# Embracing the Chaos

## Sensitivity Analysis on Chaotic Dynamical Systems by NILSS

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# Outline





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# Should the world behave nicely?



*"Chaos was the law of nature; Order was the dream of man."*
*— Henry Adams*

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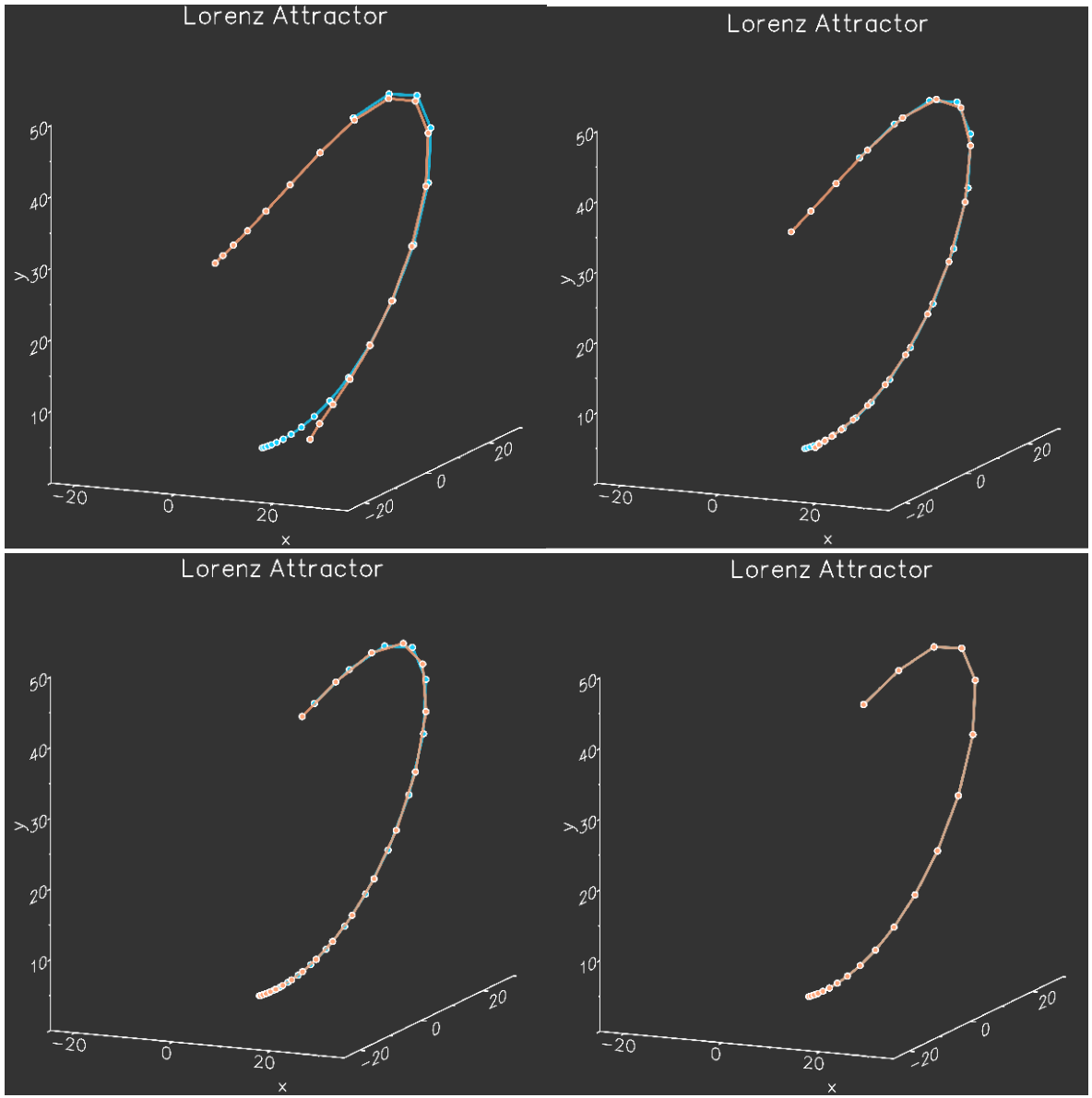
Source https://pbs.twimg.com/media/C75sWjvW0AA8Mfc.jpg

