Simple linear

SST,$\sum(y_i-\bar{y})^2$=SSE,$\sum(y_i-\hat{y})^2$+SSR,$\sum(\hat{y}-\bar{y})^2,\hat{\beta}_1^2S_{xx},\hat{\beta}_1S_{xy}$

E[]: $\beta_1^2S_{xx}+(n-1)\sigma^2=(n-2)\sigma^2+\sigma^2+\hat{\beta}_1^2S_{xx}$

$S_{xx}\sum_{i=1}^{10}(x_i-\bar{x})^2=472,S_{yy}=\sum_{i=1}^{10}(y_i-\bar{y})^2=731,\ S_{xy}\sum_{i=1}^{10}(x_i-\bar{x})(y_i-\bar{y})=274$

$\hat{\beta}_1=\frac{S_{xy}}{S_{xx}}=\frac{274}{472}=0.5805=\sum c_iy_i=r\frac{SD_x}{SD_x}=\frac{\sigma^2}{(n-1)S_x^2};\hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x}$

$\hat{\sigma}^2=\frac{SSE}{n-2}=\frac{1}{8}(S_{yy}-\frac{S_{xy}^2}{S_{xx}})=\frac{1}{8}(731-\frac{274^2}{472})=71.4926$

$\hat{\beta}_1\sim N(\beta_1,\frac{\hat{\sigma}^2}{S_{xx}});\hat{\beta}_0\sim N(\beta_0,\hat{\sigma}^2[\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}]);$

$Var(\hat{\beta}_0)=\frac{71.4926}{472}=0.1515;\ Var(\hat{\beta}_1)=71.4926(\frac{1}{10}+\frac{\bar{x}^2}{472})$

$H_0:\beta_1=0;\ t_0=\frac{\hat{\beta}_1-0}{\sqrt{Var(\hat{\beta}_1)}}=\frac{0.5805}{\sqrt{0.1515}}=1.4916<t_{\frac{0.05}{2},n-2}=2.31$Fail to reject

$\hat{\beta}_1\pm t_{\frac{0.05}{k+1},n-2}se(\hat{\beta}_1)=0.5805\pm2.31\sqrt{0.1515},\ (-0.3185,1.4795)$

$\hat{y}=\hat{\beta}_0+\hat{\beta}_1x_0=168.3609;\bar{x}=147.6923,S_{XX}=\sum_{i=1}^{13}(x_i-\bar{x})^2=6230.769$

CI $se(y_0)=\sqrt{MSE(\frac{1}{n}+\frac{(x_0-\bar{x})^2}{S_{XX}})}=\sqrt{275.06005(\frac{1}{13}+\frac{(170-147.6923)^2}{6230.769})}=6.5671$

$\hat{y}\pm t_{n-2,0.025}se(y_0)=168.3609\pm2.200985*6.5671,\ (153.9068,182.815)$

PI $se(y_0)=\sqrt{MSE(1+\frac{1}{n}+\frac{(x_0-\bar{x})^2}{S_{XX}}});\ 168.3609\pm2.2*17.8378;\ (129.1002,207.6216)$

Multi linear

least-squares estimators $SSE=\sum_{i=1}^n(y_i-\beta_0-\beta_1x_i-\beta_2x_i^2)^2$

$\frac{\partial SSE}{\partial\beta_{0,1,2}}=2\sum_{i=1}^n(y_i-\hat{\beta}_0-\hat{\beta}_1x_i-\hat{\beta}_2x_i^2)(-1,-x_i,-x_i^2)\overset{set}{=}0$

Regression with indicator ....

$y_i=\beta_0+\beta_1x_i+\beta_2w_i+\beta_3w_ix_i+\varepsilon_i,\ w_i=\begin{cases}0 & 1\le i\le\frac{n}{2}\\ 1 & \frac{n}{2}+1\le i\le n\end{cases}$

$\mathbf{Y_{n\times1}}=\mathbf{X_{n\times(k+1)}}\boldsymbol{\beta_{(k+1)\times1}}+\boldsymbol{\varepsilon_{n\times1}}$

equality of slopes $H_0:\beta_3=0,k=3,r=1$

$dfE_{Ful}=n-(k+1),dfE_{Red}-dfE_{Ful}=r$

Partitioned regression ....

(df)	SS_F	$-3,4,5$	$SS_{1,2}$	$-2,4$	$SS_{1,3,5}$
R	367(5)	-96(3)	271(2)	-153(2)	214(3)
E	336(69)	+96(3)	432(72)	+153(2)	489(71)
T	703(74)				

	df	SS_F	$-\alpha$	$SS_{\beta\gamma}$	$-\beta$	$SS_{\alpha\gamma}$	$-\alpha$	SS_γ
α	2	110	-110	0		110	-110	0
γ	11	160		160		160		160
β		250	-10	240	-250	0		0
E	22	300	+120	420	+250	550	+110	660
T	35	820						

$H_0:\beta_3=\beta_4=\beta_5=0;r=3;\ H_0':\beta_2=\beta_4=0;r=2;$

$F_{p,3,69}=\frac{96/3}{336/69}=6.5714;F'=\frac{153/2}{336/69}=15.7098$

$p<0.05$ reject H_0 at 0.05 level of significance

$R^2=\frac{SSR}{SST}=\frac{271}{703};\ R^2_{adj}=1-\frac{SSE/dfE}{SST/dfT}=1-\frac{432/72}{703/74};\ R^2_{pre}=1-\frac{PRESS}{SST}$

coefficient of determination is the proportion of variation explained by regressor x

$|r|=\sqrt{R^2}=\frac{S_xy}{\sqrt{S_{xx}S_{yy}}}=\frac{Cov(x,y)}{\sqrt{VXVY}}=\frac{Cov(\hat{\beta}_1,\hat{\beta}_0)}{se\hat{\beta}_1se\hat{\beta}_0}$

CI: $\hat{\beta}_1\pm t_{\frac{\alpha}{2},n-k-1}se(\hat{\beta}_1),\ se(\hat{\beta}_1)=\sqrt{MSE\cdot C_{22}}$; Bonferroni:$t_{\frac{\alpha}{2(k+1)}}$, Scheffe $\sqrt{2F_\alpha}$

$8.1556\pm t(0.005,9)\sqrt{8.8436*0.2483},\ 8.1556\pm3.25\times4.4071;(-6.1653,22.4765)$

$V(\hat{\beta}_1-\hat{\beta}_2)=V(\hat{\beta}_1)+V(\hat{\beta}_2)-2Cov(\hat{\beta}_1,\hat{\beta}_2)=MSE(C_{22}+C_{33}-2C_{23})$

$=78.20842[0.2483+10.7694-2(-0.49669)]=939.3676$ unbia est of var

Lack of fit H_0: There is no lack of fit, the model is appropriate ....

