

OPT CV8 - Philip Ward

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10.1

Stat (2, 1, 5)

- a) saddle point
- b) not a critical point
- c) local min

10.2 de

d) $f(x, y) = 3x^3 - 3xy^2$

$x = 0$

$y = 0$

$x = \frac{1}{2}$

$y = \pm 1$

$\nabla f = [3 - 3x^2 - 3y^2, 6xy]$

Stat bod

$[\frac{1}{2}, 0]$

$[0, 1]$

Saddle point

$[0, -1]$

Saddle

$3 - 3x^2 - 3y^2 = 0$

$6xy = 0$

Local max

$H_f = \begin{bmatrix} -6 & -6y \\ -6y & -6x \end{bmatrix}$

$H_f[\frac{1}{2}, 0] = \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}$

$H_f[0, 1] = \begin{bmatrix} -6 & 0 \\ 0 & 0 \end{bmatrix}$

$H_f[0, -1] = \begin{bmatrix} -6 & 0 \\ 0 & 0 \end{bmatrix}$

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e) $f(x,y,z) = 6x^2 - 2y^3 - 3z^4$

$\nabla = \begin{bmatrix} 12x \\ -6y^2 \\ -12z^3 \end{bmatrix}$

$\frac{\Delta}{\gamma} = 0$

$x = -2$

$x = 2$

$[0,0]$

$[1,-1]$

$[-1,1]$

$[1,1]$

$[-1,-1]$

$H = \begin{bmatrix} -12x & 12z \\ 12z & -12x - 36z^2 \end{bmatrix}$

$[0,0] = \begin{bmatrix} 0 & 0 \\ 0 & -36 \end{bmatrix}$

$[1,-1][1,1]$

$[1,1] \text{ a } [-1,-1]$

$\begin{bmatrix} -12 & -12 \\ -12 & 24 \end{bmatrix}$

$\begin{bmatrix} -12 & 12 \\ 12 & -24 \end{bmatrix}$

10.3 $f(x) = a^T x = \sum_{i=1}^n x_i \log x_i = \sum_{i=1}^n g_i(x_i)$

$g_i(x) = a_i x - x \log x$

let $x = (x_1, \dots, x_n)$ and $f_i(x) = a_i x - x \log x$

10.5

$\sin x = \frac{1}{2}x$

holden $x=0$

$x_{k+1} = x_k - \frac{(2 \sin x_k - x_k)}{(2 \cos x_k - 1)}$

$x_0 = 2 \rightarrow x_k = 1.895465 \dots$

all 10

all 10

$$10.6. \quad f(x, y) = x^2 - y + 5y(y^2 - 2x)$$

$$f' = [2x - 2\cos(y^2 - 2x) \quad | \quad 2y(\cos(y^2 - 2x) - 1)]$$

$$H = \begin{bmatrix} 2 - 4\sin(y^2 - 2x) & 4y\sin(y^2 - 2x) \\ 4y\sin(y^2 - 2x) & -4y^2(y^2 - 2x) + 2(\cos(y^2 - 2x)) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} - \begin{bmatrix} 2 - 4\sin(b^2 - 2a) & 4b\sin(b^2 - 2a) \\ 4b\sin(b^2 - 2a) & -4b^2(b^2 - 2a) + 2\cos(b^2 - 2a) \end{bmatrix}^{-1} \begin{bmatrix} 2a - 2\cos(b^2 - 2a) \\ 2b\cos(b^2 - 2a) - 1 \end{bmatrix}$$

$$(a, b) = [1, 1]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 - 4\sin(-1) & 4\sin(-1) \\ 4\sin(-1) & -4\sin(-1) + 2\cos(-1) \end{bmatrix}^{-1} \begin{bmatrix} 2 - 2\cos(-1) \\ 2\cos(-1) - 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0,6521 \\ 0,7185 \end{bmatrix}$$