SILLIPAV

Na nasledujicich radcich naleznete hodnoceni jednotlivych prikladu, kontakt na opravujiciho a jeho pripadny komentar.

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   Tohle neni LP, nemame tam jen linearni funkce neznamych

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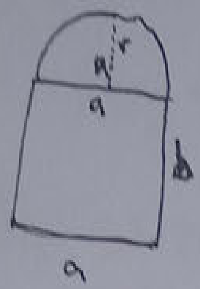
   Vypada to, jako by se minimalizovalo, protoze se vybral sloupecek se zapornym prvnim cislem. Hodnota objectivu nebude 1.5, ale -1.5 po iteraci.

celkem 22b

1/ $V_o = \pi r^2$ $\quad r = \frac{a}{2}$ $\quad 2\pi r \quad \pi r^2$ $\quad S_o = 2\pi r$ Sillinger



$V = a \cdot b + \frac{\pi r^2}{2}$

$V_\square = a \cdot b$

$\boxed{V = a \cdot b + \frac{\pi}{2} \cdot \frac{a^2}{4}}$ ✓ $\rightarrow$ $\frac{ac}{2} - \frac{a^2}{2} - \frac{\pi a^2}{4} + \frac{\pi a^2}{8} = V$

4

numerická chyba

$S = a + 2b + \frac{2\pi r}{2} = c$

$V_a' = \frac{c}{2} - \frac{2a}{2} - \frac{2\pi a}{4} - \frac{2\pi a}{8} = 0 \quad /\cdot 8$

+ formulace $\boxed{a + 2b + \frac{\pi a}{2} = c}$ ✓

max $V(a,b)$

s.t. $S(a,b) = c$

$2b = c - a - \frac{\pi a}{2}$

$b = \frac{c}{2} - \frac{a}{2} - \frac{\pi a}{4}$

$4c - 8a - 4\pi a - 2\pi a = 0$

$-8a - 6\pi a = -4c$

$-a \cdot (+8 + 6\pi) = -4c$

$a = \frac{4c}{8 + 6\pi} = \frac{2c}{4 + 3\pi}$

$a = \frac{2c}{4 + 3\pi} \quad b = \frac{c}{2} - \frac{2c - \pi c}{4 + 3\pi}$

$4b = 2c - 2a - \pi a$

$4b = 2c - \frac{4c}{4 + 3\pi} - \frac{2\pi c}{4 + 3\pi}$

$b = \frac{1}{2}c - \frac{c}{4 + \frac{8}{3}\pi} - \frac{\pi c}{8 + 6\pi}$

$a = \frac{2c}{4 + 3\pi}$

2)

Sillage

3)

Sztoyov

T_2 ?

$$T_2 = f(x_0,y_0) + f'(x_0,y_0)\begin{bmatrix} x-x_0 \\ y-y_0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} x-x_0 \\ y-y_0 \end{bmatrix}^T f''(x_0,y_0)\begin{bmatrix} x-x_0 \\ y-y_0 \end{bmatrix}$$

$$f(x,y) = 3x^2 - 2xy + x - xy - 2y^2 - 3y$$

$$\frac{f(x,y)}{\partial x} = 6x - 2y + 1 - y = 6x - 3y + 1$$

$$\frac{f(x,y)}{\partial y} = -2x - x - 4y - 3 = -3x - 4y - 3$$

$$\frac{f(x,y)}{\partial x^2} = 6$$

$$\frac{f(x,y)}{\partial y^2} = -4$$

$$\frac{f(x,y)}{\partial xy} = -3$$

$$f(\tfrac{1}{3}; -1) = \frac{3}{9} + \frac{2}{3} + \frac{1}{3} + \frac{1}{3} - 2 + 3 = \frac{5+3}{3} = \frac{8}{3}$$

$$f'(\tfrac{1}{3}; -1) = \begin{bmatrix} 6 & 0 \end{bmatrix}$$

$$f''(\tfrac{1}{3}; -1) = \begin{bmatrix} 6 & -3 \\ -3 & -4 \end{bmatrix}$$

T_2 pro $(\frac{1}{3}; -1)$ $\quad x_1 = x$, $x_2 = y$

$$T_2(x,y) = \frac{8}{3} + \begin{bmatrix} 6 & 0 \end{bmatrix} \cdot \begin{bmatrix} x-\frac{1}{3} \\ y+1 \end{bmatrix} + \frac{1}{2} \cdot \begin{bmatrix} x-\frac{1}{3} \\ y+1 \end{bmatrix}^T \cdot \begin{bmatrix} 6 & -3 \\ -3 & -4 \end{bmatrix} \cdot \begin{bmatrix} x-\frac{1}{3} \\ y+1 \end{bmatrix} =$$

