$y_i=\beta_0+\beta_1x_i+\varepsilon_i, i_{1:n}$ SLR; $\mathbf{Y}_{\mathbf{n}\times\mathbf{1}}=\mathbf{X}_{\mathbf{n}\times(\mathbf{k+1})}\boldsymbol{\beta}_{(\mathbf{k+1})\times\mathbf{1}}+\boldsymbol{\varepsilon}_{\mathbf{n}\times\mathbf{1}}$ MLR; p=k+1

Assumptions: constant variance, zero mean, independent; $\varepsilon_{ijkl}\overset{iid}{\sim}N(0,\sigma^2 I)$

Y'Y=$\sum y_i^2$=$S_{yy}+n\bar{y}^2$	(Y-$\bar{y}$1)'(Y-$\bar{y}$1)=$\sum(y_i-\bar{y})^2=\sum y_i^2-n\bar{y}^2$=(n-1)$S_y^2=S_{yy}$
X'Y=$[\sum y_i,\sum x_{1i}y_i,..,\sum x_{ki}y_i]'$	(X-$\bar{x}$1)'(Y-$\bar{y}$1):$\sum(x_i-\bar{x})(y_i-\bar{y})=\sum xy-n\bar{x}\bar{y}=\vec{S}_{xy}$
X'X=$\begin{bmatrix} n & \sum x_{ki} \\ \sum x_{ki} & \sum x_{ki}^2\end{bmatrix}_{p\times p}$	(X-$\bar{x}$1)'(X-$\bar{x}$1):$[\sum(x_i-\bar{x})^2=\sum x_i^2-n\bar{x}^2$=(n-1)$S_x^2=S_{xx}]$

$(X'X)^{-1}=\frac{1}{nS_{xx}}\begin{bmatrix}\sum x_{ki}^2 & -\sum x_{ki}\\ -\sum x_{ki} & n\end{bmatrix}$; $(X'X)^{-1}X'y=\begin{bmatrix}\bar{y}-\bar{x}\frac{S_{xy}}{S_{xx}}\\ \frac{S_{xy}}{S_{xx}}\end{bmatrix}=\begin{bmatrix}\bar{y}-\hat{\beta}_1\bar{x}\\ \{\sum c_iy_i\}\{r\frac{S_y}{S_x}\}\end{bmatrix}=\begin{bmatrix}\hat{\beta}_0\\ \hat{\beta}_1\end{bmatrix}$

$\hat{V}[\hat{\beta}]=\sigma^2(X'X)^{-1}$=E[$(\hat{\beta}-E\hat{\beta})(\hat{\beta}-E\hat{\beta})'$]=$\begin{bmatrix}\hat{V}[\hat{\beta}_1] & Cov(\hat{\beta}_0,\hat{\beta}_1)\\ .. & \hat{V}[\hat{\beta}_1]\end{bmatrix}=\begin{bmatrix}\frac{\hat{\sigma}^2\sum x_i^2}{nS_{xx}} & \frac{-\bar{x}\hat{\sigma}^2}{S_{xx}}\\ \frac{-\bar{x}\hat{\sigma}^2}{S_{xx}} & \frac{\hat{\sigma}^2}{S_{xx}}\end{bmatrix}$

$\hat{V}[\hat{\beta}_0]=\hat{\sigma}^2(\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}})$; $\hat{V}[\hat{\beta}_1]=\frac{\hat{\sigma}^2}{S_{xx}}$; Cov$(\hat{\beta}_0,\hat{\beta}_1)=\frac{-\bar{x}\hat{\sigma}^2}{S_{xx}}$; Cov$(\bar{y},\hat{\beta}_1)$=0

Cov$[\vec{a}'\hat{\beta}]=\sigma^2a'(X'X)^{-1}a$; se$(\vec{a}'\hat{\beta}\{\hat{\beta}_i\}\{\hat{\beta}_i-\hat{\beta}_j\})=\hat{\sigma}\sqrt{a'(X'X)^{-1}a\{C_{ii}\}\{C_{ii}+C_{jj}-2C_{ij}\}}$

$\hat{Y}=\hat{\beta}_0+\hat{\beta}_1x_{1i}+.+\hat{\beta}_kx_{ki}=X\hat{\beta}$=X(X'X)$^{-1}$X'y=HY; $\hat{y}_0=\hat{\beta}_0+\hat{\beta}_1x_0$; $\hat{Y}_0=X_0\hat{\beta}$

$Y'\hat{Y}=\hat{\beta}_0n\bar{y}+\hat{\beta}_1\sum x_{1i}y_i+.+\hat{\beta}_k\sum x_{ki}y_i=\hat{\beta}'X'X\hat{\beta}=\hat{\beta}'X'y$=Y'X(X'X)$^{-1}$X'Y=Y'HY

	df	SS					E[SS]; $\hat{\sigma}^2$=MSE
R	1{k}	$\sum(\hat{y}-\bar{y})^2$	$\hat{Y}'\hat{Y}$-$n\bar{y}^2$	$\hat{\beta}'X'y$-$n\bar{y}^2$		$\hat{\beta}_1S_{xy}=\hat{\beta}_1^2S_{xx}$	$k\sigma^2+\beta_{1:k}'X_c'X_c\beta_{1:k}$
E	n-2{p}	$\sum(y_i-\hat{y})^2$	Y'Y-$\hat{Y}'\hat{Y}$	Y'Y-$\hat{\beta}'X'y$	(Y-X$\hat{\beta}$)'(Y-X$\hat{\beta}$)	$S_{yy}-\hat{\beta}_1S_{xy}$	(n-p)σ^2
T	n-1	$\sum(y_i-\bar{y})^2$	Y'Y-$n\bar{y}^2$	$\sum y_i^2$-$n\bar{y}^2$	(Y-$\bar{y}$1)'(Y-$\bar{y}$1)	S_{yy};(n-1)S_y^2	(n-1)$\sigma^2+\hat{\beta}_1S_{xx}\hat{\beta}_1$

Hypothesis Cov[a'(A)Y]=a'(A)Cov[Y]a(A')

$H_0:\hat{\beta}_i$=c$\{\hat{\beta}_j\}\{\vec{a}'\hat{\beta}\}$; t$_0$=$\frac{\hat{\beta}_i-c\{\hat{\beta}_i-\hat{\beta}_j\}\{\vec{a}'\hat{\beta}\}}{se(\hat{\beta}_i\{\hat{\beta}_i-\hat{\beta}_j\}\{\vec{a}'\hat{\beta}\})}<t_{\frac{\alpha}{2},n-2\{p\}}$; t test $\hat{\beta}_i$= partial F test on a x_i

H_0:$\beta_{1:k}$=0; $\frac{SSR}{\sigma^2}\sim\chi^2_{1(k)}$; $\frac{SSE}{\sigma^2}=\frac{Y'(I-H)Y}{\sigma^2}\sim\chi^2_{n-2(p)}$; $\frac{MSR}{MSE}\sim F_{\alpha,k,n-p}$

$H_0:T\hat{\beta}_{i:j}$=c; $\beta_1=2\beta_3$, $\beta_2=\beta_3$, β_5=0; $dfE_{Red}-dfE_{Ful}$ =n-(p-r)-(n-p)=r=j-i+1

T=$\begin{bmatrix}0&1&0&-2&0&0&0\\0&0&1&-1&0&0&0\\0&0&0&0&0&1&0\end{bmatrix}_{3\times7}\begin{bmatrix}\beta_0\\ \beta_6\end{bmatrix}_{7\times1}=\begin{bmatrix}0\\0\\0\end{bmatrix}_{3\times1}$ rank(T)=3 independent;

$F_{\alpha,r,dfE_{Ful}}=\frac{(SSE_{Red}-SSE_{Ful})/r}{SSE_{Ful}/dfE_{Ful}}=\frac{(SSR_{Ful}-SSR_{Red})}{r*MSE_{Ful}}=\frac{(T\hat{\beta}-c)'[T(X'X)^{-1}T'](T\hat{\beta}-c)}{r*MSE_{Ful}}$

CI $\hat{\beta}_i\{\hat{\beta}_i-\hat{\beta}_j\}\pm\Delta_{n-2\{p\}}se(\hat{\beta}_i\{\hat{\beta}_i-\hat{\beta}_j\})$; $\Delta_{Bonf}=t_{\frac{\alpha}{2m}}$ m:# of hypo; $\Delta_{Shef}=\sqrt{2\{p\}F_{\alpha,p,n-p}}$

σ^2:$(\frac{SSE}{\chi^2_{\alpha/2,dfE}},\frac{(n-2)\hat{\sigma}^2}{\chi^2_{1-\alpha/2,dfE}})$ Joint: H_0:$\beta=\beta$, $\frac{(\hat{\beta}-\beta)'(\hat{\beta}-\beta)}{\sigma^2(X'X)^{-1}}\sim\chi^2_p$; P($\frac{(\hat{\beta}-\beta)'X'X(\hat{\beta}-\beta)}{p\cdot MSE}<F_{\alpha,p,n-p}$)=1-$\alpha$

Elliptically reg: n$(\hat{\beta}_0-\beta_0)^2$+2$\sum$ x$(\hat{\beta}_0-\beta_0)(\hat{\beta}_1-\beta_1)$+$\sum$ x$^2(\hat{\beta}_1-\beta_1)^2$ ≤cpMSE

$\hat{y}_0\pm t_{\frac{\alpha}{2},n-2}\hat{\sigma}\sqrt{\frac{1}{k}+(\frac{1}{n}+\frac{(x_0-\bar{x})^2}{S_{xx}})\{x_0'(X'X)^{-1}x_0\}}$# new valu(PI)k=1;∞(CI);$\hat{\beta}_0asx_0$=0

$R^2=\frac{SSR}{SST}=\frac{S_{xy}^2}{S_{xx}S_{yy}}$;r=$\frac{S_{xy}}{(n-1)S_xS_y}=\frac{Cov(x,y)}{\sqrt{VXVY}}=\frac{Cov(\hat{\beta}_1,\hat{\beta}_0)}{se\hat{\beta}_1se\hat{\beta}_0}$; R^2_{adj}=1-$\frac{SSE/(n-p)}{SST/(n-1)}$; R^2_{pre}=1-$\frac{PRESS}{SST}$

coefficient of determination is the proportion of variation explained by regressor x

Standardized $W=b_0+b_1Z=\frac{y_i-\bar{y}}{S_y}$=r$\frac{x_i-\bar{x}}{S_x}$; b_0=0; $b_1=\frac{S_{zw}}{S_{zz}}$=r

Multicollinearity X$\vec{Z}$=0; VIF=$\frac{1}{1-R_j^2}$; for the Reg of X_i on all of the other predictors .

