Ecuaciones diferenciales

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Métodos

```
MetodoEuler <- function(a, b, N, alfa, funcion){

  df <- data.frame(t = rep(NA, times = (N+1)), w = rep(NA, times = (N+1)))

  # Paso 1
  h <- (b - a) / N
  df[1,1] <- a
  df[1,2] <- alfa


  # Paso 2
  for (i in 1:N) {

    # Paso 3
    df[i+1,1] <- a + i*h
    df[i+1,2] <- df[i,2] + h * eval(funcion, list(y = df[i,2], t = (a + (i-1)*h) ))
  }

  return(df)
}

# Alfa = y(0)
MetodoEuler(a = 0, b = 2, N = 4, alfa = 0.5, funcion = expression(y - t^2 + 1))
```

```
##     t      w
## 1 0.0 0.5000
## 2 0.5 1.2500
## 3 1.0 2.2500
## 4 1.5 3.3750
## 5 2.0 4.4375
```

```
RungeKutta <- function(a, b, N, alfa, funcion){

  df <- data.frame(t = rep(NA, times = (N+1)), w = rep(NA, times = (N+1)))

  h <- (b - a)/N
  df[1,1] <- a
  df[1,2] <- alfa
  t <- a

  for (i in 1:N) {
```

```
    k <- rep(NA, times = 4)

    # Paso 3
    k[1] <- h * eval(funcion, list(t = t, y = df[i,2] ))
    k[2] <- h * eval(funcion, list(t = t + h/2, y = df[i,2] + k[1]/2))
    k[3] <- h * eval(funcion, list(t = t + h/2, y = df[i,2] + k[2]/2))
    k[4] <- h * eval(funcion, list(t = t + h, y = df[i,2] + k[3]))

    df[i+1,2] <- df[i,2] + (k[1] + 2*k[2] + 2*k[3] + k[4])/6

    t <- a + i * h
    df[i+1,1] <- t
  }

  return(df)
}
# Alfa = y(0)
RungeKutta(a = 0, b = 2, N = 10, alfa = 0.5, funcion = expression(y - t^2 + 1))
```

```
##      t         w
## 1  0.0 0.5000000
## 2  0.2 0.8292933
## 3  0.4 1.2140762
## 4  0.6 1.6489220
## 5  0.8 2.1272027
## 6  1.0 2.6408227
## 7  1.2 3.1798942
## 8  1.4 3.7323401
## 9  1.6 4.2834095
## 10 1.8 4.8150857
## 11 2.0 5.3053630
```

Ejercicio 1

a

```
MetodoEuler(a = 0, b = 1, N = 10, alfa = 2, funcion = expression(t - y + 2))
```

```
##      t        w
## 1  0.0 2.000000
## 2  0.1 2.000000
## 3  0.2 2.010000
## 4  0.3 2.029000
## 5  0.4 2.056100
## 6  0.5 2.090490
## 7  0.6 2.131441
## 8  0.7 2.178297
## 9  0.8 2.230467
## 10 0.9 2.287420
## 11 1.0 2.348678
```

b

```
MetodoEuler(a = 0, b = 2, N = 10, alfa = 0.5, funcion = expression(y - t^2 + 1))
```

```
##       t        w
## 1   0.0 0.500000
## 2   0.2 0.800000
## 3   0.4 1.152000
## 4   0.6 1.550400
## 5   0.8 1.988480
## 6   1.0 2.458176
## 7   1.2 2.949811
## 8   1.4 3.451773
## 9   1.6 3.950128
## 10  1.8 4.428154
## 11  2.0 4.865785
```

Ejercicio 2

a

```
RungeKutta(a = 0, b = 2, N = 10, alfa = 0.5, funcion = expression(y - t^2 + 1))
```

```
##       t         w
## 1   0.0 0.5000000
## 2   0.2 0.8292933
## 3   0.4 1.2140762
## 4   0.6 1.6489220
## 5   0.8 2.1272027
## 6   1.0 2.6408227
## 7   1.2 3.1798942
## 8   1.4 3.7323401
## 9   1.6 4.2834095
## 10  1.8 4.8150857
## 11  2.0 5.3053630
```

b

```
RungeKutta(a = 0, b = 0.1, N = 10, alfa = 5, funcion = expression(-20*y + 7 * exp(-0.5*t)))
```

```
##       t         w
## 1  0.00 5.0000000
## 2  0.01 4.1569464
## 3  0.02 3.4663946
## 4  0.03 2.9007029
## 5  0.04 2.4372397
## 6  0.05 2.0574761
## 7  0.06 1.7462416
## 8  0.07 1.4911157
## 9  0.08 1.2819293
## 10 0.09 1.1103567
## 11 0.10 0.9695813
```