# Getting Closer



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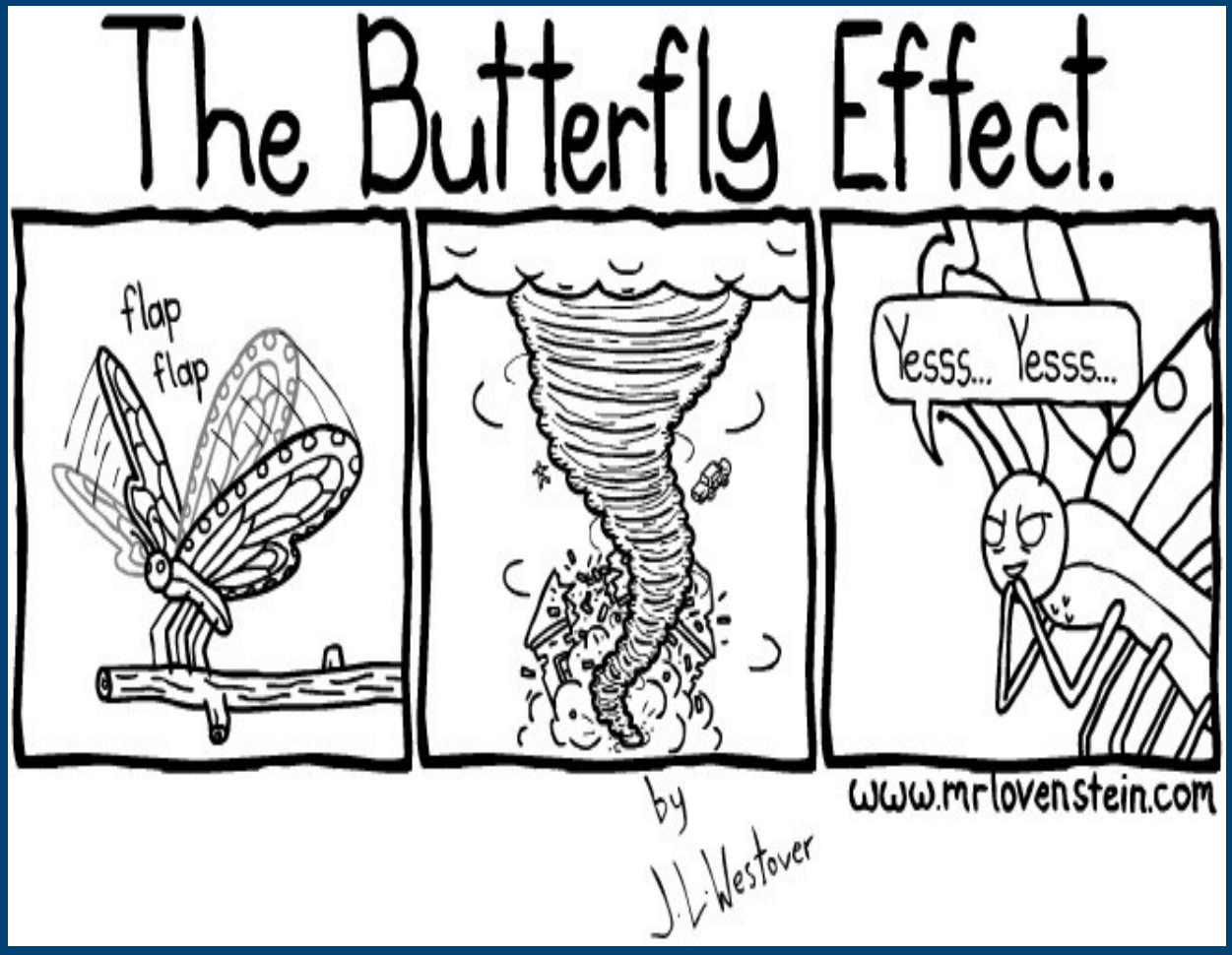
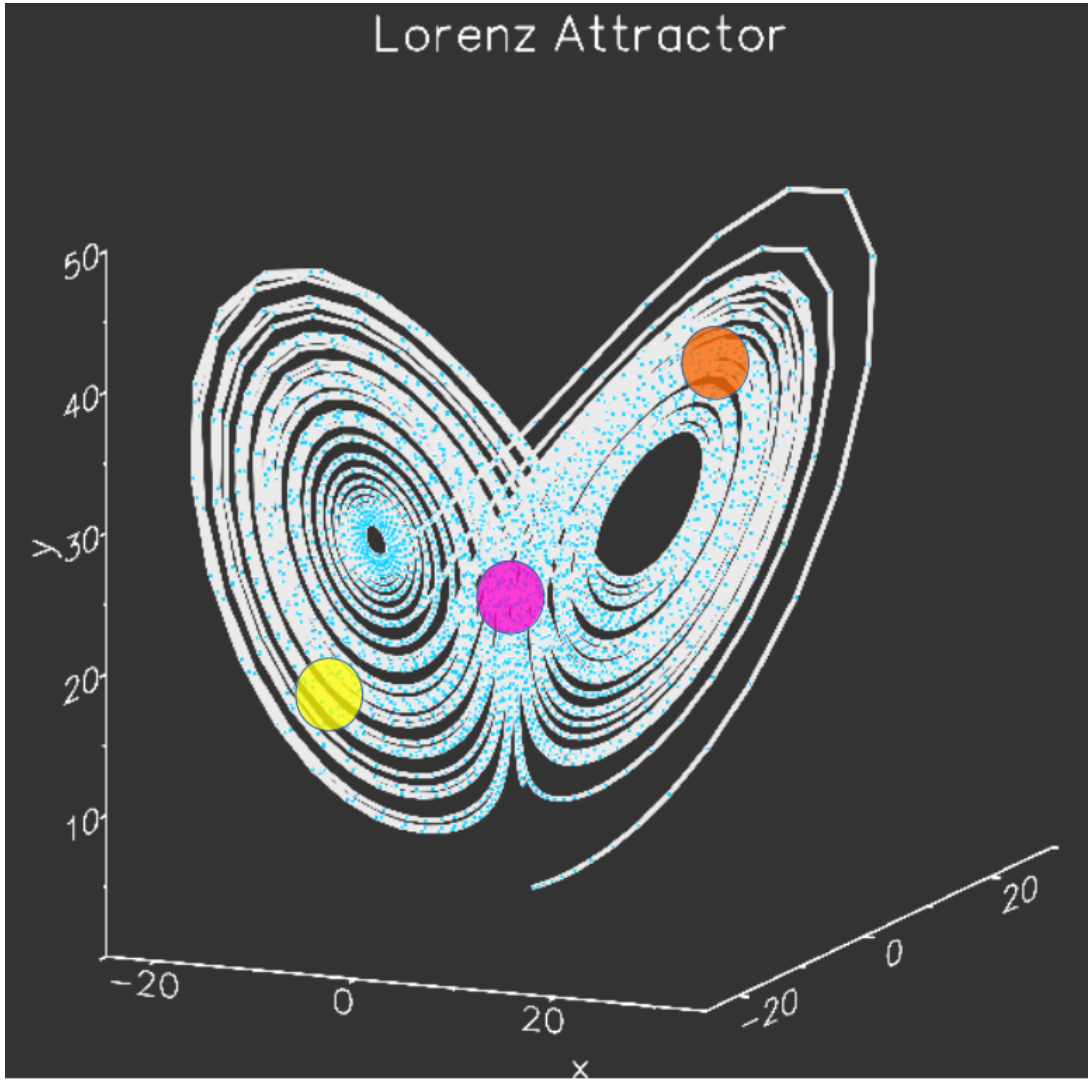



