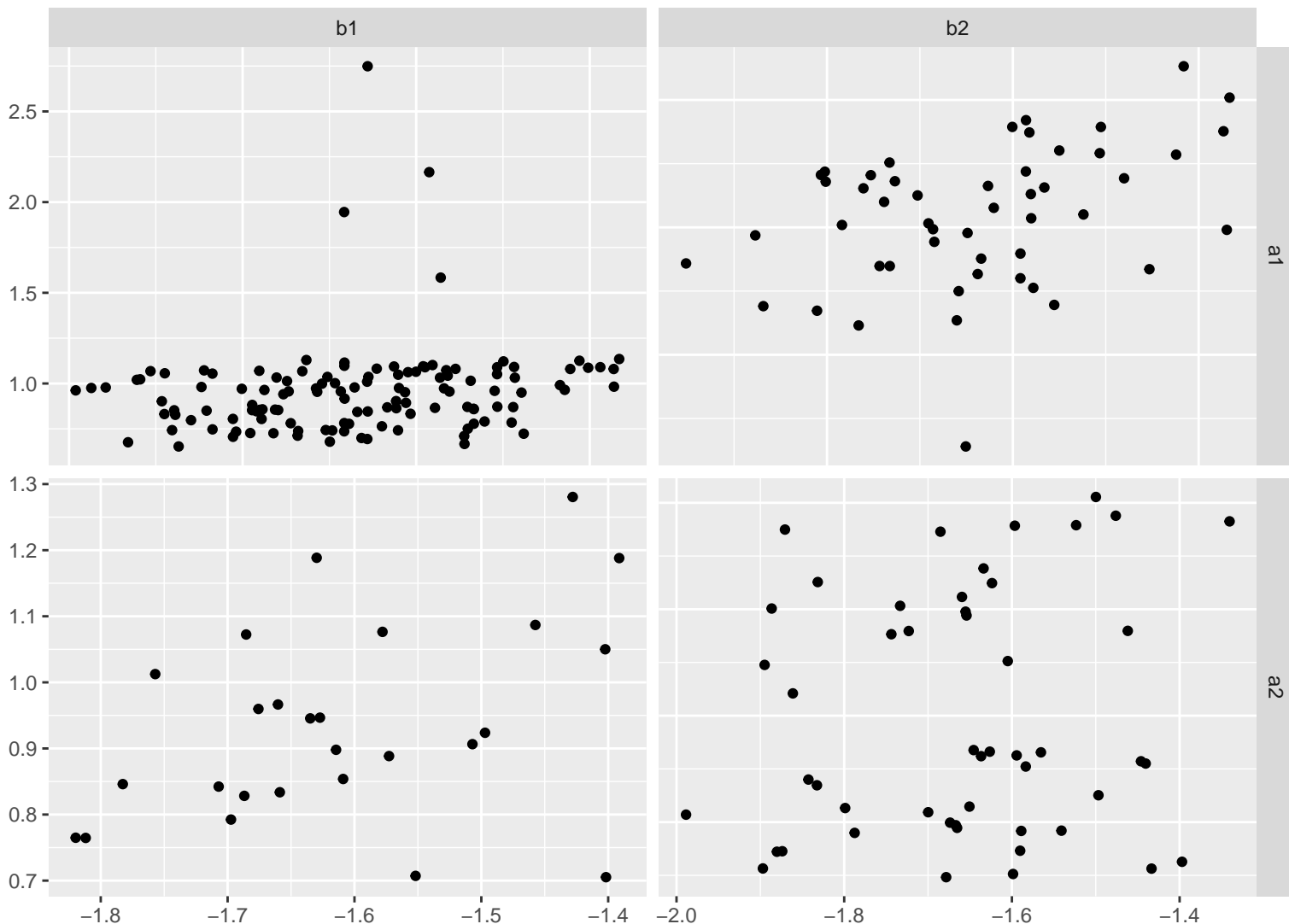
[illegible]







