# STAT 510: Spatiotemporal Stats

## Introduction and Visualizing SPT Data

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# Introduction

# Why study spatio-temporal processes

- Spatio-temporal processes are common in the real world. These can result from interactions across many processes and scales
- Snapshots of spatial events at a fixed time are informative, but provide an incomplete picture about the process
- Looking at a time series at a single location might miss out on the influence nearby locations exert

there is no history without geography and vice-versa

# Why should it be statistical?

Processes mostly follow deterministic (even if sometimes chaotic) rules, but...

There is often incomplete data and knowledge about mechanisms driving phenomena

$\Rightarrow$ uncertainty in data, model and parameter values

# Why should these models be statistical?

**Statistical spatio-temporal models allow:**

- capturing the notion of uncertainty without obscuring important trends
- building-in system components that appear random – even if they are not, models are useful if predictions are accurate
- parameter estimation and process prediction (conditional on observed data)
- can be based on our physical understanding of a process (i.e., on a mechanistic model)

# Why spatio-temporal modeling?

**To characterize processes with uncertain and (too often) incomplete data and system knowledge to:**

- make predictions in space and time (smoothing and filtering)
- make predictions in time (forecasting)
- data assimilation with mechanistic models
- conduct parameter inference
- other (computer-model emulation, monitoring network design)

# The two approaches

### Descriptive (or maginal) approach

Characterize the first and second-moment (mean and covariance) behavior of the process

- Many different processes can generate same marginal form
- More useful if knowledge is limited about driving mechanisms behind process

# The two approaches

### Dynamical (or conditional) approach

Process values at a location evolve from past values at many locations

- Relate more closely to causal (mechanistic) explanations of the process
- Most useful when good prior knowledge about process is available

Both approaches can be connected through their covariance functions

# Hierarchical Statistical Models

- The likelihood of marginal probability models for processes with complex dependencies are hard (sometimes impossible) to compute
- These also struggle with the fact that data are noisy imperfect measurements of what we are usually interested in
- Alternatively, complexity can be built-in gradually through conditioning

# Hierarchical Statistical Models

If most complex dependencies in data arise (for example) from the actual process, it is useful to set up the model with two components

$$\underbrace{[\text{data}|\text{process}]}_{\text{data model}} \quad \text{and} \quad \underbrace{[\text{process}]}_{\text{process model}}$$

{(Note: Square braket notation, common throughout the book, is used to denote pdf's or pmf's)}

the game here is to use these two components to do inference on the process through [process|data] using Bayes rule.

# Hierarchical Statistical Models

- Parameter uncertainty can be accounted for by assigning probability distributions to parameters (a.k.a. prior distributions)
- This leads to a Bayesian hierarchical model (BHM), which uses the hierarchy:
  1. $[\text{data}|\text{process}, \text{parameters}]$
  2. $[\text{process}|\text{parameters}]$
  3. $[\text{parameters}]$

  to deduce

$$[\text{process}, \text{parameters}|\text{data}] \propto [\text{data}|\text{process}, \text{parameters}] \times [\text{process}|\text{parameters}] \times [\text{parameters}]$$

# Hierarchical Statistical Models

**An aside**

- An alternative to specifying parameter priors (that disregards parameter uncertainty) is to estimate parameter values in $[\text{data}|\text{process}, \text{parameters}]$ and $[\text{process}|\text{parameters}]$ using the data
- This is known as empirical hierarchical modeling (EHM), and is preferrable in some cases (we'll see some examples later on in the course)
- There are hybrid versions of this

# Hierarchical Statistical Models

A popular type of a hierarchical models in time series, are known as *State-Space* models in one or many variables:

- Data modeled as noisy observations of an underlying (hidden) **state process** evolving through time
- Goal of these models is to understand the **state process** while accounting for **noise**
- Used for **smoothing** (inference on hidden state throughout observed time period), **filtering** (inference on state value at most current time point) and **forecasting** (inference on state values beyond observed period).

# Hierarchical Statistical Models

For this course, we will go beyond this type of models. We are interested in the temporal evolution of entire spatial processes.

As you may imagine, this adds a big layer of computational complexity

# Spatio-Temporal Data Exploration

# Flavors of temporal and spatial data

As I mentionend last time, this field is BIG, and methodology has branched off according to the specific flavor of the data.

## In time

- Time intervals type: regular or irregular?
- Is time discrete or continuous?
- Is the random event the time at which an event occurs?

# Flavors of temporal and spatial data

## In space:

Spatial data corresponds to either a fixed time-point or a temporal aggregation over multiple timepoints and can be:

- Geostatistical: point-level/coordinate observations, space is treated as a continuous surface over a given spatial domain
- Areal: space is discretized into a grid, polygons or small areas
- Point-process: the locations where events occur are themselves random, and each location can be complemented with attributes (called marks)