SILLIPAV

Na nasledujicich radcich naleznete hodnoceni jednotlivych prikladu, kontakt na opravujiciho a jeho pripadny komentar.

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celkem 21b

Sillinger

5

1/

Hadicky 200kg/hod

gumicky 140kg/hod

LP

$\max \ 25h + 30g$

2.p.

$h \leq 6000$

$g \leq 4000$

$\frac{h}{200} + \frac{g}{140} \overset{=}{\cancel{\leq}} 40$ ~~= [illegible] 24 = [illegible]~~

$h, g \geq 0$

2)

Sillinger

$x^T F y = 0 \qquad x, y \in \mathbb{R}^3$

$F = 3 \times 3$

$x = 3$

$x^T F y \qquad a^T f$

$$(x_1, x_2, x_3) \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \left((x_1 f_{11} + x_2 f_{21} + x_3 f_{31}) ; (\ldots) ; (\ldots) \right) \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$= \begin{array}{ccc} f_{11} & f_{21} & f_{31} \\ x_1 y_1 & x_2 y_1 & x_3 y_1 \end{array}$$

$$f: \begin{pmatrix} f_{11} \\ f_{21} \\ f_{31} \\ \vdots \\ f_{33} \end{pmatrix} \qquad a: (x_1 y_1 ; x_2 y_1 ; x_3 y_1 ; x_1 y_2 , x_2 y_2 , x_3 y_2 ; x_1 y_3 ; x_2 y_3 , x_3 y_3)$$

V 4,1

3)

$U = \text{span}\{(1,2,0); (0,1,-1)\} \in \mathbb{R}^3$ 5b

a) báze $U^\perp$ ortogo. doplněk U

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right)$$

$t \cdot (-2 \;\; 1 \;\; 1)$

například $\text{span}\{(-2,1,1)\}$ ✓ 2b

normalizovaní $y = \frac{1}{\sqrt{6}} \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ ← ortogonální báze

b) $UU^T x$

$\cancel{(I - U^\perp U^{\perp T})\, x}$

$\cancel{I - \begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}}$

$x = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$x_p = x + \gamma \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ $\quad x_p \in U$

$\alpha \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 2 & 1 & -1 & 1 \\ 0 & -1 & -1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & -5 & 3 \\ 0 & -1 & -1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 6 & 0 & -6 & 3 \\ 0 & -1 & -1 & 0 \end{array}\right) = \begin{array}{c} \alpha \;\; \beta \;\; \gamma \\ \left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -6 & 3 \end{array}\right) \end{array}$$

To nevede k nejméně operací

$\alpha = 0$

$\beta = \frac{1}{2}$

$\gamma = -\frac{1}{2}$

$x_p = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} - \frac{1}{2} \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1/2 \\ -1/2 \end{pmatrix}$ ✓

3b

5) $f(x,y)=x+2y \quad X=\{(x,y)\in\mathbb{R}^2 \mid x^2-4x+y^2=0\}$

4b

Silling

Lagrangeova metoda

$\mathcal{L}: x+2y+\lambda\cdot(x^2-4x+y^2)$ ✓

$\frac{\partial \mathcal{L}}{\partial x} = 1+2\lambda x-4\lambda=0 \qquad x=\frac{+4\lambda-1}{2\lambda}$

$\frac{\partial \mathcal{L}}{\partial y} = 2+2\lambda y=0 \qquad y=\frac{-2}{2\lambda}=-\frac{1}{\lambda}$

$\frac{\partial \mathcal{L}}{\partial \lambda} = x^2-4x+y^2=0$

$\frac{16\lambda^2-8\lambda+1}{4\lambda^2}+\frac{-8\lambda+2}{\lambda}+\frac{1}{\lambda^2}=0 \quad /\cdot 4\lambda^2$

$16\lambda^2-8\lambda+1-32\lambda^2+8\lambda+4$

$-16\lambda^2+5=0$

$16\lambda^2=5$

$\lambda^2=\frac{5}{16}$

$\lambda=\pm\frac{\sqrt5}{4}$

3b

$\lambda_1=\frac{\sqrt5}{4}$

$x=\frac{\sqrt5-1}{\frac{\sqrt5}{2}}=\frac{2\sqrt5-2}{\sqrt5}$

$y=-\frac{1}{\frac{\sqrt5}{4}}=-\frac{4}{\sqrt5}$

$\left(\frac{2\sqrt5-2}{\sqrt5}; -\frac{4}{\sqrt5}\right)$ Minimum

$\lambda_2=-\frac{\sqrt5}{4}$

$x=\frac{-\sqrt5-1}{-\frac{\sqrt5}{2}}=\frac{-2\sqrt5-2}{-\sqrt5}=\frac{2\sqrt5+2}{\sqrt5}$

