STAT 661: Project

LS v.s. EM

Jacob, Robin, Ryan & Shen

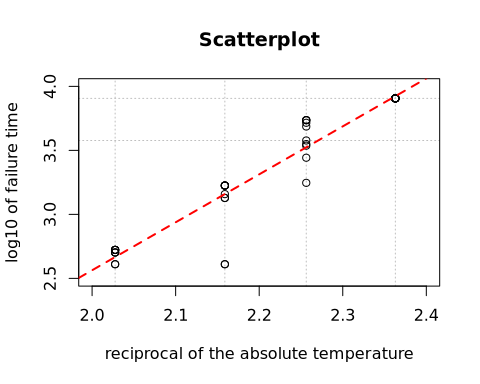
Dec, 2019

# Least Square Method v.s. EM Method

# Appendix

## LS Code

temp <- c(150,170,190,220) #temperature levels   
trec <- 1000/(temp+273.2) #reciprocal of the absolute temperature T   
x <- c(rep(trec[1],10),rep(trec[2],10),rep(trec[3],10),rep(trec[4],10))   
cen <- c(8064,5448,1680,528) #censoring times   
logcen <- log10(cen) #log10 censoring times   
y\_uncensored <-log10(c(rep(1,10),  
 1764,2772,3444,3542,3780,4860,5196,rep(1,3),  
 408,408,1344,1344,1440,rep(1,5),  
 408,408,504,504,504,rep(1,5)))  
y\_censored <- c(rep(logcen[1],10),  
 rep(0,7),rep(logcen[2],3),  
 rep(0,5),rep(logcen[3],5),  
 rep(0,5),rep(logcen[4],5))  
S <- 23; Y<-matrix(nrow=S,ncol=40)  
Y[1,] <- y\_0 <- y\_uncensored+y\_censored  
fit0 <- lm(y\_0~x) #linear model between log10 of observed life time

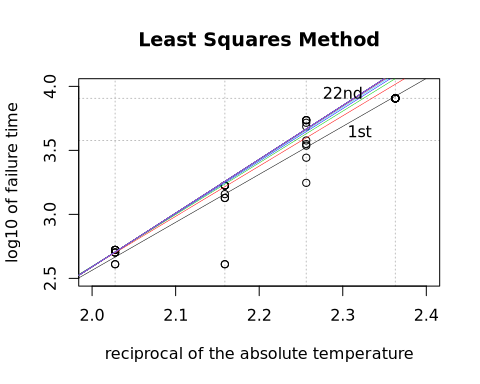


# Iteration 0   
sigma\_0 <- sigma(fit0) #standard error of residuals   
beta\_00 <- coef(fit0)[1] #intercept   
beta\_10 <- coef(fit0)[2] #slope   
mu\_0 <- beta\_00 + beta\_10\*trec #mean log time to failure   
z <- (logcen-mu\_0)/sigma\_0 #z-vector   
ex\_mu\_0 <- mu\_0 + sigma\_0\*dnorm(z)/(1-pnorm(z)) #new expected mean log times to failure  
  
delta = 1e-006; iteration <- 1  
  
PHI<-matrix(nrow=S,ncol=8,dimnames=list(NULL,   
 c('mu150','mu170','mu190','mu220','Intercept','Slope','Sigma','Iteration')))  
  
PHI[1,]<-phi<-c(ex\_mu\_0, beta\_00, beta\_10,sigma\_0,iteration)  
# Subsequent iteration  
repeat {   
 phi[8] <- phi[8]+1   
 y\_censored <- c(rep(phi[1],10),  
 rep(0,7),rep(phi[2],3),  
 rep(0,5),rep(phi[3],5),  
 rep(0,5),rep(phi[4],5))  
   
 y<- y\_uncensored+y\_censored  
 Y[phi[8],]<-y # Replace the new censored values  
 fit <- lm(y~x) # fit a new model  
 phi[5] <- coef(fit)[1] #intercept   
 phi[6] <- coef(fit)[2] #slope   
 phi[7] <- sigma(fit) #standard error of residuals   
 mu <- phi[5] + phi[6]\*trec   
 z <- (logcen-mu)/phi[7] #z-vector   
 #new expected mean log times to failure  
 phi[1:4] <- mu + phi[7]\*dnorm(z)/(1-pnorm(z))  
 conv <- dist(rbind(PHI[phi[8]-1,1:4],phi[1:4]))   
 if(conv < delta) break   
 PHI[phi[8],]<-phi   
}