$$= \frac{8}{3} + 6x - 2 + \frac{1}{2} \cdot \begin{bmatrix} \overset{6x-3y-5}{6x-2-3y-3} & \overset{-3x-4y-3}{-3x+1-4y-4} \end{bmatrix} \cdot \begin{bmatrix} x-\frac{1}{3} \\ y+1 \end{bmatrix} =$$

$$= \frac{8}{3} + 6x - 2 + \frac{1}{2} \cdot \left((6x-3y-5)\cdot(x-\tfrac{1}{3}) + (-3x-4y-3)\cdot(y+1) \right) =$$

$$6x^2 - 3xy - 5x - 2x + y + \frac{5}{3} - 3xy - 4y^2 - 3y - 3x - 4y - 3 =$$

$$= \frac{2}{3} + 6x + \frac{1}{2} \cdot \left(6x^2 - 6xy - 10x - 6y - \frac{4}{3} - 4y^2\right) =$$

$$= \frac{2}{3} + 6x + 3x^2 - 3xy - 5x - 3y - \frac{2}{3} - 2y^2$$

$$= 3x^2 - 2y^2 + x - 3y - 3xy$$

$= f(x,y)$!

mebol f už měl stupeň 2

3b

S:lli

4) $f(x,y) = x^2 - 2xy + 2y$

$$\begin{bmatrix} x \\ y \end{bmatrix}^T \cdot \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}^T \begin{bmatrix} x \\ y \end{bmatrix} + 6$$

3b $\cancel{\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}}$

$b = -2Ax_0$

$\begin{bmatrix} 0 \\ 2 \end{bmatrix} = -2 \cdot \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

$0 = -4x_0 + 2y_0$

$2 = 2x_0$

$2y_0 = 4x_0$

$0 = -4x_0 + 2y_0$

$2 = 2x_0$

$x_0 = 2$

$y_0 = 4$

a) $f(x,y) = \frac{1}{2} \begin{bmatrix} x-2 \\ y-4 \end{bmatrix}^T \cdot \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x-\cancel{2} \\ y-\cancel{4} \end{bmatrix} + \cancel{8}$

$x_0 = -2$

$y_0 = +2x_0$

$y_0 = -4$

b) $A = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} = A\lambda I$ $\begin{vmatrix} 2-\lambda & -1 \\ -1 & 0-\lambda \end{vmatrix} = -2\lambda + \lambda^2 - 1$

$\lambda^2 - 2\lambda - 1 = 0$

$D: 4+4 = 8$

$\lambda_1 \lambda_2 = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$ $\to 1+\sqrt{2}$, $\to 1-\sqrt{2}$

Vlastní čísla

2b

29b

c) $\frac{f(x,y)}{\partial x} = 2x - 2y$ $\quad 2x - 2y = 0$

$2x = 2y$

$y = 1$

$\frac{f(x,y)}{\partial y} = -2x + 2 = 0$

$x = 1$

Stac. bod $\Rightarrow$ podezřelý

$\frac{f''(x,y)}{x^2} = 2$

$\frac{f''(x,y)}{y^2} = 0$

$\frac{f''(x,y)}{xy} = -2$

$H = \begin{bmatrix} 2 & -2 \\ -2 & 0 \end{bmatrix}$

minory $2; -4$ indefinitní

sedlový bod

2b

e) Přes vlastní čísla $\lambda_1 = 1+\sqrt{2} > 0$ $\sqrt{2} > 1$

$\lambda_2 = 1-\sqrt{2} < 0$ má sedlový bod

~~sedlový bod~~

$\Downarrow$ vrstevnice je hyperbola 2b

d) ~~$f = x^2 - 2xy + 2y$~~

e) ~~$f = x^2 - 2xy + 2y$~~

d) $y^T \Lambda y = 2$

$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}^T \begin{bmatrix} 1+\sqrt{2} & \\ & 1-\sqrt{2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 + \sqrt{2}y_1 & + y_2 - \sqrt{2}y_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} > y_1^2 + \sqrt{2}y_1^2 + y_2^2 \ldots$

$y_1 = x \quad y_2 = y$

$= x^2 + \sqrt{2}x^2 + y^2 - \sqrt{2}y^2 = x^2(1+\sqrt{2}) + y^2(1-\sqrt{2}) = 2$

$\frac{x^2}{(1-\sqrt{2})} + \frac{y^2}{(1+\sqrt{2})} = \frac{2}{(1+\sqrt{2})\cdot(1-\sqrt{2})}$ #

Elipsa 0b

$\leftarrow$ ~~rovnice elipsy~~ $\frac{(x-x_0)}{a^2} + \frac{(y-y_0)}{b^2} = 1$

5)