Calibration given a y*, $\hat{x}^*=\frac{y^*-\hat{\beta}_0}{\hat{\beta}_1}$ V$[\hat{x}^*]=\frac{\hat{\sigma}^2}{\hat{\beta}_1^2}[1+\frac{1}{n}+\frac{(\hat{x}^*-\bar{x})^2}{S_{xx}}]$

least-squares estimators $\frac{\partial\vec{a}\vec{x}}{\partial\vec{x}}=\vec{a}$; $\frac{\partial\vec{x}'A\vec{x}}{\partial\vec{x}}=2A\vec{x}$

SSE=$\sum_i^n(y_i-\beta_0-\beta_1x_i-\beta_2x_i^2)^2$=e'e=$(Y-\hat{Y})'(Y-\hat{Y})=(Y-X\hat{\beta})'(Y-X\hat{\beta})$;

$\frac{\partial SSE}{\partial\beta_{0,1,2}}=2\sum_i^n(y_i-\hat{\beta}_0-\hat{\beta}_1x_i-\hat{\beta}_2x_i^2)(-1,-x_i,-x_i^2)=0=-2X'Y+2X'X\hat{\beta}$

$\beta_0=\beta_1,h(y_i)=\beta_1f(1+x_i)+\beta_2g(x_i^2)$; $\beta_0=0,h(y_i)=\beta_1f(x_i)+\beta_2g(x_i^2)$,

$\hat{\beta}_1=\frac{\sum g(x^2)^2\sum f(x)h(y)-\sum g(x^2)f(x)\sum g(x^2)h(y)}{\sum g(x^2)^2\sum f(x)^2-[\sum g(x^2)f(x)]^2}$, $\hat{\beta}_2=\frac{\sum f(x)^2\sum g(x^2)h(y)-\sum g(x^2)f(x)\sum f(x)h(y)}{\sum g(x^2)^2\sum f(x)^2-[\sum g(x^2)f(x)]^2}$

$d\beta_0=\beta_2,\beta_1=c,h(y_i-cx_i)=\beta_0g(1+dx_i^2)$, $\hat{\beta}_0=\frac{\sum g(1+dx^2)h(y-cx)}{\sum g(1+dx^2)^2}$

Generalized least-squares solution V=PΛP'; V^{-1}=L'L; L=$\Lambda^{-\frac{1}{2}}$P'

V[ε]=σ^2V; Cov[γ]=LCov[ε]L'=$\Lambda^{-\frac{1}{2}}$P'σ^2V$(\Lambda^{-\frac{1}{2}}$P'$)^{-1}=\sigma^2$I; W{LY}=Z{LX}β+{Lε}γ

$\hat{\beta}$=(Z'Z)$^{-1}$Z'W=(X'V^{-1}X)$^{-1}$X'V^{-1}y; Cov($\hat{\beta}$)=$\sigma^2(X'V^{-1}X)^{-1}$

$H^2=X(X'X)^{-1}X'X(X'X)^{-1}X'=X(X'X)^{-1}X'$=H'=H,idempotent matrix, symmetric

Residual: e=(I-H)Y; Cov($\vec{e}$) $=\sigma^2(I-H)$; Var(e_i) $=\sigma^2(1-h_{ii})$; Cov(e_i,e_j) $=-\sigma^2h_{ij}$

Standardized $d_i=\frac{e_i}{\sqrt{MSE}}$; PRESS $e_{(i)}=y_i-\hat{y}_{(i)}=\frac{e_i}{1-h_{ii}}$; Studentized $r_i=\frac{e_i}{\sqrt{MSE(1-h_{ii})}}$

$h_{ii}=h_i'h_i=\sum h_{ii}^2=h_{ii}^2+\sum_{j\neq i}h_{ij}^2$; $h_{ii}-h_{ii}^2=h_{ii}(1-h_{ii})=\sum_{j\neq i}h_{ij}^2\geq0$; $0\leq h_{ij}\leq1$

Leverage of the i^{th} data point h_{ii}: how much y_i contributes to $\hat{y}_i$; $\sum_i^n h_{ii}=p$

$\hat{y}_i$=(i^{th} row of H)y=$\sum_{j=1}^n h_{ij}y_j=h_{ii}y_i+\sum_{i\neq j}h_{ij}y_j$. examine the point with $>\frac{2p}{n}$

Omit the i^{th} point: Cook's $D_i=\frac{(\hat{\beta}_{(i)}-\hat{\beta})'X'X(\hat{\beta}_{(i)}-\hat{\beta})}{p\cdot MSE}$; $\frac{e_i^2h_{ii}}{(1-h_{ii})^2p\cdot MSE}$; $\frac{r_i^2h_{ii}}{(1-h_{ii})p}$ examine if >1

DFFITS$_i$=$\frac{e_i\sqrt{h_{ii}}}{(1-h_{ii})\sqrt{MSE_{(i)}}}$; examie if >2$\sqrt{p/n}$ X=$[X_1|X_2]$; $\hat{\beta}=\frac{(X_1'X_1)^{-1}X_1'y}{(X_2'X_2)^{-2}X_2'y}$

Lack of fit y_{ij} denote the $j^{th}_{1:n_i}$ observation on the response at $x_{i_{1:m}}$ m distinct x values

$\sum_i^m\sum_j^{n_i}(y_{ij}-\hat{y}_i)^2(SSE)=\sum_i^m\sum_j^{n_i}(y_{ij}-\bar{y}_i)^2(SS_{PE})+\sum^m n_i(\bar{y}_i-\hat{y}_i)^2(SS_{Lof})$

n-p(dfE)=n-m(dfPE)+m-p(dfLof); F=$\frac{SS_{Lof}/df_{Lof}}{MS_{PE}/df_{PE}}$ H_0:no LOF, model is appropriate

Indicator $y_i=\beta_0+\beta_1x_i+w_i(\beta_2+\beta_3x_i)+\varepsilon_i$, $w_{i=m+1..n}$=1, 0o.w.

$y_i=\beta_0+\beta_1x_i+w_{1i}(\gamma_0+\gamma_1x_i)+w_{2i}(\delta_0+\delta_1x_i)+\varepsilon_i$, $w_{1i=m+1..k}$=1,$w_{2i=k+1..n}$=1, 0o.w.

$y_i=\beta_0+\beta_1x_{1i}+\beta_2x_{2i}+\beta_3x_{3i}+\varepsilon_i$, $x_{1i=\frac{k}{2}+1..k}$=1,$x_{2i=1,4,7..}$=-1,$x_{2i=3,6,9..}$=1, 0o.w.

$$\mathbf{Y}_{1:n}=\begin{bmatrix}1&x_1&0&0&0&0\\1&x_m&0&0&0&0\\1&x_{m+1}&1&x_{m+1}&0&0\\1&x_k&1&x_k&0&0\\1&x_{k+1}&0&0&1&x_{k+1}\\1&x_n&0&0&1&x_n\end{bmatrix}\begin{bmatrix}\beta_0\\ \beta_1\\ \gamma_0\\ \gamma_1\\ \delta_0\\ \delta_1\end{bmatrix}+\varepsilon_{1:n}';\begin{bmatrix}1&0&-1&x_{31}\\1&0&0&.\\1&0&1&.\\1&1&-1&.\\1&1&0&.\\1&1&1&x_{3n}\end{bmatrix}\beta_{1:4}'$$

μ overall mean; $y_{ijkl}(\varepsilon_{ijkl})$ is response(random error) (for the k^{th} replicate EU) when i^{th} (Latin(Greek) letter) treatment(level of Factor X) is applied (at j^{th} block) (in the k^{th} row and k^{th} column);

τ_i is fixed(random) (main) effect of i^{th} (Latin(Greek) letter) treatment (block; level of Factor X; row; column);

$(\tau\beta)_{ij}$ is interaction effect of i^{th} level of Factor A and j^{th} level of Factor B;

RCD	df	SS; $y_{ij}=\mu+\tau_i+\varepsilon_{ij}$; $i_{1:a}$,$j_{1:n}$; $\sum_i^a\tau_i$=0	EMS
Tre	a-1	$n\sum_i^a(\bar{y}_{i.}-\bar{y}_{..})^2$; $\frac{1}{n}\sum_i^a y_{i.}^2-\frac{1}{N}y_{..}^2$	$\sigma^2+\frac{n\sum\tau^2}{a-1}$
E	a(n-1)	$\sum_i^a\sum_j^n(y_{ij}-\bar{y}_{i.})^2$;$\sum_i^a\sum_j^n y_{ij}^2-\frac{1}{n}\sum_i^a y_{i.}^2$	σ^2 ;(n-1)$\sum_i^a S_i^2$
T	an-1	$\sum_i^a\sum_j^n(y_{ij}-\bar{y}_{..})^2$; $\sum_i^a\sum_j^n y_{ij}^2-\frac{1}{N}y_{..}^2$	N×a; a× 1

y_{11}	100. 0	μ_1
y_{1n}	100. 0	μ_2
y_{21}	010. 0	:
y_{2n}	010. 0	μ_a
:	:	
y_{a1}	00. 01	
y_{an}	00. 01	

$\hat{\mu}$=$\bar{y}_{..}$; $\bar{y}_{i.}-\bar{y}_{..}$=$\hat{\mu}_i$; y_{ij}~N(μ_i,σ^2); $\bar{y}_{i.}$~$(\mu_i,\frac{\sigma^2}{n})$; $y_{ij}-\bar{y}_{i.}$~$(0,\frac{n-1}{n}\sigma^2)$; $\bar{y}_{i.}-\bar{y}_{j.}$~$(\tau_i-\tau_j,\frac{2\sigma^2}{n})$