## LS Results

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| mu150 | mu170 | mu190 | mu220 | Intercept | Slope | Sigma | Iteration |
| 4.038 | 3.809 | 3.329 | 2.83 | -4.931 | 3.747 | 0.1572 | 1 |
| 4.099 | 3.84 | 3.366 | 2.858 | -5.26 | 3.926 | 0.1799 | 2 |
| 4.131 | 3.856 | 3.382 | 2.869 | -5.486 | 4.04 | 0.1911 | 3 |
| 4.149 | 3.865 | 3.39 | 2.874 | -5.623 | 4.108 | 0.1969 | 4 |
| 4.159 | 3.87 | 3.395 | 2.877 | -5.704 | 4.148 | 0.2001 | 5 |
| 4.165 | 3.873 | 3.397 | 2.878 | -5.752 | 4.172 | 0.2019 | 6 |
| 4.168 | 3.874 | 3.399 | 2.879 | -5.779 | 4.185 | 0.2029 | 7 |
| 4.17 | 3.875 | 3.4 | 2.879 | -5.795 | 4.193 | 0.2035 | 8 |
| 4.171 | 3.876 | 3.4 | 2.879 | -5.805 | 4.198 | 0.2039 | 9 |
| 4.172 | 3.876 | 3.401 | 2.88 | -5.81 | 4.2 | 0.2041 | 10 |
| 4.172 | 3.877 | 3.401 | 2.88 | -5.814 | 4.202 | 0.2042 | 11 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.815 | 4.203 | 0.2042 | 12 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.817 | 4.203 | 0.2043 | 13 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.817 | 4.204 | 0.2043 | 14 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.818 | 4.204 | 0.2043 | 15 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.818 | 4.204 | 0.2043 | 16 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.818 | 4.204 | 0.2043 | 17 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.818 | 4.204 | 0.2043 | 18 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.818 | 4.204 | 0.2043 | 19 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.818 | 4.204 | 0.2043 | 20 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.818 | 4.204 | 0.2043 | 21 |
| 4.173 | 3.877 | 3.401 | 2.88 | -5.818 | 4.204 | 0.2043 | 22 |
| NA | NA | NA | NA | NA | NA | NA | NA |

## LS figure



## EM Method

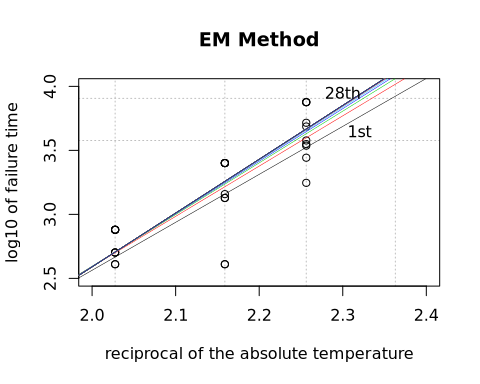
* E-step
* M-step

## the EM algorithm’s pseudo code

## EM Code

temp <- c(150,170,190,220) #temperature levels   
trec <- 1000/(temp+273.2) #reciprocal of the absolute temperature T   
nu <- c(rep(trec[1],10),rep(trec[2],10),rep(trec[3],10),rep(trec[4],10))  
# index\_nu <- c(rep(-2,7),rep(-3,5),rep(-4,5),rep(1,10),rep(2,3),rep(3,5),rep(4,5))  
  
cen <- c(8064,5448,1680,528) #censoring times   
w <- log10(cen) #log10 censoring times: last time still working  
y\_uncensored <-log10(c(rep(1,10),  
 1764,2772,3444,3542,3780,4860,5196,rep(1,3),  
 408,408,1344,1344,1440,rep(1,5),  
 408,408,504,504,504,rep(1,5)))  
y\_censored <- c(rep(w[1],10),  
 rep(0,7),rep(w[2],3),  
 rep(0,5),rep(w[3],5),  
 rep(0,5),rep(w[4],5))  
  