Source http://www.mrlovenstein.com/comic/50

# Let's Focus

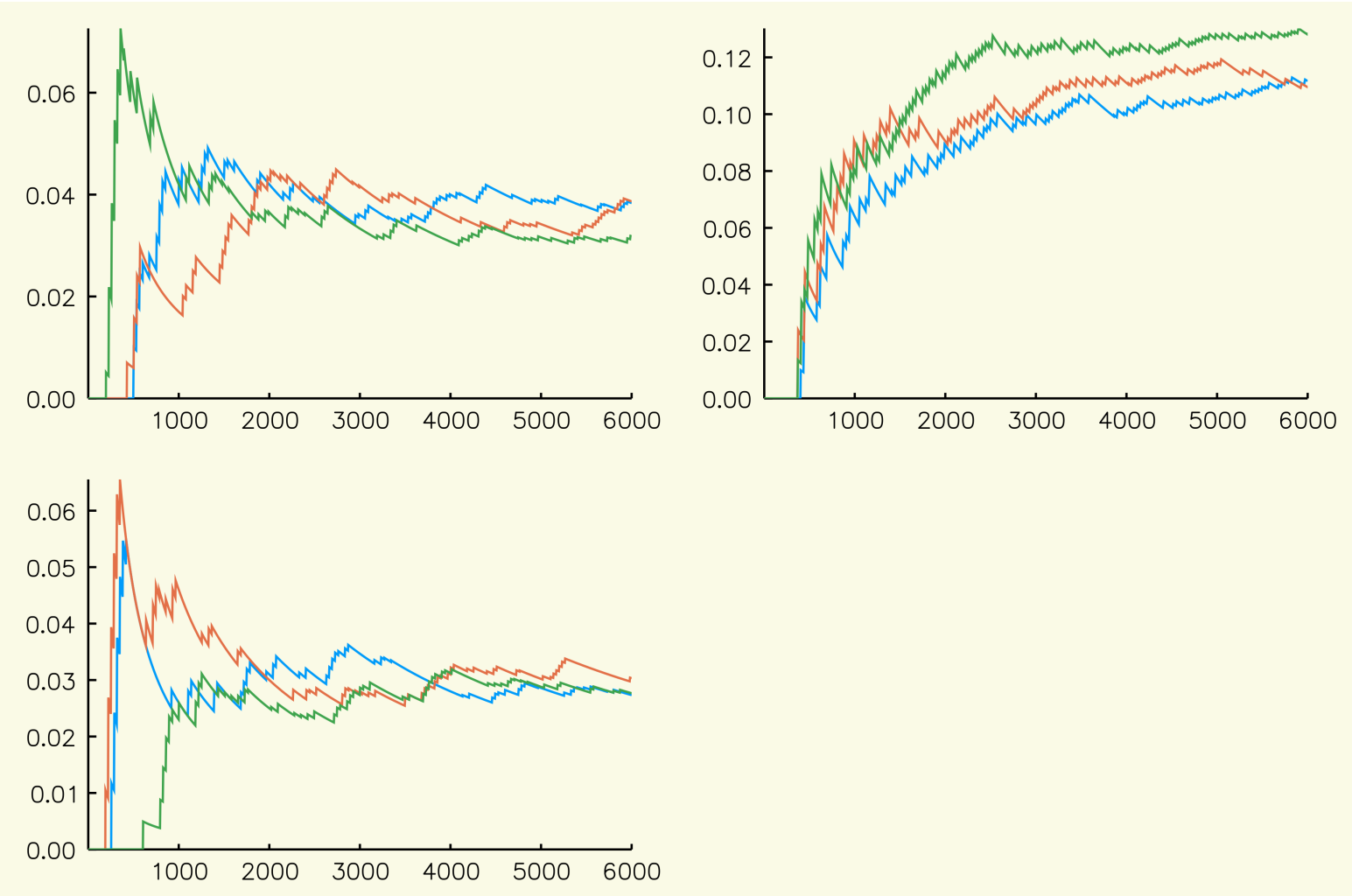





Three highlighted zones

# There is hope after all






Time spent on average around this zones

Source https://www.onlinecollegecourses.com/2012/06/21/why-optimism-matters-for-student-success-now-and-after-graduation-2/

# Dynamical systems and sensitivities



The governing equation of a dynamical system is

$$\frac{du}{dt} = f(u, s), \quad u(t = 0) = u_0, \tag{1}$$

We want to analyze the changes of a long-time averaged quantity represented by $J(u, s)$.

$$\langle J \rangle_\infty := \lim_{t \to \infty} \frac{1}{T} \int_0^T J(u, s) dt. \tag{2}$$

# It doesn't look that hard



We want to calculate $\frac{d}{ds}\langle J \rangle_\infty$ the problem is...

[1] This competes hand in hand with string theory for the prize for worst predicted results of all time.

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The usual methods diverge most of the time, sometimes they exceed by $10^{100}$ the expected value.[1]

