Embracing the Chaos

Sensitivity Analysis on Chaotic Dynamical Systems by NILSS

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Outline

Should the world behave nicely?

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"Chaos was the law of nature; Order was the dream of man."
— Henry Adams

Source https://pbs.twimg.com/media/C75sWjvW0AA8Mfc.jpg

The Lorenz equations

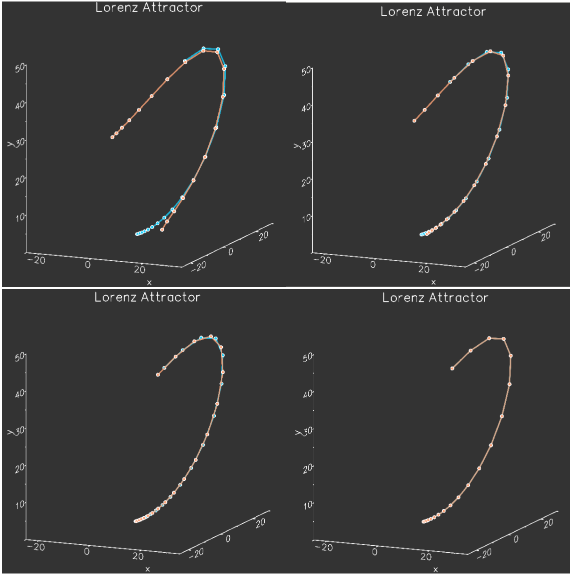
The Lorenz 63 system is a 3 variable system with three parameters σ, ρ, and β.

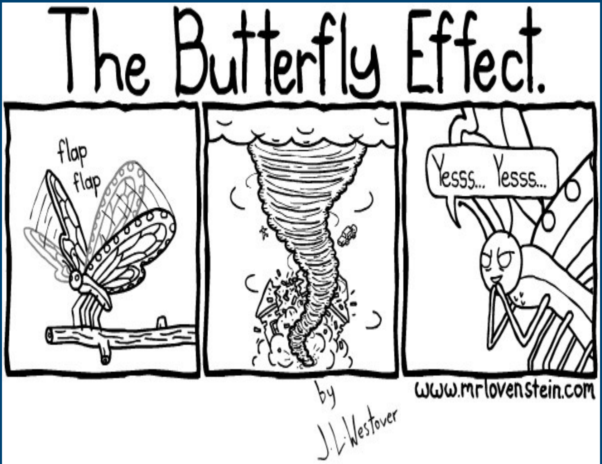
$$\frac{dx}{dt} = \sigma(y - x) \tag{1}$$

