

Embracing the Chaos

Sensitivity Analysis on Chaotic Dynamical Systems by NILSS

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June 20, 2018

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Outline

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Chaos as a way of living

Predictability in chaos?

So what about butterflies?
I have seen this before

Oh no... Nevermind Should we give up?

Non-Intrusive Least Squares Shadowing

In average I'm not that sensitive

two

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Should the world behave nicely?

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"Chaos was the law of nature; Order was the dream of man."

— Henry Adams



Source https://pbs.twimg.com/media/C75sWjvW0AA8Mfc.jpg

Getting Closer

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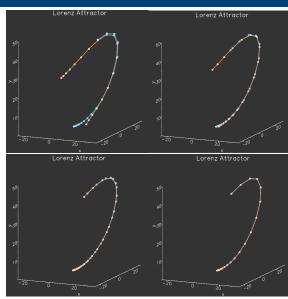
sensitive

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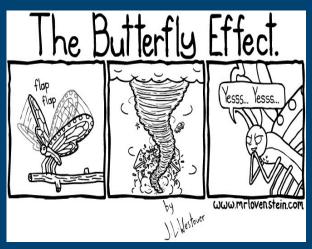
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Source http://www.mrlovenstein.com/comic/50

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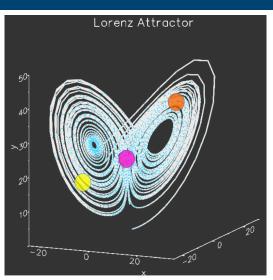
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Three highlighted zones zones

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There is hope after all

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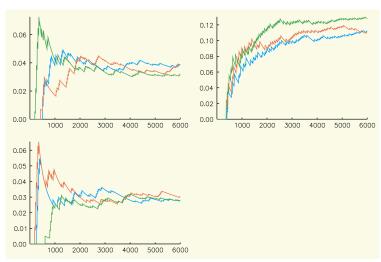
sensitive

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Time spent on average around this zones



Source https://www.onlinecollegecourses.com/2012/06/21/why-optimism-matters-for-student-success-now-and-after-graduation-2/

Dynamical systems and sensitivities

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The governing equation of a dynamical system is

$$\frac{du}{dt}=f(u,s),\quad u(t=0)=u_0,$$

We want to analyze the changes of a long-time averaged quantity represented by J(u, s).

$$\langle J \rangle_{\infty} := \lim_{t \to \infty} \frac{1}{T} \int_{0}^{T} J(u, s) dt.$$
 (2)

It doesn't look that hard

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RWITHAACHEN UNIVERSITY We want to calculate $\frac{d}{ds}\langle J\rangle_{\infty}$ the problem is...

It doesn't look that hard

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We want to calculate $\frac{d}{ds}\langle J\rangle_{\infty}$ the problem is...

$$\frac{d}{ds}\langle J\rangle_{\infty} \neq \lim_{T \to \infty} \frac{\partial}{\partial s} \langle J\rangle_{T}(s, \phi, T). \tag{3}$$



It doesn't look that hard

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$$\frac{d}{ds}\langle J\rangle_{\infty} \neq \lim_{T \to \infty} \frac{\partial}{\partial s} \langle J\rangle_{T}(s, \phi, T). \tag{3}$$

The usual methods diverge most of the time, e.g. the transient method.

More to know

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