$\sum^m\sum_{j=1}^{n_i}(y_{ij}-\hat{y}_i)^2(SSE)=\sum^m\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_i)^2(SS_{PE})+\sum^m n_i(y_{ij}-\hat{y}_i)^2(SS_{LOF})$

$SS_{PE}=\sum_{i=1}^m\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_i)^2=28;\ SS_{LOF}=703.87576-28=675.8758;$

$df_{PE}=n-m=12-9=3;\ MS_{PE}=28/3;\ df_{LOF}=dfE-df_{PE}=m-(k+1)=6$

$F=\frac{SS_{LOF}/df_{LOF}}{MS_{PE}}=\frac{675.87576/7}{28/3}=10.34504>F(0.05,6,3)=8.94$. Reject

CRD

$\mathbf{A_{f,r}}\ y_{ij}=\mu+\tau_i+\varepsilon_{ij};\varepsilon_{ij}\sim iidN(0,\sigma^2)$; f $\sum_{i=1}^a\tau_i=0$; r$\tau_i\sim iidN(0,\sigma^2_\tau)$

	SS	df	MS	F	EMS	EMS_r
Trt	$n\sum^a(\bar{y}_{i.}-\bar{y}_{..})^2$	a-1	$\frac{SS_{Trt}}{a-1}$	$\frac{MS_{Trt}}{MS_E}$	$\sigma^2+\frac{n\sum\tau^2}{a-1}$	$\sigma^2+n\sigma^2_\tau$
E	$\sum^a\sum^n(y_{ij}-\bar{y}_{i.})^2$	a(n-1)	$\frac{SS_E}{N-a}$		σ^2	
T	$\sum^a\sum^n(y_{ij}-\bar{y}_{..})^2$	an-1				

$\hat{\sigma}^2=MSE=\frac{SSE}{N-a};\ \hat{\sigma}^2_\tau=\frac{MS_{Trt}-MSE}{n}$

$y_{ij}\sim n(\mu+\tau_i,\sigma^2);\bar{y}_{i.}\sim n(\mu+\tau_i,\frac{\sigma^2}{n});\bar{y}_{i.}-\bar{y}_{j.}\sim n(\mu_i-\mu_j,\frac{2\sigma^2}{n})$

$E[y_{ij}-\bar{y}_{i.}]=V[]=\frac{n-1}{n}\sigma^2$

CI of μ_i: $\bar{y}_{i.}\pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n}}$; unbalanved $\bar{y}_{i.}\pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n_i}}$

$\mu_i-\mu_j$: $\bar{y}_{i.}-\bar{y}_{j.}\pm t_{\frac{\alpha}{2}}\sqrt{\frac{2MS_E}{n}}$; unbalanved $\bar{y}_{i.}-\bar{y}_{j.}\pm t_{\frac{\alpha}{2}}\sqrt{MS_E(\frac{1}{n_i}+\frac{1}{n_j})}$

σ^2: $\frac{SSE}{\chi^2_{\alpha/2,df_E}},\frac{SSE}{\chi^2_{1-\alpha/2,df_E}}$ Γ: $\sum_{i=1}^a c_i\bar{y}_i\pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n}\sum_{i=1}^a c_i^2}$ $\frac{MS_{Trt}}{MS_E}\cdot\frac{\sigma^2}{n\sigma^2_\tau+\sigma^2}\sim F_{a-1,N-a}$

CI of prop of var $\frac{\sigma^2_\tau}{\sigma^2_\tau+\sigma^2}$: $\frac{L}{1+L},\frac{U}{1+U}$; $L=\frac{1}{n}\left(\frac{MS_{Trt}}{MS_EF_{\alpha/2}}-1\right)$; $U=\frac{1}{n}\left(\frac{MS_{Trt}}{MS_EF_{1-\alpha/2}}-1\right)$

$H_0:\beta_1=2\beta_3,\beta_2=\beta_3,\beta_5=0$

$\mathbf{T}=\begin{bmatrix}0&1&0&-2&0&0&0\\0&0&1&-1&0&0&0\\0&0&0&0&0&1&0\end{bmatrix}_{3\times7}\ \mathbf{fi}=\begin{bmatrix}\beta_0\\ \vdots\\ \beta_6\end{bmatrix}_{7\times1}\ \mathbf{C}=\begin{bmatrix}0\\0\\0\end{bmatrix}_{3\times1}\ rank(T)=3$

contrast $\Gamma=\sum_{i=1}^a c_i\mu_i,C=\sum_{i=1}^a c_i\bar{y}_i;\sum_{i=1}^a c_i=0$;Orthogonal $\sum_{i=1}^a c_id_i=0$

$SS_C=\frac{(\sum_{i=1}^a c_i\bar{y}_i)^2}{\frac{1}{n}\sum_{i=1}^a c_i^2},\sum_1^{a-1}SS_C=SS_{Trt};\ V[\sum_{i=1}^a c_iy_{i.}]=\sigma^2\sum_{i=1}^a n_ic_i^2$

		(a)	Contrast	(b)		
$\hat{\tau}_1+\hat{\tau}_2+\hat{\tau}_3$=0	$\hat{\mu}$=13.8				$\hat{\mu}$=8.4	$\hat{\tau}_3$=0
$\hat{\mu}+\hat{\tau}_1=\bar{y}_{1.}$=10.8	$\hat{\tau}_1$=-3.0	10.8	$\mu+\tau_1$	10.8	$\hat{\tau}_1$=2.4	$\hat{\mu}+\hat{\tau}_1=\bar{y}_{1.}$=10.8
$\hat{\mu}+\hat{\tau}_2=\bar{y}_{2.}$=22.2	$\hat{\tau}_2$=8.4	-9.0	$2\tau_1-\tau_2-\tau_3$	-9.0	$\hat{\tau}_2$=13.8	$\hat{\mu}+\hat{\tau}_2=\bar{y}_{2.}$=22.2
$\hat{\mu}+\hat{\tau}_3=\bar{y}_{3.}$=8.4	$\hat{\tau}_3$=-5.4	19.2	$\mu+\tau_1+\tau_2$	24.6	$\hat{\tau}_3$=0	$\hat{\mu}+\hat{\tau}_3=\bar{y}_{3.}$=8.4