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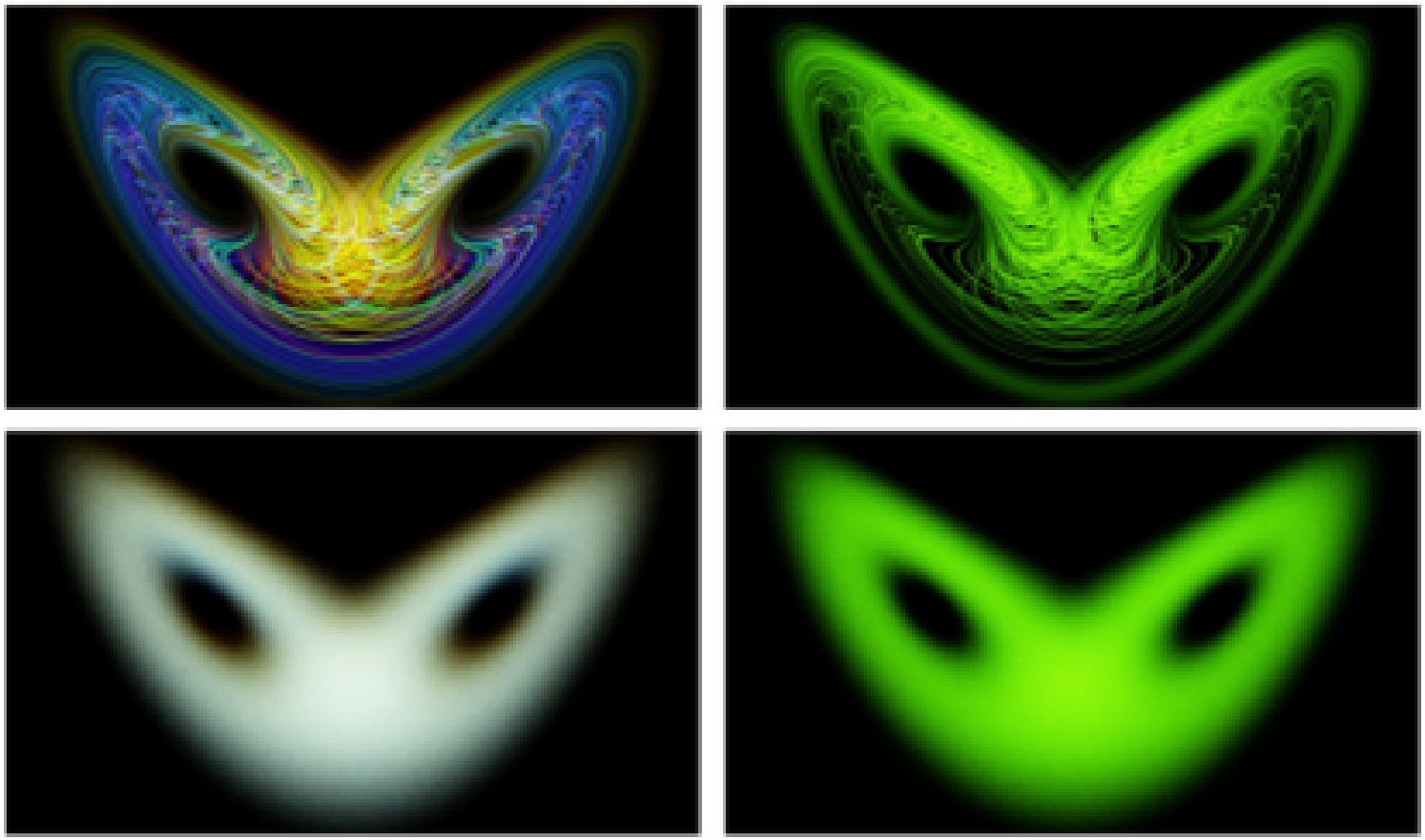
Source http://knowyourmeme.com/

# Vary initial conditions vs time evolution





Ni, A., Wang, Q., (2017), *Sensitivity analysis on chaotic dynamical systems by Non-Intrusive Least Squares Shadowing (NILSS)* , Journal of Computational Physics, **347**, 56-77.

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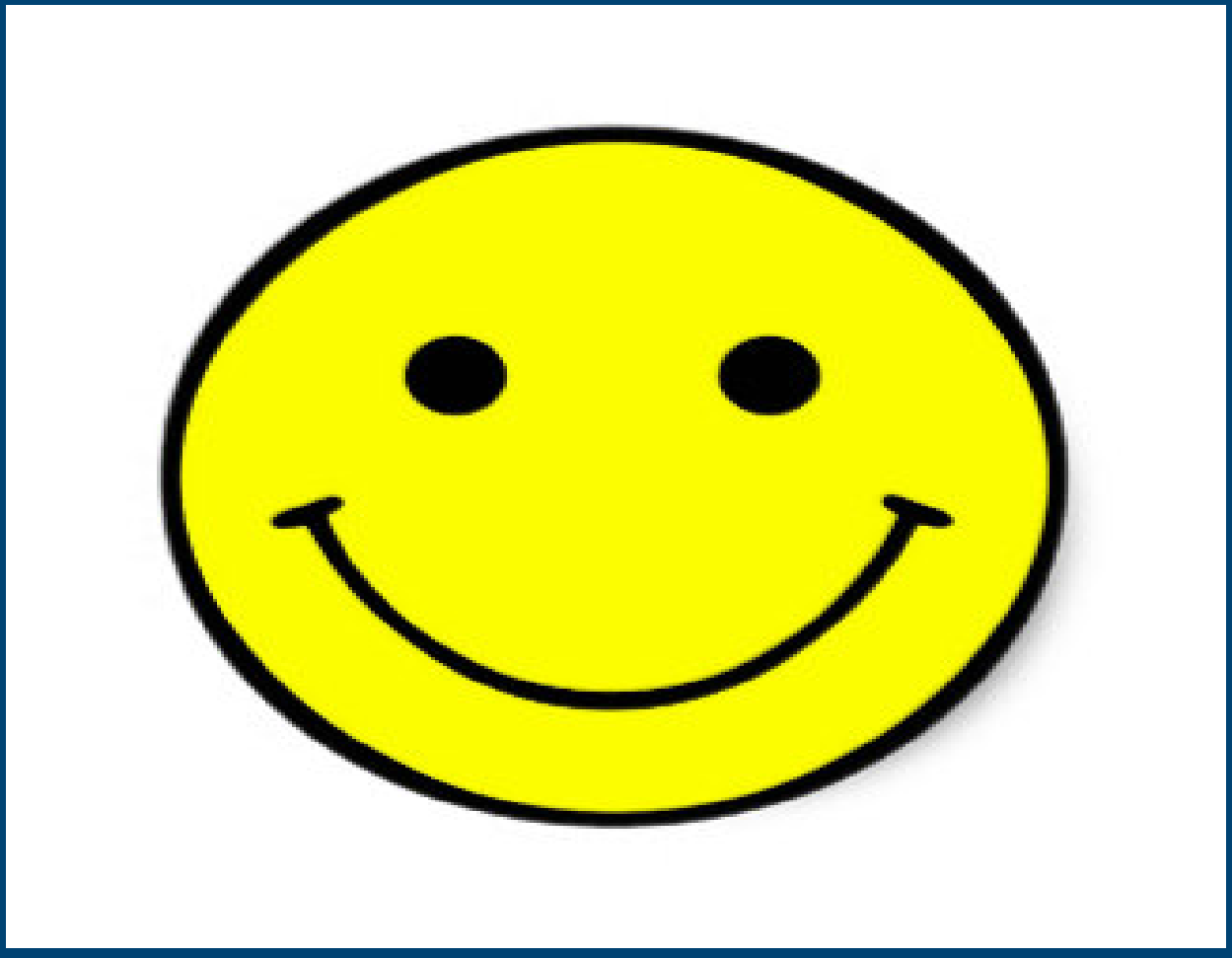

