Discrete Choice Modeling

USP 657

SIC

`A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models`

1 Introduction

2 Elements of the choice decision process

3 Utility-based choice theory

4 The Multinomial Logit Model

Extreme Value Type I Distribution

- Probability Density Function

The extreme value type I distribution has two forms. One is based on the smallest extreme and the other is based on the largest extreme. We call these the minimum and maximum cases, respectively. Formulas and plots for both cases are given. The extreme value type I distribution is also referred to as the Gumbel distribution.

The general formula for the probability density function of the Gumbel (minimum) distribution is $f(x) = \frac{1}{\beta}e^{\frac{x-\mu}{\beta}}e^{-e^{\frac{x-\mu}{\beta}}}$

where μ is the location parameter and β is the scale parameter.

The standard Gumbel distribution (minimum) $f(x) = e^{x}e^{-e^{x}}$

The general formula for the probability density function of the Gumbel (maximum) distribution is $f(x) = \frac{1}{\beta}e^{-\frac{x-\mu}{\beta}}e^{-e^{-\frac{x-\mu}{\beta}}}$

the standard Gumbel distribution (maximum) $f(x) = e^{-x}e^{-e^{-x}}$

The following is the plot of the Gumbel probability density function for the maximum case.

- Cumulative Distribution Function

The formula for the cumulative distribution of the Gumbel distribution (minimum) is

$F(x) = 1 - e^{-e^{x}}$

The formula for the cumulative distribution function of the Gumbel distribution (maximum) is $F(x) = e^{-e^{-x}}$

Gumbel_pmf	Gumbel_cdf
Gumbel_pdf	Gumbel_cdf

`p.8-10` Example:

