Definitions

1. A **triangulated graph** is a planar graph such that any edge is in a face and any face is a triangle.
2. A **chordal graph** is a graph such that every cycle of length at least 4 contains a **chord**, i.e., an edge connecting two nonadjacent vertices of the cycle.
3. Is a triangulated graph a chordal graph?
4. A **weakly pancyclic graph** is a graph which contains cycles of every length between the girth and the circumference.
5. A graph is ***t*-tough** if for every integer $k > 1$, the graph cannot be split into k components by removal of fewer than tk vertices
6. Hence a t-tough graph with $t > 0$ is connected.

Conjectures

1. A triangulated graph is weakly pancyclic.
2. A chordal graph is weakly pancyclic.
3. There is a 9-tough triangulated non-Hamiltonian graph.
4. A 9.1-tough triangulated graph is Hamiltonian.

[1] Adam Kabela, Tomáš Kaiser, 10-tough chordal graphs are Hamiltonian, Journal of Combinatorial Theory, Series B, Volume 122, January 2017, Pages 417-427.

Locally property P

Let P be a property on graphs. A graph G has **local P** if $G_1(v)$ has property P for every vertex v in G.

Hence a triangulated graph is locally connected.

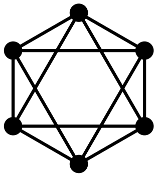
Lemma

If G is 1-tough and locally 1-tough then $G - u$ is 1-tough for every vertex u in G.

Proof.

Assume that $G - u$ is split into k components by removal fewer than k vertices, and t of these components intersect $G_1(u)$. Then among the k removal vertices there are at least t from $G_1(u)$ and the remaining at most $k - t$ from $G - u - G_1(u)$ since $G_1(u)$ is 1-tough. Then G is split into $1 + (k - t)$ components by by removal fewer than $1 + k - t$ vertices in $G - G_1(u)$, a contradiction to the 1-tough assumption of G. □

A 1-tough and locally 1-tough graph

$G:$ 

More conjectures

1. A 1-tough and locally 1-tough graph is weakly pancyclic.
2. A 1-tough and locally 1-tough graph is Hamiltonian.