X'X=n$I_{a\times a}$; (X'X)$^{-1}$=$\frac{1}{n}I_{a\times a}$; X'y=$[y_{1.},..,y_{a.}]$; β=$[\bar{y}_{1.},..,\bar{y}_{a.}]$

CI μ_i: $\bar{y}_{i.}\pm t_{\frac{\alpha}{2}}\sqrt{\frac{MSE}{n}}$; $\sqrt{\frac{MSE}{n_i}}$(unb)

paired $\mu_i-\mu_j$: $\bar{y}_{i.}-\bar{y}_{j.}\pm t_{\frac{\alpha}{2},df_E}\sqrt{\frac{2MSE}{n}}$; $\sqrt{\frac{MSE}{1}(\frac{1}{n_i}+\frac{1}{n_j})}$(unb);Fisher huge Type I

Multiple Comparison:Type I experimentwise error rate(Prob of making at least one I error when performing a set of comparisons)

Contrast $\sum_i^a c_i$=0; C=$\sum_i^a c_i\bar{y}_i$; E[C]=Γ = $\sum_i^a c_i\mu_i$; V[C]=$\sigma^2\sum_i^a\frac{c_i^2}{n\{n_i\}}$

$SS_C=\frac{n\{\text{of mean}\}C^2}{\sum_i^a c_i^2}\sim\chi_1^2$; $\frac{C-0}{\sqrt{\frac{MSE}{n}\sum_i^a c_i^2}}\sim t_{N-a}$; $\frac{C^2}{\frac{MSE}{n}\sum_i^a c_i^2}\sim F_{1,N-a}$

Compare 'a' means, a-1 contrast.Orthogonal $\vec{c}\perp\vec{d}$; $\sum_i^a c_id_i$=0

μ_1-μ_2=0;$\mu_1+\mu_2-2\mu_3$=0; $\mu_1+\mu_2+\mu_3-3\mu_4$=0 $\vec{c}$=[1,-1,0,0]; $\vec{d}$=[1,1,-2,0]; $\vec{e}$=[1,1,1,-3]

$\sum_1^{a-1}SS_C=SS_{Trt}$; $\sum_1^{a-1}df_C=df_{Trt}$=a-1; Type I error rate:1-(1-α)$^{a-1}$;

Simultaneous Scheffe CI: C±$\sqrt{(a-1)F_\alpha\hat{V}[C]}$; control α_e, higher Type II;

Bonferroni:$t_{\frac{\alpha}{2m}}\sqrt{\frac{2MSE}{n}}$, m=$\binom{a}{2}$# of comparisons; control α_e;

Tukey: $q_{\alpha(a,df_E)}\sqrt{\frac{MSE}{n}}$;a=# of compared means, $\sqrt{\frac{MSE}{2}(\frac{1}{n_i}+\frac{1}{n_j})}$(unb)

Dunnett: $d_{\alpha(a-1,df_E)}\sqrt{\frac{2MSE}{n}}$ One control group vs. each other group

Hypothesis H_0main:$\tau_i\{\beta_j\}$\{inter:$(\tau\beta)_{ij}$\}=0∀i, j; H_1 at least one ≠ 0.

RCBD	df	SS; $y_{ij}=\mu+\tau_i+\beta_j+\varepsilon_{ij}$; $i_{1:a}$,$j_{1:b}$; $\sum_j^b\beta_j$=0	EMS	EMS$_r$
Tre	a-1	$b\sum_i^a(\bar{y}_{i.}-\bar{y}_{..})^2$;$\frac{1}{b}\sum_i^a y_{i.}^2-\frac{1}{N}y_{..}^2$	$\sigma^2+\frac{b\sum^a\tau^2}{a-1}$	$\sigma^2+b\sigma_\tau^2$
B	b-1	$a\sum_j^b(\bar{y}_{.j}-\bar{y}_{..})^2$;$\frac{1}{a}\sum_j^b y_{.j}^2-\frac{1}{N}y_{..}^2$	$\sigma^2+\frac{a\sum^b\beta^2}{b-1}$	$\sigma^2+a\sigma_\beta^2$
E	(a-1)(b-1)	$\sum_i^a\sum_j^b(y_{ij}-\bar{y}_{i.}-\bar{y}_{.j}+\bar{y}_{..})^2$	σ^2	
T	ab-1	$\sum_i^a\sum_j^b(y_{ij}-\bar{y}_{..})^2$; $\sum_i^a\sum_j^b y_{ij}^2-y_{..}^2/N$		$\hat{\sigma}_\tau^2=\frac{MS_{Trt}-MSE}{n}$

Missing:blk 1st ANOVA; Exact:full reg model & no Trt $F_0=\frac{(SSE_{red}-SSE_{ful})/r}{MSE_{ful}}$;

Approx: $y_{..}'=y_{..}$-x,$y_{i.}'=y_{i.}$-x,$y_{.j}'=y_{.j}$-x; $\hat{x}=\frac{ay_{i.}'+by_{.j}'-y_{..}'}{(a-1)(b-1)}$,$df_E$-1;LaSq:$\frac{p(y_{i..}'+y_{.j.}'+y_{..k}')-2y_{...}'}{(p-2)(p-1)}$

Unbalanced: n_{ij}=1 $i^{th}_{1:a}$ Trt appears in $j^{th}_{1:b}$ Blk; =0 o.w.; proportional:$n_{ij}=\frac{n_{i.}n_{.j}}{n_{..}}$

Yates:V[y_{ijk}]:$\hat{\sigma}^2=\frac{\sum\sum\sum(y_{ijk}-\bar{y}_{ij.})^2}{n_{..}-ab}$; cell mean $\bar{V}(\bar{y}_{ij.})=\frac{\hat{\sigma}^2}{ab}\sum_j\sum_j\frac{1}{n_{ij}}$; ma'ly MSE, dfE:$n_{..}$-ab

Relative Efficiency

$\frac{(df_{E(rcbd)}+1)(df_{E(crd)}+3)MSE_{crd}}{(df_{E(rcbd)}+3)(df_{E(crd)}+1)MSE_{rcbd}}$; $\frac{(df_{E(La)}+1)(df_{E(rcbd)}+3)MSE_{rcbd}}{(df_{E(La)}+3)(df_{E(rcbd)}+1)MSE_{La}}$; $\frac{(df_{E(La)}+1)(df_{E(crd)}+3)MSE_{crd}}{(df_{E(La)}+3)(df_{E(crd)}+1)MSE_{La}}$

eliminate two nuisance sources of variability;blocking in two directions. the rows and columns actually represent two restrictions on randomization.

LaSq	df	SS $y_{ijk}=\mu+\tau_i+\alpha_j+\beta_k+\varepsilon_{ijk}$, i, j, $k_{1:p}$
Tre	p-1	$p\sum_i^p(\bar{y}_{i..}-\bar{y}_{...})^2$; $\frac{\sum^p y_{i..}^2}{p}-\frac{y_{...}^2}{p}$
RC	p-1	$p\sum_{j\{k\}}^p(\bar{y}_{.j.\{..k\}}-\bar{y}_{...})^2$; $\frac{1}{p}\sum^p y_{.j.\{..k\}}^2-\frac{1}{N}y_{...}^2$
E	(p-1)(p-2)	$\sum_i^p\sum_j^p\sum_k^p(y_{ijk}-\bar{y}_{i..}-\bar{y}_{.j.}-\bar{y}_{..k}+2\bar{y}_{...})^2$
T	p^2-1	$\sum^p\sum^p\sum^p(y_{ijk}-\bar{y}_{...})^2$; 9× 7; 7× 1

	1	2	3
1	A	B	C
2	B	C	A
3	C	A	B

y_{111}	1 1 0	1 0	1 0		μ
y_{123}	1 1 0	0 1	-1-1		τ_1
y_{132}	1 1 0	-1-1	0 1		τ_2
y_{212}	1 0 1	1 0	0 1		α_1
y_{221}	1 0 1	0 1	1 0		α_2
y_{233}	1 0 1	-1-1	-1-1		β_1
y_{313}	1-1-1	1 0	-1-1		β_2
y_{322}	1-1-1	0 1	0 1		
y_{221}	1-1-1	-1-1	1 0		

rep123	df	SS
Tre	p-1	$np\sum_i^p(\bar{y}_{i...}-\bar{y}_{....})^2$;$\frac{\sum^p y_{i...}^2}{np}-\frac{y_{....}^2}{np^2}$
RC	p-1	$np\sum_{j\{k\}}^p(\bar{y}_{.j..\{..k.\}}-\bar{y}_{....})^2$; $\frac{1}{np}\sum^p y_{.j..\{..k.\}}^2-\frac{1}{N}y_{....}^2$
Rep-RC	n(p-1)	$p\sum_l^n\sum_{j\{k\}}^p(\bar{y}_{.j.l\{..kl\}}-\bar{y}_{....})^2$; $\frac{1}{p}\sum^n\sum^p y_{.j.l\{..kl\}}^2-\frac{1}{p^2}y_{...l}^2$
Rep	n-1	$p^2\sum_l^n(\bar{y}_{...l}-\bar{y}_{....})^2$; $\frac{1}{p^2}\sum^n y_{...l}^2-\frac{1}{N}y_{....}^2$
E		(p-1)(np+n-3);(p-1)(np-2);(p-1)(np-n-1)
T	np^2-1	Rep each point; change R∣C; change R&C

Graeco-Latin	$y_{ijkl}=\mu+\tau_i+\gamma_j+\alpha_k+\beta_l+\varepsilon_{ijkl}$;i, j, k, $l_{1:p}$	
Gr;La	p-1	$p\sum_{i\{j\}}^p(\bar{y}_{i...\{.j..\}}-\bar{y}_{....})^2$
R;C	p-1	$p\sum_{k\{l\}}^p(\bar{y}_{..k.\{...l\}}-\bar{y}_{....})^2$
E	(p-1)(p-3)	$SS_T-SS_.$
T	p^2-1	$\sum\sum\sum\sum(y_{ijkl}-\bar{y}_{....})^2$

Aα	Bβ	Cγ	Dδ
Bδ	Aγ	Dβ	Cα
Cβ	Dα	Aδ	Bγ
Dγ	Cδ	Bα	Aβ

a1	b2	c3	d4	e5
b3	c4	d5	e1	a2
c5	d1	e2	a3	b4
d2	e3	a4	b5	c1
e4	a5	b1	c2	d3

Only two of the four subscripts are necessary to completely identify an observation. The Latin-Squre design can only use 2 blocking factors, as we distribute the levels of the treatment factor on a table with rows of one blocking factor (each row is fore one block) and columns of the other blocking factor (each column is one block) To test three blocking factors by turning the Latin-square design into a Graeco-Latin square design, which allows to add a Greek letter to each entry in the table, where each Greek letter stands for a block of the factor G.