# A vs B

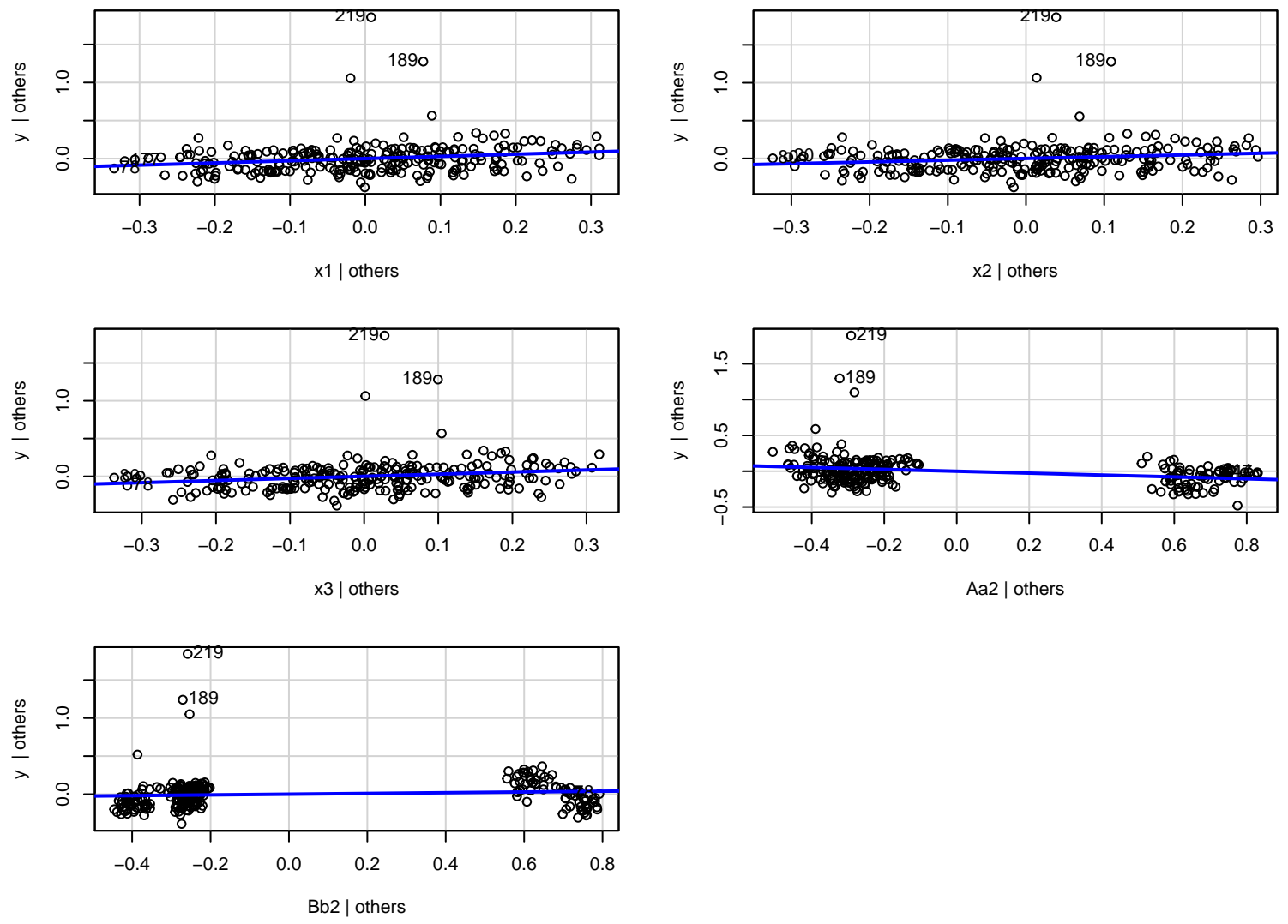


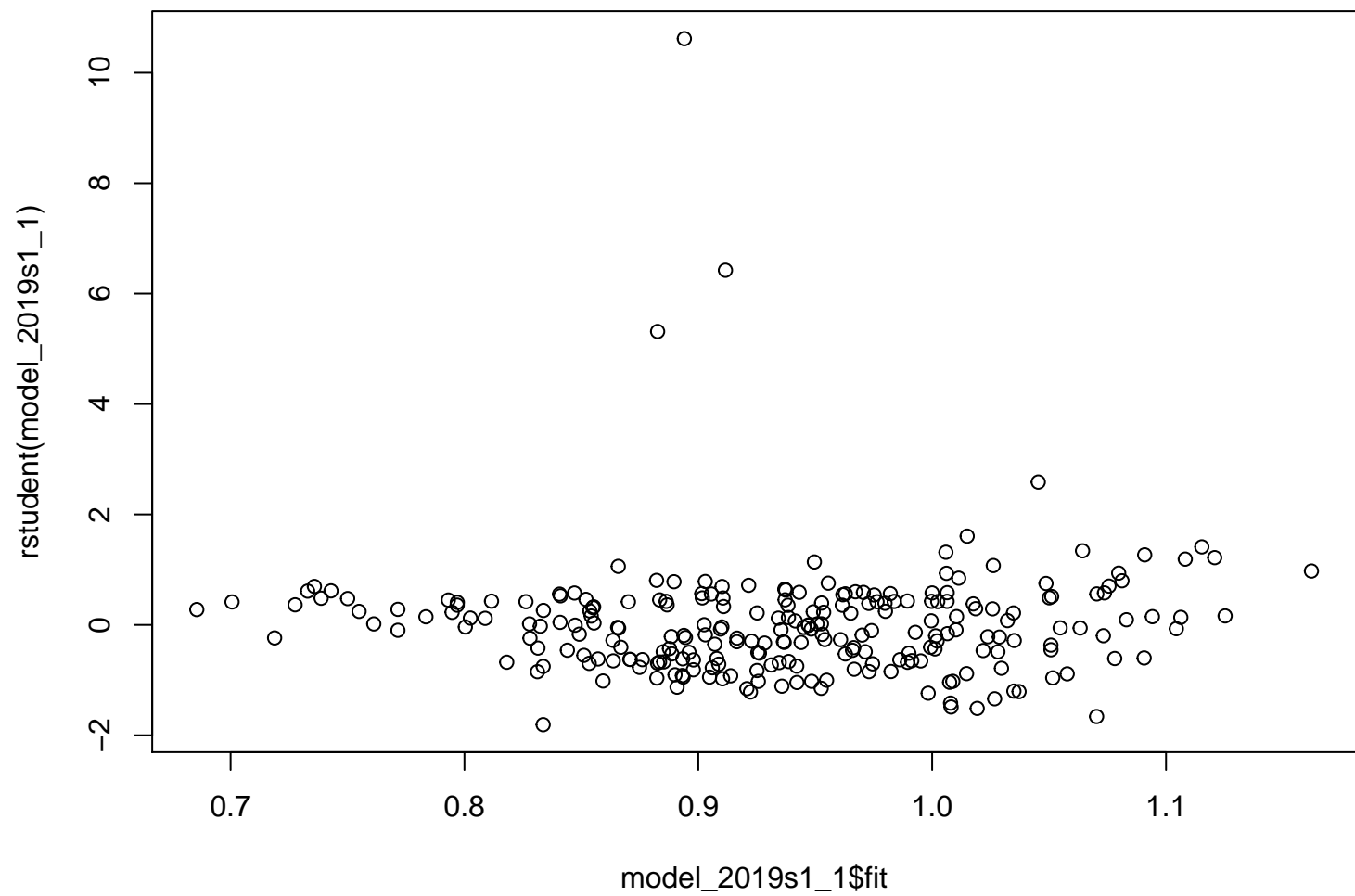
```
model_2019s1 <- lm(log(y) ~ x1*x2*x3*A*B,table_2019s1_250)
car::vif(model_2019s1)
summary(model_2019s1)
library(olsrr)
# ols_plot_diagnostics(model_2019s1_1)
ols_step_both_aic(model_2019s1)

##
## Call:
## lm(formula = y ~ x1 + x2 + x3 + A + B, data = table_2019s1_250)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.37764 -0.12857 -0.01063  0.08906  1.85501
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.39962    0.15936   8.783 2.87e-16 ***
## x1             0.28627    0.09490   3.017  0.00283 **
## x2             0.22936    0.09579   2.394  0.01740 *
## x3             0.27945    0.09504   2.940  0.00359 **
## Aa2            -0.13200    0.02971  -4.443 1.35e-05 ***
## Bb2             0.04808    0.02956   1.627  0.10513
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2125 on 244 degrees of freedom
