STAT 661: Project

LS v.s. EM

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1 Least Square Method v.s. EM Method

Schmee, J., & Hahn, G. (1979). A Simple Method for Regression Analysis with Censored Data. Technometrics, 21(4), 417-432. doi:10.2307/1268280

Aitkin, M. (1981). A Note on the Regression Analysis of Censored Data. Technometrics, 23(2), 161-163. doi:10.2307/1268032

1.1 Introduction

Problems requiring regression analysis of censored data arise frequently in practice. For example, in accelerated testing one wishes to relate stress and average time to failure from data including unfailed units, i. e., censored observations. Maximum likelihood is one method for obtaining the desired estimates; in this paper, we propose an alternative approach. An initial least squares fit is obtained treating the censored values as failures. Then, based upon this initial fit, the expected failure time for each censored observation is estimated. These estimates are then used, instead of the censoring times, to obtain a revised least squares fit and new expected failure times are estimated for the censored values. These are then used in a further least squares fit. The procedure is iterated until convergence is achieved. This method is simpler to implement and explain to non-statisticians than maximum likelihood and appears to have good statistical and convergence properties. The method is illustrated by an example, and some simulation results are described. Variations and areas for further study also are discussed.

1.2 Least Square Method

Description of method for simple situation

$\mu_x = \beta_0 + \beta_1 x$

$\mu_x^\star = \mu_x + \frac{\sigma f(z)}{1-F(z)}$ where $z = \frac{(c_x - \mu_x)}{\sigma}$

- Iteration 0
  - Step 1:$\hat{\beta}_0^{(0)} = -4.9307$,$\hat{\beta}_1^{(0)} = 3.7471$,$\hat{\sigma}^{(0)} = 0.1572$.
  - Step 2:$x = \frac{1000}{170+273.2} = 2.256318$

$\hat{\mu}_{2.26}^{(0)} = -4.9307 + 3.7471 \frac{1000}{170+273.2} = 3.523948$

$C_{2.26} = \log_{10}(5448) = 3.736237$

$z = \frac{C_{2.26} - \hat{\mu}_{2.26}^{(0)}}{\sigma} = \frac{3.736237-3.523948}{0.1572178} = 1.350286$

$\hat{\mu}_{2.26}^{\star(0)} = \hat{\mu}_{2.26}^{(0)} + \hat{\sigma}^{(0)} \frac{f(z)}{1-F(z)} = 3.8089$ Or 6440 hours

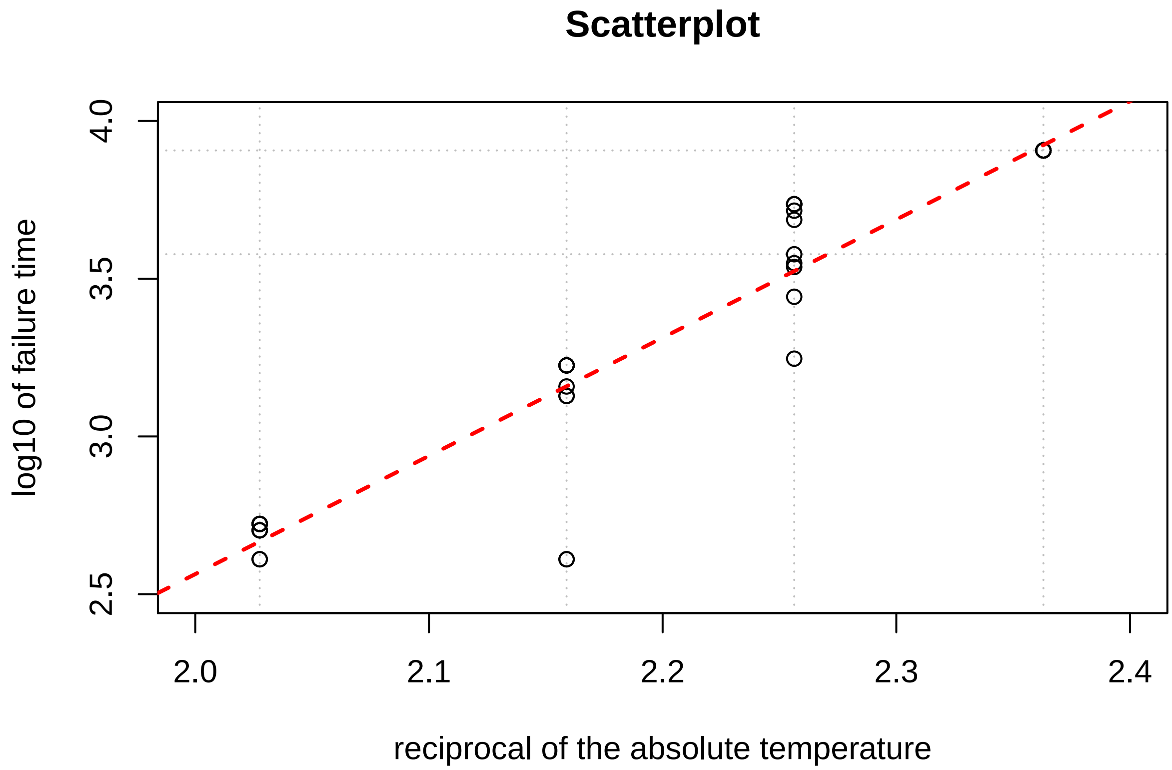
- Iteration 1
  - Step 1:$\hat{\beta}_0^{(1)} = -5.2603$,$\hat{\beta}_1^{(1)} = 3.9263$,$\hat{\sigma}^{(1)} = 0.1799$.
  - Step 2:$\hat{\mu}_{2.26}^{\star(1)} = 3.83972$
- Subsequent Iterations