BIBD $y_{ij}=\mu+\tau_i+\beta_j+\varepsilon_{ij}$; a: # Trt; k: # of EUs in each Blk; b:# Blk; r: # of each Trt λ: # of times each pair of Trt in the same block; a=7; k=5; N=ar=bk=105;b=$\binom{a}{k}$

Tre$_{adj}$	a-1	$\frac{k}{\lambda a}\sum^a Q_i^2$; $\hat\tau_i=\frac{k}{\lambda a}Q_i$	$\hat\tau=\bar y_{i.}-\bar y_{..}-\frac{1}{r}\sum_j n_{ij}\beta_j$; min(r)=3*5
Blk	b-1	$k\sum_j^b(\bar y_{.j}-\bar y_{..})^2$; $\frac{1}{k}\sum_j^b y_{.j}^2-\frac{1}{N}y_{..}^2$	$\hat\beta=\bar y_{.j}-\bar y_{..}-\frac{1}{k}\sum_i n_{ij}\tau_i$
E	N-a-b+1	T-Trt$_{adj}$-Blk; $\lambda=\frac{k-1}{a-1}r=\frac{(k-1)bk}{(a-1)a}$	$Q_i=\frac{y_{i.}}{r}-\sum_j^b n_{ij}\frac{y_{.j}}{k}=\bar y_{i.}-\sum^b n_{ij}\bar y_{.j}$

Factorial model k factors,p generators; 2^p blocks/fraction;2^{k-p}Run, Blk size; 2^p-1 alias; 2^p-p-1 auto confounded; dfT: $2^k n$-1; dfB:2^p-1; dfE: 2^k(n-1);ow=1
AD+,ABC-,BCD-;(1),bc,abd,acd;ABC+BCD+;b,c,ad,abcd
AD-,ABC-,BCD+;ab,ac,bcd,d;ABC+BCD-;a,abc,bd,cd; I=ABC=BCD=AD
I=ABCDF=ABDEG=CEFG;CE=FG,CF=EG,CG=EF minimum aberration

2^{k-p}	2^k	2^p	2^{k-p}	Generator	2^{6-2}_{IV}	64	4	16	E=ABC;F=BCD	
2^{3-1}_{III}	8	2	4	C=AB	2^{6-3}_{III}	64	8	8	D=AB;E=AC;F=BC	Day1: (1), abcd, abef, cdef
2^{4-1}_{IV}	16	2	8	D=ABC	2^{7-1}_{VII}	128	2	64	G=ABCDEF	Day2: adf, ace, bde, bcf
2^{5-1}_{V}	32	2	16	E=ABCD	2^{7-2}_{IV}	128	4	32	F=ABCD;G=ABDE	Day3: bdf, acf, bce, ade Day4: ab, cd, ef, abcdef
2^{5-2}_{III}	32	4	8	D=AB;E=AC	2^{7-3}_{IV}	128	8	16	E=ABC;F=BCD;G=ACD	$\binom{6}{1}$=6;$\binom{6}{2}$=15;$\binom{6}{3}$=20
2^{6-1}_{VI}	64	2	32	F=ABCDE	2^{7-4}_{III}	128	16	8	D=AB;E=AC;F=BC;G=ABC	AB=,ACE=,ACF= Confed

Run	A	B	C	D	E	F	ABD	ABF		2^{6-2}_{IV} ABC=E,BCD=F
(1)	-	-	-	-	-	-	-	-	1	I=ABCE=BCDF=ADEF
ae	+	-	-	-	+	-	+	+	4	A=BCE=DEF=ABCDF
bef	-	+	-	-	+	+	+	-	2	B=ACE=CDF=ABDEF
abf	+	+	-	-	-	+	-	+	3	C=ABE=BDF=ACDEF
cef	-	-	+	-	+	+	-	+	3	D=BCF=AEF=ABCDE
acf	+	-	+	-	-	+	+	-	2	E=ABC=ADF=BCDEF
bc	-	+	+	-	-	-	+	+	4	F=BCD=ADE=ABCEF
abce	+	+	+	-	+	-	-	-	1	AB=CE=ACDF=BDEF
df	-	-	-	+	-	+	+	+	4	AC=BE=ABDF=CDEF
adef	+	-	-	+	+	+	-	-	1	AD=EF=BCDE=ABCF
bde	-	+	-	+	+	-	-	+	3	AF=DE=BCEF=ABCD
abd	+	+	-	+	-	-	+	-	2	BD=CF=ACDE=ABEF
cde	-	-	+	+	+	-	+	-	2	BF=CD=ACEF=ABDE
acd	+	-	+	+	-	-	-	+	3	AE=BC=DF=ABCDEF
bcdf	-	+	+	+	-	+	-	-	1	ABD=BEF=ACF=CDE
a-f	+	+	+	+	+	+	+	+	4	ABF=BDE=ACD=CEF

Run	A	B	C	E	D	F	ACE	ACF		2^{6-2}_{IV} ABC=D,ABE=F
(1)	-	-	-	-	-	-	-	-	1	I=ABCD=ABEF=CDEF
adf	+	-	-	-	+	+	+	-	2	A=BCD=BEF=ACDEF
bdf	-	+	-	-	+	+	-	+	3	B=ACD=AEF=BCDEF
ab	+	+	-	-	-	-	+	+	4	C=ABD=DEF=ABCEF
cd	-	-	+	-	+	-	+	+	4	D=ABC=CEF=ABDEF
acf	+	-	+	-	-	+	-	+	3	E=ABF=CDF=ABCDE
bcf	-	+	+	-	-	+	+	-	2	F=ABE=CDE=ABCDF
abcd	+	+	+	-	+	-	-	-	1	AB=CD=EF=ABCDEF
ef	-	-	-	+	-	+	+	+	4	AC=BD=BCEF=ADEF
ade	+	-	-	+	+	-	-	+	3	AD=BC=BDEF=ACEF
bde	-	+	-	+	+	-	+	-	2	AE=BF=BCDE=ACDF
abef	+	+	-	+	-	+	-	-	1	AF=BE=BCDF=ACDE
cdef	-	-	+	+	+	+	-	-	1	CE=DF=ABDE=ABCF
ace	+	-	+	+	-	-	+	-	2	CF=DE=ABDF=ABCE
bce	-	+	+	+	-	-	-	+	3	ACE=BDE=BCF=ADF
a-f	+	+	+	+	+	+	+	+	4	ACF=BDF=BCE=ADE

Fold-over Rep: reverse the sign of every letter in each generator

SS$_C$=$\frac{n(\sum_i^a c_i\bar y)^2}{\sum_i^a c_i^2}=\frac{(\sum_i^a c_i y)^2}{n\sum_i^a c_i^2}=\frac{(n)C^2}{(n)2^k}$; SS$_{AB}$=$\frac{n}{4}C_{AB}^2=\frac{n}{4}(\bar y_{(1)}$-$\bar y_a$-$\bar y_b$+$\bar y_{ab})^2$; SS$_{ABC}$=$\frac{1}{2^3}C_{ABC}^2$

I=ABC=$A^2B^2C^2$; res=3	I=AB^2C^2=BC=A^2BC	I=AB^2C=A^2BC^2; res=3
A=A^2BC=B^2C^2=A^2=BC=AB^2C^2	A=$A^2B^2C^2$=ABC=BC	A=A^2B^2C=BC^2=B^2C=ABC^2
B=AB^2C=A^2C^2=B^2=AC=A^2BC^2	B=AC^2=ABC^2	B=AC=$A^2B^2C^2$=ABC=A^2C^2
C=ABC^2=A^2B^2=C^2=AB=A^2B^2C	C=AB^2=AB^2C	C=AB^2C^2=A^2B=AB^2=A^2BC
A^2B=B^2C=AC^2=AB^2=A^2C=BC^2	AB=AC=BC^2	AB=A^2C=B^2C^2=BC=AC^2
dfA=dfB=dfC=dfE=2		dfA=dfB=dfC=dfAB=2