## Multiple R-squared:  0.1533, Adjusted R-squared:  0.1359
## F-statistic: 8.835 on 5 and 244 DF, p-value: 1.007e-07
## Anova Table (Type II tests)
##
## Response: y
##          Sum Sq Df F value    Pr(>F)
## x1         0.4110  1   9.0998 0.002827 **
## x2         0.2590  1   5.7330 0.017405 *
## x3         0.3905  1   8.6459 0.003593 **
## A          0.8916  1  19.7399 1.347e-05 ***
## B          0.1195  1   2.6456 0.105129
## Residuals 11.0213 244
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## x1 x2 x3 A B  
## 48.920081 50.351228 49.831957 1.033647 1.023466

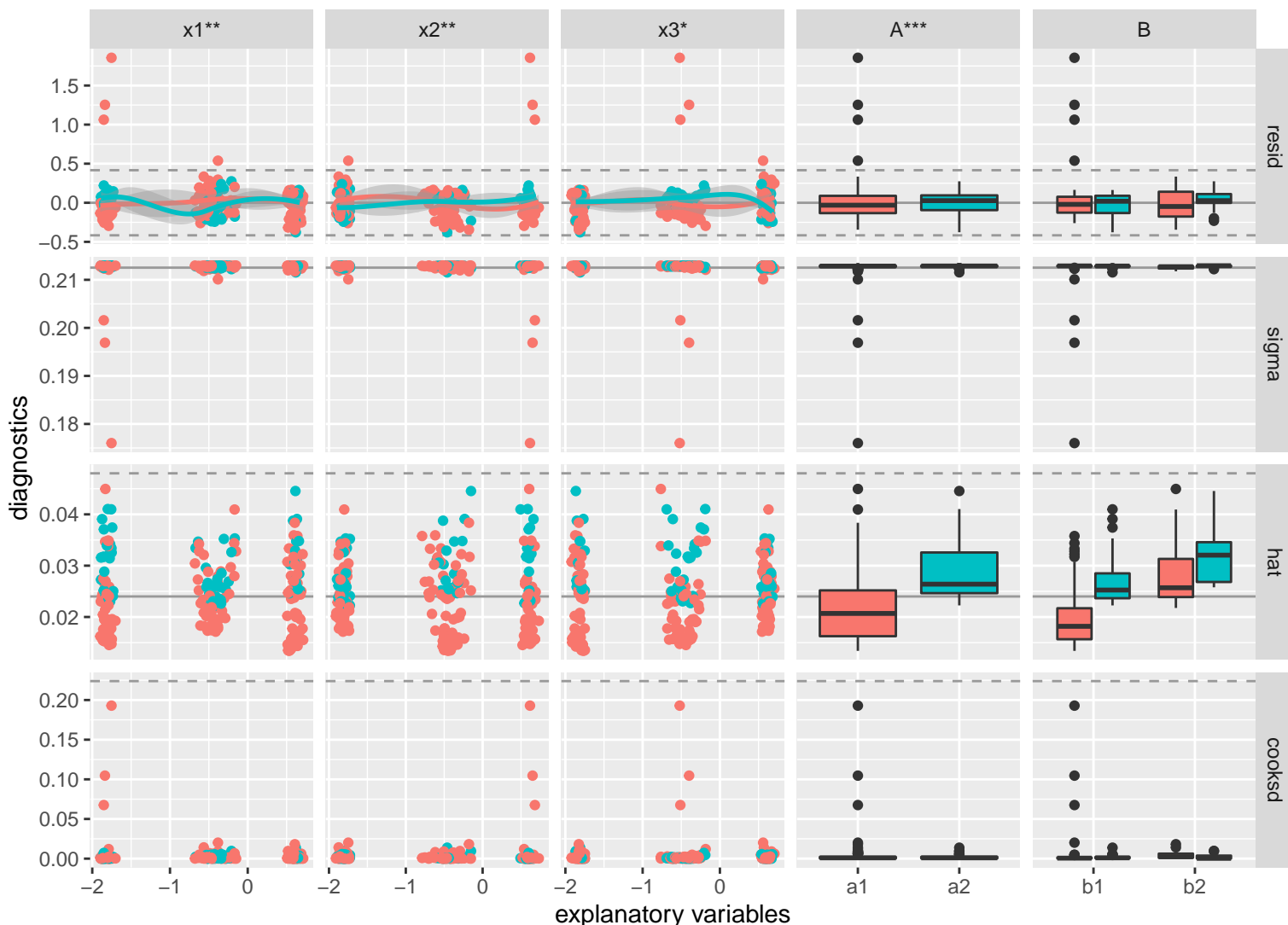
Added-Variable Plots





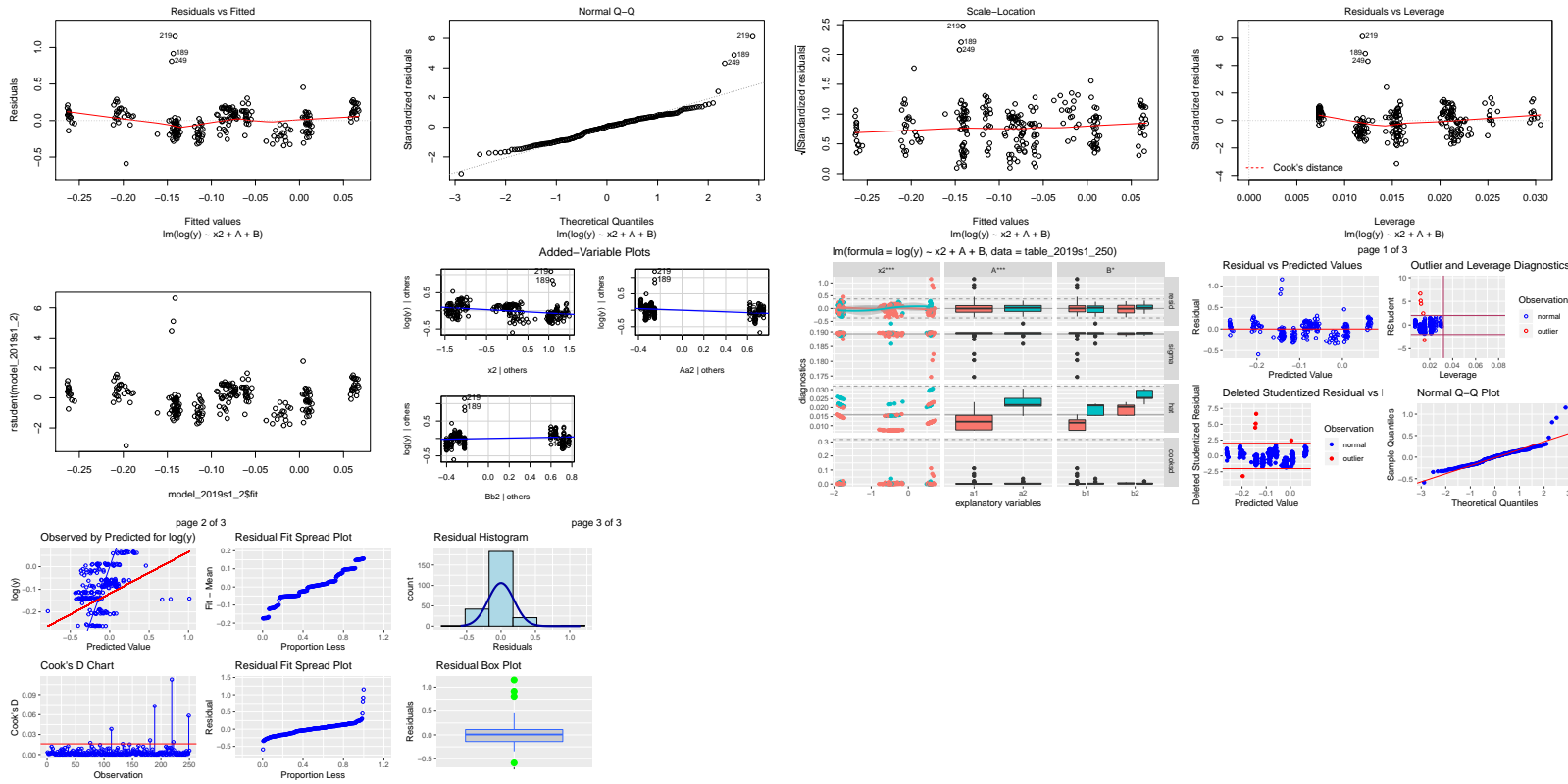
```
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'  
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'  
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```