Source http://www.insurancechat.co.za/2017-09/could-sending-a-smiley-face-get-me-into-legal-hot-water/

# Substract instabilities



- We are looking to build trajectories with parameters $\rho$ and $\rho + \delta\rho$ such that they don't diverge from each other.

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- We are looking to build trajectories with parameters $\rho$ and $\rho + \delta\rho$ such that they don't diverge from each other.
- Their difference contains only the long-time effect.
- Therefore we can reveal the long time effect with shorter trajectories.

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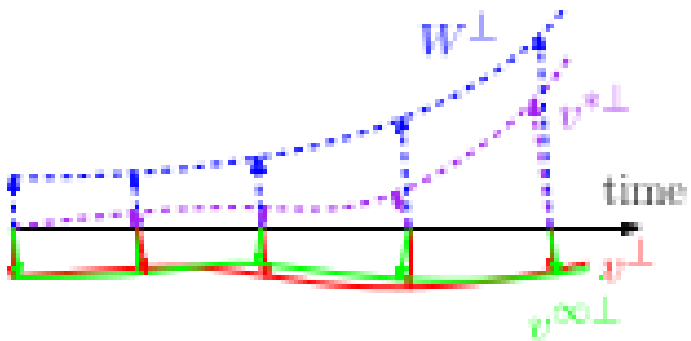
# Idea






Now we only need to solve a minimization problem.


Ni, A., Wang, Q., (2017), *Sensitivity analysis on chaotic dynamical systems by Non-Intrusive Least Squares Shadowing (NILSS)* , Journal of Computational Physics, **347**, 56-77.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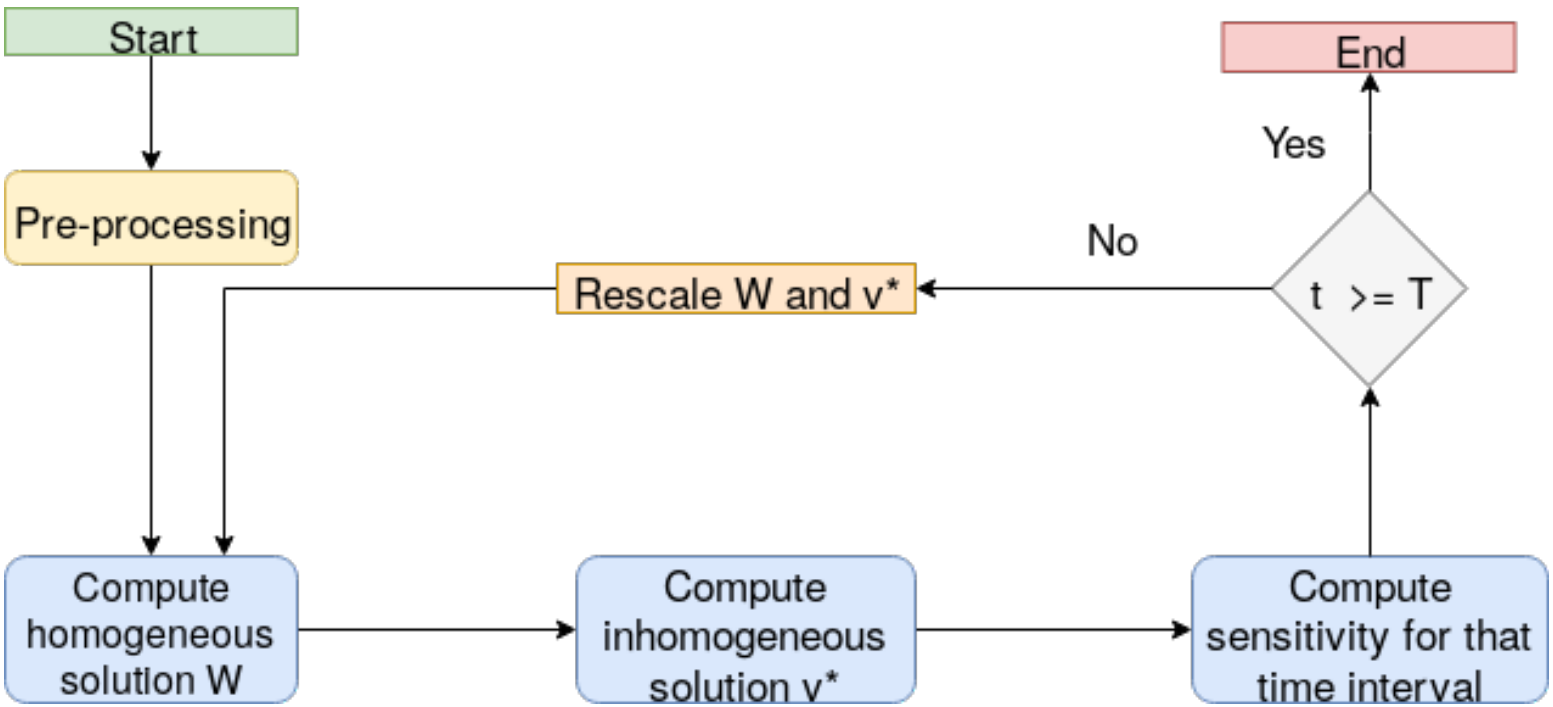
# Flowchart