$\mathbf{A_{f,r}B_{f,r}}$ $y_{ijk}=\mu+\tau_i+\beta_j+(\tau\beta)_{ij}+\varepsilon_{ijk}$, $i_{1:a}$,$j_{1:b}$,$k_{1:n}$
ff:$\sum_i^a(\tau\beta)_{ij}$=$\sum_j^b(\tau\beta)_{ij}$=0; rr:E[τ_i=β_j=γ_k=$(\tau\beta)_{ij}$=$(\tau\gamma)_{ik}$]=0; V[]=σ^2_τ; σ^2_β; σ^2_γ; $\sigma^2_{\tau\beta}$; $\sigma^2_{\tau\gamma}$
$\bar y_{...}$=μ+$\bar\tau_.$+$\bar\beta_.$+$(\bar{\tau\beta})_{..}$; $\bar y_{i..}$=μ+$\tau_i+\bar\beta_.$+$(\bar{\tau\beta})_{i.}$; $\bar y_{.j.}$=μ+$\bar\tau_.+\beta_j$+$(\bar{\tau\beta})_{.j}$; $\bar y_{ij.}$=μ+$\tau_i+\beta_j$+$(\tau\beta)_{ij}$
$\hat\tau_i=\bar y_{i..}-\bar y_{...}$; $\hat\beta_j=\bar y_{.j.}-\bar y_{...}$; $(\hat{\tau\beta})_{ij}=\bar y_{ij.}-\bar y_{i..}-\bar y_{.j.}+\bar y_{...}$
Cov(y_{ijk},$y_{i'j'k'}$)=0; Cov(y_{ijk},$y_{i'jk'}$)=σ^2_β; Cov(y_{ijk},$y_{ij'k'}$)=σ^2_τ; Cov(y_{ijk},$y_{ijk'}$)=σ^2_τ+σ^2_β+$\sigma^2_{\tau\beta}$
$\hat\sigma^2_\tau$=$\frac{MSA-MSAB}{bn}$; $\hat\sigma^2_\beta$=$\frac{MSB-MSAB}{an}$; $\hat\sigma^2_{\tau\beta}=\frac{MSAB-MSE}{n}$

fr:$\sum_i^a\tau_i$=$\sum_i^a(\tau\beta)_{ij}$(rest'd)=E[β_j=$(\tau\beta)_{ij}$]=0; V[]=σ^2_β; ;$\frac{a-1}{a}\sigma^2_{\tau\beta}$) Cov($\tau\beta_{ij},_{i'j}$)=$\frac{-\sigma^2_{\tau\beta}}{a}$; ij')=0

AB	df	SS;	E	MS$_{ff}$	rr	fr; restricted
A	a-1	$bn\sum^a(\bar y_{i..}-\bar y_{...})^2$	σ^2+$\frac{bn\sum\tau_i^2}{a-1}$	$n\sigma^2_{\tau\beta}+bn\sigma^2_\tau$	$n\sigma^2_{\tau\beta}+\frac{bn\sum\tau_i^2}{a-1}$
B	b-1	$an\sum^b(\bar y_{.j.}-\bar y_{...})^2$	σ^2+$\frac{an\sum\beta_j^2}{b-1}$	$n\sigma^2_{\tau\beta}+an\sigma^2_\beta$	{$n\sigma^2_{\tau\beta}$+}$an\sigma^2_\beta$
AB	(a-1)(b-1)	$n\sum^a\sum^b(\bar y_{ij.}-\bar y_{i..}-\bar y_{.j.}+\bar y_{...})^2$	σ^2+$\frac{n\sum\sum(\tau\beta)_{ij}}{(a-1)(b-1)}$	$n\sigma^2_{\tau\beta}$	$n\sigma^2_{\tau\beta}$
E	ab(n-1)	$\sum^a\sum^b\sum^n(y_{ijk}-\bar y_{ij.})^2$	σ^2		{unrestricted}
Blk	n-1	$ab\sum^n\bar y_{..k}^2-\bar y_{...}^2$; $\frac{\sum^n y_{..k}^2}{ab}-\frac{y_{...}^2}{abn}$	σ^2+$ab\sigma^2_\delta$		
E	(ab-1)(n-1)	$(n-1)\sum^a\sum^b S_{ij}^2$	σ^2		
T	abn-1	$\sum\sum\sum(y_{ijk}-\bar y_{...})^2$			

Γ: $\sum_i^a c_i\bar y_i\{C\}\pm t_{\frac{\alpha}{2}}\sqrt{\frac{MS_E}{n}\sum_i^a c_i^2}$ $\frac{MS_{Trt}}{MS_E}\cdot\frac{\sigma^2}{n\sigma^2_\tau+\sigma^2}\sim F_{a-1,N-a}$
CI of prop of var $\frac{\sigma^2_\tau}{\sigma^2_\tau+\sigma^2}$: $\frac{L}{1+L}$, $\frac{U}{1+U}$; L=$\frac{1}{n}\left(\frac{MS_{Trt}}{MS_E F_{\alpha/2}}-1\right)$; U=$\frac{1}{n}\left(\frac{MS_{Trt}}{MS_E F_{1-\alpha/2}}-1\right)$
r(r+r) $y_{ijk}=\mu+\tau_i+\beta_j+\gamma_k+(\tau\beta)_{ij}+(\tau\gamma)_{ik}+\varepsilon_{ijk}$

R1	1	b	c	a-1	$\sigma^2+b\sigma^2_{\tau\gamma}+c\sigma^2_{\tau\beta}+bc\sigma^2_\tau$	$\frac{1}{bc}\sum_{i=1}^a y_{i..}^2-\frac{1}{abc}y_{...}^2$	$\frac{A+E}{AB+AC}$
R2	a	1	c	b-1	$\sigma^2+c\sigma^2_{\tau\beta}+ac\sigma^2_\beta$	$\frac{1}{ac}\sum_{j=1}^b y_{.j.}^2-\frac{1}{abc}y_{...}^2$	$\frac{B}{AB}$
R3	a	b	1	c-1	$\sigma^2+b\sigma^2_{\tau\gamma}+ab\sigma^2_\gamma$	$\frac{1}{ab}\sum_{k=1}^c y_{..k}^2-\frac{1}{abc}y_{...}^2$	$\frac{C}{AC}$
R1R2	1	1	c	(a-1)(b-1)	$\sigma^2+c\sigma^2_{\tau\beta}$	$\frac{\sum^a\sum^b y_{ij.}^2}{c}-\frac{\sum^a y_{i..}^2}{bc}-\frac{\sum^b y_{.j.}^2}{ac}+\frac{y_{...}^2}{abc}$	$\frac{AB}{E}$
R1R3	1	b	1	(a-1)(c-1)	$\sigma^2+b\sigma^2_{\tau\gamma}$	$\frac{\sum^a\sum^c y_{i.k}^2}{b}-\frac{\sum^a y_{i..}^2}{bc}-\frac{\sum^c y_{..k}^2}{ab}+\frac{y_{...}^2}{abc}$	$\frac{AC}{E}$
E	1	1	1	a(b-1)(c-1)	σ^2	$\sum^a\sum^b\sum^c y_{ijk}^2-\frac{\sum^a\sum^b y_{ij.}^2}{c}-\frac{\sum^a\sum^c y_{i.k}^2}{b}+\frac{\sum^a y_{i..}^2}{bc}$	

frr) $F_{df_n,df_d}=\frac{MS'}{MS''}=\frac{MS_A+MS_{E\{ABC\}}}{MS_{AB}+MS_{AC}}$; $df_n=\frac{(MS_A+MS_{E\{ABC\}})^2}{\frac{MS_A^2}{df_A}+\frac{MS^2_{E\{ABC\}}}{df_{E\{ABC\}}}}$;$df_d=\frac{(MS_{AB}+MS_{AC})^2}{\frac{MS^2_{AB}}{df_{AB}}+\frac{MS^2_{AC}}{df_{AC}}}$

Fixed v.s. random H$_0$:$\tau_{1:a}$=0 or σ^2_τ=0The levels of the factor are chosen so that you can test for differences between these specific levels(Fixed were selected at random from a larger number of levels (Random effect)
There are only 3 specific factories. Each factories 4 suppliers who randomly selected from a large amount of suppliers. In this test, the factories have fixed effects. The suppliers has random effects. Fixed(Random) effects across individuals arecon-stant(vary); interest in themselves(underlying population); a sample exhausts the population(a small part of the population); a realized value of a random variable

Nested model$\mathbf{C_r(B_r(A_f))}$ $y_{ijkl}=\mu+\tau_i+\beta_{j(i)}+\gamma_{k(ij)}+\varepsilon_{(ijk)l}$ No interaction terms

df	k		i	j	EMS	/		i	j	EMS	den		i	j	EMS	den
a-1	n	f	0	b	$\sigma^2_\varepsilon+\frac{bn\sum^a\alpha_i^2}{a-1}$	E	f	0	b	$\sigma^2_\varepsilon+n\sigma^2_\beta+\frac{bn\sum^a\alpha_i^2}{a-1}$	B(A)	r	1	b	$\sigma^2_\varepsilon+n\sigma^2_\beta+bn\sigma^2_\alpha$	B(A)
a(b-1)	n	f(f)	a	0	$\sigma^2_\varepsilon+\frac{n\sum^a\sum^b\beta_{ij}^2}{a(b-1)}$	E	r(f)	0	1	$\sigma^2_\varepsilon+n\sigma^2_\beta$	E	r(f)	a	1	$\sigma^2_\varepsilon+n\sigma^2_\beta$	E

ab(n-1)	1	E	1	1	σ^2	$\sum_i^a\sum_j^b\sum_k^n(\bar y_{ijk}-\bar y_{ij.})^2$	$Cov(\beta_{ij},\bar\beta_{i.})=\frac{1}{b}\sigma^2_\beta$; $Cov(\bar\varepsilon_{ij.},\bar\varepsilon_{i..})=\frac{\sigma^2_\varepsilon}{bn}$

y_{ijk}~ N(μ_i,σ^2_β+σ^2); $\bar y_{ij.}$~ (μ_i,σ^2_β+$\frac{1}{n}\sigma^2$); $\bar y_{i..}$-$\bar y_{j..}$~ τ_i-τ_j, $\frac{2}{b}(\sigma^2_\beta+\frac{\sigma^2}{n})$; $\bar y_{ij.}$-$\bar y_{i..}$~ (0,$\frac{b-1}{b}(\sigma^2_\beta+\frac{\sigma^2}{n})$)
$f(\bar y_{i1.},..\bar y_{ib.})=\prod_j^b f(\bar y_{ij.})=(2\pi(\sigma^2_\beta+\frac{1}{n}\sigma^2_\varepsilon))^{-\frac{b}{2}}\exp[\frac{-1}{2(\sigma^2_\beta+\frac{1}{n}\sigma^2_\varepsilon)}\sum_j^b(\bar y_{ij.}-\mu-\alpha_i)^2]$
the levels of factor B are similar but not identical for different levels of factor A
Suppose we have 3 factories+4 suppliers at each. **Crossed**: same 4 suppliers for each factory;**Nested**: each fact'y has a different set of 4 sup'ers. Every(each) category of one factor co-occurs with every(only one) category of the other factor. There at least one observation in every combinations of categories specific (are not represented).
r(r(f)):$\sum_i^a\tau_i$=E[$\beta_{j(i)}$=$\gamma_{k(ji)}$]=0; V[]=σ^2_β; σ^2_γ

