Reliability Polynomials

Reliability Polynomials

Let G be a connected graph with m edges. Let $Re(G, p)$ denote the probability that G remains connected if each edge of G is independently removed with probability $1 - p$.

Since each subgraph G' of G with m' edges has probability $p^{m'}(1-p)^{m-m'}$ to appear,

$$Re(G, p) = \sum_{m' \geq 0} \sum_{G'} p^{m'}(1-p)^{m-m'},$$

where the second sum is over all spanning subgraphs G' of G with m' edges.

Example

$$Re(C_n, p) = np^{n-1}(1-p) + p^n = (1-n)p^n + np^{n-1}$$
$$Re(P_n, p) = p^{n-1}.$$

Uniformly most reliable graphs

A connected graph G is called **uniformly most reliable** if

$$Re(G, p) \geq Re(H, p)$$

for all $p \in [0, 1]$ and all connected graphs H with the same order and same size of G.

Example

C_n is a uniformly most reliable graph from the definition.

Conjectures

1. If G is a uniformly most reliable graph of order n and size m with $2m$ a multiple of n then G is regular.
2. Taking absolute values of the coefficients of polynomial $Re(G, p)$ in p form a unimodal sequence.
3. If G is obtained from H by adding an edge, then the zeros of $Re(H, p)$ interlace the zeros of $Re(G, p)$.