Ejercicio 3

No me da como en la guía

a

```
df_a <- MetodoEuler(a = 0, b = 4*pi, N = 100, alfa = 0.01, funcion = expression(exp(sin(t*y)) + sin(log

df_a
```

```
##              t          w
## 1    0.0000000  0.0100000
## 2    0.1256637  0.2606058
## 3    0.2513274  0.2679855
## 4    0.3769911  0.2807649
## 5    0.5026548  0.3004011
## 6    0.6283185  0.3292204
## 7    0.7539822  0.3709207
## 8    0.8796459  0.4313526
## 9    1.0053096  0.5197086
## 10   1.1309734  0.6501921
## 11   1.2566371  0.8435356
## 12   1.3823008  1.1229146
## 13   1.5079645  1.4789803
## 14   1.6336282  1.8038909
## 15   1.7592919  2.0262879
## 16   1.8849556  2.1911746
## 17   2.0106193  2.3344529
## 18   2.1362830  2.4749142
## 19   2.2619467  2.6281190
## 20   2.3876104  2.8216652
## 21   2.5132741  3.1246807
## 22   2.6389378  3.5804171
## 23   2.7646015  3.8233591
## 24   2.8902652  3.9962640
## 25   3.0159289  4.1734804
## 26   3.1415927  4.4261510
## 27   3.2672564  4.8839383
## 28   3.3929201  5.1077931
## 29   3.5185838  5.2795215
## 30   3.6442475  5.5005963
## 31   3.7699112  5.9438113
## 32   3.8955749  6.1505255
## 33   4.0212386  6.3223992
## 34   4.1469023  6.6108427
## 35   4.2725660  6.9983016
## 36   4.3982297  7.1615391
## 37   4.5238934  7.4137968
## 38   4.6495571  7.8222526
## 39   4.7752208  7.9809477
## 40   4.9008845  8.2783442
## 41   5.0265482  8.5499407
## 42   5.1522120  8.7093954
## 43   5.2778757  9.0869530
```

```
## 44   5.4035394  9.2478848
## 45   5.5292031  9.4416837
## 46   5.6548668  9.8592291
## 47   5.7805305 10.0154002
## 48   5.9061942 10.4418389
## 49   6.0318579 10.5818467
## 50   6.1575216 10.9613769
## 51   6.2831853 11.0930631
## 52   6.4088490 11.3955949
## 53   6.5345127 11.5397261
## 54   6.6601764 11.7469826
## 55   6.7858401 11.9951317
## 56   6.9115038 12.1668120
## 57   7.0371675 12.4873280
## 58   7.1628313 12.6749797
## 59   7.2884950 12.9178385
## 60   7.4141587 13.1011760
## 61   7.5398224 13.3306162
## 62   7.6654861 13.5195784
## 63   7.7911498 13.7144961
## 64   7.9168135 13.9077480
## 65   8.0424772 14.0772956
## 66   8.1681409 14.2787172
## 67   8.2938046 14.4228502
## 68   8.4194683 14.6394220
## 69   8.5451320 14.7592822
## 70   8.6707957 15.0091532
## 71   8.7964594 15.1093886
## 72   8.9221231 15.4468018
## 73   9.0477868 15.5804346
## 74   9.1734505 15.8149032
## 75   9.2991143 16.0760559
## 76   9.4247780 16.1687267
## 77   9.5504417 16.5543449
## 78   9.6761054 16.8905395
## 79   9.8017691 17.0644657
## 80   9.9274328 17.1653636
## 81  10.0530965 17.4529721
## 82  10.1787602 17.5676499
## 83  10.3044239 17.7633818
## 84  10.4300876 18.0588968
## 85  10.5557513 18.1990684
## 86  10.6814150 18.3090268
## 87  10.8070787 18.5934405
## 88  10.9327424 18.7320729
## 89  11.0584061 18.8305128
## 90  11.1840698 19.1296203
## 91  11.3097336 19.3253031
## 92  11.4353973 19.3952142
## 93  11.5610610 19.7430061
## 94  11.6867247 20.0673117
## 95  11.8123884 20.3912992
## 96  11.9380521 20.7037232
## 97  12.0637158 21.0128757
```

```
## 98  12.1893795 21.3126075
## 99  12.3150432 21.6087977
## 100 12.4407069 21.8961939
## 101 12.5663706 22.1806379
```