Fτ_i	0	b	c	n	a-1	$\sigma^2+n\sigma^2_\gamma+cn\sigma^2_\beta+\frac{bcn\sum^a\tau_i^2}{a-1}$	$\frac{\sum^a y_{i...}^2}{bcn}-\frac{y_{....}^2}{abcn}$	$\frac{A}{B(A)}$
R(f)$\beta_{j(i)}$	1	1	c	n	a(b-1)	$\sigma^2+n\sigma^2_\gamma+cn\sigma^2_\beta$	$\frac{\sum^a\sum^b y_{ij..}^2}{cn}-\frac{\sum^a y_{i...}^2}{bcn}$	$\frac{B(A)}{C(B)}$
R(r(f))$\gamma_{k(ji)}$	1	1	1	n	ab(c-1)	$\sigma^2+n\sigma^2_\gamma$	$\frac{1}{n}\sum^a\sum^b\sum^c y_{ijk.}^2-\frac{1}{cn}\sum^a\sum^b y_{ij..}^2$	$\frac{C(B)}{E}$

fr(f):$\sum_i^a\tau_i$=$\sum_k^c\gamma_k$=$\sum_i^a(\tau\gamma)_{ik}$=$\sum_k^c(\tau\gamma)_{ik}$=$\sum_k^c(\beta\gamma)_{j(i)k}$=E[$\beta_{j(i)}$;$(\beta\gamma)_{j(i)k}$]=0; V:$\sigma^2_\beta$; $\frac{c-1}{c}\sigma^2_{\beta\gamma}$

F1 τ_i	0	b	c	n	a-1	$\sigma^2+cn\sigma^2_\beta+\frac{bcn\sum_{i=1}^a\tau_i^2}{a-1}$	$\frac{A}{B(A)}$
R(f1) $\beta_{j(i)}$	1	1	c	n	a(b-1)	$\sigma^2+cn\sigma^2_\beta$	$\frac{B(A)}{E}$
F2 $(\gamma)_k$	a	b	0	n	c-1	$\sigma^2+n\sigma^2_{\gamma\beta}+\frac{abn\sum^c\gamma_k^2}{c-1}$	$\frac{C}{CB(A)}$
F2F1 $(\tau\gamma)_{ik}$	0	b	0	n	(a-1)(c-1)	$\sigma^2+n\sigma^2_{\gamma\beta}+\frac{bn\sum^a\sum^c(\tau\gamma)_{ik}^2}{(a-1)(c-1)}$	$\frac{AC}{CB(A)}$
FR(f1)$(\gamma\beta)_{kj(i)}$	1	1	0	n	a(b-1)(c-1)	$\sigma^2+n\sigma^2_{\beta\gamma}$	$\frac{CB(A)}{E}$

ff(r):$\sum_j^b\beta_{j(i)}$=$\sum_k^c\gamma_k$=$\sum_k^c(\tau\gamma)_{ik}$=$\sum_j^b(\beta\gamma)_{(i)jk}$=$\sum_k^c(\beta\gamma)_{(i)jk}$=0;
E[τ_i=$\beta_{(i)j}$=$(\tau\gamma)_{ik}$=$(\beta\gamma)_{(i)jk}$]=0; V[]=σ^2_τ; $\frac{b-1}{b}\sigma^2_\beta$; $\frac{c-1}{c}\sigma^2_{\tau\gamma}$; $\frac{(b-1)(c-1)}{bc}\sigma^2_{\beta\gamma}$

R τ_i	1	b	c	n	a-1	$\sigma^2+bcn\sigma^2_\tau$	$\frac{A}{E}$
F1(r)$\beta_{j(i)}$	1	0	c	n	a(b-1)	$\sigma^2+cn\sigma^2_\beta$	$\frac{B(A)}{E}$
F2 $(\gamma)_k$	a	b	0	n	c-1	$\sigma^2+bn\sigma^2_{\tau\gamma}+\frac{abn\sum^c\gamma_k^2}{c-1}$	$\frac{C}{AC}$
F2R $(\tau\gamma)_{ik}$	1	b	0	n	(a-1)(c-1)	$\sigma^2+bn\sigma^2_{\tau\gamma}$	$\frac{AC}{E}$
F2F1(r)$(\beta\gamma)_{kj(i)}$	1	0	0	n	a(b-1)(c-1)	$\sigma^2+n\sigma^2_{\beta\gamma}$	$\frac{CB(A)}{E}$
R$\varepsilon_{(ijk)l}$	1	1	1	1	abc(n-1)	σ^2; $\sum\sum\sum\sum y_{ijkl}^2-\frac{1}{n}\sum\sum\sum y_{ijk.}^2$	

Split-Plot$y_{ijkl}=\mu+\tau_i+\beta_j+(\tau\beta)_{ij}+\gamma_k+(\tau\gamma)_{ik}+(\beta\gamma)_{jk}+(\tau\beta\gamma)_{ijk}+\varepsilon_{ijk}$
Rep+A(HTC)+RA(whole-plot error)+B(ETC)+AB(sub-plot)+RAB(sub-plot error)
unable to completely randomize the order of the runs. a given level of fertilizer is applied to a larger area. A comparison of the effect of the different fertilizer levels is made with reference to the natural variation between these larger areas, which are called whole plots. The effect of the different varieties and the interaction between variety and fertilizer level are compared to the random variation between the smaller units, which are called subplots.
$\sum_j^a\beta_j$=$\sum_k^b\gamma_k$=$\sum_j^a(\beta\gamma)_{jk}$=$\sum_k^b(\beta\gamma)_{jk}$=$\sum_j^a(\tau\beta)_{ij}$=$\sum_k^b(\tau\gamma)_{ik}$=E[$(\tau\beta)_{ij}$=$(\tau\gamma)_{ik}$]=0;
$\sum_j^a(\tau\beta\gamma)_{ijk}$=$\sum_k^b(\tau\beta\gamma)_{ijk}$=E[$\tau_i$=$(\tau\beta\gamma)_{ijk}$]=0; V[]=$\sigma^2_\tau$;$\frac{a-1}{a}\sigma^2_{\tau\beta}$;$\frac{b-1}{b}\sigma^2_{\tau\gamma}$; $\frac{(a-1)(b-1)}{ab}\sigma^2_{\tau\beta\gamma}$

$\mathbf{R_rA_fB_f}$	$i_{1:r}$	$j_{1:a}$	$k_{1:b}$	l	df	EMS	F
Repτ_i	1	a	b	1	r-1	$\sigma^2+ab\sigma^2_\tau$	$\frac{R}{E}$
F β_j	r	0	b	1	a-1	$\sigma^2+b\sigma^2_{\tau\beta}+\frac{rb\sum^a\beta_j^2}{a-1}$	$\frac{A}{RA}$
RF $(\tau\beta)_{ij}$	1	0	b	1	(r-1)(a-1)	$\sigma^2+b\sigma^2_{\tau\beta}$	$\frac{RA}{E}$
F $(\gamma)_k$	r	a	0	1	b-1	$\sigma^2+a\sigma^2_{\tau\gamma}+\frac{ra\sum^b\gamma_k^2}{b-1}$	$\frac{B}{RB}$
RF $(\tau\gamma)_{ik}$	1	a	0	1	(r-1)(b-1)	$\sigma^2+a\sigma^2_{\tau\gamma}$	$\frac{RB}{E}$
FF $(\beta\gamma)_{jk}$	r	0	0	1	(a-1)(b-1)	$\sigma^2+\sigma^2_{\tau\beta\gamma}+\frac{r\sum^a\sum^b(\beta\gamma)_{jk}^2}{(a-1)(b-1)}$	$\frac{AB}{RAB}$
RFF$(\tau\beta\gamma)_{ijk}$	1	0	0	1	(r-1)(a-1)(b-1)	$\sigma^2+\sigma^2_{\tau\beta\gamma}$	$\frac{RAB}{E}$
R $\varepsilon_{(ijk)h}$	1	1	1	1	0	not estimatable	

randomization restrictions:A+B+Rep(CRD);A+B+Blk(RCBD);Day+A+B(Split plot)

$\mathbf{R_rA_fB_fC_fD_f}$	Blk	A	B	TA	TB	AB	TAB	C..subplot	E	T	2-way	3-way	4-way	5-way
df	1	1	1	1	1	1	1	24	0	31	10	10	5	1

ANOCOVA $y_{ij}=\mu+\tau_i+\beta(x_{ij}-\bar x_{..})+\varepsilon_{ij}$; $\sum_i^a\tau_i$=E[ε_{ij}]=0;V[ε_{ij}]=σ^2; ε_{ij} indep $\forall i,j$
$\hat\mu$=$\bar y_{..}$; $\hat\beta$=$\frac{E_{xy}}{E_{xx}}=\frac{S_{xy}-T_{xy}}{S_{xx}-T_{xx}}$; $\hat\tau_i$=$\bar y_{i.}-\bar y_{..}-\hat\beta(\bar x_{i.}-\bar x_{..})$; V[$\hat\beta$]=$\frac{\sigma^2}{E_{xx}}$; V[$\hat\mu_i$]=$\sigma^2[\frac{1}{n}+\frac{(\bar x_{i.}-\bar x_{..})^2}{E_{xx}}]$

	df	SS	S,T,E_{yy}	S,T,E_{xx}	S,T,E_{xy}
Reg	1	$\frac{S_{xy}^2}{S_{xx}}$	$\sum_i\sum_j(y_{ij}-\bar y_{..})^2$	$\sum_i\sum_j(x_{ij}-\bar x_{..})^2$	$\sum_i\sum_j(x_{ij}-\bar x_{..})(y_{ij}-\bar y_{..})$
Trt	a-1	$T_{yy}-\frac{S_{xy}^2}{S_{xx}}+\frac{E_{xy}^2}{E_{xx}}$	$n\sum_i(\bar y_{i.}-\bar y_{..})^2$	$n\sum_i(\bar x_{i.}-\bar x_{..})^2$	$n\sum_i(\bar x_{i.}-\bar x_{..})(\bar y_{i.}-\bar y_{..})$
E	an-1-a	$E_{yy}-\frac{E_{xy}^2}{E_{xx}}$	$\sum_i\sum_j(y_{ij}-\bar y_{i.})^2$	$\sum_i\sum_j(x_{ij}-\bar x_{i.})^2$	$\sum_i\sum_j(x_{ij}-\bar x_{i.})(y_{ij}-\bar y_{i.})$
T	an-1	S_{yy} =	$\sum_i\sum_j y_{ij}^2-an\bar y_{..}^2$		