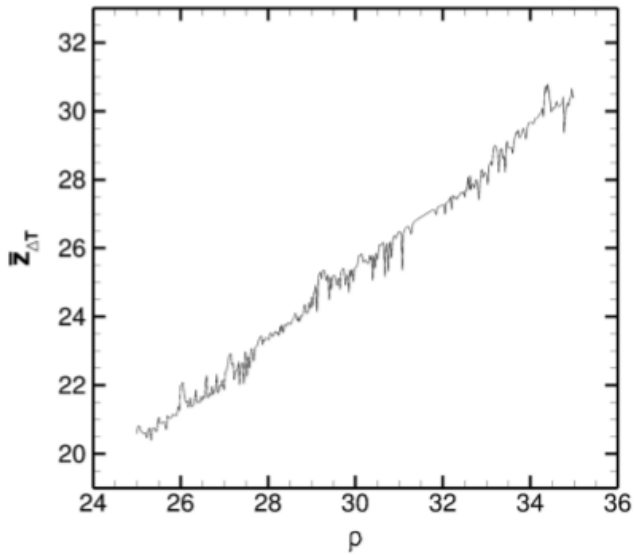


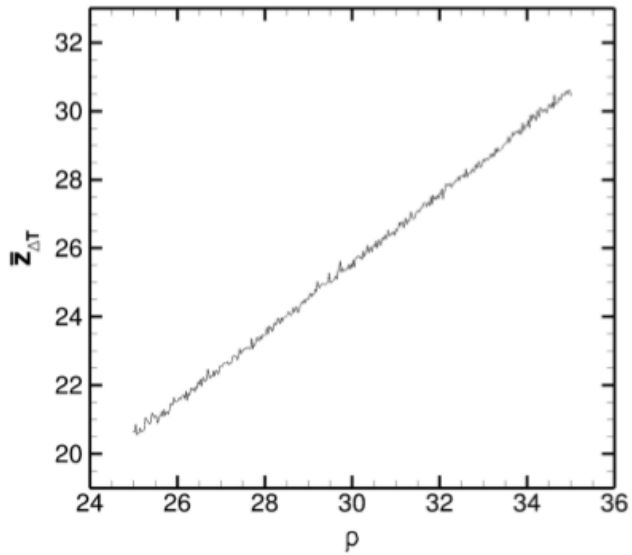


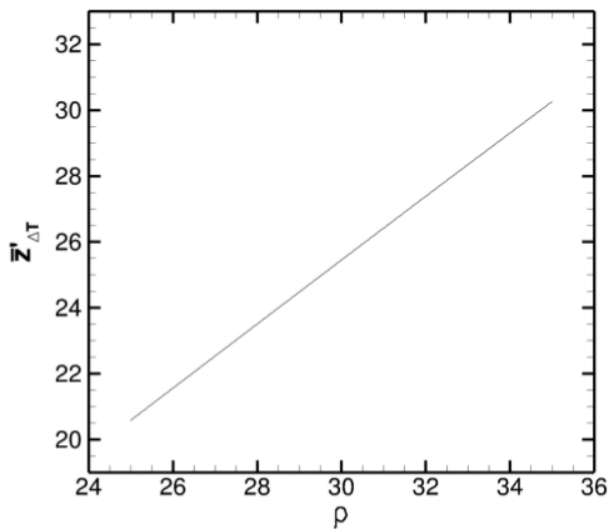
Source https://www.pinterest.de/pin/12821192062040872 4

# Lorenz attractor





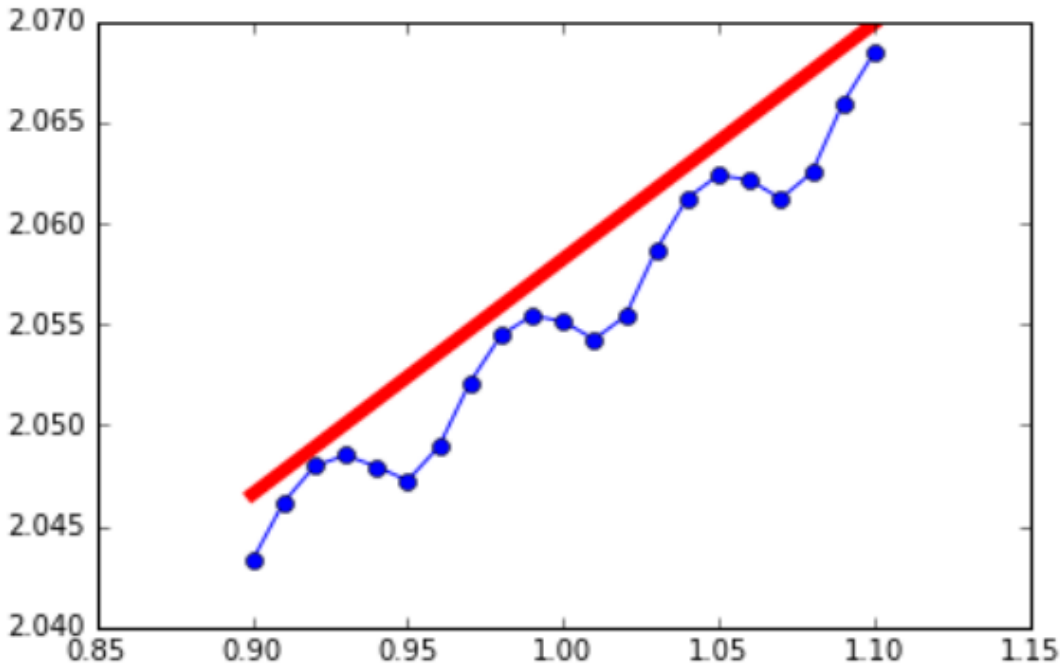




Patrick J. Blonigan, Qiqi Wang, Eric J. Nielsen, and Boris Diskin. Ëast-Squares Shadowing Sensitivity Analysis of Chaotic Flow Around a Two-Dimensional Airfoil, AIAA Journal, Vol. 56, No. 2 (2018), pp. 658-672.

