Embracing the Chaos

Sensitivity Analysis on Chaotic Dynamical Systems by NILSS

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Outline

Should the world behave nicely?

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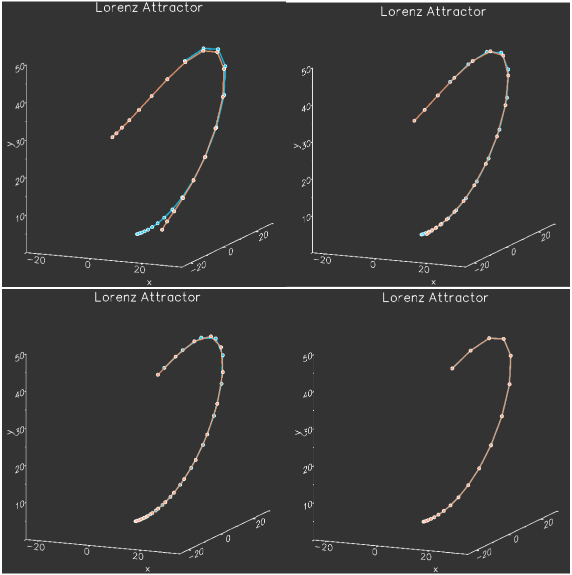
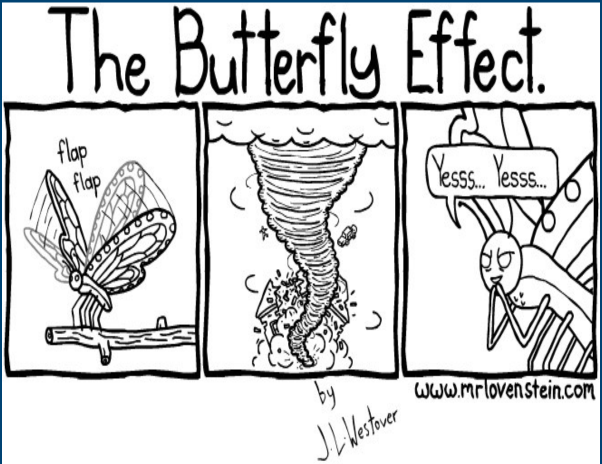
"Chaos was the law of nature; Order was the dream of man."
— Henry Adams

Source https://pbs.twimg.com/media/C75sWjvW0AA8Mfc.jpg

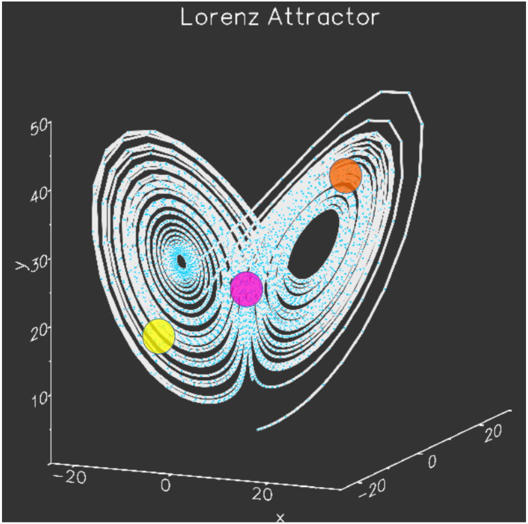
Getting Closer





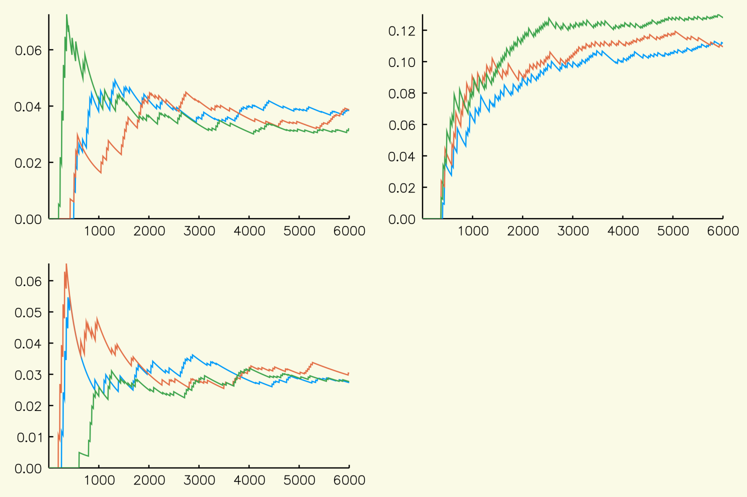
Source http://www.mrlovenstein.com/comic/50

Let's Focus



Three highlighted zones zones

There is hope after all



Time spent on average around this zones

Source https://www.onlinecollegecourses.com/2012/06/21/why-optimism-matters-for-student-success-now-and-after-graduation-2/

Dynamical systems and sensitivities

The governing equation of a dynamical system is

$$\frac{du}{dt} = f(u, s), \quad u(t = 0) = u_0, \tag{1}$$

We want to analyze the changes of a long-time averaged quantity represented by $J(u, s)$.

$$\langle J \rangle_\infty := \lim_{t \to \infty} \frac{1}{T} \int_0^T J(u, s) dt. \tag{2}$$

It doesn't look that hard

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The usual methods diverge most of the time, e.g. the transient method.

More to know

1. Ni A., Wang Q., (2017), *Sensitivity analysis on chaotic dynamical systems by Non-Intrusive Least Squares Shadowing (NILSS)* , Journal of Computational Physics, **347**, 56-77.
2. Ni, A., Wang, Q., Fernandez, P., and Talnikar, C., *Sensitivity analysis on chaotic dynamical systems by Finite Difference Non-Intrusive Least Squares Shadowing (FD-NILSS)*, `arXiv:1711.06633`
3. Safiran N., Lotz J., Naumann U., (2016), *Algorithmic Differentiation of Numerical Methods:Tangent and Adjoint Solvers for Parameterized Systems of Nonlinear Equations*, Procedia Computer Science, **80**, 2231-2235.
4. Strogatz, Steven H., (2015). *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering.* Boulder, CO: Westview Press.
5. Gleick, J. (1988). *Chaos: Making a new science*. New York, N.Y., U.S.A: Penguin.
6. Alvarez A., Ghys É., and Leys J., *Chaos a Mathematical Adventure*, `http://www.chaos-math.org/en`