lm(formula = y ~ x1 + x2 + x3 + A + B, data = table\_2019s1\_250)



```
##
## Call:
## lm(formula = log(y) ~ x2 + A + B, data = table_2019s1_250)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.58842 -0.13906  0.00879  0.11250  1.15248
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.10330    0.01704  -6.064 4.97e-09 ***
## x2           -0.06149    0.01214  -5.066 7.96e-07 ***
## Aa2           0.02183    0.02614  -4.661 5.15e-06 ***
## Bb2           0.05454    0.02624   2.079 0.0387 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1893 on 246 degrees of freedom
## Multiple R-squared:  0.1695, Adjusted R-squared:  0.1594
## F-statistic: 16.74 on 3 and 246 DF, p-value: 6.312e-10
## Anova Table (Type II tests)
##
## Response: log(y)
##           Sum Sq Df F value    Pr(>F)
## x2         0.9202  1 25.6682 7.965e-07 ***
## A          0.7789  1 21.7278 5.154e-06 ***
## B          0.1549  1  4.3212 0.03868 *
## Residuals 8.8191 246
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##           x2          A          B
## 1.018554 1.007957 1.015740
##           48.8 %          51.2 %
## (Intercept) -0.10383914 -0.10277031
## x2          -0.06187532 -0.06111380
## Aa2          -0.12265219 -0.12101237
## Bb2          0.05371814  0.05536428
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
## Analysis of Variance Table
##
## Response: log(y)
```

```
##          Df Sum Sq Mean Sq F value    Pr(>F)
## x2         1  0.8964  0.89643   25.0051 1.088e-06 ***
## A          1  0.7486  0.74864   20.8825 7.733e-06 ***
## B          1  0.1549  0.15491    4.3212 0.03868 *
## Residuals 246  8.8191  0.03585
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