$$\frac{dy}{dt} = x(\rho - z) - y \tag{2}$$

$$\frac{dz}{dt} = xy - \beta z \tag{3}$$

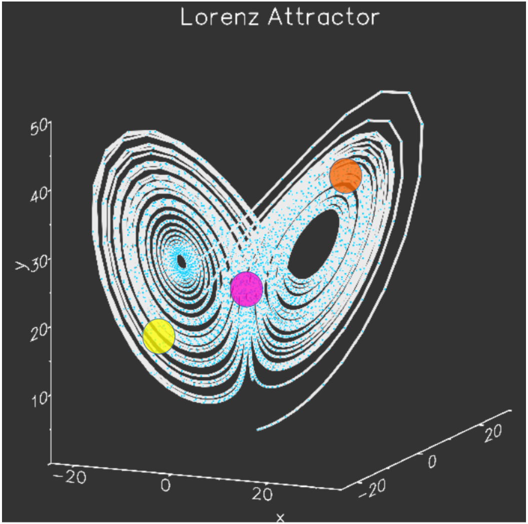
Getting Closer





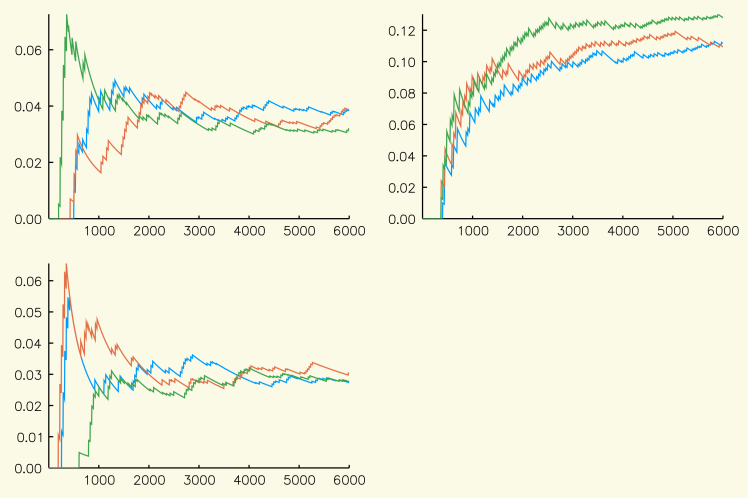
Source http://www.mrlovenstein.com/comic/50

Let's Focus



Three highlighted zones

There is hope after all



Time spent on average around this zones

Source https://www.onlinecollegecourses.com/2012/06/21/why-optimism-matters-for-student-success-now-and-after-graduation-2/

The governing equation of a dynamical system is

$$\frac{du}{dt} = f(u, s), \quad u(t = 0) = u_0, \tag{4}$$

We want to analyze the changes of a long-time averaged quantity represented by $J(u, s)$.

$$\langle J \rangle_\infty := \lim_{t \to \infty} \frac{1}{T} \int_0^T J(u, s) dt. \tag{5}$$

It doesn't look that hard

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$$\frac{d}{ds}\langle J\rangle_\infty \neq \lim_{T\to\infty} \frac{\partial}{\partial s}\langle J\rangle_T(s, \phi, T). \tag{6}$$

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The usual methods diverge most of the time, sometimes they exceed by 10^{100} the expected value.[1]

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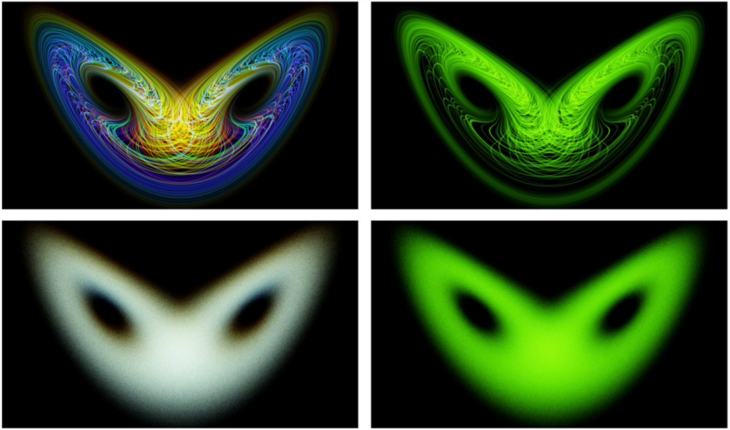
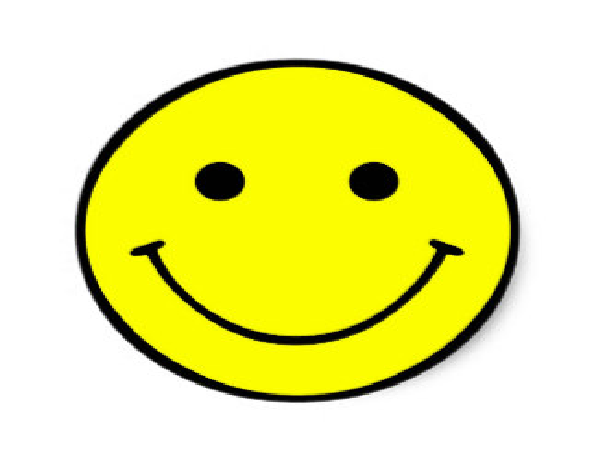
Source http://knowyourmeme.com/

Vary initial conditions vs time evolution



Ni, A., Wang, Q., (2017), *Sensitivity analysis on chaotic dynamical systems by Non-Intrusive Least Squares Shadowing (NILSS)* , Journal of Computational Physics, **347**, 56-77.



Source http://www.insurancechat.co.za/2017-09/could-sending-a-smiley-face-get-me-into-legal-hot-water/

Subtract instabilities

- We are looking to build trajectories with parameters ρ and $\rho + \delta\rho$ such that they don't diverge from each other.