# Van der Pol attractor



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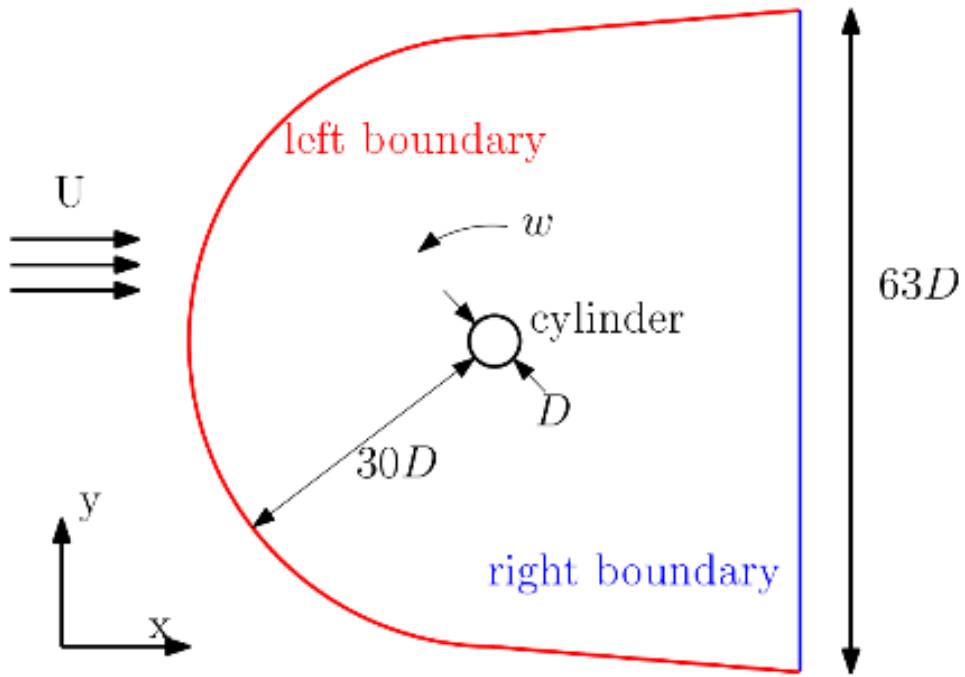
# One more example






Cylinder can rotate

Angxiu Ni, Qiqi Wang, Pablo Fernandez, Chaitanya Talnikar. Sensitivity analysis on chaotic dynamical systems by Finite Difference Non-Intrusive Least Squares Shadowing (FD-NILSS). arXiv:1711.06633 [physics.comp-ph]

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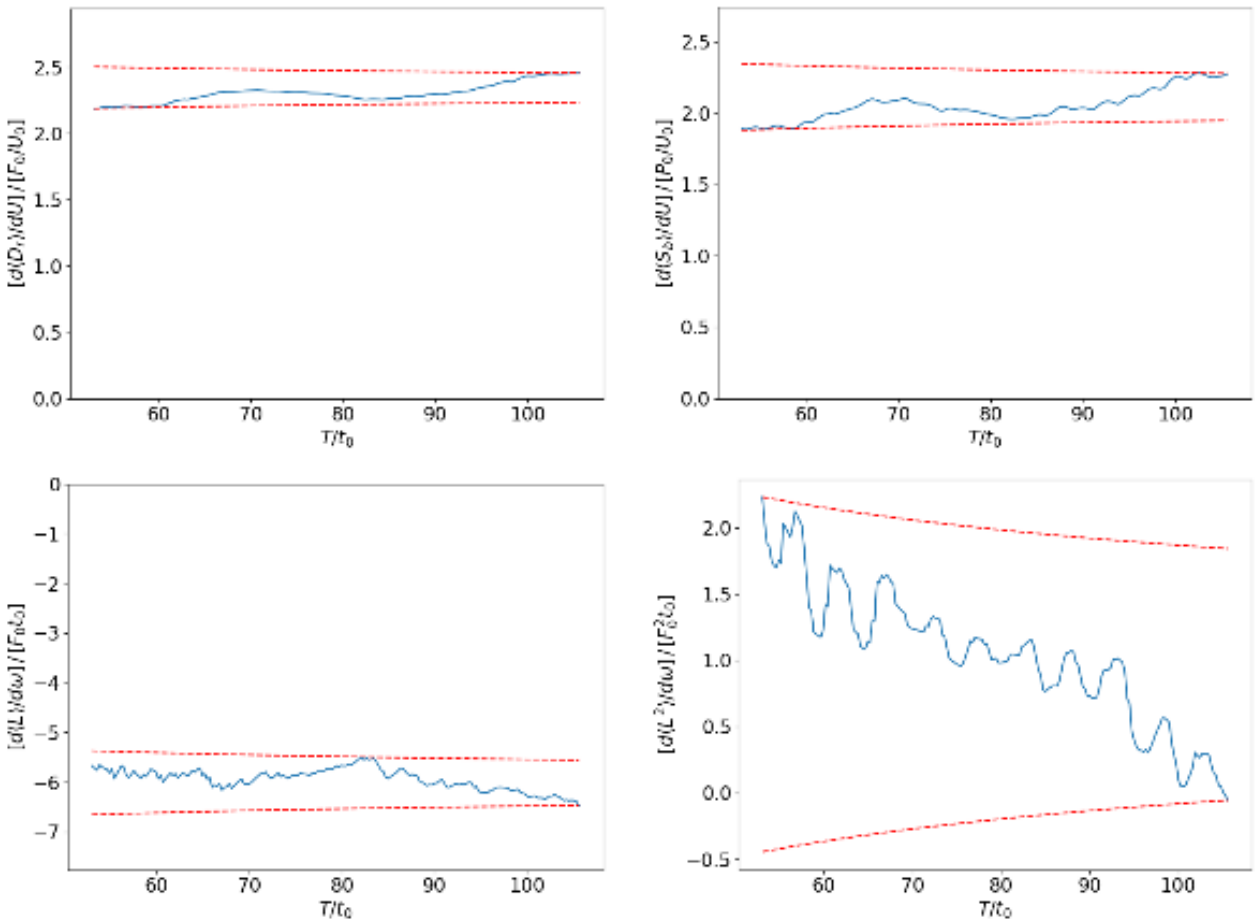


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# Advantages



- Easy to implement if you already have a solver.
- Low cost in comparison to other methods.