b

```
df_b <- RungeKutta(a = 0, b = 4*pi, N = 100, alfa = 0.01, funcion = expression(exp(sin(t*y)) + sin(log(y

df_b
```

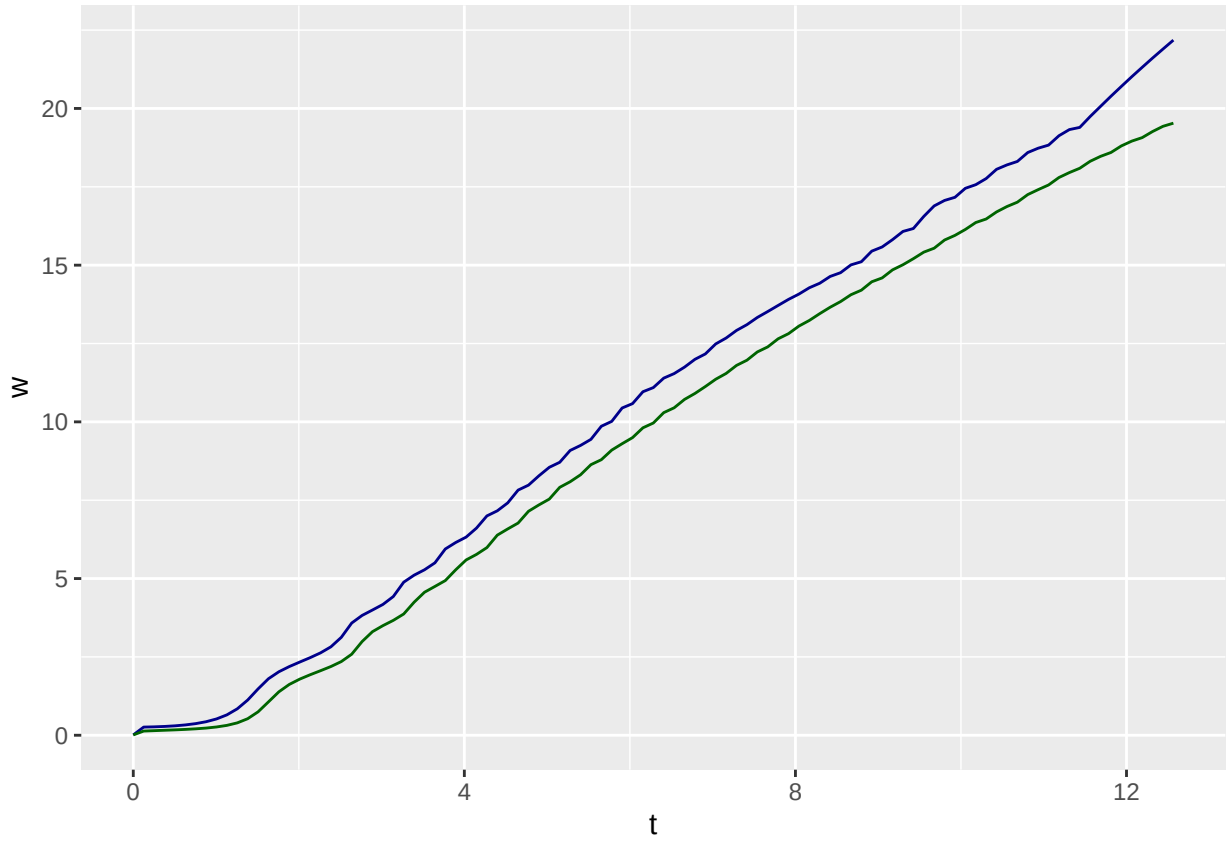
```
##              t         w
## 1    0.0000000  0.0100000
## 2    0.1256637  0.1334719
## 3    0.2513274  0.1465485
## 4    0.3769911  0.1587271
## 5    0.5026548  0.1715703
## 6    0.6283185  0.1864088
## 7    0.7539822  0.2048354
## 8    0.8796459  0.2292066
## 9    1.0053096  0.2634352
## 10   1.1309734  0.3144452
## 11   1.2566371  0.3948665
## 12   1.3823008  0.5273948
## 13   1.5079645  0.7467082
## 14   1.6336282  1.0651789
## 15   1.7592919  1.3860183
## 16   1.8849556  1.6207772
## 17   2.0106193  1.7904368
## 18   2.1362830  1.9294677
## 19   2.2619467  2.0587871
## 20   2.3876104  2.1928889
## 21   2.5132741  2.3510477
## 22   2.6389378  2.5855316
## 23   2.7646015  2.9900436
## 24   2.8902652  3.3058230
## 25   3.0159289  3.4970199
## 26   3.1415927  3.6653410
## 27   3.2672564  3.8690312
## 28   3.3929201  4.2450162
## 29   3.5185838  4.5632425
## 30   3.6442475  4.7478059
## 31   3.7699112  4.9351165
## 32   3.8955749  5.2793010
## 33   4.0212386  5.5928837
## 34   4.1469023  5.7705893
## 35   4.2725660  5.9876813
## 36   4.3982297  6.3854471
## 37   4.5238934  6.5822477
## 38   4.6495571  6.7704930
## 39   4.7752208  7.1466554
## 40   4.9008845  7.3495636
## 41   5.0265482  7.5359000
## 42   5.1522120  7.9104406
```

```