Subtract instabilities

- We are looking to build trajectories with parameters ρ and $\rho + \delta\rho$ such that they don't diverge from each other.
- Their difference contains only the long-time effect.
- Therefore we can reveal the long time effect with shorter trajectories.

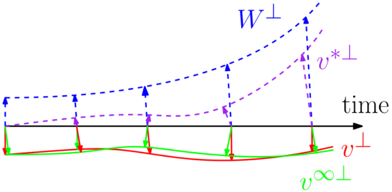
Decomposing our equation

To detect variations due changes on the parameters we differentiate the governing equation and decouple into homogeneous and inhomogeneous part

$$\frac{dv^*}{dt} - \partial_u f v^* = \partial_s f, \tag{7}$$

$$\frac{dw}{dt} - \partial_u f w = 0. \tag{8}$$

Idea



Now we only need to solve a minimization problem.

$$v^\perp = v^{*\perp} + W^\perp a \approx v^{\infty\perp}, \tag{9}$$

$$\min_a \frac{1}{2} \int_0^T (v^{*\perp} + W^\perp a)^T (v^{*\perp} + W^\perp a) \tag{10}$$

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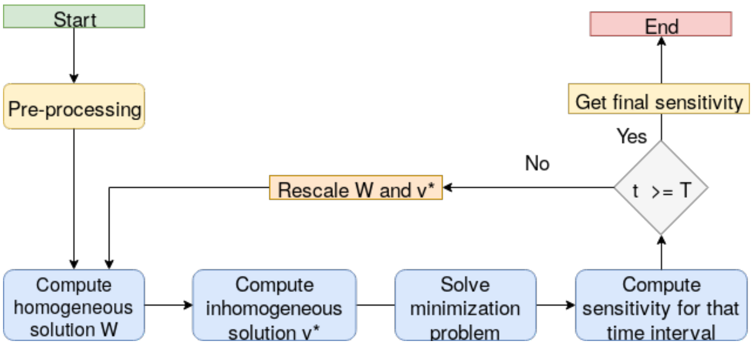
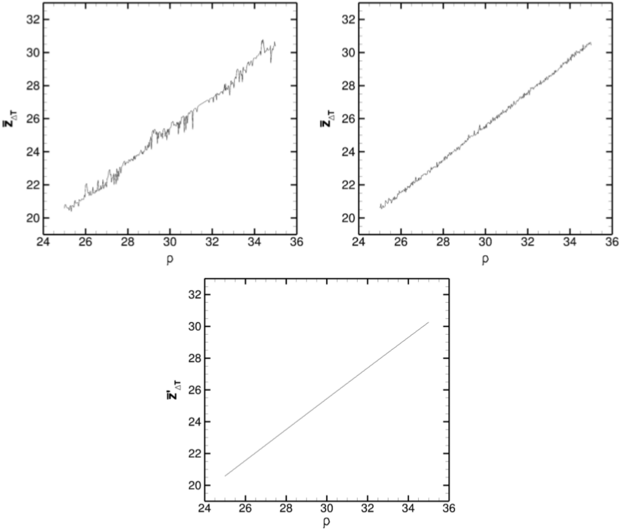
Flowchart


Start
Pre-processing
Compute homogeneous solution W
Compute inhomogeneous solution v*
Solve minimization problem
Compute sensitivity for that time interval
t >= T
No
Rescale W and v*
Yes
Get final sensitivity
End

Source https://www.pinterest.de/pin/128211920620408724

Lorenz attractor

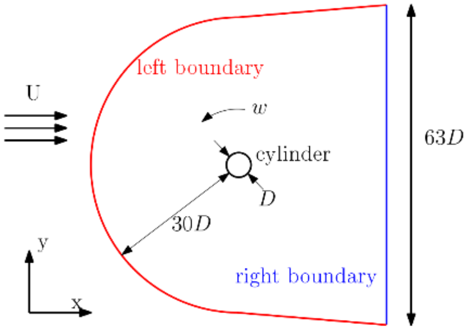
Introduction
Chaos as a way of living
Predictability in chaos?
So what about butterflies?
I have seen this before
Oh no... Nevermind
Should we give up?
Non-Intrusive Least Squares Shadowing
We have to be careful
Exploit similarities
Algorithm
Examples
Why should I care?
References



Patrick J. Blonigan, Qiqi Wang, Eric J. Nielsen, and Boris Diskin. Least-Squares Shadowing Sensitivity Analysis of Chaotic Flow Around a Two-Dimensional Airfoil, AIAA Journal, Vol. 56, No. 2 (2018), pp. 658-672.

