# Definitions

1. A **triangulated graph** is a planar graph such that any edge is in a face and any face is a triangle.
2. A **chordal graph** is a graph such that every cycle of length at least $4$ contains a **chord**, i.e., an edge connecting two nonadjacent vertices of the cycle.
3. Is a triangulated graph a chordal graph?
4. A **weakly pancyclic graph** is a graph which contains cycles of every length between the girth and the circumference.
5. A graph is ***t*-tough** if for every integer $k > 1$, the graph cannot be split into $k$ components by removal of fewer than $tk$ vertices
6. Hence a $t$-tough graph with $t > 0$ is connected.

# Conjectures

1. A triangulated graph is weakly pancyclic.
2. A chordal graph is weakly pancyclic.
3. There is a $9$-tough triangulated non-Hamiltonian graph.
4. A $9.1$-tough triangulated graph is Hamiltonian.

[1] Adam Kabela, Tomáš Kaiser, $10$-tough chordal graphs are Hamiltonian, Journal of Combinatorial Theory, Series B, Volume 122, January 2017, Pages 417-427.

# Locally property $P$

Let $P$ be a property on graphs. A graph $G$ has **local** $P$ if $G_1(v)$ has property $P$ for every vertex $v$ in $G$.

Hence a triangulated graph is locally connected.

# Lemma

If $G$ is $1$-tough and locally $1$-tough then $G - u$ is $1$-tough for every vertex $u$ in $G$.

Proof.

Assume that $G - u$ is split into $k$ components by removal fewer than $k$ vertices, and $t$ of these components intersect $G_1(u)$. Then among the $k$ removal vertices there are at least $t$ from $G_1(u)$ and the remaining at most $k - t$ from $G - u - G_1(u)$ since $G_1(u)$ is $1$-tough. Then $G$ is split into $1 + (k - t)$ components by by removal fewer than $1 + k - t$ vertices in $G - G_1(u)$, a contradiction to the $1$-tough assumption of $G$. □

# More conjectures

1. If $G$ is $1$-tough and locally $1$-tough then $G - u$ is locally $1$-tough for every vertex $u$ in $G$.
2. A $1$-tough and locally $1$-tough graph is weakly pancyclic.
3. A $1$-tough and locally $1$-tough graph is Hamiltonian.