## 43   5.2778757  8.0883362
## 44   5.4035394  8.3102889
## 45   5.5292031  8.6337656
## 46   5.6548668  8.7925229
## 47   5.7805305  9.0983446
## 48   5.9061942  9.3031319
## 49   6.0318579  9.4970546
## 50   6.1575216  9.8102379
## 51   6.2831853  9.9627876
## 52   6.4088490 10.2944206
## 53   6.5345127 10.4481339
## 54   6.6601764 10.7159003
## 55   6.7858401 10.9061961
## 56   6.9115038 11.1264618
## 57   7.0371675 11.3601870
## 58   7.1628313 11.5446580
## 59   7.2884950 11.8008856
## 60   7.4141587 11.9672089
## 61   7.5398224 12.2311953
## 62   7.6654861 12.3920196
## 63   7.7911498 12.6498895
## 64   7.9168135 12.8146261
## 65   8.0424772 13.0561404
## 66   8.1681409 13.2356405
## 67   8.2938046 13.4529733
## 68   8.4194683 13.6571922
## 69   8.5451320 13.8372431
## 70   8.6707957 14.0568813
## 71   8.7964594 14.2011320
## 72   8.9221231 14.4699056
## 73   9.0477868 14.5926519
## 74   9.1734505 14.8501292
## 75   9.2991143 15.0131403
## 76   9.4247780 15.2061935
## 77   9.5504417 15.4144931
## 78   9.6761054 15.5409024
## 79   9.8017691 15.8024409
## 80   9.9274328 15.9504608
## 81  10.0530965 16.1392215
## 82  10.1787602 16.3598285
## 83  10.3044239 16.4723379
## 84  10.4300876 16.6986843
## 85  10.5557513 16.8709853
## 86  10.6814150 17.0091124
## 87  10.8070787 17.2519983
## 88  10.9327424 17.4094331
## 89  11.0584061 17.5583420
## 90  11.1840698 17.7967104
## 91  11.3097336 17.9540529
## 92  11.4353973 18.0918476
## 93  11.5610610 18.3149924
## 94  11.6867247 18.4718108
## 95  11.8123884 18.5954666
## 96  11.9380521 18.8057226
```

```
## 97  12.0637158 18.9582509
## 98  12.1893795 19.0694833
## 99  12.3150432 19.2641769
## 100 12.4407069 19.4302189
## 101 12.5663706 19.5304345
```

c

```
ggplot() +
  geom_line(aes(x = df_a$t, y = df_a$w), colour = "darkblue") +
  geom_line(aes(x = df_b$t, y = df_b$w), colour = "darkgreen") +
  xlab("t") + ylab("w")
```



