Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India

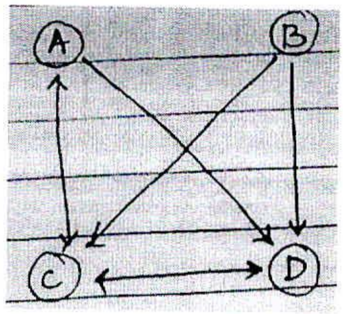
(Autonomous College Affiliated to University of Mumbai)

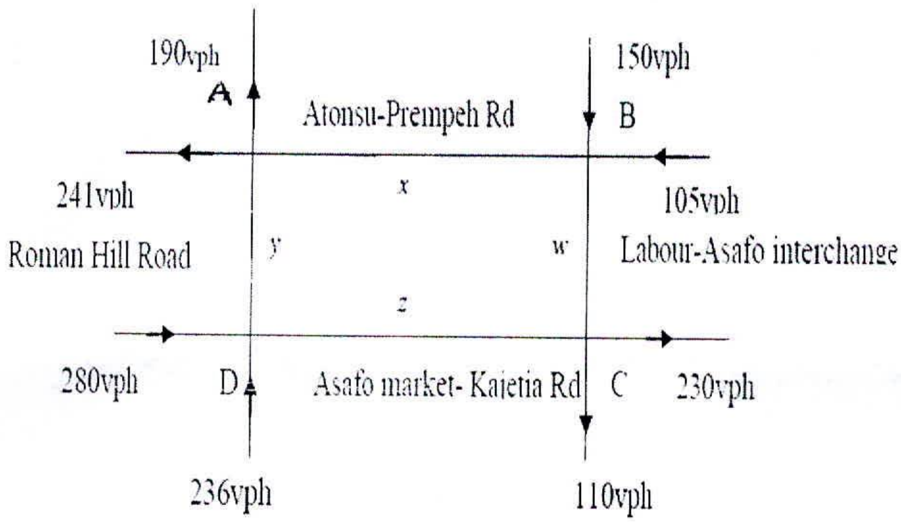
End Semester Examination-May 2023

Max. Marks: - 100	**Duration: 3 Hours**
Class: S.E	**Semester: IV**
Course Code: - MA201	**Branch: - COMPS, AIML, DS**
Name of the Course: Linear Algebra	

Instructions:
1) **All Questions are Compulsory.**
2) **Assume suitable data if necessary.**

Q No.		Max. Mks	CO	BL
Q.1	a) If V is the vector space and inner product for the vector space is defined by. $$< u, v > = u.v = \int_{-1}^{1} u(x).v(x)dx$$ Apply Gram-Schmidt orthogonalisation process to transform the standard basis $S = \{1, x, x^2\}$ into an orthonormal basis.	8	4	3
	b) State only one axiom that fails to hold for each of the following sets W to be subspaces of the respective real vector spaces V with Standard operations. 1) $W = \{(x, y) \mid x^2 = y^2\}$, $V = R^2$ 2) $W = \{(x, y) \mid x\, y \geq 0\}$, $V = R^2$ 3) $W = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$, $V = R^3$ 4) $W = \{f \mid f(x) \leq 0 \text{ for all } x\}$, $V = F(-\infty, \infty)$	8	4	2
	c) Verify whether (1, 2), (3, 6) are linearly independent when placed with Initial points at the origin by analytical and geometrical method.	6	4	2
	OR			
	c) Show that the vectors v_1, v_2, v_3 are linearly independent and when combined with v_4 becomes dependent where $v_1 = (1, 2, 4)$, $v_2 = (2, -1, 3)$, $v_3 = (0, 1, 2)$, $v_4 = (-3, 7, 2)$.	6	4	2
	d) Find the basis of row space, column space and null space for the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$	7	4	1
	e) Find the least square solution of AX= B where $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$	5	4	2

Q.2	a) Find the highest Page Rank from the given directed graph.  **OR**	8	6	3
	a) Solve the following system of differential equation $y' = \begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix} y$ using diagonalisation with initial conditions $y_1(0)$ =3 and $y_2(0)$ = - 2.	8	6	3
	b) If A = $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ find e^{At}	7	6	2
	c) Find the singular value decomposition of the matrix A = $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$	8	6	3
	d) i) If λ *is an eigen value of a non singular matrix A then prove that* $\frac{\lvert A \rvert}{\lambda}$ is an eigen value of adj.A.	3	6	1
Q.3	a) Apply Crout's Method to solve the following equations $x + y + 4z = 43$ $2x + 10y + z = 63$ $25x + 2y + z = 69$	8	2	3
	b) Show that the matrix A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalisable. Find the diagonal matrix and the transforming matrix.	8	5	3
Q.4	a) Given the Hill 2-cipher key A = $\begin{bmatrix} 1 & 1 \\ 2 & 6 \end{bmatrix}$ Compute A^{-1} *modulo* 27 and hence decode the following message X , N , U , F , Y , V , C , R , S , Q , E , J	8	3	3

Q.5	a) Show that the following system of equations is consistent if a, b, c are in arithmetic progression. $3x + 4y + 5z = a$, $4x + 5y + 6z = b$, $5x + 6y + 7z = c$ Find the solution when $a = 1$, $b = 2$, $c = 3$.	8	1	3
	OR			
	a) Investigate for what values of λ and μ, the equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have i) no solution ii) a unique solution iii) infinite number of solutions.	8	1	3
	b) The diagram in the Figure below describes the four one-way streets in Kumasi (vph is number of vehicles per hour). Determine the amount of traffic between each of four intersections. 	8	1	3

*********** All the Best **********