$\hat{\beta}_0 = -5.81829$,$\hat{\beta}_1 = 4.20426$,$\hat{\sigma} = 0.204322$.

$\hat{\mu}_{2.26}^{\star(17)} = 3.87676$

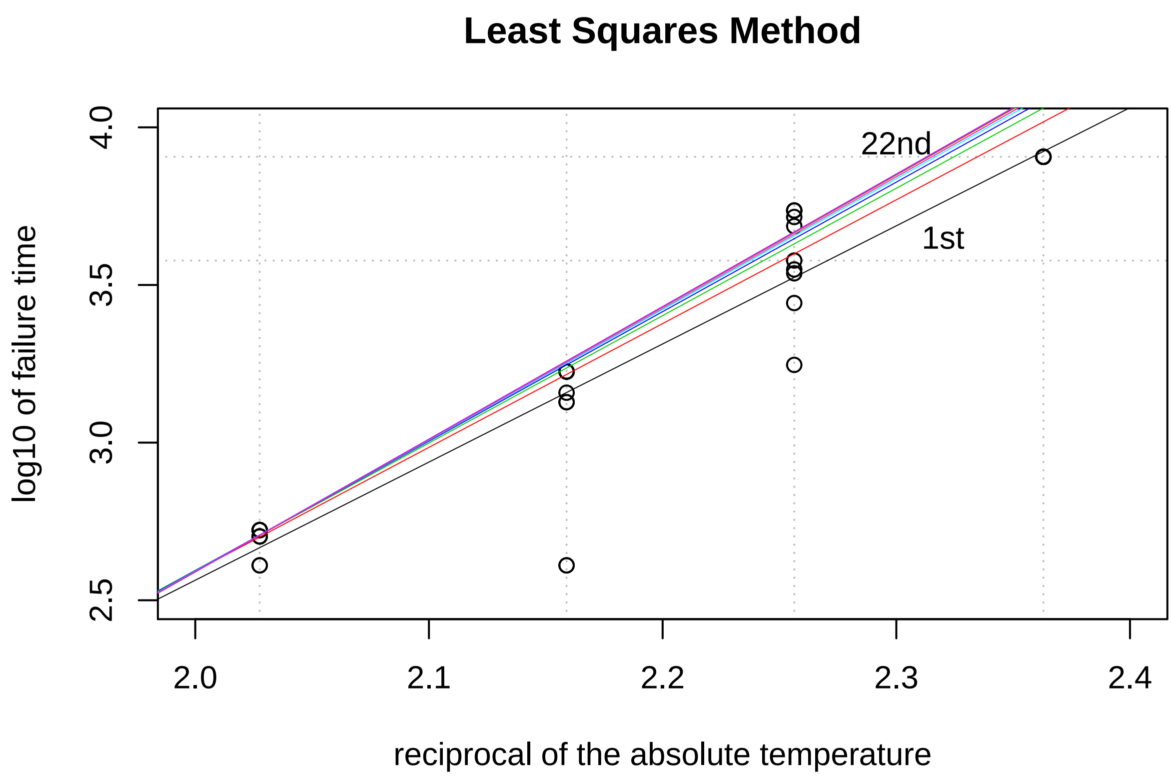
1.3 LS Code

```
temp <- c(150,170,190,220) #temperature levels
trec <- 1000/(temp+273.2) #reciprocal of the absolute temperature T
x <- c(rep(trec[1],10),rep(trec[2],10),rep(trec[3],10),rep(trec[4],10))
cen <- c(8064,5448,1680,528) #censoring times
logcen <- log10(cen) #log10 censoring times
y_uncensored <-log10(c(rep(1,10),1764,2772,3444,3542,3780,4860,5196,rep(1,3),408,408,1344,1344,1440,rep
y_censored <- c(rep(logcen[1],10),rep(0,7),rep(logcen[2],3),rep(0,5),rep(logcen[3],5),rep(0,5),rep(logce
S <- 23; Y<-matrix(nrow=S,ncol=40)
Y[1,] <- y_0 <- y_uncensored+y_censored
fit0 <- lm(y_0~x) #linear model between log10 of observed life time for different r eciprocal values
plot(x,y_0,main="Scatterplot",xlab="reciprocal of the absolute temperature",ylab="log10 of failure time
     xlim=c(2,2.4),ylim=c(2.5,4),
     panel.first=abline(h=c(3.577492,3.906551),v=c(2.027575,2.158895,2.256318,2.362949),lty=3,col="gray
abline(fit0,lwd=2,lty=2,col="red")
```



```r