One more example

3D Flow past a cylinder



Cylinder can rotate

Angxiu Ni, Qiqi Wang, Pablo Fernandez, Chaitanya Talnikar. Sensitivity analysis on chaotic dynamical systems by Finite Difference Non-Intrusive Least Squares Shadowing (FD-NILSS). arXiv:1711.06633 [physics.comp-ph]

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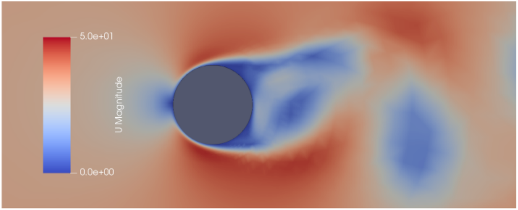

Snapshot of the flow

Angxiu Ni, Qiqi Wang, Pablo Fernandez, Chaitanya Talnikar. Sensitivity analysis on chaotic dynamical systems by Finite Difference Non-Intrusive Least Squares Shadowing (FD-NILSS). arXiv:1711.06633 [physics.comp-ph]

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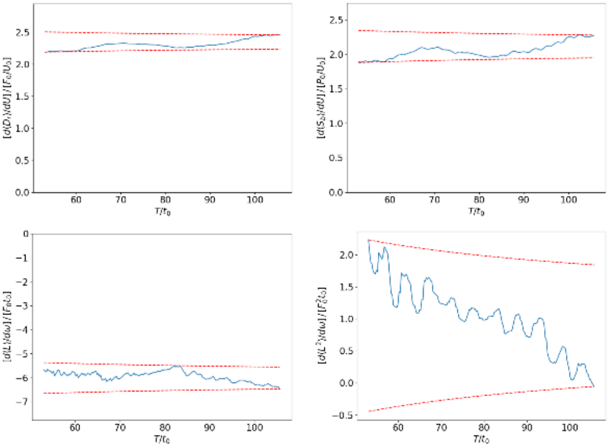

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Advantages (self proclaimed)

- Easy to implement if you already have a solver.
- Low cost in comparison to other methods.

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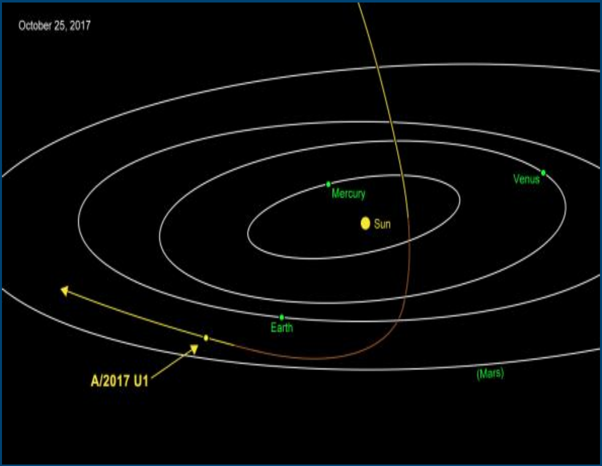
- Easy to implement if you already have a solver.
- Low cost in comparison to other methods.
- Therefore faster.
- Uses less memory.

Where are sensitivities used?

Sensitivities help us to

- Control processes and systems.
- Solve inverse problems (e.g. CAT scan images).
- Estimate simulation errors.
- Quantify uncertainties.



Source https://news.nationalgeographic.com/

More to know

1. Ni A., Wang Q., (2017), *Sensitivity analysis on chaotic dynamical systems by Non-Intrusive Least Squares Shadowing (NILSS)* , Journal of Computational Physics, **347**, 56-77.
2. Ni A., Wang Q., Fernandez P., and Talnikar C., *Sensitivity analysis on chaotic dynamical systems by Finite Difference Non-Intrusive Least Squares Shadowing (FD-NILSS)*, `arXiv:1711.06633`
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4. Strogatz Steven H., (2015). *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering.* Boulder, CO: Westview Press.
5. Gleick J. (1988). *Chaos: Making a new science*. New York, N.Y., U.S.A: Penguin.
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Source http://www.sednacomics.com/