Sillinger

$c_i > 0$

$$f(y) = \sum^{n} \frac{c_i^2}{y_i}$$

$$\min \sum^{n} \frac{c_i^2}{y_i} \qquad c_i^2 \cdot y_i^{-1}$$

θ

z.p. $y_1 + \dots + y_i = 1$ $\qquad y_i + v \geq 0$

$y_i \geq 0$

$$\begin{array}{cc} c^T & d \\ A & b \end{array}$$

$$\begin{array}{ccc|c} o_1 & \dots & o_n & 0 \\ \hline 1 & \dots & 1 & 1 \end{array}$$

$$\begin{array}{cccccc|c} c_1 & \dots & c_n & v_1 & \dots & v_n & d \\ \hline 1 & \dots & 1 & 1 & & 0 & 1 \\ 0 & \ddots & 0 & & \ddots & & 0 \\ 0 & \dots & 0 & 0 & \dots & 1 & 0 \end{array}$$

9

Sillinger

3b.

a) $\min |x-a| \quad a \in \mathbb{R}$

$x \geq 0$

pokud $a > 0$ tak $x = 0$ ✓ a se bude odčítat

$a = 0$ tak $x = 0$ ✓ a se nepřičte ani neodečte, $x < 0$ nie nesmí

$a < 0$ tak $x = 0$ ✓ nemáme jak snížit a, $x \geq 0$

$\min -a$

Špatně, minimum vyjde $\max\{-a, 0\}$

$\|x\|_1 = |x_1| + \dots |x_n|$

b) $\min \|x-a\|_1 \quad x \in \mathbb{R}^n \quad a = (a_1, \dots a_n)$

$x \geq 0$

$\min \begin{pmatrix} x-a \\ \vdots \\ x-a \end{pmatrix}$

$\min |x-a| = \cancel{\min\{|x_1-a_1|, \dots |x_n-a_n|\}}$

stejně jako v a

$\min -a_i$

7)

$\|x\|_{\frac{1}{2}} = \left(|x_1|^{\frac{1}{2}} + \dots + |x_n|^{\frac{1}{2}} \right)^2$

např. $n=2$

$\|x\|=0$ pak $x=0$

$|x|_{\frac{1}{2}} = (\sqrt{x} + \sqrt{y})^2 = x^2 + 2\sqrt{xy} + y^2 = 0$

platí

$n=1$ $\quad \sqrt{x} = \quad a=2$

$\|x\|_{\frac{1}{2}} \geq 0$ platí

$\|ax\|_{\frac{1}{2}} = |a| \, \|x\|_{\frac{1}{2}}$

$n=1$

$\|x+y\| \leq \|x\| + \|y\|$

$(\sqrt{2 \cdot x})^2 = 2x = 2 \cdot x$ platí

$(\sqrt{x+y})^2 \leq x + y$

Axiomy $n=1$

$\|x\|_{\frac{1}{2}} = 0 \Rightarrow x = 0$

$(\sqrt{x})^2 = x \overset{?}{=} 0$ ✓

$\|ax\|_{\frac{1}{2}} = |a| \|x\|_{\frac{1}{2}}$

$(\sqrt{ax})^2 = a \cdot \sqrt{x}^2$

$ax = ax$ ✓

$\|x+y\| \leq \|x\| + \|y\|$

$(\sqrt{x+y})^2 \leq x + y$ ✓

X

$\|0\| = 0$

$(\sqrt{0})^2 = 0$ ✓

$\|x\| \geq 0 \quad x \in \mathbb{R}$

nelze $\cancel{(\sqrt{ax})^2} = x = -5$ X

není norma

$\|x\|_p = \left(\sum_i |x_i|^p \right)^{1/p}$

8 tady hledáme pivot

$Ax=b$ nemusíme sledovat prom.

Sillinger

-1	0	-3	0	1	0	0	
1	0	3	0	-1	1	4	4/3
3	1	2	0	-2	0	1	1/2
-1	0	4	1	-1	0	2	1/2

} jeden z nich

-1	0	-3	0	1	0	0	/3 · 2 řádek
1	0	3	0	-1	1	4	/ -3 · 2 řádek
3	1	2	0	-2	0	1	/:2
-1	0	4	1	-1	0	2	/ -4 · 2 řádek

-1	0	-3	0	1	0	0
1	0	3	0	-1	1	4
3/2	1/2	1	0	-1	0	1/2
-1	0	4	1	-1	0	2

k zbavení se záporu

-1	0	-3	0	1	0	0
-7/2	-3/2	0	0	2	1	5/2
3/2	1/2	1	0	-1	0	1/2
-7	-2	0	1	3	0	0

starý nový průběh báze

7/2	3/2	0	0	-2	0	3/2
-7/2	-3/2	0	0	2	1	5/2
3/2	1/2	1	0	-1	0	1/2
-7	-2	0	1	3	0	0

Báz. řeš.

$x_1, x_2, x_5 = 0$

$x_3 = 5/2$

$x_4 = 1/2$

$x_6 = 0$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_3 \\ x_4 \\ x_6 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 1/2 \\ 0 \end{bmatrix}$$

hodnota kritéria je $3/2$

Bázové sloupce

x_3, x_4, x_6