# Iteration 0
sigma_0 <- sigma(fit0) #standard error of residuals
beta_00 <- coef(fit0)[1] #intercept
beta_10 <- coef(fit0)[2] #slope
mu_0 <- beta_00 + beta_10*trec #mean log time to failure
z <- (logcen-mu_0)/sigma_0 #z-vector
ex_mu_0 <- mu_0 + sigma_0*dnorm(z)/(1-pnorm(z)) #new expected mean log times to failure
delta = 1e-006; iteration <- 1
PHI<-matrix(nrow=S,ncol=8,dimnames=list(NULL, c('mu150','mu170','mu190','mu220','Intercept','Slope','Si
PHI[1,]<-phi<-c(ex_mu_0, beta_00, beta_10,sigma_0,iteration)
# Subsequent iteration
repeat {
 phi[8] <- phi[8]+1
 y_censored <- c(rep(phi[1],10),rep(0,7),rep(phi[2],3),rep(0,5),rep(phi[3],5),rep(0,5),rep(phi[4],5))
 y<- y_uncensored+y_censored
 Y[phi[8],]<-y   # Replace the new censored values
 fit <- lm(y~x)   # fit a new model
 phi[5] <- coef(fit)[1] #intercept
 phi[6] <- coef(fit)[2] #slope
 phi[7] <- sigma(fit) #standard error of residuals
 mu <- phi[5] + phi[6]*trec
 z <- (logcen-mu)/phi[7] #z-vector
 phi[1:4] <- mu + phi[7]*dnorm(z)/(1-pnorm(z)) #new expected mean log times to failure
 conv <- dist(rbind(PHI[phi[8]-1,1:4],phi[1:4]))
 if(conv < delta) break
  PHI[phi[8],]<-phi
}
```