```
model_2019s1_3 <- lm(table_2019s1_500,formula=log(y)~ x2+A+B)
summary(model_2019s1_3)
```

```
## Call:
## lm(formula = log(y) ~ x2 + A + B, data = table_2019s1_500)
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.34814 -0.10156 -0.00912  0.09695  0.32199
## Coefficients:
##      Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.137812   0.012796 -10.770 < 2e-16 ***
## x2          -0.074053   0.008689  -8.523 1.59e-15 ***
## Aa2         -0.098649   0.019104  -5.164 5.00e-07 ***
## Bb2          0.055665   0.018112   3.073 0.00235 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.1357 on 246 degrees of freedom
## Multiple R-squared:  0.3167, Adjusted R-squared:  0.3084
## F-statistic: 38.01 on 3 and 246 DF, p-value: < 2.2e-16
```

```
ols_regress(log(y)~ x2+A+B, data = table_2019s1_500)
```

```
##          Model Summary
## -----
## R              0.563      RMSE              0.136
## R-Squared      0.317      Coef. Var      -129.304
## Adj. R-Squared 0.308      MSE              0.018
## Pred R-Squared 0.294      MAE              0.111
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
```

```
##          ANOVA
## -----
##      Sum of      DF      Mean Square      F      Sig.
## Regression    2.099        3         0.700    38.007    0.0000
## Residual      4.528       246         0.018
## Total         6.627       249
```

```
##          Parameter Estimates
## -----
##      model      Beta      Std. Error      Std. Beta      t      Sig      lower      upper
## -----
```

```
## (Intercept) -0.138 0.013 -10.770 0.000 -0.163 -0.113
## x2 -0.074 0.009 -0.452 -8.523 0.000 -0.091 -0.057
## Aa2 -0.099 0.019 -0.273 -5.164 0.000 -0.136 -0.061
## Bb2 0.056 0.018 0.163 3.073 0.002 0.020 0.091
## -----
library(Metrics)
Metrics::rmse(table_2019s1_500$y,exp(predict(model_2019s1_2,table_2019s1_500)))
## [1] 0.1285634
ols_press(model_2019s1_3)
## [1] 4.681989
MPV::PRESS(model_2019s1_3)
## [1] 4.681989
sum((residuals(model_2019s1_3)/(1 - lm.influence(model_2019s1_3)$hat))^2)
## [1] 4.681989
ols_pred_rsqr(model_2019s1_3)
## [1] 0.2935096
# str(model_2019s1_3)
# From 564-lab caculate prediction power
deviation <- table_2019s1_500$y-mean(table_2019s1_500$y)
SST <- deviation**deviation
1-(MPV::PRESS(model_2019s1_3)/SST)
## [1]
## [1,] 0.2378794
# by definition PRESS
sum((table_2019s1_500$y-exp(model_2019s1_2$fit))^2)
## [1] 8.358063
sum((table_2019s1_500$y-exp(predict(model_2019s1_2,table_2019s1_500)))^2)
## [1] 4.13214
# one method of RMSE
sqrt(mean(model_2019s1_3$residuals^2))
## [1] 0.1345847
# remove outlier
table_2019s1_250[c(189,219,249),]
table_2019s1_250_noouter <- table_2019s1_250[-c(189,219,249), ]
table_2019s1_250_noouter <- table_2019s1_250[-c(113,189,219,249), ]
model_2019s1_noouter <- lm(y ~ sqrt(!is.na(x1))+x2+x3+A+B, data = table_2019s1_250_noouter)
summary(model_2019s1_noouter)
plot(model_2019s1_noouter)
```

- calculate for each observation the square of the prediction errors,
- obtain the square root of the average of all squared prediction errors.

<https://blog.minitab.com/blog/adventures-in-statistics-2/multiple-regression-analysis-use-adjusted-r-squared-and-predicted-r-squared-to>

## 2019S2

[14.4] [566-fe-4] [Example 8.4]

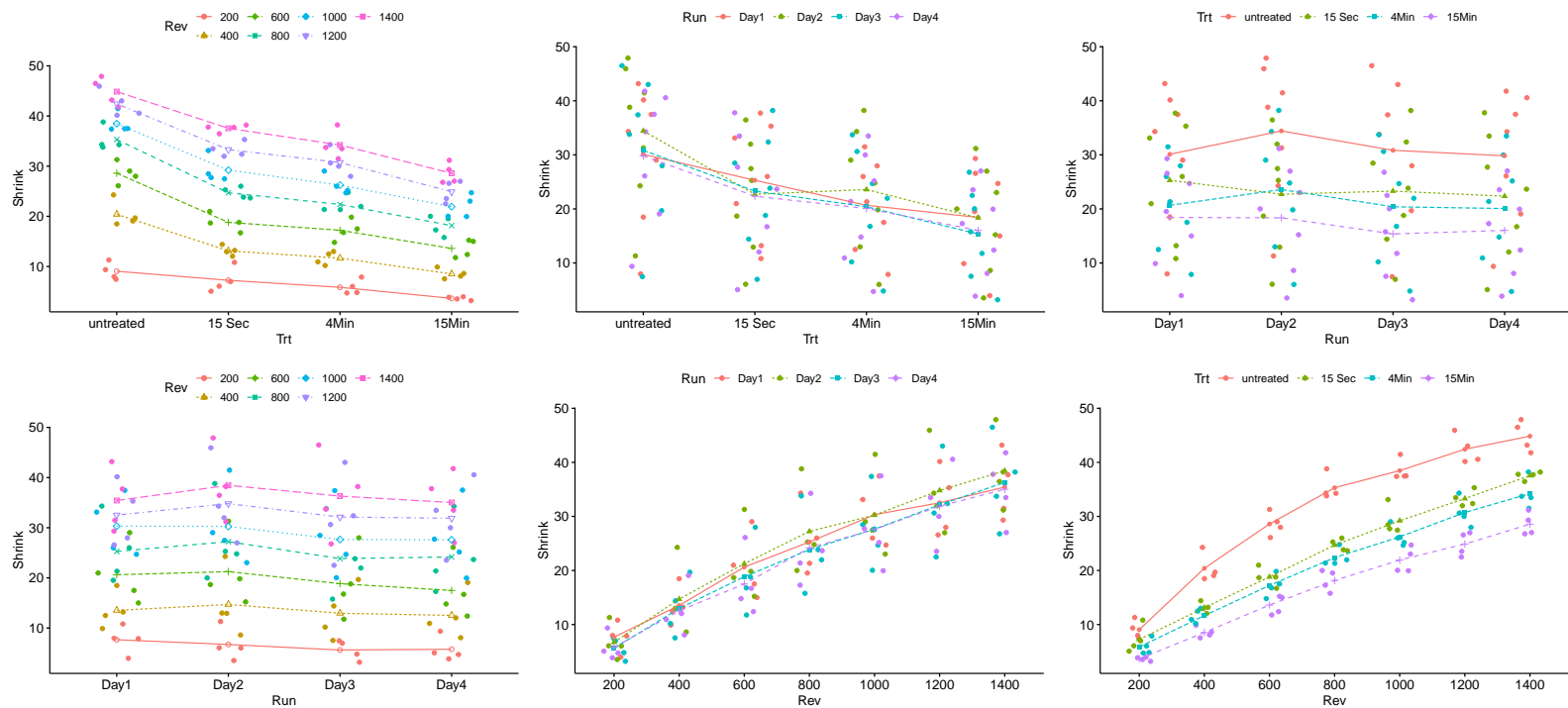
An experiment was conducted to compare 4 wool fiber treatments (Trt) at 7 dry cycle revolutions (Rev) over 4 experimental runs (Run) (i.e., the blocks). The outcome measured from this experiment was the top shrinkage (Shrink) of the fiber. A restriction on the randomization: within each experimental run (blocks), wool fiber treatments were randomized to whole plots, and within each whole plot, measurements were obtained for all of 7 dry cycle revolutions (split plot treatments). In other words, the experiment was set as a **split-plot** design with:

- whole plot (wool fiber treatment) treatments: untreated, alcoholic potash 15 Sec, alcoholic potash 4Min, and alcoholic potash 15Min;
- subplot treatments: dry cycle revolutions (200 to 1400 by 200); and
- blocks: 4 experimental runs (possibly different days).

Do a full analysis and report your findings for the experiment above (data in “Wool-Shrink.xlsx”), using a split plot design where both Trt and Rev are treated as categorical variables.

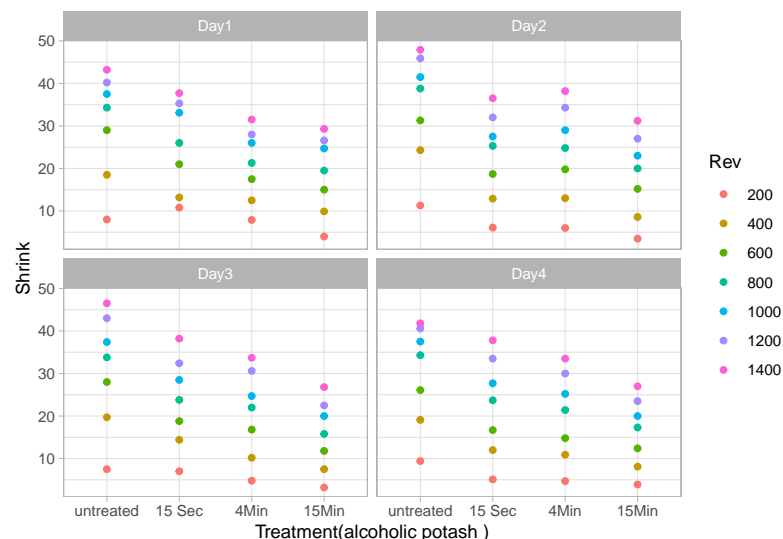
```
table_2019s2 <- readxl::read_xlsx("~/qushen26/stat2019_website/static/stat566/qe_lab/WoolShrink.xlsx")
table_2019s2$Run <- factor(table_2019s2$Run,labels=c("Day1","Day2","Day3","Day4"))
table_2019s2$Trt <- factor(table_2019s2$Trt,labels=c("untreated","15 Sec","4Min","15Min"))
table_2019s2$Rev <- as.factor(table_2019s2$Rev)
str(table_2019s2)
## Classes 'tbl_df', 'tbl' and 'data.frame': 112 obs. of 4 variables:
## $ Run : Factor w/ 4 levels "Day1","Day2",...: 1 1 1 1 1 1 1 1 1 1 ...
## $ Trt : Factor w/ 4 levels "untreated","15 Sec",...: 1 1 1 1 1 1 1 2 2 2 ...
## $ Rev : Factor w/ 7 levels "200","400","600",...: 1 2 3 4 5 6 7 1 2 3 ...
## $ Shrink: num 8 18.5 29 34.3 37.5 40.2 43.2 10.8 13.2 21 ...
```





The above plots show that: There is not much difference in the average shrink from different days. The average shrink are lower when the treatment is longer. The average shrink are higher when the revolutions are faster.

The tables show the same thing with the numerical summaries for each factor level and their combinations.