$H_{C0,D0};\sum_{i=1}^3 c_id_i$=0	C,D$\sum_{i=1}^a c_i,d_i\bar{y}_{i.}$	$F_{1,12}$	$SS_{C,D}/MS_E$
$\mu_1-2\mu_2+\mu_3=0$	$\bar{y}_{1.}-2\bar{y}_{2.}+\bar{y}_{3.}$	$C^2/\frac{MS_E}{n}\sum_{i=1}^a c_i^2$	$(-25.2)^2/\frac{16.9}{5}6$
$\mu_1-\mu_3=0$	$\bar{y}_{1.}-\bar{y}_{3.}$	$D^2/\frac{MS_E}{n}\sum_{i=1}^a d_i^2$	$2.4^2/\frac{16.9}{5}2$

RCBD

$\mathbf{Trt_f+Blk_f}$

$y_{ij}=\mu+\tau_i+\beta_j+\varepsilon_{ij}$, for $\varepsilon_{ij}\sim iidN(0,\sigma^2),i=1,..,a,j=1,..,b$

f: $\sum_{i=1}^a\tau_i=0,\sum_{j=1}^b\beta_j=0$; r: $\beta_j\sim iidN(0,\sigma^2_\beta)$

Trt	$b\sum^a(\bar{y}_{i.}-\bar{y}_{..})^2$	a-1	$\frac{SS_{Trt}}{a-1}$	$\frac{MS_{Trt}}{MS_E}$	$\sigma^2+\frac{b\sum^a\tau^2}{a-1}$	$\sigma^2+b\sigma^2_\tau$
Blk	$a\sum^b(\bar{y}_{.j}-\bar{y}_{..})^2$	b-1	$\frac{SS_{Blk}}{b-1}$	$\frac{MS_{Blk}}{MS_E}$	$\sigma^2+\frac{a\sum^b\tau^2}{b-1}$	$\sigma^2+a\sigma^2_\tau$
E	T-Trt-Blk	(a-1)(b-1)	$\frac{SS_E}{df_E}$	σ^2		
T	$\sum^a\sum^b(y_{ij}-\bar{y}_{..})^2$	ab-1				

Missing Values: Exact(partial) method $F_0=\frac{(SSE_{red}-SSE_{ful})/r}{MSE_{ful}}=\frac{1403.7-921.5}{184.30*(a-1)}$;

Apporximation method $\hat{x}=\frac{ay_{1.}'+by_{.1}'-y_{..}'}{(a-1)(b-1)},F_{adj}=\frac{MS_{Trt}}{SSE/dfE_{adj}}=\frac{162.08}{921.5/(6-1)}$

Latin Square

$y_{ijk}=\mu+\tau_i+\alpha_j+\beta_k+\varepsilon_{ijk},\ i,j,k=1,..,6$; where μ overall mean

τ_i is effect of i^{th} treatment; α_j is effect of j^{th} block of factor R; β_k effect of k^{th} block of factor C; ε_{ijkl} is random error when i^{th} treatment is applied at j^{th} block of factor R and k^{th} block of factor C; y_{ijkl} is response ;

Assumptions: $\varepsilon_{ijk}\sim iidN(0,\sigma^2)$;$\sum\alpha=0$;$\sum\beta=0$; Indep

The Latin-Squre design can only use 2 blocking factors, as we distribute the levels of the treatment factor on a table with rows of one blocking factor (each row is fore one block) and columns of the other blocking factor (each column is one block)

To test three blocking factors by turning the Latin-square design into a Graeco-Latin square design, which allows to add a Greek letter to each entry in the table, where each Greek letter stands for a block of the factor G.

Graeco-Latin square: $y_{ijkl}=\mu+\theta_i+\tau_j+\omega_k+\psi_l+\varepsilon_{ijkl}$;$i,j,k,l=1,2,..,p$ where y_{ijk} is the observation in row i and column l for Latin letter j and Greek letter k, θ_i is the effect of the ith row, τ_j is the effect of Latin letter treatment j, ω_k is the effect of Greek letter treatment k, ψ_l is the effect of column l, and $\varepsilon_{ijkl}\sim iidN(0,\sigma^2)$ is a random error component. Only two of the four subscripts are necessary to completely identify an observation.

LSD	SS	df
Trt	$p\sum_{i=1}^p(\bar{y}_{i..}-\bar{y}_{...})^2$	p-1
Row	$p\sum_{j=1}^p(\bar{y}_{.j.}-\bar{y}_{...})^2$	p-1
Col	$p\sum_{k=1}^p(\bar{y}_{..k}-\bar{y}_{...})^2$	p-1
E	$SS_T-SS_.$	(p-1)(p-2)
T	$\sum^p\sum^p\sum^p(y_{ijk}-\bar{y}_{..})^2$	p^2-1

Grae	SS	df
Trt1	$b\sum_{i=1}^a(\bar{y}_{i.}-\bar{y}_{..})^2$	p-1
Trt2	$b\sum_{i=1}^a(\bar{y}_{i.}-\bar{y}_{..})^2$	p-1
Row	$a\sum_{j=1}^b(\bar{y}_{.j}-\bar{y}_{..})^2$	p-1
Col	$a\sum_{j=1}^b(\bar{y}_{.j}-\bar{y}_{..})^2$	p-1
E	$SS_T-SS_.$	(p-1)(p-3)
T	$\sum^p\sum^p(y_{ijkl}-\bar{y}_{..})^2$	p^2-1

df	Case1	Case2	Case3
Trt	p-1	p-1	p-1
Row	p-1	n(p-1)	n(p-1)
Col	p-1	p-1	n(p-1)
Rep	n-1	n-1	n-1
E	(p-1)(np+n-3)	(p-1)(np-2)	(p-1)(np-n-1)
T	np^2-1	np^2-1	np^2-1

Aα	Bβ	Cγ	Dδ
Bδ	Aγ	Dβ	Cα
Cβ	Dα	Aδ	Bγ
Dγ	Cδ	Bα	Aβ

Relative Efficiency $\frac{(df_{E(LS)}+1)(df_{E(CRD)}+3)MS_{CRD}}{(df_{E(LS)}+3)(df_{E(CRD)}+1)MS_{LS}}=2.3$