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- Easy to implement if you already have a solver.
- Low cost in comparison to other methods.
- Therefore faster.
- Uses less memory.

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# Where are sensitivities used?



Sensitivities help us to

- Design products.
- Control processes and systems.
- Solve inverse problems (e.g. CAT scan images).
- Estimate simulation errors.
- Quantify uncertainties.

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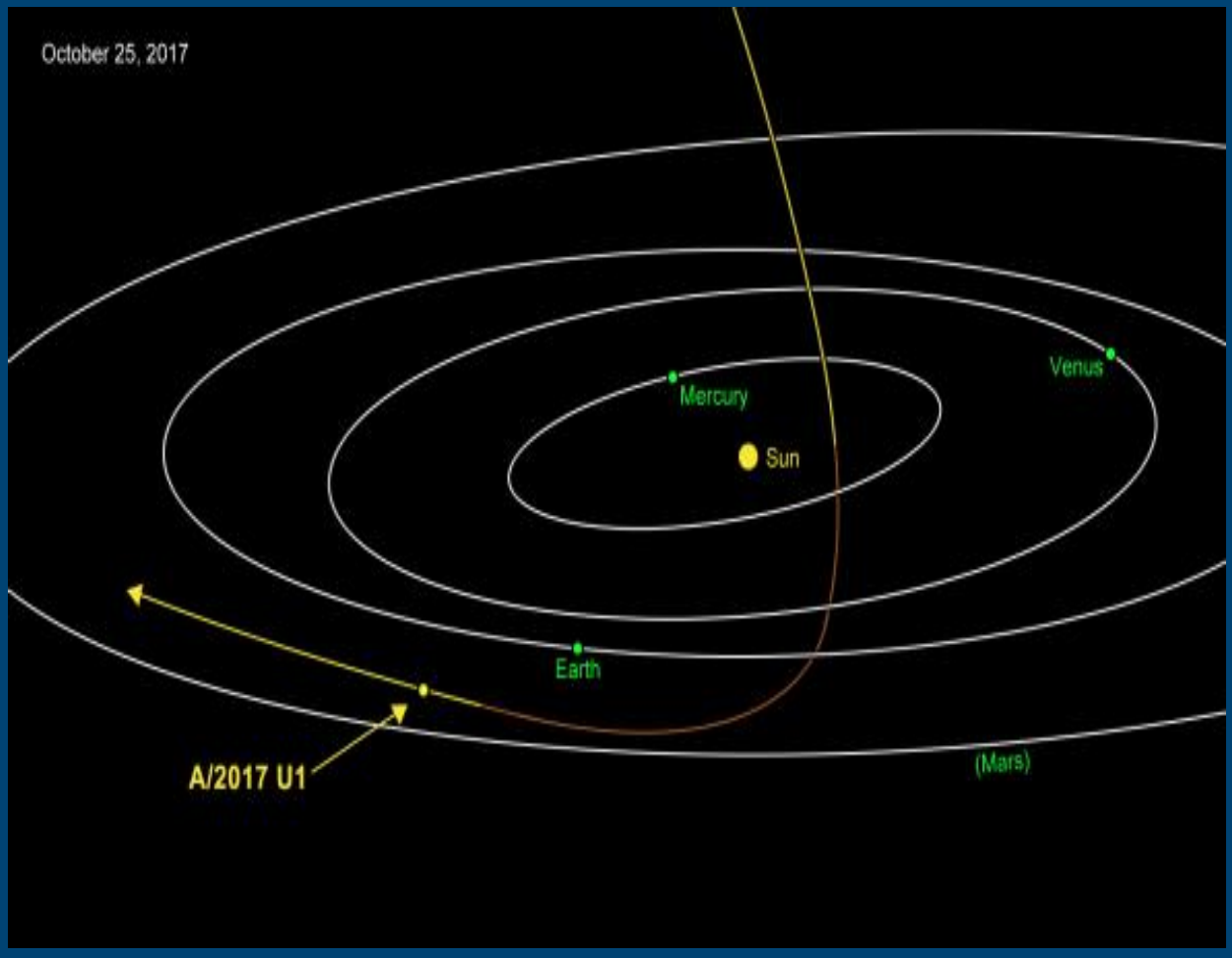


Source https://news.nationalgeographic.com/

# More to know




1. Ni, A., Wang, Q., (2017), *Sensitivity analysis on chaotic dynamical systems by Non-Intrusive Least Squares Shadowing (NILSS)* , Journal of Computational Physics, **347**, 56-77.
2. Ni, A., Wang, Q., Fernandez, P., and Talnikar, C., *Sensitivity analysis on chaotic dynamical systems by Finite Difference Non-Intrusive Least Squares Shadowing (FD-NILSS)*, `arXiv:1711.06633`
3. Safiran, N., Lot,z J., Naumann, U., (2016), *Algorithmic Differentiation of Numerical Methods:Tangent and Adjoint Solvers for Parameterized Systems of Nonlinear Equations*, Procedia Computer Science, **80**, 2231-2235.
4. Strogatz, Steven H., (2015). *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering.* Boulder, CO: Westview Press.
5. Gleick, J. (1988). *Chaos: Making a new science*. New York, N.Y., U.S.A: Penguin.
6. Alvarez A., Ghys É., and Leys J., *Chaos a Mathematical Adventure*, `http://www.chaos-math.org/en`


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Source http://www.sednacomics.com/