S <- 30  
Y<-matrix(nrow=S,ncol=40)  
Y[1,] <- y <- (y\_uncensored+y\_censored)  
  
index\_censored <- which(y %in% w)  
index\_uncensored <- which(!(y %in% w))  
  
m <- length(index\_uncensored)  
n<-length(index\_censored)+m  
  
# initial value  
fit0 <- lm(y~nu) #linear model   
sigma\_0 <- sigma(fit0) #standard error of residuals   
beta0\_0 <- as.numeric(coef(fit0)[1]) #intercept   
beta1\_0 <- as.numeric(coef(fit0)[2]) #slope   
  
mu\_i0 <- beta0\_0 + beta1\_0 \* nu[index\_censored]  
mu\_j0 <- beta0\_0 + beta1\_0 \* nu[index\_uncensored]  
D\_j <- y[index\_uncensored]-mu\_j0   
  
beta0\_star <- beta0\_0  
beta1\_star<- beta1\_0  
sigma\_star<- sigma\_0 # should be replaced by solutions  
  
par=c(beta0\_star,beta1\_star,sigma\_star)  
  
# define mu\_i function  
mu\_i\_F <- function (beta0,beta1) {  
 beta0+beta1\*nu[index\_censored]  
}  
  
k=1  
  
# define H function  
HF <- function (beta0,beta1,sigma) {  
 dnorm((y\_censored[index\_censored]-mu\_i\_F(beta0,beta1))/sigma  
 )/(   
1-pnorm((y\_censored[index\_censored]-mu\_i\_F(beta0,beta1))/sigma  
 ) )  
}  
  
  
ET <- mu\_i\_F(par[1],par[2])+ par[3]\*HF(par[1],par[2],par[3])  
ET\_sq <- mu\_i\_F(par[1],par[2])^2+   
 sigma\_star^2+  
 sigma\_star\*(y[index\_censored]+mu\_i\_F(par[1],par[2]))\*HF(par[1],par[2],par[3])  
ER <- ET\_sq-2\*mu\_i0\*ET+mu\_i0^2  
  
Q <- 0  
Q[1] <- -n\*log(2\*pi)/2-n\*log(sigma\_0)-(1/(2\*sigma\_0^2))\*(sum((D\_j)^2)+sum(ER))  
  
THETA<-matrix(nrow=S,ncol=8,dimnames=list(NULL,  
 c('Intercept','Slope','Sigma','mu150','mu170','mu190','mu220','Iteration')))  
THETA[1,]<-c(par,unique(ET),k)  
delta = 1e-6  
repeat {   
 # load THETA  
beta0 <- THETA[k,1]  
beta1 <- THETA[k,2]  
sigma <- THETA[k,3]  
  
# Update y  
y[index\_censored] <- ET  
y[index\_uncensored] <- y\_uncensored[index\_uncensored]  
  
# E step  
  
mu\_i <- beta0 + beta1 \* nu[index\_censored]  
mu\_j <- beta0 + beta1 \* nu[index\_uncensored]  
D\_j <- y[index\_uncensored]-mu\_j # difference between uncensored y and mean\_j  
  
  
Q\_theta <- function(par){   
 -n\*log(2\*pi)/2-  
 n\*log(sigma)-  
 (1/(2\*sigma^2))\*sum((D\_j)^2)- # 1/2sigma^2\*sum(j uncensored) -  
 (1/(2\*sigma^2))\*sum( # 1/2sigma^2\*sum(i censored   
 (mu\_i\_F(par[1],par[2]))^2+ # mu\_i\_star^2+  
 par[3]\*( # sigma\_star\*(  
 y[index\_censored]+ # w\_i+  
 mu\_i\_F(par[1],par[2])- # mu\_i\_star-  
 2\*mu\_i # 2mu\_i  
 )\*HF(par[1],par[2],par[3])- # )\*H)-  
 2\*mu\_i\*mu\_i\_F(par[1],par[2])+ # 2\*mu\_i\*mu\_i\_star+  
 mu\_i^2 # mu\_i^2  
 )+ # )+  
 (1/(2\*sigma^2))\*(n-m)\*(par[3]^2) # 1/2sigma^2\*(n-m)\*sigma\_star^2+   
}  
  