$df_{E(LS)}=(p-1)(p-2)$=20,$df_{E(GS)}=(p-1)(p-3)$=15, $df_{E(CRD)}$=a(n-1)=7.2,32

BIBD

$y_{ij}=\mu+\tau_i+\beta_j+\varepsilon_{ij}$;$i=1,..,a,j=1,..,b;\ y_{i.}=\sum_j y_{ij},\bar{y}_{.j}=\frac{1}{k}\sum_i y_{ij}=\frac{y_{.j}}{k}$;

$Q_i=(y_{i.}-\sum_{j=1}^b n_{ij}\bar{y}_{.j})$ where $n_{ij}=\begin{cases}1 & i^{th}\text{ trt appears in }j^{th}\text{ blk}\\ 0 & \text{o.w.}\end{cases}$

	SS	df	
Trt	$\frac{k}{\lambda a}\sum^a Q_i^2=\frac{k}{\lambda a}\sum^a(y_{i.}-\sum^b n_{ij}\bar{y}_{.j})^2$	a-1	Rep$r=15$
Blk	$k\sum_{j=1}^b(\bar{y}_{.j}-\bar{y}_{..})^2$	b-1	Trt$a=7$
E	T-Trt$_{adj}$-Blk	N-a-b+1	Blk$b=\binom{7}{5}=21$
T	$\sum_{i=1}^a\sum_{j=1}^b(y_{ij}-\bar{y}_{..})^2$	N-1	Size$k=5$
			$N=ar=bk=105$

replications of each pair$\lambda=\frac{(k-1)}{a-1}r=\frac{k(k-1)}{a(a-1)}b=10=\frac{2}{3}r\in\mathbf{N}^+$

the smallest number of observations per treatment is a multiple of 3 and 5

Fixed v.s. random; crossed v.s. nested

(a)Fixed effects are constant across individuals, random effects vary. Effects are fixed if they are interesting in themselves or random if there is interest in the underlying population. When a sample exhausts the population, the corresponding variable is fixed; when the sample is a small part of the population the corresponding variable is random. If an effect is assumed to be a realized value of a random variable, it is called a random effect.

Example: A lab want to test the quality of products. There are only two specific test machines. Each machine assigned two operators who randomly sellected from a large amont of operators. Each poerator test the products in three specific temperature. In this test, the factor mathine and temperature have fixed effects. The factor operator has random effects.

(b)Two factors are crossed when every category of one factor co-occurs in the design with every category of the other factor. There is at least one observation in every combination of categories for the two factors.

A factor is nested within another factor when each category of the first factor co-occurs with only one category of the other. An observation has to be within one category of Factor 2 in order to have a specific category of Factor 1. All combinations of categories are not represented.

You can calculate an interaction between two crossed factors. If they are nested, you cannot because you do not have every combination of one factor along with every combination of the other.

In **Example** (a), Three temperature are applied on each machine. Thus, temperature and machine have crossed effect. The operators were assigned to each machine are different. The operators are nested in the levels of factor machine. Thus, the effects of the factor operator are nested effect.

Factorial model

k factors,p generators; 2^p blocks/fraction;2^{k-p}Run, Blk size;

2^p-1 alias; 2^p-p-1 auto confounded;

I=ABC=BCD=AD

AD+,ABC-,BCD-;(1),bc,abd,acd;ABC+BCD+;b,c,ad,abcd

AD-,ABC-,BCD+;ab,ac,bcd,d;ABC+BCD-;a,abc,bd,cd

I=ABCDF=ABDEG=CEFG;CE=FG,CF=EG,CG=EF minimum aberration

2^{k-p}	2^k	2^p	2^{k-p}	Generator
2_{III}^{3-1}	8	2	4	C=AB
2_{IV}^{4-1}	16	2	8	D=ABC
2_{V}^{5-1}	32	2	16	E=ABCD
2_{III}^{5-2}	32	4	8	D=AB;E=AC
2_{VI}^{6-1}	64	2	32	F=ABCDE
2_{IV}^{6-2}	64	4	16	E=ABC;F=BCD
2_{III}^{6-3}	64	8	8	D=AB;E=AC;F=BC
2_{VII}^{7-1}	128	2	64	G=ABCDEF
2_{IV}^{7-2}	128	4	32	F=ABCD;G=ABDE
2_{IV}^{7-3}	128	8	16	E=ABC;F=BCD;G=ACD
2_{III}^{7-4}	128	16	8	D=AB;E=AC;F=BC;G=ABC

A	B	C	D	E	F	ABF	ABD	Run	
-	-	-	-	-	-	-	-	(1)	I=ABCE=BCDF=ADEF
+	-	-	-	+	-	+	+	ae	A = BCE = DEF = ABCDF
-	+	-	-	+	+	-	+	bef	B = ACE = CDF = ABDEF
+	+	-	-	-	+	+	-	abf	C = ABE = BDF = ACDEF
-	-	+	-	+	+	+	-	cef	D = BCF = AEF = ABCDE
+	-	+	-	-	+	-	+	acf	E = ABC = ADF = BCDEF
-	+	+	-	-	-	+	+	bc	F = BCD = ADE = ABCEF
+	+	+	-	+	-	-	-	abce	BF = CD = ACEF = ABDE
-	-	-	+	-	+	+	+	df	AB = CE = ACDF = BDEF
+	-	-	+	+	+	-	-	adef	AC = BE = ABDF = CDEF
-	+	-	+	+	-	+	-	bde	AD = EF = BCDE = ABCF
+	+	-	+	-	-	-	+	abd	AE = BC = DF = ABCDEF
-	-	+	+	+	-	-	+	cde	AF = DE = BCEF = ABCD
+	-	+	+	-	-	+	-	acd	BD = CF = ACDE = ABEF
-	+	+	+	-	+	-	-	bcdf	ABD = CDE = ACF = BEF
+	+	+	+	+	+	+	+	abcdef	ACD = BDE = ABF = CEF

ANOVA-df: T= $2^k n-1$;Blk= 2^p-1;E= $2^k(n-1)$;others=1

$SS_{Block}=\frac{1}{2^3}C_{ABC}^2=SS_{ABC}$; $SS_{AB}=\frac{n}{4}C_{AB}^2=\frac{n}{4}(\bar{y}_{(1)}-\bar{y}_a-\bar{y}_b+\bar{y}_{ab})^2$

