

In[110]:=

(* Define the variables *)

Clear[λ, μ, θ, ρ, z, g1, g2, π0, π11, π12]

$$\pi_0 = 1 - \rho$$

$$\pi_{11} = \left(\lambda / \left(\mu (1 - \rho) + (\theta \mu \rho) / (\lambda + \theta) \right) \right) \pi_0$$

$$\pi_{12} = \left(\lambda \rho / \left((1 - \rho) (\lambda + \theta) + \theta \rho \right) \right) \pi_0;$$

(* Define the equations *)

$$\text{eq1} = (\lambda + \mu) g1[z] == \lambda z (\pi_0 + g1[z]) + \mu (1 - \rho) / z (g1[z] - z \pi_{11}) + \theta / z (g2[z] - z \pi_{12})$$

$$\text{eq2} = g2[z] == g1[z] \rho \mu / (\theta + \lambda (1 - z))$$

(* Solve the equations *)

sol = Solve[{eq1, eq2}, {g1[z], g2[z]}]

simplifiedSolution = FullSimplify[sol]

Out[111]=

$$1 - \rho$$

Out[112]=

$$\frac{\lambda (1 - \rho)}{(1 - \rho) \mu + \frac{\rho \theta \mu}{\theta + \lambda}}$$

Out[114]=

$$(\lambda + \mu) g1[z] == \frac{(1 - \rho) \mu \left(-\frac{z \lambda (1 - \rho)}{(1 - \rho) \mu + \frac{\rho \theta \mu}{\theta + \lambda}} + g1[z] \right)}{z} + z \lambda (1 - \rho + g1[z]) + \frac{\theta \left(-\frac{\rho z \lambda (1 - \rho)}{\rho \theta + (1 - \rho) (\theta + \lambda)} + g2[z] \right)}{z}$$

Out[115]=

$$g2[z] == \frac{\rho \mu g1[z]}{\theta + (1 - z) \lambda}$$

Out[116]=

$$\left\{ \left\{ g1[z] \rightarrow \frac{z (-\theta - \lambda + z \lambda) (-\lambda + \lambda \rho)}{-z \theta \lambda - z \lambda^2 + z^2 \lambda^2 + \theta \mu + \lambda \mu - \rho \lambda \mu - z \lambda \mu}, g2[z] \rightarrow \frac{\rho z \mu (-\lambda + \lambda \rho)}{z \theta \lambda + z \lambda^2 - z^2 \lambda^2 - \theta \mu - \lambda \mu + \rho \lambda \mu + z \lambda \mu} \right\} \right\}$$

Out[117]=

$$\left\{ \left\{ g1[z] \rightarrow -\frac{z \lambda (\theta + \lambda - z \lambda) (-1 + \rho)}{z^2 \lambda^2 + (\theta + \lambda - \rho \lambda) \mu - z \lambda (\theta + \lambda + \mu)}, g2[z] \rightarrow \frac{\rho z \lambda \mu (-1 + \rho)}{z \lambda (\theta + \lambda - z \lambda) - \theta \mu + (-1 + \rho + z) \lambda \mu} \right\} \right\}$$