mu150	mu170	mu190	mu220	Intercept	Slope	Sigma	Iteration
4.038	3.809	3.329	2.83	-4.931	3.747	0.1572	1
4.099	3.84	3.366	2.858	-5.26	3.926	0.1799	2
4.131	3.856	3.382	2.869	-5.486	4.04	0.1911	3
4.149	3.865	3.39	2.874	-5.623	4.108	0.1969	4
4.159	3.87	3.395	2.877	-5.704	4.148	0.2001	5
4.165	3.873	3.397	2.878	-5.752	4.172	0.2019	6
4.168	3.874	3.399	2.879	-5.779	4.185	0.2029	7
4.17	3.875	3.4	2.879	-5.795	4.193	0.2035	8
4.171	3.876	3.4	2.879	-5.805	4.198	0.2039	9
4.172	3.876	3.401	2.88	-5.81	4.2	0.2041	10
4.172	3.877	3.401	2.88	-5.814	4.202	0.2042	11
4.173	3.877	3.401	2.88	-5.815	4.203	0.2042	12
4.173	3.877	3.401	2.88	-5.817	4.203	0.2043	13
4.173	3.877	3.401	2.88	-5.817	4.204	0.2043	14
4.173	3.877	3.401	2.88	-5.818	4.204	0.2043	15
4.173	3.877	3.401	2.88	-5.818	4.204	0.2043	16
4.173	3.877	3.401	2.88	-5.818	4.204	0.2043	17
4.173	3.877	3.401	2.88	-5.818	4.204	0.2043	18
4.173	3.877	3.401	2.88	-5.818	4.204	0.2043	19
4.173	3.877	3.401	2.88	-5.818	4.204	0.2043	20
4.173	3.877	3.401	2.88	-5.818	4.204	0.2043	21
4.173	3.877	3.401	2.88	-5.818	4.204	0.2043	22
NA	NA	NA	NA	NA	NA	NA	NA



1.4 EM Method

- E-step

$$Q(\vec{\theta},\vec{\theta}^{\star}) = -\frac{n}{2}\ln(2\pi) - n\ln(\sigma) - \frac{1}{2\sigma^2}\sum_{j=1}^{m}(t_j-\beta_0-\beta_1\nu_j)^2 - \frac{1}{2\sigma^2}\sum_{i=m+1}^{n}E[(T_i-\beta_0-\beta_1\nu_i)^2|T_i>w_i,\vec{\theta}^{\star}]$$

$$E[(T_i-\beta_0-\beta_1\nu_i)^2|T_i>w_i,\vec{\theta}^{\star}] = \mu_i^{\star 2}+\sigma^{\star 2}+\sigma^{\star}(w_i+\mu_i^{\star})H(\frac{w_i-\mu_i^{\star}}{\sigma^{\star}})-2(\beta_0+\beta_1\nu_i)[\mu_i^{\star}+\sigma^{\star}H(\frac{w_i-\mu_i^{\star}}{\sigma^{\star}})]+(\beta_0+\beta_1\nu_i)^2$$

- M-step

$$\frac{\partial Q}{\partial \beta_0} = -\frac{1}{\sigma^2}\left\{\sum_{j=1}^{m}[t_j-\beta_0-\beta_1\nu_j] + \sum_{i=m+1}^{n}[\mu_i^{\star}+\sigma^{\star}H(\frac{w_i-\mu_i^{\star}}{\sigma^{\star}})-\beta_0-\beta_1\nu_i]\right\} = 0$$

$$\frac{\partial Q}{\partial \beta_1} = -\frac{1}{\sigma^2}\left\{\sum_{j=1}^{m}[t_j-\beta_0-\beta_1\nu_j]\nu_j + \sum_{i=m+1}^{n}[\mu_i^{\star}+\sigma^{\star}H(\frac{w_i-\mu_i^{\star}}{\sigma^{\star}})-\beta_0-\beta_1\nu_i]\nu_i\right\} = 0$$

$$\frac{\partial Q}{\partial \sigma^2} = \frac{1}{2\sigma^2}\left\{-n+\frac{1}{\sigma^2}\sum_{j=1}^{m}[t_j-\beta_0-\beta_1\nu_j]^2 + \frac{1}{\sigma^2}\sum_{i=m+1}^{n}E[(T_i-\beta_0-\beta_1\nu_i)^2|T_i>w_i,\vec{\theta}^{\star}]\right\} = 0$$

$$\sum_{j=1}^{m}[t_j-\beta_0-\beta_1\nu_j]^2 + \sum_{i=m+1}^{n}\left\{\mu_i^{\star 2}+\sigma^{\star 2}+\sigma^{\star}(w_i+\mu_i^{\star}-2\mu_i)H(\frac{w_i-\mu_i^{\star}}{\sigma^{\star}})-2\mu_i\mu_i^{\star}+\mu_i^2\right\} = n\sigma^2$$

1.5 EM Code