# Get Q  
Q[k]<- Q\_theta(par)  
  
# M step   
  
############ from Jacob  
# par <- optim(par,  
# Q\_theta,  
# control=list(fnscale=-1),  
# method="L-BFGS-B",   
# lower=c(-10,2,0.1),  
# upper=c(-2,10,1))$par  
  
############ cheat  
fit <- lm(y~nu) # fit a new model  
par[1] <- coef(fit)[1] #intercept   
par[2] <- coef(fit)[2] #slope   
par[3] <- sigma(fit)   
############   
  
k <- k+1   
  
# Update ET  
ET <- mu\_i\_F(par[1],par[2])+ par[3]\*HF(par[1],par[2],par[3])  
  
# Update Q  
Q[k]<- Q\_theta(par)  
  
# Update THETA  
 THETA[k,1] <- par[1]  
 THETA[k,2] <- par[2]  
 THETA[k,3] <- par[3]  
 THETA[k,4:7]<-unique(ET)   
 THETA[k,8] <- k  
  
# Update Y  
Y[k,] <- y   
  
# par=c(beta0,beta1,sigma)  
 if(abs(Q[k]-Q[k-1])<=delta) break  
}

## EM figure



## The results of THETA

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Intercept | Slope | Sigma | mu150 | mu170 | mu190 | mu220 | Iteration |
| -4.931 | 3.747 | 0.1572 | 4.038 | 3.809 | 3.329 | 2.83 | 1 |
| -5.26 | 3.926 | 0.1799 | 4.099 | 3.84 | 3.366 | 2.858 | 2 |
| -5.486 | 4.04 | 0.1911 | 4.131 | 3.856 | 3.382 | 2.869 | 3 |
| -5.623 | 4.108 | 0.1969 | 4.149 | 3.865 | 3.39 | 2.874 | 4 |
| -5.704 | 4.148 | 0.2001 | 4.159 | 3.87 | 3.395 | 2.877 | 5 |
| -5.752 | 4.172 | 0.2019 | 4.165 | 3.873 | 3.397 | 2.878 | 6 |
| -5.779 | 4.185 | 0.2029 | 4.168 | 3.874 | 3.399 | 2.879 | 7 |
| -5.795 | 4.193 | 0.2035 | 4.17 | 3.875 | 3.4 | 2.879 | 8 |
| -5.805 | 4.198 | 0.2039 | 4.171 | 3.876 | 3.4 | 2.879 | 9 |
| -5.81 | 4.2 | 0.2041 | 4.172 | 3.876 | 3.401 | 2.88 | 10 |
| -5.814 | 4.202 | 0.2042 | 4.172 | 3.877 | 3.401 | 2.88 | 11 |
| -5.815 | 4.203 | 0.2042 | 4.173 | 3.877 | 3.401 | 2.88 | 12 |
| -5.817 | 4.203 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 13 |
| -5.817 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 14 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 15 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 16 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 17 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 18 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 19 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 20 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 21 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 22 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 23 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 24 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 25 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 26 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 27 |
| -5.818 | 4.204 | 0.2043 | 4.173 | 3.877 | 3.401 | 2.88 | 28 |
| NA | NA | NA | NA | NA | NA | NA | NA |
| NA | NA | NA | NA | NA | NA | NA | NA |

## The results of Q

*20.08*, *21.4*, *20.64*, *20.09*, *19.75*, *19.55*, *19.43*, *19.36*, *19.32*, *19.29*, *19.28*, *19.27*, *19.27*, *19.26*, *19.26*, *19.26*, *19.26*, *19.26*, *19.26*, *19.26*, *19.26*, *19.26*, *19.26*, *19.26*, *19.26*, *19.26*, *19.26* and *19.26*