DCM

`Discrete choice methods with simulation`

Common Distributions

D	$E; EX^2; V$	$f(x); F(x), P(X \le x)$	$MLE; T; I$	$M_x(t); I$
$Bern(p)$	p;p;pq	$p^x q^{1-x}, x = 1, 0; 0 \le p \le 1$	$\bar{X}$;$\sum x_i \sim Bin(n,p)$;$\frac{1}{pq}$; $Ep_{mle} = p$,$Vp_{mle} = \frac{pq}{n}$	$pe^t + q$
$Bino(n,p)$	np;$np(np+q)$;npq	$\binom{n}{x} p^x q^{n-x}$,$x = 0, 1..n; 0 \le p \le 1;$	$\bar{X}$or$k \ge X_{(n)}$;$\sum x_i \sim Bino(n,p)$;$1/pq$	$(pe^t + q$
$Geom(p)$	$1/p$;$(p+2q)/p^2$;q/p^2	pq^{x-1},$x = 1, 2, ..; 0 \le p \le 1$;$1 - q^x$	$1/\bar{X}$;$\sum x_i$;	$\frac{pe^t}{1-qe^t}, t$
$NBino(r,p)$	r/p; ;rq/p^2;$0 \le p \le 1$	$\binom{x-1}{r-1} p^r q^{x-r}$,$x = r, r+1..$	$\bar{X}$;$\sum x_i$	$(\frac{pe^t}{1-qe^t})^r$
$HGeom(N,m,k)$	$\frac{km}{N}$;;$\mu\frac{(N-m)(N-k)}{N(N-1)}$;$N, m, k \ge 0$	$\binom{m}{x}\binom{N-m}{k-x}/\binom{N}{k}$;	$m-(N-k) \le x \le m$	
$Pois(\mu)$	μ;$\mu^2+\mu$;μ;$\mu \ge 0$	$\frac{\mu^x}{x!}e^{-\mu}$,$x = 0, 1..$;$e^{-\mu}\sum_{i=0}^{x}\frac{\mu^i}{i!}$	$\bar{X}$;$\sum x_i$;$\frac{1}{\mu}$;$P(x+1) = \frac{\lambda}{\lambda+1}P(x)$	$e^{\mu(e^t-1)}$
$Unif(n)$	$\frac{n+1}{2}$;$\frac{(n+1)(2n+1)}{6}$;$\frac{n^2-1}{12}$	$\frac{1}{n}$,$x = 1, 2..n$,$n = b-a+1$;$\frac{x-a+1}{n}$	;X_nComp	$\frac{1}{n}\sum_{i=1}^{n}$
$Unif(a,b)$	$\frac{a+b}{2}$; ;$\frac{(b-a)^2}{12}$	$\frac{1}{b-a}$,$a \le x \le b$;$\frac{x-a}{b-a}$	$\min x_{(1)}, x_{(n)}$;R ancillary	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
$Norm(\mu,\sigma^2)$	μ;$\mu^2+\sigma^2$;σ^2	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$,$\sigma > 0$	$\bar{X}, \frac{\sum(x_i-\bar{x})^2}{n}$;;$I_\mu = \frac{1}{\sigma^2}$,$I_{\sigma^2} = \frac{1}{2\sigma^4}$; $E\hat{\sigma}^2_{mle} = \frac{(n-1)\sigma^2}{n}$,$V\hat{\sigma}_{mle}$ [illegible]	$e^{\mu t+\frac{\sigma^2 t^2}{2}}$
$SNorm(0,1)$	0;1;1	$\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$		$e^{\frac{t^2}{2}}$
$LNorm(\mu,\sigma^2)$	$e^{\mu+\frac{\sigma^2}{2}}$;$e^{2\mu+2\sigma^2}$;$E^2X(e^{\sigma^2}-1)$	$\frac{1}{x\sigma\sqrt{2\pi}}e^{\frac{-(\ln x-\mu)^2}{2\sigma^2}}$,$x \ge 0$,$\sigma > 0$	$\hat{\mu} = \frac{1}{n}\sum \ln x_i$,$\hat{\sigma}^2 = \frac{1}{n}\sum \ln(x_i-\hat{\mu})^2$	$EX^n =$
$Cauchy(\theta,\sigma)$		$\frac{1}{\pi\sigma}\frac{1}{1+(\frac{x-\theta}{\sigma})^2}$;;$\sigma > 0$	$=t_1$;$=\frac{Z_1}{Z_2}$	
$DExpo(\mu,\sigma^2)$	μ;$\mu^2+2\sigma^2$;$2\sigma^2$	$\frac{1}{2\sigma}e^{-\lvert\frac{x-\mu}{\sigma}\rvert}$,$\sigma > 0;$	$\hat{\mu} = median$,$\hat{\sigma} = \frac{1}{n}\sum\lvert x_i\rvert$	$\frac{e^{\mu t}}{1-\sigma^2 t^2}$
$Expo(\beta)$	β; ;β^2,$\beta > 0$	$\frac{1}{\beta}e^{-\frac{x}{\beta}}, x \ge 0$;$1-e^{-\lambda x}$	$\frac{1}{\bar{X}}$;$\sum x_i$;$\frac{1}{\lambda^2}$;$E\lambda_{mle} = \frac{\lambda}{n-1}$, $V\lambda_{mle} = \frac{n^2\lambda^2}{(n-1)^2(n-2)}$,$V\lambda$ [illegible]	$\frac{\lambda}{\lambda-t}, t$ [illegible]
$Gamm(\alpha,\beta)$	$\alpha\beta$; ;$\alpha\beta^2$;$\alpha, \beta > 0$	$\frac{1}{\Gamma(a)\beta^\alpha}x^{a-1}e^{-x/\beta}$,$x \ge 0$	$\prod x_i$,$\sum x_i$	$(\frac{1}{1-\beta t})^a$
$Beta(a,b)$	$\frac{a}{a+b}$;$\frac{a(a+1)}{(a+b)(a+b+1)}$;$\frac{ab}{(a+b)^2(a+b+1)}$	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$;$0 \le x \le 1$	$\alpha > 0, \beta > 0$	$B(\alpha,\beta)$
χ^2_p	p;$2p+p^2$;$2p$	$\frac{1}{\Gamma\frac{p}{2}2^{\frac{p}{2}}}x^{\frac{p}{2}-1}e^{-\frac{x}{2}}$,$x \ge 0; p = 1, 2..$	;$\sum \ln x_i$;	$(1-2t)$
t_p	$0, p > 1$; ;$\frac{p}{p-2}, p > 2$	$\frac{\Gamma(\frac{p+1}{2})}{\Gamma(\frac{p}{2})\sqrt{p\pi}}(1+\frac{x^2}{p})^{-\frac{p+1}{2}}$		
$F_{p,q}$	$\frac{q}{q-2}, q > 2$;;$2(\frac{q}{q-2})^2\frac{p+q-2}{p(q-4)}, q > 4$	$\frac{\Gamma(\frac{p+q}{2})}{\Gamma(\frac{p}{2})\Gamma(\frac{q}{2})}(\frac{p}{q})^{\frac{p}{2}}\frac{x^{\frac{p}{2}-1}}{(1+\frac{p}{q}x)^{\frac{p+q}{2}}}$,$x \ge 0$	$F_{p,q} = \frac{\chi^2_p/p}{\chi^2_q/q}$;$F_{1,q} = t^2_q$	
$Arcsine$	$\frac{1}{2}$; ;$\frac{1}{8}$	$\frac{1}{\pi\sqrt{x(1-x)}}$,$x \in [0,1]$;$\frac{1}{\pi \arcsin\sqrt{x}}$		$Beta(\frac{1}{2}$,
$Dirichlet$	$\frac{a_i}{\sum_k a_k}\sum_{i=1}^{k} x_i = 1$;;$\frac{a_i(a_0-a_i)}{a_0^2(a_0+1)}$	$\frac{1}{B(a)}\Pi_{i=1}^{k}x_i^{a_i-1}$,$x \in (0,1)$	$B(a) = \frac{\Pi_{i=1}^{k}\Gamma(a_i)}{\Gamma(\sum_{i=1}^{k}a_i)}$	$Cov(X_i$
$Weibull(\gamma,\beta)$	$\beta^{1/\gamma}\Gamma(1+1/\gamma)$;; $\beta^{2/\gamma}[\Gamma(1+2/\gamma)-\Gamma^2(1+1/\gamma)]$	$\frac{\gamma}{\beta}x^{\gamma-1}e^{-\frac{x^\gamma}{\beta}}$,$x \ge 0$,$\gamma > 0, \beta > 0;$ $1-e^{-(\frac{x}{\beta})^\gamma}$	x^γ,$\sum x^\gamma$,$\sum \ln x$	;;$\beta^{\frac{n}{\gamma}}\Gamma(1$
$Pareto(\alpha,\beta)$	$\frac{\beta\alpha}{\beta-1}$,$\beta > 1$;;$\frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}$,$\beta > 2$	$\frac{\beta\alpha^\beta}{x^{\beta+1}}$;$1-(\frac{\alpha}{x})^\beta$,$x > \alpha$,$\alpha, \beta > 0$	$\hat{\alpha} = \min x_i$,$\hat{\beta} = \frac{n}{\sum \ln(x_i/x_{(1)})}$; $\sum x_i$comp/suff;$1/\beta^2$	