```
library(GAD)
table_2019s2$Run_r <- as.random(table_2019s2$Run)
table_2019s2$Trt_f <- as.fixed(table_2019s2$Trt)
table_2019s2$Rev_f <- as.fixed(table_2019s2$Rev)
model_2019s2_1 <- aov(formula = Shrink ~ Run_r+Trt_f + Trt_f%in%Run_r+ Rev_f%in%Run_r + Rev_f + Trt_f:Rev_f, data=table_2019s2)
pander::pander(gad(model_2019s2_1))
```

Table 3: Analysis of Variance Table

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
<b>Run_r</b>	3	124.3	41.43	36.47	5.099e-13
<b>Trt_f</b>	3	3013	1004	78.84	8.81e-07
<b>Rev_f</b>	6	11052	1842	876.8	3.405e-21
<b>Run_r:Trt_f</b>	9	114.6	12.74	11.21	1.218e-09
<b>Run_r:Rev_f</b>	18	37.81	2.101	1.849	0.04245
<b>Trt_f:Rev_f</b>	18	269.5	14.97	13.18	8.477e-14
<b>Residual</b>	54	61.35	1.136	NA	NA

The results show all the main effects and the interaction effect of Runs and Recolutions are significant at 0.05 significance level (P-value=0.5082).

```
library("lme4")
model_2019s2_2 <- lmer(formula = Shrink ~ (1|Run) + Trt + (1|Run:Trt) + Rev + (1|Run:Rev) + Trt:Rev, data=table_2019s2)
summary(model_2019s2_2)$varcor
## Groups Name Std.Dev.
## Run:Rev (Intercept) 0.49104
## Run:Trt (Intercept) 1.28736
```

```
## Run (Intercept) 0.99516
## Residual 1.06587
pander::pander(confint(model_2019s2_2)[1:4,1:2])
```

Computing profile confidence intervals ...

	2.5 %	97.5 %
.sig01	0	0.726
.sig02	0.7415	1.82
.sig03	0	2.512
.sigma	0.7906	1.097

The results of variance components show the variance of interaction term of Runs and revolutions is negligible and hence dropping interaction term of them.