Ejercicio 4

a

```
MetodoEuler(a = 2, b = 3, N = 20, alfa = 1, funcion = expression(1 + (t - y)^2))
```

```
##       t        w
## 1  2.00 1.000000
## 2  2.05 1.100000
## 3  2.10 1.195125
## 4  2.15 1.286065
## 5  2.20 1.373384
## 6  2.25 1.457549
```

```
## 7  2.30 1.538948
## 8  2.35 1.617908
## 9  2.40 1.694706
## 10 2.45 1.769578
## 11 2.50 1.842726
## 12 2.55 1.914327
## 13 2.60 1.984531
## 14 2.65 2.053471
## 15 2.70 2.121263
## 16 2.75 2.188010
## 17 2.80 2.253802
## 18 2.85 2.318718
## 19 2.90 2.382831
## 20 2.95 2.446205
## 21 3.00 2.508895
```

```
MetodoEuler(a = 2, b = 3, N = 80, alfa = 1, funcion = expression(1 + (t - y)^2))
```

```
##         t        w
## 1  2.0000 1.000000
## 2  2.0125 1.025000
## 3  2.0250 1.049689
## 4  2.0375 1.074080
## 5  2.0500 1.098182
## 6  2.0625 1.122007
## 7  2.0750 1.145563
## 8  2.0875 1.168861
## 9  2.1000 1.191910
## 10 2.1125 1.214718
## 11 2.1250 1.237293
## 12 2.1375 1.259643
## 13 2.1500 1.281776
## 14 2.1625 1.303699
## 15 2.1750 1.325418
## 16 2.1875 1.346940
## 17 2.2000 1.368272
## 18 2.2125 1.389419
## 19 2.2250 1.410388
## 20 2.2375 1.431183
## 21 2.2500 1.451809
## 22 2.2625 1.472273
## 23 2.2750 1.492579
## 24 2.2875 1.512731
## 25 2.3000 1.532735
## 26 2.3125 1.552593
## 27 2.3250 1.572312
## 28 2.3375 1.591893
## 29 2.3500 1.611342
## 30 2.3625 1.630663
## 31 2.3750 1.649857
## 32 2.3875 1.668930
## 33 2.4000 1.687885
## 34 2.4125 1.706723
## 35 2.4250 1.725450
## 36 2.4375 1.744067
```

```
## 37 2.4500 1.762578
## 38 2.4625 1.780985
## 39 2.4750 1.799290
## 40 2.4875 1.817498
## 41 2.5000 1.835609
## 42 2.5125 1.853627
## 43 2.5250 1.871553
## 44 2.5375 1.889390
## 45 2.5500 1.907141
## 46 2.5625 1.924807
## 47 2.5750 1.942390
## 48 2.5875 1.959892
## 49 2.6000 1.977316
## 50 2.6125 1.994663
## 51 2.6250 2.011934
## 52 2.6375 2.029132
## 53 2.6500 2.046259
## 54 2.6625 2.063315
## 55 2.6750 2.080303
## 56 2.6875 2.097224
## 57 2.7000 2.114079
## 58 2.7125 2.130870
## 59 2.7250 2.147599
## 60 2.7375 2.164266
## 61 2.7500 2.180874
## 62 2.7625 2.197423
## 63 2.7750 2.213914
## 64 2.7875 2.230349
## 65 2.8000 2.246730
## 66 2.8125 2.263056
## 67 2.8250 2.279329
## 68 2.8375 2.295551
## 69 2.8500 2.311723
## 70 2.8625 2.327845
## 71 2.8750 2.343918
## 72 2.8875 2.359943
## 73 2.9000 2.375922
## 74 2.9125 2.391856
## 75 2.9250 2.407744
## 76 2.9375 2.423588
## 77 2.9500 2.439390
## 78 2.9625 2.455149
## 79 2.9750 2.470866
## 80 2.9875 2.486543
## 81 3.0000 2.502180
```