$\mathbf{A_f}\times\mathbf{B_f}$ $y_{ijkl}=\mu+\tau_i+\beta_j+(\tau\beta)_{ij}+\varepsilon_{ijk}$,
$i=1,2; j=1,2,3; k=1,2,3,4; l=1,2,[a=2,b=3,n=4]$ where μ overall mean
τ_i is fixed main effect of i^{th} level of Factor A; β_j j^{th} B;
$(\tau\beta)_{ij}$ is fixed interaction effect of i^{th} level of Factor A and j^{th} level of Factor B;
ε_{ijkl} is random error for the k^{th} replicate EU when i^{th} level of Factor A and j^{th} level of Factor B are applied; y_{ijkl} is response for the;
Assumptions: $\varepsilon_{ijk}\sim iidN(0,\sigma^2)$ (constant variance, zero mean, independent); $\sum_i^2\tau_i=0;\sum_j^3\beta_j=0;\sum_i^2(\tau\beta)_{ij}=0;\sum_j^3(\tau\beta)_{ij}=0$;indep

term	i(f)	j(f)	k(r)	df	SS	MS	EMS
Aτ_if	0	b	n	a-1	$bn\sum^a(\bar{y}_{i..}-\bar{y}_{...})^2;\frac{\sum^a y_{i..}^2}{bn}-\frac{y_{...}^2}{abn}$;96	96	$\sigma^2+\frac{bn\sum\tau_i^2}{a-1}$
Bβ_jf	a	0	n	b-1	$an\sum^b(\bar{y}_{.j.}-\bar{y}_{...})^2;\frac{\sum^b y_{.j.}^2}{an}-\frac{y_{...}^2}{abn}$;208	104	$\sigma^2+\frac{an\sum\beta_j^2}{b-1}$
AB $(\tau\beta)_{ij}$f	0	0	n	(a-1)(b-1)	$n\sum^a\sum^b(y_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})^2$;112 $n\sum\sum y_{ij.}^2-\frac{1}{abn}y_{...}^2-SS_A-SS_B$	56	$\sigma^2+\frac{n\sum\sum(\tau\beta)_{ij}}{(a-1)(b-1)}$
Eε_{ijk}r	1	1	1	ab(n-1),18	$SST-\sum SS;(n-1)\sum^a\sum^b S_{ij}^2$;126	7	σ^2
Total				abn-1,23	$\sum\sum\sum(y_{ijk}-\bar{y}_{...})^2\sum\sum\sum y_{ijk}^2-\frac{y_{...}^2}{abn}$; 542		
if Blk			1	n-1	df_E=(ab-1)(n-1); $\frac{\sum^n y_{..k}^2}{ab}-\frac{y_{...}^2}{abn}$		$\sigma^2+ab\sigma_\delta^2$

$H_0:(\tau\beta)_{ij}=0\forall i,j$; $F_{p,2,18}\frac{MS_{AB}}{MSE}=\frac{56}{7}=8$; $F_{0.05,2,8}=3.55$. There is enough evidence to reject H_0. The model may not be reduced, as the interaction effects is significant at 5% significance level.

$\bar{y}_{12.}-\bar{y}_{22.}\pm t_{\frac{\alpha}{2},18}\sqrt{\frac{2MSE}{n}}=14-8\pm 2.1\sqrt{\frac{2*7}{4}}=6\pm 3.9287;[2.0713,9.9287]$
$E[SS_A]=bn\sum_{i=1}^a E[(\bar{y}_{i..}-\bar{y}_{...})^2]=bn\sum_{i=1}^a(E[]^2+V[])=bn\sum_{i=1}^a\tau_i^2+(a-1)\sigma^2$

$\mathbf{A_r}\times\mathbf{B_r}+\mathbf{A_r}\times\mathbf{C_r}$ $y_{ijk}=\mu+\tau_i+\beta_j+\gamma_k+(\tau\beta)_{ij}+(\tau\gamma)_{ik}+\varepsilon_{ijk}$
iid $\varepsilon_{ijk}\sim iidN(0,\sigma^2)$; $\tau_i\sim iidN(0,\sigma_\tau^2)$; $\beta_j\sim iidN(0,\sigma_\beta^2)$; $\gamma_k\sim iidN(0,\sigma_\gamma^2)$; $(\tau\beta)_{ij}\sim iidN(0,\sigma_{\tau\beta}^2)$; $(\tau\gamma)_{ik}\sim iidN(0,\sigma_{\tau\gamma}^2)$; indep

	i	j	k	df	SS ∣ EMS	F
τ_i	1	b	c	a-1	$\sigma^2+b\sigma_{\tau\gamma}^2+c\sigma_{\tau\beta}^2+bc\sigma_\tau^2$ ∣ $\frac{1}{bc}\sum_{i=1}^a y_{i..}^2-\frac{1}{abc}y_{...}^2$	A/(AB+AC-E)
β_j	a	1	c	b-1	$\sigma^2+c\sigma_{\tau\beta}^2+ac\sigma_\beta^2$ ∣ $\frac{1}{ac}\sum_{j=1}^b y_{.j.}^2-\frac{1}{abc}y_{...}^2$	MS_B/MS_{AB}
γ_k	a	b	1	c-1	$\sigma^2+b\sigma_{\tau\gamma}^2+ab\sigma_\gamma^2$ ∣ $\frac{1}{ab}\sum_{k=1}^c y_{..k}^2-\frac{1}{abc}y_{...}^2$	C/AC
$(\tau\beta)_{ij}$	1	1	c	(a-1)(b-1)	$\sigma^2+c\sigma_{\tau\beta}^2$ ∣ $\frac{\sum^a\sum^b y_{ij.}^2}{c}-\frac{\sum^a y_{i..}^2}{bc}-\frac{\sum^b y_{.j.}^2}{ac}+\frac{y_{...}^2}{abc}$	AB/E
$(\tau\gamma)_{ik}$	1	b	1	(a-1)(c-1)	$\sigma^2+b\sigma_{\tau\gamma}^2$ ∣ $\frac{\sum^a\sum^c y_{i.k}^2}{b}-\frac{\sum^a y_{i..}^2}{bc}-\frac{\sum^c y_{..k}^2}{ab}+\frac{y_{...}^2}{abc}$	AC/E
ε_{ijk}	1	1	1	a(b-1)(c-1)	σ^2 ∣ $\sum^a\sum^b\sum^c y_{ijk}^2-\frac{\sum^a\sum^b y_{ij.}^2}{c}-\frac{\sum^a\sum^c y_{i.k}^2}{b}+\frac{\sum^a y_{i..}^2}{bc}$	
Total				$abc-1$	$\sum_{i=1}^a\sum_{j=1}^b\sum_{k=1}^c y_{ijk}^2-\frac{1}{abc}y_{...}^2$	