```
model_2019s2_3 <- aov(formula = Shrink ~ Run_r+Trt_f + Trt_f%in%Run_r+ Rev_f + Trt_f:Rev_f, data=table_2019s2)
pander::pander(gad(model_2019s2_3))
```

Table 5: Analysis of Variance Table

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Run_r	3	124.3	41.43	30.08	1.024e-12
Trt_f	3	3013	1004	78.84	8.81e-07
Rev_f	6	11052	1842	1337	1.01e-71
Run_r:Trt_f	9	114.6	12.74	9.249	3.546e-09
Trt_f:Rev_f	18	269.5	14.97	10.87	4.62e-14
Residual	72	99.16	1.377	NA	NA

```
model_2019s2_4<- lmer(formula = Shrink ~ (1|Run)+Trt+Rev+(1|Run:Trt)+Rev*Trt, data=table_2019s2, REML = TRUE)
```

The ANOVA table of new model shows that the interaction effects are significant. This means that the effects of day v.s.revolutions and treatment v.s.revolutions on the shrink are not independent. Hence, the simple effects must be tested.

When the day2, the mean shrinks between the 15-Sec and 4-Min treatment don't have significant difference. For all the rest of days, the mean shrinks are significantly different between any different treatment.

The changes of days for a given treatment don't give consistent results.

For untreated cases, the mean shrinks are not significantly different between 1200 and 1400 revolutions. For all the rest of treatments, the mean shrinks are significantly different between any different revolutions.

For a given revolution, 15-Sec and 4-Min treatment don't have significant difference on the mean shrinks.

Run_r	Trt_f	contrast	estimate	SE	df	t.ratio	p.value
Day1	.	untreated - 15 Sec	4.8000000	0.6272872	72	7.6519974	0.0000000
Day1	.	untreated - 4Min	9.4285714	0.6272872	72	15.0307091	0.0000000
Day1	.	untreated - 15Min	11.6714286	0.6272872	72	18.6061960	0.0000000
Day1	.	15 Sec - 4Min	4.6285714	0.6272872	72	7.3787118	0.0000000
Day1	.	15 Sec - 15Min	6.8714286	0.6272872	72	10.9541986	0.0000000
Day1	.	4Min - 15Min	2.2428571	0.6272872	72	3.5754869	0.0219854
Day2	.	untreated - 15 Sec	11.7142857	0.6272872	72	18.6745174	0.0000000
Day2	.	untreated - 4Min	10.8428571	0.6272872	72	17.2853155	0.0000000
Day2	.	untreated - 15Min	16.0714286	0.6272872	72	25.6205270	0.0000000
Day2	.	<b>15 Sec - 4Min</b>	<b>-0.8714286</b>	<b>0.6272872</b>	<b>72</b>	<b>-1.3892019</b>	<b>0.9337766</b>
Day2	.	15 Sec - 15Min	4.3571429	0.6272872	72	6.9460095	0.0000001
Day2	.	4Min - 15Min	5.2285714	0.6272872	72	8.3352114	0.0000000
Day3	.	untreated - 15 Sec	7.5428571	0.6272872	72	12.0245673	0.0000000
Day3	.	untreated - 4Min	10.4428571	0.6272872	72	16.6476491	0.0000000
Day3	.	untreated - 15Min	15.4714286	0.6272872	72	24.6640273	0.0000000
Day3	.	15 Sec - 4Min	2.9000000	0.6272872	72	4.6230818	0.0006908
Day3	.	15 Sec - 15Min	7.9285714	0.6272872	72	12.6394600	0.0000000
Day3	.	4Min - 15Min	5.0285714	0.6272872	72	8.0163782	0.0000000
Day4	.	untreated - 15 Sec	7.4714286	0.6272872	72	11.9106983	0.0000000
Day4	.	untreated - 4Min	9.7571429	0.6272872	72	15.5545066	0.0000000
Day4	.	untreated - 15Min	13.8000000	0.6272872	72	21.9994925	0.0000000
Day4	.	15 Sec - 4Min	2.2857143	0.6272872	72	3.6438083	0.0179467
Day4	.	15 Sec - 15Min	6.3285714	0.6272872	72	10.0887942	0.0000000
Day4	.	4Min - 15Min	4.0428571	0.6272872	72	6.4449859	0.0000005
.	untreated	Day1 - Day2	-4.3285714	0.6272872	72	-6.9004619	0.0000001
.	<b>untreated</b>	<b>Day1 - Day3</b>	<b>-0.7428571</b>	<b>0.6272872</b>	<b>72</b>	<b>-1.1842377</b>	<b>0.9760367</b>
.	<b>untreated</b>	<b>Day1 - Day4</b>	<b>0.2714286</b>	<b>0.6272872</b>	<b>72</b>	<b>0.4327022</b>	<b>0.9999934</b>
.	untreated	Day2 - Day3	3.5857143	0.6272872	72	5.7162242	0.0000106
.	untreated	Day2 - Day4	4.6000000	0.6272872	72	7.3331642	0.0000000
.	<b>untreated</b>	<b>Day3 - Day4</b>	<b>1.0142857</b>	<b>0.6272872</b>	<b>72</b>	<b>1.6169399</b>	<b>0.8470570</b>
.	15 Sec	Day1 - Day2	2.5857143	0.6272872	72	4.1220581	0.0039382
.	<b>15 Sec</b>	<b>Day1 - Day3</b>	<b>2.0000000</b>	<b>0.6272872</b>	<b>72</b>	<b>3.1883322</b>	<b>0.0644078</b>
.	15 Sec	Day1 - Day4	2.9428571	0.6272872	72	4.6914032	0.0005393
.	<b>15 Sec</b>	<b>Day2 - Day3</b>	<b>-0.5857143</b>	<b>0.6272872</b>	<b>72</b>	<b>-0.9337259</b>	<b>0.9956923</b>
.	<b>15 Sec</b>	<b>Day2 - Day4</b>	<b>0.3571429</b>	<b>0.6272872</b>	<b>72</b>	<b>0.5693450</b>	<b>0.9999261</b>
.	<b>15 Sec</b>	<b>Day3 - Day4</b>	<b>0.9428571</b>	<b>0.6272872</b>	<b>72</b>	<b>1.5030709</b>	<b>0.8959656</b>
.	4Min	Day1 - Day2	-2.9142857	0.6272872	72	-4.6458556	0.0006362
.	<b>4Min</b>	<b>Day1 - Day3</b>	<b>0.2714286</b>	<b>0.6272872</b>	<b>72</b>	<b>0.4327022</b>	<b>0.9999934</b>
.	<b>4Min</b>	<b>Day1 - Day4</b>	<b>0.6000000</b>	<b>0.6272872</b>	<b>72</b>	<b>0.9564997</b>	<b>0.9948267</b>
.	4Min	Day2 - Day3	3.1857143	0.6272872	72	5.0785578	0.0001277
.	4Min	Day2 - Day4	3.5142857	0.6272872	72	5.6023552	0.0000166
.	<b>4Min</b>	<b>Day3 - Day4</b>	<b>0.3285714</b>	<b>0.6272872</b>	<b>72</b>	<b>0.5237974</b>	<b>0.9999643</b>
.	<b>15Min</b>	<b>Day1 - Day2</b>	<b>0.0714286</b>	<b>0.6272872</b>	<b>72</b>	<b>0.1138690</b>	<b>1.0000000</b>
.	15Min	Day1 - Day3	3.0571429	0.6272872	72	4.8735936	0.0002759
.	15Min	Day1 - Day4	2.4000000	0.6272872	72	3.8259987	0.0102635
.	15Min	Day2 - Day3	2.9857143	0.6272872	72	4.7597246	0.0004202
.	15Min	Day2 - Day4	2.3285714	0.6272872	72	3.7121297	0.0145964
.	<b>15Min</b>	<b>Day3 - Day4</b>	<b>-0.6571429</b>	<b>0.6272872</b>	<b>72</b>	<b>-1.0475949</b>	<b>0.9898501</b>

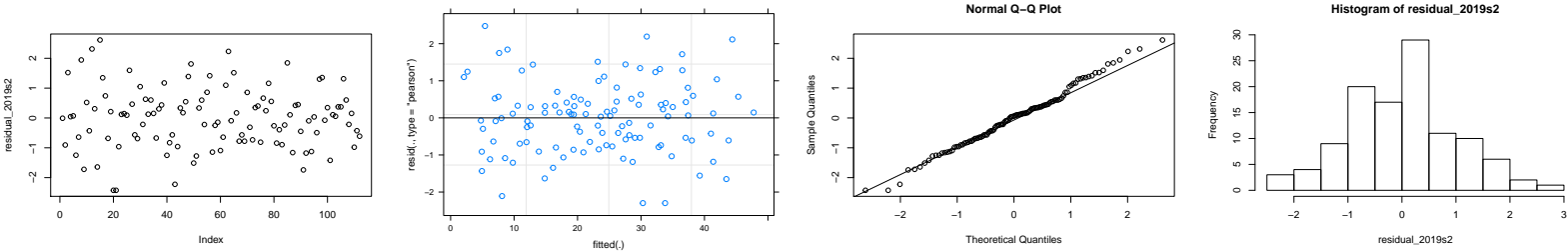
Trt	Rev	contrast	estimate	SE	df	t.ratio	p.value
untreated	.	200 - 400	-11.350	0.8298202	72.00000	-13.677662	0.0000000
untreated	.	200 - 600	-19.550	0.8298202	72.00000	-23.559322	0.0000000
untreated	.	200 - 800	-26.250	0.8298202	72.00000	-31.633360	0.0000000
untreated	.	200 - 1000	-29.425	0.8298202	72.00000	-35.459490	0.0000000
untreated	.	200 - 1200	-33.375	0.8298202	72.00000	-40.219558	0.0000000
untreated	.	200 - 1400	-35.800	0.8298202	72.00000	-43.141878	0.0000000
untreated	.	400 - 600	-8.200	0.8298202	72.00000	-9.881659	0.0000000
untreated	.	400 - 800	-14.900	0.8298202	72.00000	-17.955698	0.0000000
untreated	.	400 - 1000	-18.075	0.8298202	72.00000	-21.781828	0.0000000
untreated	.	400 - 1200	-22.025	0.8298202	72.00000	-26.541896	0.0000000
untreated	.	400 - 1400	-24.450	0.8298202	72.00000	-29.464216	0.0000000
untreated	.	600 - 800	-6.700	0.8298202	72.00000	-8.074039	0.0000000
untreated	.	600 - 1000	-9.875	0.8298202	72.00000	-11.900169	0.0000000
untreated	.	600 - 1200	-13.825	0.8298202	72.00000	-16.660236	0.0000000
untreated	.	600 - 1400	-16.250	0.8298202	72.00000	-19.582556	0.0000000
untreated	.	800 - 1000	-3.175	0.8298202	72.00000	-3.826130	0.0233929
untreated	.	800 - 1200	-7.125	0.8298202	72.00000	-8.586198	0.0000000
untreated	.	800 - 1400	-9.550	0.8298202	72.00000	-11.508518	0.0000000
untreated	.	1000 - 1200	-3.950	0.8298202	72.00000	-4.760067	0.0010353
untreated	.	1000 - 1400	-6.375	0.8298202	72.00000	-7.682387	0.0000000
untreated	.	1200 - 1400	-2.425	0.8298202	72.00000	-2.922320	0.2371347
15 Sec	.	200 - 400	-5.875	0.8298202	72.00000	-7.079847	0.0000000
15 Sec	.	200 - 600	-11.750	0.8298202	72.00000	-14.186679	0.0000000
15 Sec	.	200 - 800	-17.450	0.8298202	72.00000	-21.028653	0.0000000
15 Sec	.	200 - 1000	-21.950	0.8298202	72.00000	-26.451514	0.0000000
15 Sec	.	200 - 1200	-26.050	0.8298202	72.00000	-31.392344	0.0000000
15 Sec	.	200 - 1400	-30.300	0.8298202	72.00000	-36.513936	0.0000000
15 Sec	.	400 - 600	-5.675	0.8298202	72.00000	-6.838831	0.0000003