b

```
RungeKutta(a = 2, b = 3, N = 20, alfa = 1, funcion = expression(1 + (t - y)^2))
```

```
##       t        w
## 1  2.00 1.000000
## 2  2.05 1.097619
## 3  2.10 1.190909
```

```
## 4  2.15 1.280435
## 5  2.20 1.366667
## 6  2.25 1.450000
## 7  2.30 1.530769
## 8  2.35 1.609259
## 9  2.40 1.685714
## 10 2.45 1.760345
## 11 2.50 1.833333
## 12 2.55 1.904839
## 13 2.60 1.975000
## 14 2.65 2.043939
## 15 2.70 2.111765
## 16 2.75 2.178571
## 17 2.80 2.244444
## 18 2.85 2.309459
## 19 2.90 2.373684
## 20 2.95 2.437179
## 21 3.00 2.500000
```

```
RungeKutta(a = 2, b = 3, N = 80, alfa = 1, funcion = expression(1 + (t - y)^2))
```

```
##         t        w
## 1  2.0000 1.000000
## 2  2.0125 1.024846
## 3  2.0250 1.049390
## 4  2.0375 1.073645
## 5  2.0500 1.097619
## 6  2.0625 1.121324
## 7  2.0750 1.144767
## 8  2.0875 1.167960
## 9  2.1000 1.190909
## 10 2.1125 1.213624
## 11 2.1250 1.236111
## 12 2.1375 1.258379
## 13 2.1500 1.280435
## 14 2.1625 1.302285
## 15 2.1750 1.323936
## 16 2.1875 1.345395
## 17 2.2000 1.366667
## 18 2.2125 1.387758
## 19 2.2250 1.408673
## 20 2.2375 1.429419
## 21 2.2500 1.450000
## 22 2.2625 1.470421
## 23 2.2750 1.490686
## 24 2.2875 1.510801
## 25 2.3000 1.530769
## 26 2.3125 1.550595
## 27 2.3250 1.570283
## 28 2.3375 1.589836
## 29 2.3500 1.609259
## 30 2.3625 1.628555
## 31 2.3750 1.647727
## 32 2.3875 1.666779
## 33 2.4000 1.685714
```

```
## 34 2.4125 1.704535
## 35 2.4250 1.723246
## 36 2.4375 1.741848
## 37 2.4500 1.760345
## 38 2.4625 1.778739
## 39 2.4750 1.797034
## 40 2.4875 1.815231
## 41 2.5000 1.833333
## 42 2.5125 1.851343
## 43 2.5250 1.869262
## 44 2.5375 1.887093
## 45 2.5500 1.904839
## 46 2.5625 1.922500
## 47 2.5750 1.940079
## 48 2.5875 1.957579
## 49 2.6000 1.975000
## 50 2.6125 1.992345
## 51 2.6250 2.009615
## 52 2.6375 2.026813
## 53 2.6500 2.043939
## 54 2.6625 2.060996
## 55 2.6750 2.077985
## 56 2.6875 2.094907
## 57 2.7000 2.111765
## 58 2.7125 2.128558
## 59 2.7250 2.145290
## 60 2.7375 2.161960
## 61 2.7500 2.178571
## 62 2.7625 2.195124
## 63 2.7750 2.211620
## 64 2.7875 2.228059
## 65 2.8000 2.244444
## 66 2.8125 2.260776
## 67 2.8250 2.277055
## 68 2.8375 2.293282
## 69 2.8500 2.309459
## 70 2.8625 2.325587
## 71 2.8750 2.341667
## 72 2.8875 2.357699
## 73 2.9000 2.373684
## 74 2.9125 2.389624
## 75 2.9250 2.405519
## 76 2.9375 2.421371
## 77 2.9500 2.437179
## 78 2.9625 2.452946
## 79 2.9750 2.468671
## 80 2.9875 2.484355
## 81 3.0000 2.500000
```

Ejercicio 5

```
MetodoEuler(a = 1, b = 2, alfa = 0, N = 10, funcion = expression((2/t) * y + (t^2) * exp(t) ))
```

```
##      t          w
```

```
## 1  1.0  0.0000000
## 2  1.1  0.2718282
## 3  1.2  0.6847556
## 4  1.3  1.2769783
## 5  1.4  2.0935477
## 6  1.5  3.1874451
## 7  1.6  4.6208178
## 8  1.7  6.4663964
## 9  1.8  8.8091197
## 10 1.9 11.7479965
## 11 2.0 15.3982357
```