$F_{df_n,df_d}=\frac{MS'}{MS''}$; $df_n=df_c$;$df_d=\frac{(AC+BC+ABC)^2}{\frac{AC^2}{df_{AC}}+\frac{BC^2}{df_{BC}}+\frac{ABC^2}{df_{ABC}}}$

Mixed model
$\mathbf{A_f}\times\mathbf{B_r}$:$y_{ijk}=\mu+\tau_i+\beta_j+(\tau\beta)_{ij}+\varepsilon_{ijk}$
iid$\varepsilon_{ijk}\sim iidN(0,\sigma^2)$; $\sum_{i=1}^a\tau_i=0$; $\sum_{i=1}^a(\tau\beta)_{ij}=0$ (restricted model); $\beta_j\sim iidN(0,\sigma_\beta^2)$; $(\tau\beta)_{ij}\sim iidN(0,\frac{a-1}{a}\sigma_{\tau\beta}^2)$
$E(SS_{AB})=E[\sum_{i=1}^a\sum_{j=1}^b\sum_{k=1}^n(\widehat{\tau\beta})_{ij}^2]=n\sum_{i=1}^a\sum_{j=1}^b E[(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})^2]$=
$V[\sum^a\sum^b\sum^n(\widehat{\tau\beta})_{ij}^2]$+0=$(a-1)(b-1)(n\sigma_{\tau\beta}^2+\sigma^2)$ =$n\sum^a\sum^b(\bar{y}_{ij.}-\bar{y}_{...})^2-SS_A-SS_B$

Nested model

	i	j	k	df	EMS $B_r$inA_f	F	$B_f$inA_f	$B_r$inA_r
α_if	0	b	n	a-1	$\sigma_\varepsilon^2+n\sigma_\beta^2+\frac{bn\sum^a\alpha_i^2}{a-1}$	A/AB	$\sigma_\varepsilon^2+\frac{bn\sum^a\alpha_i^2}{a-1}$	$\sigma_\varepsilon^2+n\sigma_\beta^2+bn\sigma_\alpha^2$
β_{ij}r	0	1	n	a(b-1)	$\sigma_\varepsilon^2+n\sigma_\beta^2$	AB/E	$\sigma_\varepsilon^2+\frac{n\sum^a\sum^b\beta_{ij}^2}{a(b-1)}$	$\sigma_\varepsilon^2+n\sigma_\beta^2$
ε_{ijk}r	1	1	1	ab(n-1)	σ_ε^2		σ_ε^2	σ_ε^2
T				abn-1				

$\varepsilon_{(ijk)l}\sim iidN(0,\sigma^2)$ | $\sum_{i=1}^2\tau_i=0$ | $\sum_{k=1}^3\gamma_k=0$ | $\beta_{j(i)}\sim iidN(0,\sigma_\beta^2)$
$\sum_{i=1}^2(\tau\gamma)_{ik}=0$ | $\sum_{k=1}^3(\tau\gamma)_{ik}=0$ | $\sum_{i=1}^2(\beta\gamma)_{j(i)k}=0$ | $(\beta\gamma)_{j(i)k}\sim iidN(0,\frac{2-1}{2}\sigma_{\beta\gamma}^2)$

$\mathbf{B_r}$-in-$\mathbf{A_f}$ $y_{ijk}=\mu+\alpha_i+\beta_{ij}+\varepsilon_{ijk}$; $\sum_{i=1}^a\alpha_i=0$, $\beta_{ij}\sim N(0,\sigma_\beta^2)$, $\varepsilon_{ij}\sim N(0,\sigma_\varepsilon^2)$
$Cov(y_{111},y_{112})=Cov(\beta_{11}+\varepsilon_{111},\beta_{11}+\varepsilon_{112})=Var(\beta_{11})+Cov(\varepsilon_{111},\varepsilon_{112})=\sigma_\beta^2$
$Cov(y_{111},y_{121})=Cov(\beta_{11}+\varepsilon_{111},\beta_{12}+\varepsilon_{121})=Cov(\beta_{11},\beta_{12})+Cov(\varepsilon_{111},\varepsilon_{121})=0$
$Cov(y_{111},y_{211})=Cov(\beta_{11}+\varepsilon_{111},\beta_{21}+\varepsilon_{211})=Cov(\beta_{11},\beta_{21})+Cov(\varepsilon_{111},\varepsilon_{211})=0$
$Var(y_{111})=\sigma_\beta^2+\sigma_\varepsilon^2=Var(y_{112})$;

$Cor(y_{111},y_{112})=\frac{\sigma_\beta^2}{\sigma_\beta^2+\sigma_\varepsilon^2}$ $Cor(y_{111},y_{121})=Cor(y_{111},y_{211})=0$

A linear combination of normal distributed random variables and constants are normal distributed.
$E[\bar{y}_{ij.}]=E[\frac{1}{n}\sum_{k=1}^n y_{ijk}]=\frac{\sum_{k=1}^n}{n}E[\mu+\alpha_i+\beta_{ij}+\varepsilon_{ijk}]=\mu+\alpha_i,\forall i=1,..,a;j=1,..,b$

$Var[\bar{y}_{ij.}]=Var[\frac{1}{n}\sum_{k=1}^n y_{ijk}]=\frac{\sum_{k=1}^n}{n^2}Var[\mu+\alpha_i+\beta_{ij}+\varepsilon_{ijk}]=\frac{1}{n}(\sigma_\beta^2+\sigma_\varepsilon^2),\forall i$
$f(\bar{y}_{i1.},..\bar{y}_{ib.})=\prod_{j=1}^b f(\bar{y}_{ij.})=(2\pi\frac{\sigma_\beta^2+\sigma_\varepsilon^2}{n})^{-\frac{b}{2}}\exp[\frac{-n}{2(\sigma_\beta^2+\sigma_\varepsilon^2)}\sum_{j=1}^b(\bar{y}_{ij.}-\mu-\alpha_i)^2]$