TypeI:Sequential;II: Conditional

H$_0$: τ_i=0; $F_{a-1,an-(a+1)}$=$\frac{T_{yy}-\frac{S_{xy}^2}{S_{xx}}+\frac{E_{xy}^2}{E_{xx}}}{(a-1)MSE_{Ful}}$

V[$\hat\mu_i$-$\hat\mu_j$]=V[$\hat\mu_i$]+V[$\hat\mu_j$]+Cov[$\hat\mu_i$,$\hat\mu_j$]=$\sigma^2[\frac{2}{n}+\frac{(\bar x_{i.}-\bar x_{j.})^2}{E_{xx}}]$

$\frac{\hat\mu_i-\hat\mu_j-(\tau_i-\tau_j)}{\sqrt{MSE[\frac{2}{n}+\frac{(\bar x_{i.}-\bar x_{j.})^2}{E_{xx}}]}}\approx t_{df_E}$; N$\times$(a+1); (a+1)$\times$ 1

$$\left|\begin{array}{cccc|c} 1 & 1 & 0\text{-}0 & x_{11}-\bar x_{..} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 0\text{-}0 & x_{1n}-\bar x_{..} \\ \hline 1 & 0 & 10\text{-}0 & x_{21}-\bar x_{..} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 10\text{-}0 & x_{2n}-\bar x_{..} \\ \hline 1 & \text{-}1 & .\ \text{-}1 & x_{a1}-\bar x_{..} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \text{-}1 & .\ \text{-}1 & x_{an}-\bar x_{..} \end{array}\right| \left|\begin{array}{c} \mu \\ \tau_1 \\ \vdots \\ \tau_{a-1} \\ \beta \end{array}\right|$$

$2^1\times 3^1$	A	X	AX	E	T	$2^1\times 3^2$	A	X	W	AX	AW	XW	AXW	E	T	$2^2\times 4^1$	A	B	X	AB	AX	BX	ABX	E	T
$2^1\times 2^2$	1	2	2	2	7	$2^1\times 2^4$	1	2	2	2	2	4	4	14	31	$2^2\times 2^2$	1	1	3	1	3	3	3	0	15

Kruskal-Wallis H_0:pop'n medians are all equal. $\bar R_{..}$:$\frac{1}{N}\sum^a\sum^n R_{ij}$=$\frac{N+1}{2}$; SST:$\frac{N(N^2-1)}{12}$;
σ^2:$\frac{1}{N}$SST; $\frac{N}{N-1}H\{\frac{a}{a-1}Q\}\sim\chi^2_{a-1}$; H:$\frac{12}{N(N+1)}\sum^a\frac{R_{i.}^2}{n}$-$3(N+1)$; Q:$\frac{12}{a(a+1)}\sum^a\frac{R_{i.}^2}{b}$-$N(\frac{a+1}{2})^2$
AIC: $n\ln\frac{SSE}{n}+2p$; Mallow's Cp $\frac{SSE_p}{MSE_{full}}-(n-2p)$
Box Cox $x_{ij}=\frac{y_{ij}^\lambda-1}{\lambda}$ Let $w_{ij}=\frac{y_{ij}^\lambda-1}{\lambda\dot y^{\lambda-1}}$; $\dot y=\sqrt[an]{\prod_i\prod_j|y_{ij}|}$ geom. mean

$\hat\beta_0=\hat\beta_1=\frac{\sum x_i^4\sum(1+x_i)y_i-\sum x_i^2(1+x_i)\sum x_i^2y_i}{\sum x_i^4\sum(1+x_i)^2-[\sum x_i^2(1+x_i)]^2};\ \hat\beta_2=\frac{\sum(1+x_i)^2\sum x_i^2y_i-\sum x_i^2(1+x_i)\sum(1+x_i)y_i}{\sum x_i^4\sum(1+x_i)^2-[\sum x_i^2(1+x_i)]^2}$

$y_i=\beta_1(1+x_i)+\beta_2x_i^2+\varepsilon_i$

$\beta_1=0;\ \hat\beta_2=2\hat\beta_0=\frac{2\sum_{i=1}^n(1+2x_i^2)y_i}{\sum_{i=1}^n(1+2x_i^2)^2};\ y_i=\beta_0(1+2x_i^2)+\varepsilon_i$

$\beta_1=1;\ \hat\beta_0=\hat\beta_2=\frac{\sum_{i=1}^n(1+x_i^2)(y_i-x_i)}{\sum_{i=1}^n(1+x_i^2)^2};\ y_i-x_i=\beta_2(1+x_i^2)+\varepsilon_i$

$\hat\beta_1=1;\ \hat\beta_0=2\hat\beta_2=\frac{2\sum_{i=1}^n(2+x_i^2)(y_i-x_i)}{\sum_{i=1}^n(2+x_i^2)^2}$

$\beta_0=1;\ \hat\beta_1=\hat\beta_2=\frac{\sum_{i=1}^n(x_i+x_i^2)(y_i-1)}{\sum_{i=1}^n(x_i+x_i^2)^2};\ y_i-1=\beta_2(x_i+x_i^2)+\varepsilon_i$

$\beta_1=0;\ \hat\beta_0=2\hat\beta_2=\frac{2\sum_{i=1}^n(2+x_i^2)y_i}{\sum_{i=1}^n(2+x_i^2)^2};\ y_i=\beta_2(2+x_i^2)+\varepsilon_i$

$\beta_0=0;\ \hat\beta_1=\frac{\sum_{i=1}^n x_i^4\sum_{i=1}^n x_iy_i-\sum_{i=1}^n x_i^3\sum_{i=1}^n x_i^2y_i}{\sum_{i=1}^n x_i^4\sum_{i=1}^n x_i^2-(\sum_{i=1}^n x_i^3)^2};\ \hat\beta_2=\frac{\sum_{i=1}^n x_i^2\sum_{i=1}^n x_i^2y_i-\sum_{i=1}^n x_i^3\sum_{i=1}^n x_iy_i}{\sum_{i=1}^n x_i^4\sum_{i=1}^n x_i^2-(\sum_{i=1}^n x_i^3)^2}$

$y_i=\beta_1x_i+\beta_2x_i^2+\varepsilon_i$

y_i-3=$\beta_1x_i+\varepsilon_i;\ \hat\beta=\frac{\sum(y+3)x}{\sum x^2}$; E[$\hat\beta$]=$\hat\beta$;V[$\hat\beta$]=$\frac{\sigma^2}{\sum x^2}$;se[$\hat\beta$]=$\frac{\sigma}{\sqrt{\sum x^2}}$

SSE=$\sum(y+3-\beta x)^2$=$\sum(y+3)^2$-$2\beta\sum(y+3)x+\beta^2\sum x^2$=$\sum(y+3)^2$-$\beta^2\sum x^2$

E[]=V[$\sum(y$+3)]+E[$\sum(y$+3)]^2-$\sum x^2$(E[$\beta]^2$+V[β])=nσ^2+$\beta\sum x^2$-$\sum x^2(\beta^2+\frac{\sigma^2}{\sum x^2}$)=(n-1)$\sigma^2$

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fr(f):$y_{ijkl}=\mu+\tau_i+\beta_{j(i)}+\gamma_k+\tau\gamma_{ik}+\beta\gamma_{jk(i)}+\varepsilon_{(ijk)l}$ where

$\varepsilon_{(ijk)l}\overset{iid}{\sim}N(0,\sigma^2);\ \beta_{j(i)}\overset{iid}{\sim}N(0,\sigma_\beta^2);\ \beta\gamma_{jk(i)}\overset{iid}{\sim}N(0,\frac{(a-1)}{a}\sigma_{\beta\gamma}^2)$

$\sum_{i=1}^a\tau_i=0;\sum_{k=1}^c\gamma_k=0;\sum_{i=1}^a\tau\gamma_{ik}=\sum_{k=1}^c\tau\gamma_{ik}=0$

An exact F test is available for each factor. The SS and df for AB go to B(A) and for ABC to BC(A).