## The results of Y

Table continues below

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 3.907 | 3.907 | 3.907 | 3.907 | 3.907 | 3.907 | 3.907 | 3.907 | 3.907 | 3.907 |
| 4.038 | 4.038 | 4.038 | 4.038 | 4.038 | 4.038 | 4.038 | 4.038 | 4.038 | 4.038 |
| 4.099 | 4.099 | 4.099 | 4.099 | 4.099 | 4.099 | 4.099 | 4.099 | 4.099 | 4.099 |
| 4.131 | 4.131 | 4.131 | 4.131 | 4.131 | 4.131 | 4.131 | 4.131 | 4.131 | 4.131 |
| 4.149 | 4.149 | 4.149 | 4.149 | 4.149 | 4.149 | 4.149 | 4.149 | 4.149 | 4.149 |
| 4.159 | 4.159 | 4.159 | 4.159 | 4.159 | 4.159 | 4.159 | 4.159 | 4.159 | 4.159 |
| 4.165 | 4.165 | 4.165 | 4.165 | 4.165 | 4.165 | 4.165 | 4.165 | 4.165 | 4.165 |
| 4.168 | 4.168 | 4.168 | 4.168 | 4.168 | 4.168 | 4.168 | 4.168 | 4.168 | 4.168 |
| 4.17 | 4.17 | 4.17 | 4.17 | 4.17 | 4.17 | 4.17 | 4.17 | 4.17 | 4.17 |
| 4.171 | 4.171 | 4.171 | 4.171 | 4.171 | 4.171 | 4.171 | 4.171 | 4.171 | 4.171 |
| 4.172 | 4.172 | 4.172 | 4.172 | 4.172 | 4.172 | 4.172 | 4.172 | 4.172 | 4.172 |
| 4.172 | 4.172 | 4.172 | 4.172 | 4.172 | 4.172 | 4.172 | 4.172 | 4.172 | 4.172 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 | 4.173 |
| NA | NA | NA | NA | NA | NA | NA | NA | NA | NA |
| NA | NA | NA | NA | NA | NA | NA | NA | NA | NA |

Table continues below

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.736 | 3.736 | 3.736 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.809 | 3.809 | 3.809 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.84 | 3.84 | 3.84 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.856 | 3.856 | 3.856 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.865 | 3.865 | 3.865 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.87 | 3.87 | 3.87 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.873 | 3.873 | 3.873 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.874 | 3.874 | 3.874 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.875 | 3.875 | 3.875 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.876 | 3.876 | 3.876 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.876 | 3.876 | 3.876 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| 3.246 | 3.443 | 3.537 | 3.549 | 3.577 | 3.687 | 3.716 | 3.877 | 3.877 | 3.877 |
| NA | NA | NA | NA | NA | NA | NA | NA | NA | NA |
| NA | NA | NA | NA | NA | NA | NA | NA | NA | NA |

Table continues below

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.225 | 3.225 | 3.225 | 3.225 | 3.225 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.329 | 3.329 | 3.329 | 3.329 | 3.329 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.366 | 3.366 | 3.366 | 3.366 | 3.366 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.382 | 3.382 | 3.382 | 3.382 | 3.382 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.39 | 3.39 | 3.39 | 3.39 | 3.39 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.395 | 3.395 | 3.395 | 3.395 | 3.395 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.397 | 3.397 | 3.397 | 3.397 | 3.397 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.399 | 3.399 | 3.399 | 3.399 | 3.399 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.4 | 3.4 | 3.4 | 3.4 | 3.4 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.4 | 3.4 | 3.4 | 3.4 | 3.4 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| 2.611 | 2.611 | 3.128 | 3.128 | 3.158 | 3.401 | 3.401 | 3.401 | 3.401 | 3.401 |
| NA | NA | NA | NA | NA | NA | NA | NA | NA | NA |
| NA | NA | NA | NA | NA | NA | NA | NA | NA | NA |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.723 | 2.723 | 2.723 | 2.723 | 2.723 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.83 | 2.83 | 2.83 | 2.83 | 2.83 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.858 | 2.858 | 2.858 | 2.858 | 2.858 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.869 | 2.869 | 2.869 | 2.869 | 2.869 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.874 | 2.874 | 2.874 | 2.874 | 2.874 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.877 | 2.877 | 2.877 | 2.877 | 2.877 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.878 | 2.878 | 2.878 | 2.878 | 2.878 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.879 | 2.879 | 2.879 | 2.879 | 2.879 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.879 | 2.879 | 2.879 | 2.879 | 2.879 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.879 | 2.879 | 2.879 | 2.879 | 2.879 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| 2.611 | 2.611 | 2.702 | 2.702 | 2.702 | 2.88 | 2.88 | 2.88 | 2.88 | 2.88 |
| NA | NA | NA | NA | NA | NA | NA | NA | NA | NA |
| NA | NA | NA | NA | NA | NA | NA | NA | NA | NA |

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