$\hat{\alpha}_1-\hat{\alpha}_2\sim N(\alpha_1-\alpha_2,\frac{2}{b}\sigma_\beta^2+\frac{2}{bn}\sigma_\varepsilon^2)$
$SSE=\sum^a\sum_{j=1}^b\sum_{k=1}^n(y_{ijk}-\mu-\alpha_i-\beta_{ij})^2$; $\frac{\partial SSE}{\partial\alpha_i}=2\sum_{j=1}^b\sum_{k=1}^n(y_{ijk}-\mu-\alpha_i-\beta_{ij})(-1)\overset{set}{=}0$;
$\hat{\alpha}_i=\frac{\sum_{j=1}^b\sum_{k=1}^n y_{ijk}}{bn}-\mu-\frac{\sum_{j=1}^b\beta_{ij}}{b}=\bar{y}_{i..}-\mu$
$\hat{\alpha}_1-\hat{\alpha}_2=\bar{y}_{1..}-\mu-(\bar{y}_{2..}-\mu)=\frac{1}{bn}\sum_{j=1}^b\sum_{k=1}^n(y_{1jk}-y_{2jk})$
$=\frac{1}{bn}\sum_{j=1}^b\sum_{k=1}^n(\alpha_1-\alpha_2+\beta_{1j}-\beta_{2j}+\varepsilon_{1jk}-\varepsilon_{2jk})=\alpha_1-\alpha_2+\bar{\beta}_{1.}-\bar{\beta}_{2.}+\bar{\varepsilon}_{1..}-\bar{\varepsilon}_{2..}$
$E[\hat{\alpha}_1-\hat{\alpha}_2]=\alpha_1-\alpha_2$;
$Var[\hat{\alpha}_1-\hat{\alpha}_2]=Var[\bar{\beta}_{1.}-\bar{\beta}_{2.}+\bar{\varepsilon}_{1..}-\bar{\varepsilon}_{2..}]$
$=\frac{1}{b^2}\sum_{j=1}^b(Var[\beta_{1.}]+Var[\beta_{2.}])+\frac{1}{b^2n^2}\sum_{j=1}^b\sum_{k=1}^n(Var[\varepsilon_{1..}]+Var[\varepsilon_{2..}])=\frac{2}{b}\sigma_\beta^2+\frac{2}{bn}\sigma_\varepsilon^2$

$\bar{y}_{ij.}-\bar{y}_{i..}=\mu+\alpha_i+\beta_{ij}+\bar{\varepsilon}_{ij.}-(\mu+\alpha_i+\bar{\beta}_{i.}+\bar{\varepsilon}_{i..})=\beta_{ij}-\bar{\beta}_{i.}+\bar{\varepsilon}_{ij.}-\bar{\varepsilon}_{i..}$
$E[\bar{y}_{ij.}-\bar{y}_{i..}]=E[\beta_{ij}-\bar{\beta}_{i.}+\bar{\varepsilon}_{ij.}-\bar{\varepsilon}_{i..}]=0$
$Cov(\beta_{ij},\bar{\beta}_{i.})=\frac{1}{b}Cov(\beta_{ij},\sum_{j=1}^b\beta_{ij})=\frac{1}{b}[1\cdot\sigma_\beta^2+(b-1)\cdot 0]$

$Cov(\bar{\varepsilon}_{ij.},\bar{\varepsilon}_{i..})=Cov(\frac{1}{n}\sum_{k=1}^n\varepsilon_{ijk},\frac{1}{bn}\sum_{j=1}^b\sum_{k=1}^n\varepsilon_{ijk})=\frac{\sum_{k=1}^n Cov(\varepsilon_{ijk},\sum_{j=1}^b\varepsilon_{ijk})}{bn^2}=\frac{\sigma_\varepsilon^2}{bn}$
$Var[\bar{y}_{ij.}-\bar{y}_{i..}]=Var[\beta_{ij}-\bar{\beta}_{i.}+\bar{\varepsilon}_{ij.}-\bar{\varepsilon}_{i..}]=Var[\beta_{ij}-\bar{\beta}_{i.}]+Var[\bar{\varepsilon}_{ij.}-\bar{\varepsilon}_{i..}]$
$=Var[\beta_{ij}]+Var[\bar{\beta}_{i.}]-2Cov(\beta_{ij},\bar{\beta}_{i.})+Var[\bar{\varepsilon}_{ij.}]+Var[\bar{\varepsilon}_{i..}]-2Cov(\bar{\varepsilon}_{ij.},\bar{\varepsilon}_{i..})$
$=\sigma_\beta^2+\frac{1}{b}\sigma_\beta^2-\frac{2}{b}\sigma_\beta^2+\frac{1}{n}\sigma_\varepsilon^2+\frac{1}{bn}\sigma_\varepsilon^2-\frac{2}{bn}\sigma_\varepsilon^2=\frac{b-1}{b}(\sigma_\varepsilon^2+\frac{1}{n}\sigma_\varepsilon^2)$
$E[\sum_{i=1}^a\sum_{j=1}^b(\bar{y}_{ij.}-\bar{y}_{i..})^2]=\sum_{i=1}^a\sum_{j=1}^b(Var[\bar{y}_{ij.}-\bar{y}_{i..}]+E[\bar{y}_{ij.}-\bar{y}_{i..}]^2)$
$=\sum_{i=1}^a\sum_{j=1}^b[\frac{b-1}{b}(\sigma_\varepsilon^2+\frac{1}{n}\sigma_\varepsilon^2)+0]=a(b-1)(\sigma_\beta^2+\frac{1}{n}\sigma_\varepsilon^2)$
$\hat{\sigma}_\beta^2=\frac{MS_{AB}-MS_E}{n}$; $E[\hat{\sigma}^2]=\frac{1}{n}(n\sigma_\beta^2+\sigma_\varepsilon^2-\sigma_\varepsilon^2)=\sigma_\beta^2$
$E(MS_{B_f(A_f)})=\frac{n}{a(b-1)}\sum_{i=1}^a\sum_{j=1}^b E[(\bar{y}_{ij.}-\bar{y}_{i..})^2]=\sigma^2+\frac{n}{a(b-1)}\sum_{i=1}^a\sum_{j=1}^b\beta_{j(i)}^2$