Factor	i_3^F	j_5^R	k_2^F	l_3^R	EMS	F_{den}
τ_i	0	5	2	3	$15\sum_i\tau_i^2+6\sigma_\beta^2+\sigma^2$	B(A)
$\beta_{j(i)}$	1	1	2	3	$6\sigma_\beta^2+\sigma^2$	E
γ_k	3	5	0	3	$45\sum_k\gamma_k^2+3\sigma_{\beta\gamma}^2+\sigma^2$	B(A)C
$\tau\gamma_{ik}$	0	5	0	3	$7.5\sum_i\sum_k\tau\gamma_{ik}^2+3\sigma_{\beta\gamma}^2+\sigma^2$	B(A)C
$\beta\gamma_{jk(i)}$	1	1	0	3	$3\sigma_{\beta\gamma}^2+\sigma^2$	E
$\varepsilon_{(ijk)l}$	1	1	1	1	σ^2	

Source	SS	df	MS	F	Pr(>F)
A	17	2	8.5	$\frac{8.5}{4.75}$=1.79	.209
B(A)	57	12	4.75	$\frac{4.75}{2.3}$=2.07	.033
C	4	1	4	$\frac{4}{2}$=2	.183
AC	5	2	2.5	$\frac{2.5}{2}$=1.25	.321
BC(A)	24	12	2	$\frac{2}{2.3}$=0.87	.581
Err	138	60	2.3		
Total	245	89			

ff(r)	i_5^R	j_3^F	k_4^F	l_2^R	EMS	F_{den}
τ_i	1	3	4	2	$24\sigma_\tau^2+\sigma^2$	MSE
$\beta_{j(i)}$	1	0	4	2	$8\sigma_\beta^2+\sigma^2$	MSE
γ_k	5	3	0	2	$\frac{30}{3}\sum_k\gamma^2+6\sigma_{\tau\gamma}^2+\sigma^2$	MSAC
$\tau\gamma_{ik}$	1	3	0	2	$6\sigma_{\tau\gamma}^2+\sigma^2$	MSE
$\beta\gamma_{jk(i)}$	1	0	0	2	$2\sigma_{\beta\gamma}^2+\sigma^2$	MSE
$\varepsilon_{(ijk)l}$	1	1	1	1	σ^2	

Source	SS	df	MS	F	Pr(>F)
A	34	4	8.5	$\frac{8.5}{2.3}$=3.70	.009
B(A)	44	10	4.4	$\frac{4.4}{2.3}$=1.91	.06
C	24	3	8	$\frac{8}{2.5}$=3.2	.06
AC	30	12	2.5	$\frac{2.5}{2.3}$=1.09	.385
BC(A)	54	30	1.8	$\frac{1.8}{2.3}$=0.78	.768
Err	138	60	2.3		
Total	324	119			

(df)	SS_F	$-3,4,5$	$SS_{1,2}$	$-2,4$	$SS_{1,3,5}$
R	367(5)	-96(3)	271(2)	-153(2)	214(3)
E	336(69)	+96(3)	432(72)	+153(2)	489(71)
T	703(74)				

	df	SS_F	$-\alpha$	$SS_{\beta\gamma}$	$-\beta$	$SS_{\alpha\gamma}$	$-\alpha$	SS_γ
α	2	110	-110	0		110	-110	0
γ	11	160		160		160		160
β		250	-10	240	-250	0		0
E	22	300	+120	420	+250	550	+110	660
T	35	820						

Calc the F-stat for $H_0:X_3=X_4=X_5=0$.
reduced Model: $Y_i=\beta_0+\beta_1X_{1i}+\beta_2X_{2i}+\varepsilon_i$.
$SSR_{full}=\sum_{i=1}^5SSR(X_i)=108+163+29+41+26=367;$
$SSE_{full}=SST-SSR_{full}=703-367=336$
$SSR_{red}=SSR(X_1)+SSR(X_2|X_1)=108+163=271;$
$SSE_{red}=SST-SSR_{red}=703-271=432$
$dfe_{red}=75-3=72;dfe_{full}=75-6=69$

F = $\frac{\frac{(SSE_{red}-SSE_{full})}{(dfe_{red}-dfe_{full})}}{MSE_{full}}=\frac{\frac{432-336}{72-69}}{\frac{336}{69}}=\frac{32}{4.87}=6.57\sim F_{3,69}$

Calculate R^2 and R^2_{adj} for $Y_i=\beta_0+\beta_1X_{1i}+\beta_2X_{2i}+\varepsilon_i$.
$R^2=\frac{SSR}{SST}=\frac{272}{703}=.39$
$R^2_{adj}=1-\frac{MSE}{MST}=1-\frac{SSE}{SST}\frac{n-1}{n-p}=1-\frac{431}{703}\frac{74}{72}=.37$
Calculate the F-stat for testing $H_0:\beta_2=\beta_4=0$.
The model under the null hypothesis is given by Model B. That makes the F-stat:

$F=\frac{\frac{489-336}{71-69}}{\frac{336}{69}}=\frac{76.5}{4.87}=15.71\sim F_{2,69}$

$y_{ijk}=\mu+\tau_{i_{1,2}}+\beta_{j(i)_{1,2;1,2,3}}+\varepsilon_{(ij)k_{1:3;1:2;1:4;1:3;1:3}}$
$y_{ijk}=\mu+\tau_i+\gamma_{ij}+\varepsilon_{ijk}$ 27×9
$y_{ijk}=\mu+\tau_i+\beta_j+(\tau\beta)_{ij}+\varepsilon_{ijk}$; i,j=1,,3;k=1,,$n_{ij}$ 20×9

$$\left|\begin{array}{c|c|c|cc}1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\\\hline1&1&0&0&0\\1&1&0&0&0\\\hline1&0&0&1&0\\1&0&0&1&0\\1&0&0&1&0\\1&0&0&1&0\\\hline1&0&0&0&1\\1&0&0&0&1\\1&0&0&0&1\\1&0&0&-1&-1\\1&0&0&-1&-1\\1&0&0&-1&-1\end{array}\right|\left|\begin{array}{c}\mu\\\tau_1\\\beta_1\\\gamma_1\\\gamma_2\end{array}\right|$$

$$\left|\begin{array}{c|cc|cc|cc|cc}1&1&0&1&0&0&0&0&0\\1&1&0&1&0&0&0&0&0\\1&1&0&1&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&1&0&-1&-1&0&0&0&0\\1&1&0&-1&-1&0&0&0&0\\1&1&0&-1&-1&0&0&0&0\\\hline1&0&1&0&0&1&0&0&0\\1&0&1&0&0&1&0&0&0\\1&0&1&0&0&1&0&0&0\\1&0&1&0&0&0&1&0&0\\1&0&1&0&0&0&1&0&0\\1&0&1&0&0&0&1&0&0\\1&0&1&0&0&-1&-1&0&0\\1&0&1&0&0&-1&-1&0&0\\1&0&1&0&0&-1&-1&0&0\\\hline1&-1&-1&0&0&0&0&1&0\\1&-1&-1&0&0&0&0&1&0\\1&-1&-1&0&0&0&0&1&0\\1&-1&-1&0&0&0&0&0&1\\1&-1&-1&0&0&0&0&0&1\\1&-1&-1&0&0&0&0&0&1\\1&-1&-1&0&0&0&0&-1&-1\\1&-1&-1&0&0&0&0&-1&-1\\1&-1&-1&0&0&0&0&-1&-1\end{array}\right|\left|\begin{array}{l}\mu\\\tau_1\\\tau_2\\\gamma_{11}\\\gamma_{12}\\\gamma_{21}\\\gamma_{22}\\\gamma_{31}\\\gamma_{32}\\\text{df}\\\text{A:2}\\\text{AB:6}\\\text{E:18}\\\text{T:26}\end{array}\right|$$

$$\left|\begin{array}{c|cc|cc|cccc}1&1&0&1&0&1&0&0&0\\1&1&0&1&0&1&0&0&0\\1&1&0&1&0&1&0&0&0\\1&1&0&1&0&1&0&0&0\\\hline1&1&0&0&1&0&1&0&0\\1&1&0&0&1&0&1&0&0\\1&1&0&0&1&0&1&0&0\\1&1&0&0&1&0&1&0&0\\\hline1&1&0&-1&-1&-1&-1&0&0\\1&1&0&-1&-1&-1&-1&0&0\\\hline1&0&1&1&0&0&0&1&0\\1&0&1&1&0&0&0&1&0\\\hline1&0&1&0&1&0&0&0&1\\1&0&1&0&1&0&0&0&1\\\hline1&0&1&-1&-1&0&0&-1&-1\\\hline1&-1&-1&1&0&-1&0&-1&0\\1&-1&-1&1&0&-1&0&-1&0\\\hline1&-1&-1&0&1&0&-1&0&-1\\1&-1&-1&0&1&0&-1&0&-1\\\hline1&-1&-1&-1&-1&1&1&1&1\end{array}\right|\left|\begin{array}{l}\mu\\\tau_1\\\tau_2\\\beta_{11}\\\beta_{12}\\\tau\beta_{11}\\\tau\beta_{12}\\\tau\beta_{21}\\\tau\beta_{22}\end{array}\right|$$

ssa=$5(\bar y_{1..}-\bar y_{...})^2+10(\bar y_{2..}-\bar y_{...})^2$; sst=$\sum(y_{ijk}-\bar y_{...})^2)$
ssb=$3(\bar y_{11.}-\bar y_{1..})^2+2(\bar y_{12.}-\bar y_{1..})^2+4(\bar y_{21.}-\bar y_{2..})^2+3(\bar y_{22.}-\bar y_{2..})^2+3(\bar y_{23.}-\bar y_{2..})^2$
sse=$\sum_k[(y_{11k}-\bar y_{11.})^2+(y_{12k}-\bar y_{12.})^2+(y_{21k}-\bar y_{21.})^2+(y_{22k}-\bar y_{22.})^2+(y_{23k}-\bar y_{23.})^2]$
dfT=$n_{..}-1$; dfE=$n_{..}-b_.$; dfB(A)=$b_.-a$; dfA=$a-1$
0,0,0,0,0,1,1,1,1,0,0,0,-1,-1,-1=$c1_{in}a2$; 1,1,1,1,1,0,0,0,0,0,0,0,0,0,0=a1
0,0,0,0,0,0,0,0,0,1,1,1,-1,-1,-1=$c2_{in}a2$; 1,1,1,0,0,0,0,0,0,0,0,0,0,0,0=$b1_{in}a1$

$E[MS_{A_f}]=\frac{bn}{a-1}\sum_i^aE[(\bar y_{i..}-\bar y_{...})^2]=\frac{bn}{a-1}\sum_i^a(E[]^2+V[])=\frac{bn}{a-1}\sum_i^a\tau_i^2+\sigma^2$

$E(MS_{A_rB_r})=E[\frac{\sum_i^a\sum_j^b\sum_k^n(\widehat{\tau\beta})_{ij}^2}{(a-1)(b-1)}]=\frac{n\sum_i^a\sum_j^bE[(\bar y_{ij.}-\bar y_{i..}-\bar y_{.j.}+\bar y_{...})^2]}{(a-1)(b-1)}=\frac{n\sum_i^a\sum_j^bV[]+0}{(a-1)(b-1)}$=$n\sigma^2_{\tau\beta}+\sigma^2$