15 Sec	.	400 - 800	-11.575	0.8298202	72.00000	-13.948806	0.0000000
15 Sec	.	400 - 1000	-16.075	0.8298202	72.00000	-19.371667	0.0000000
15 Sec	.	400 - 1200	-20.175	0.8298202	72.00000	-24.312497	0.0000000
15 Sec	.	400 - 1400	-24.425	0.8298202	72.00000	-29.434089	0.0000000
15 Sec	.	600 - 800	-5.900	0.8298202	72.00000	-7.109974	0.0000001
15 Sec	.	600 - 1000	-10.400	0.8298202	72.00000	-12.532836	0.0000000
15 Sec	.	600 - 1200	-14.500	0.8298202	72.00000	-17.473666	0.0000000
15 Sec	.	600 - 1400	-18.750	0.8298202	72.00000	-22.595257	0.0000000
15 Sec	.	800 - 1000	-4.500	0.8298202	72.00000	-5.422862	0.0000856
15 Sec	.	800 - 1200	-8.600	0.8298202	72.00000	-10.363691	0.0000000
15 Sec	.	800 - 1400	-12.850	0.8298202	72.00000	-15.485283	0.0000000
15 Sec	.	1000 - 1200	-4.100	0.8298202	72.00000	-4.940830	0.0005344
15 Sec	.	1000 - 1400	-8.350	0.8298202	72.00000	-10.062421	0.0000000
15 Sec	.	1200 - 1400	-4.250	0.8298202	72.00000	-5.121592	0.0002718
4Min	.	200 - 400	-5.800	0.8298202	72.00000	-6.989466	0.0000001
4Min	.	200 - 600	-11.375	0.8298202	72.00000	-13.707789	0.0000000
4Min	.	200 - 800	-16.525	0.8298202	72.00000	-19.913953	0.0000000
4Min	.	200 - 1000	-20.375	0.8298202	72.00000	-24.553513	0.0000000
4Min	.	200 - 1200	-24.875	0.8298202	72.00000	-29.976375	0.0000000
4Min	.	200 - 1400	-28.375	0.8298202	72.00000	-34.194156	0.0000000
4Min	.	400 - 600	-5.275	0.8298202	72.00000	-6.371823	0.0000004
4Min	.	400 - 800	-10.725	0.8298202	72.00000	-12.924487	0.0000000
4Min	.	400 - 1000	-14.575	0.8298202	72.00000	-17.564047	0.0000000
4Min	.	400 - 1200	-19.075	0.8298202	72.00000	-22.986908	0.0000000
4Min	.	400 - 1400	-22.575	0.8298202	72.00000	-27.204690	0.0000000
4Min	.	600 - 800	-5.150	0.8298202	72.00000	-6.206164	0.0000037
4Min	.	600 - 1000	-9.000	0.8298202	72.00000	-10.845724	0.0000000
4Min	.	600 - 1200	-13.500	0.8298202	72.00000	-16.268585	0.0000000
4Min	.	600 - 1400	-17.000	0.8298202	72.00000	-20.486367	0.0000000
4Min	.	800 - 1000	-3.850	0.8298202	72.00000	-4.639559	0.0015948
4Min	.	800 - 1200	-8.350	0.8298202	72.00000	-10.062421	0.0000000
4Min	.	800 - 1400	-11.850	0.8298202	72.00000	-14.280203	0.0000000
4Min	.	1000 - 1200	-4.500	0.8298202	72.00000	-5.422862	0.0000856
4Min	.	1000 - 1400	-8.000	0.8298202	72.00000	-9.640643	0.0000000
4Min	.	1200 - 1400	-3.500	0.8298202	72.00000	-4.217781	0.0006740
15Min	.	200 - 400	-4.875	0.8298202	72.00000	-5.874267	0.0000143
15Min	.	200 - 600	-9.950	0.8298202	72.00000	-11.990550	0.0000000
15Min	.	200 - 800	-14.500	0.8298202	72.00000	-17.473666	0.0000000
15Min	.	200 - 1000	-18.275	0.8298202	72.00000	-22.022844	0.0000000
15Min	.	200 - 1200	-21.250	0.8298202	72.00000	-25.607958	0.0000000
15Min	.	200 - 1400	-24.925	0.8298202	72.00000	-30.036629	0.0000000
15Min	.	400 - 600	-5.075	0.8298202	72.00000	-6.115783	0.0000054
15Min	.	400 - 800	-9.625	0.8298202	72.00000	-11.598899	0.0000000
15Min	.	400 - 1000	-13.400	0.8298202	72.00000	-16.148077	0.0000000
15Min	.	400 - 1200	-16.375	0.8298202	72.00000	-19.733191	0.0000000
15Min	.	400 - 1400	-20.050	0.8298202	72.00000	-24.161862	0.0000000
15Min	.	600 - 800	-4.550	0.8298202	72.00000	-5.483116	0.0000677
15Min	.	600 - 1000	-8.325	0.8298202	72.00000	-10.032294	0.0000000
15Min	.	600 - 1200	-11.300	0.8298202	72.00000	-13.617408	0.0000000
15Min	.	600 - 1400	-14.975	0.8298202	72.00000	-18.046079	0.0000000
15Min	.	800 - 1000	-3.775	0.8298202	72.00000	-4.549179	0.0021940
15Min	.	800 - 1200	-6.750	0.8298202	72.00000	-8.134293	0.0000000
15Min	.	800 - 1400	-10.425	0.8298202	72.00000	-12.562963	0.0000000
15Min	.	1000 - 1200	-2.975	0.8298202	72.00000	-3.585114	0.0471045
15Min	.	1000 - 1400	-6.650	0.8298202	72.00000	-8.013785	0.0000000
15Min	.	1200 - 1400	-3.675	0.8298202	72.00000	-4.428670	0.0033332
.	200	untreated - 15 Sec	1.800	1.2247951	23.24089	1.469634	0.9779005
.	200	untreated - 4Min	3.200	1.2247951	23.24089	2.612682	0.4439814
.	200	untreated - 15Min	5.400	1.2247951	23.24089	4.408901	0.0141095
.	200	15 Sec - 4Min	1.400	1.2247951	23.24089	1.143048	0.9980039
.	200	15 Sec - 15Min	3.600	1.2247951	23.24089	2.939267	0.2741847
.	200	4Min - 15Min	2.200	1.2247951	23.24089	1.796219	0.9003941
.	400	untreated - 15 Sec	7.275	1.2247951	23.24089	5.939769	0.0004029
.	400	untreated - 4Min	8.750	1.2247951	23.24089	7.144052	0.0000259
.	400	untreated - 15Min	11.875	1.2247951	23.24089	9.695499	0.0000001
.	400	15 Sec - 4Min	1.475	1.2247951	23.24089	1.204283	0.9965681
.	400	15 Sec - 15Min	4.600	1.2247951	23.24089	3.755730	0.0589409
.	400	4Min - 15Min	3.125	1.2247951	23.24089	2.551447	0.4805075
.	600	untreated - 15 Sec	9.800	1.2247951	23.24089	8.001338	0.0000040
.	600	untreated - 4Min	11.375	1.2247951	23.24089	9.287268	0.0000003
.	600	untreated - 15Min	15.000	1.2247951	23.24089	12.246946	0.0000000
.	600	15 Sec - 4Min	1.575	1.2247951	23.24089	1.285929	0.9934208
.	600	15 Sec - 15Min	5.200	1.2247951	23.24089	4.245608	0.0203740
.	600	4Min - 15Min	3.625	1.2247951	23.24089	2.959679	0.2652318
.	800	untreated - 15 Sec	10.600	1.2247951	23.24089	8.654509	0.0000010
.	800	untreated - 4Min	12.925	1.2247951	23.24089	10.552786	0.0000000
.	800	untreated - 15Min	17.150	1.2247951	23.24089	14.002342	0.0000000
.	800	15 Sec - 4Min	2.325	1.2247951	23.24089	1.898277	0.8595571
.	800	15 Sec - 15Min	6.550	1.2247951	23.24089	5.347833	0.0016047
.	800	4Min - 15Min	4.225	1.2247951	23.24089	3.449557	0.1095576
.	1000	untreated - 15 Sec	9.275	1.2247951	23.24089	7.572695	0.0000101
.	1000	untreated - 4Min	12.250	1.2247951	23.24089	10.001673	0.0000001
.	1000	untreated - 15Min	16.550	1.2247951	23.24089	13.512464	0.0000000
.	1000	15 Sec - 4Min	2.975	1.2247951	23.24089	2.428978	0.5561910
.	1000	15 Sec - 15Min	7.275	1.2247951	23.24089	5.939769	0.0004029
.	1000	4Min - 15Min	4.300	1.2247951	23.24089	3.510791	0.0971176
.	1200	untreated - 15 Sec	9.125	1.2247951	23.24089	7.450226	0.0000132
.	1200	untreated - 4Min	11.700	1.2247951	23.24089	9.552618	0.0000002
.	1200	untreated - 15Min	17.525	1.2247951	23.24089	14.308516	0.0000000
.	1200	15 Sec - 4Min	2.575	1.2247951	23.24089	2.102393	0.7561134
.	1200	15 Sec - 15Min	8.400	1.2247951	23.24089	6.858290	0.0000491
.	1200	4Min - 15Min	5.825	1.2247951	23.24089	4.755898	0.0063729
.	1400	untreated - 15 Sec	7.300	1.2247951	23.24089	5.960181	0.0003842
.	1400	untreated - 4Min	10.625	1.2247951	23.24089	8.674920	0.0000010
.	1400	untreated - 15Min	16.275	1.2247951	23.24089	13.287937	0.0000000

- Conclusion

Choosing a higher revolution for a given treatment can get a larger shrink.

In most of the cases, longer alcoholic potash have less shrink. This effect will be more significant when higher revolution.

- Model Adequacy Checking



In the plots of residuals versus predicted value of shrink, there is no significant pattern on this plot. Therefore, the fitted model is good enough to describe the relationship between the mean value of shrink and the days, revolutions, and treatment.

The residuals in this plot are almost symmetrically distributed about zero and hence zero mean assumption is not violated. Further, the vertical deviation of the residuals from zero is about same for each predicted value and hence the constant variance assumption is not violated.

The points are along the straight line in the normal qq plot shown at bottom left and the histogram of residuals shown at the top right is about normal. These plots show no violation of normal distribution assumption of residuals.