Nested-Three-stages $\mathbf{C_r}$-in-$\mathbf{B_r}$-in-$\mathbf{A_f}$ $y_{ijkl}=\mu+\tau_i+\beta_{j(i)}+\gamma_{k(ij)}+\varepsilon_{(ijk)l}$
$\varepsilon_{ijkl}\sim iid\ N(0,\sigma^2)$; $\sum_{i=1}^2\tau_i=0$; $\beta_{j(i)}\sim iid\ N(0,\sigma_\beta^2)$; $\gamma_{k(ij)}\sim iid\ N(0,\sigma_\gamma^2)$

	i	j	k	l	df	SS ∣ EMS	F
A$_f\tau_i$	0	b	c	n	a-1	$\sigma^2+n\sigma_\gamma^2+cn\sigma_\beta^2+\frac{bcn\sum^a\tau_i^2}{a-1}$ ∣ $\frac{\sum^a y_{i...}^2}{bcn}-\frac{y_{....}^2}{abcn}$	A/B(A)
B(A)$_r\beta_{j(i)}$	1	1	c	n	a(b-1)	$\sigma^2+n\sigma_\gamma^2+cn\sigma_\beta^2$ ∣ $\frac{\sum^a\sum^b y_{ij..}^2}{cn}-\frac{\sum^a y_{i...}^2}{bcn}$	B(A)/C(A(B))
C(B(A))$r\gamma_{k(ji)}$	1	1	1	n	ab(c-1)	$\sigma^2+n\sigma_\gamma^2$ ∣ $\frac{1}{n}\sum^a\sum^b\sum^c y_{ijk.}^2-\frac{1}{cn}\sum^a\sum^b y_{ij..}^2$	C(A(B))/E
$\varepsilon_{(ijk)l}$	1	1	1	1	abc(n-1)	σ^2 ∣ $\sum^a\sum^b\sum^c\sum^n y_{ijkl}^2-\frac{1}{n}\sum^a\sum^b\sum^c y_{ijk.}^2$	
T					abcn-1	$\sum^a\sum^b\sum^c\sum^n y_{ijkl}^2-\frac{1}{abcn}y_{....}^2$	

Nested-Factorial Factors $\mathbf{B_r}$-in-$\mathbf{A_f}\times\mathbf{C_f}$

	i	j	k	l	df	EMS	F
A$_f\tau_i$	0	b	c	n	a-1	$\sigma^2+cn\sigma_\beta^2+\frac{bcn\sum_{i=1}^a\tau_i^2}{a-1}$	MS_A/B(A)
B(A)$_r\beta_{j(i)}$	1	1	c	n	a(b-1)	$\sigma^2+cn\sigma_\beta^2$	B(A)/E
C$_f(\gamma)_k$	a	b	0	n	c-1	$\sigma^2+n\sigma_{\gamma\beta}^2+\frac{abn\sum^c\gamma_k^2}{c-1}$	C/CB(A)
AC$_f(\tau\gamma)_{ik}$	0	b	0	n	(a-1)(c-1)	$\sigma^2+n\sigma_{\gamma\beta}^2+\frac{bn\sum^a\sum^c(\tau\gamma)_{ik}^2}{(a-1)(c-1)}$	AC/CB(A)
CB(A)$_r(\gamma\beta)_{kj(i)}$	1	1	0	n	a(b-1)(c-1)	$\sigma^2+n\sigma_{\gamma\beta}^2$	CB(A)/E
$\varepsilon_{(ijk)l}$	1	1	1	1	abc(n-1)	σ^2	
T					abcn-1		

Split-Plot
$\mathbf{Run_r}\times\mathbf{A_f}\times\mathbf{B_f}$ $y_{ijkl}=\mu+\tau_i+\beta_j+(\tau\beta)_{ij}+\gamma_k+(\tau\gamma)_{ik}+(\beta\gamma)_{jk}+(\tau\beta\gamma)_{ijk}+\varepsilon_{ijk}$;
i=1,2,..,r;j=1,2,..,a;k=1,2,..,b

$\varepsilon_{ijk}\sim iidN(0,\sigma^2)$ | $\tau_i\sim iidN(0,\sigma_\tau^2)$ |
$\sum_{j=1}^3\beta_j=0$ | $\sum_{j=1}^3(\tau\beta)_{ij}=0$ | $(\tau\beta)_{ij}\sim iidN(0,\frac{3-1}{3}\sigma_{\tau\beta}^2)$
$\sum_{k=1}^4\gamma_k=0$ | $\sum_{k=1}^4(\tau\gamma)_{ik}=0$ | $(\tau\gamma)_{ik}\sim iidN(0,\frac{4-1}{4}\sigma_{\tau\gamma}^2)$
$\sum_{j=1}^3(\beta\gamma)_{jk}=0$ | $\sum_{k=1}^4(\beta\gamma)_{jk}=0$ |
$\sum_{j=1}^3(\tau\beta\gamma)_{ijk}=0$ | $\sum_{k=1}^4(\tau\beta\gamma)_{ijk}=0$ | $(\tau\beta\gamma)_{ijk}\sim iidN(0,\frac{(3-1)(4-1)}{3\times 4}\sigma_{\tau\beta\gamma}^2)$

	i	j	k	df	EMS	F
$R_r\tau_i$	1	a	b	r-1	$\sigma^2+ab\sigma_\tau^2$	MS_R/MS_E
$A_f\beta_j$	r	0	b	a-1	$\sigma^2+b\sigma_{\tau\beta}^2+\frac{rb\sum^a\beta_j^2}{a-1}$	A/RA
$RA_r(\tau\beta)_{ij}$	1	0	b	(r-1)(a-1)	$\sigma^2+b\sigma_{\tau\beta}^2$=E(Whole-plot error)	RA/E
$B_f(\gamma)_k$	r	a	0	b-1	$\sigma^2+a\sigma_{\tau\gamma}^2+\frac{ra\sum^b\gamma_k^2}{b-1}$	B/RB
$RB_r(\tau\gamma)_{ik}$	1	a	0	(r-1)(b-1)	$\sigma^2+a\sigma_{\tau\gamma}^2$	RB/E
$AB_f(\beta\gamma)_{jk}$	r	0	0	(a-1)(b-1)	$\sigma^2+\sigma_{\tau\beta\gamma}^2+\frac{r\sum^a\sum^b(\beta\gamma)_{jk}^2}{(a-1)(b-1)}$	AB/RAB
$RAB_r(\tau\beta\gamma)_{ijk}$	1	0	0	(r-1)(a-1)(c-1)	$\sigma^2+\sigma_{\tau\beta\gamma}^2$	RAB/E
$\varepsilon_{(ijk)h}$				0	σ^2not estimatable	