$y=\frac{4}{\sqrt5}$ → $\left(\frac{2\sqrt5+2}{\sqrt5}; \frac{4}{\sqrt5}\right)$

Maximum

$\frac{\partial \mathcal{L}(x,y,\lambda)}{\partial(x,y,\lambda)^2}=\begin{bmatrix} 2\lambda & 0 & 2x \\ 0 & 2\lambda & \\ 2x & & \end{bmatrix}$

Rovina seče v kouli

vyjdou dva body

nahoře a dole

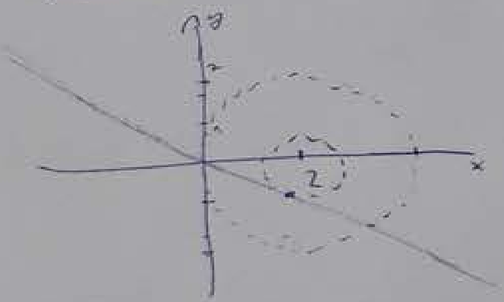
za daných podmínek to jsou globální ~~minima~~ extrémy na množině X

ve 2D koule ?

b) Vrstevnice

Hodnoty leží ve kouli

bude to bod kde se posunutí ~~...~~ přímka stane tečnou

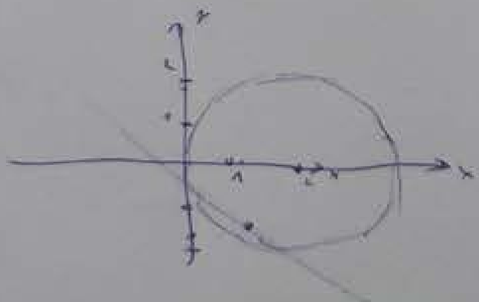


$y=-\frac{x}{2}$

1b

$\left(\frac{2\sqrt3+2}{\sqrt5}; \frac{4}{\sqrt5}\right)$

$\left(\frac{2\sqrt5-2}{\sqrt5}; -\frac{4}{\sqrt5}\right)$



5)

$(x_1, y_1, z_1) \dots (x_n, y_n, z_n)$

$a, b \in \mathbb{R}$?

a) pokud $n < 3$ pak to nestačí na jednoznačné řešení, nekonečně mnoho

pokud $n = 3$ mělo by být jedno řešení za podmínky LN sloupce

pokud $n > 3$ není řešení

X Co bylo bude ∅ 1. termín

$\Downarrow \Rightarrow$? ?

b) $M = \begin{bmatrix} x_1 & y_1 \\ \vdots & \vdots \\ x_n & y_n \end{bmatrix}$

$z = \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix}$

$tmp = A \backslash z$

$a = tmp[1]$

$b = tmp[2]$

[illegible]

6) Newtonova metoda

Silliger

$$x_{k+1} = x_k - g'(x_k)^{-1} \cdot g(x_k) \checkmark$$

Toto je špatně

$$g(x) = \begin{bmatrix} x^T A x - a \\ x^T B x - b \end{bmatrix} \checkmark \qquad \frac{g'(x)}{\partial x} = \begin{bmatrix} x^T(A+A^T) & 0 \\ 0 & x^T(B+B^T) \end{bmatrix}$$

$$g'(x) = \begin{bmatrix} x^T(A+A^T) \\ x^T(B+B^T) \end{bmatrix} \in \mathbb{R}^{2\times 2}$$

$$x_{k+1} = [x_k] - \begin{bmatrix} x_k^T(A+A^T) & x_k^T(A+A^T) \\ x_k^T(B+B^T) & x_k^T(B+B^T) \end{bmatrix}^{-1} \cdot \begin{bmatrix} x_k^T A x_k - a \\ x_k^T B x_k - b \end{bmatrix}$$

Tato matice je z $\mathbb{R}^{2\times 4}$, nelze takto invertovat.

1b.

7)

$f: \mathbb{R}^n \to \mathbb{R}$ Silly

$a \neq 0 \in \mathbb{R}^n$

$b \in \mathbb{R}$

a) $\min\{\|\cancel{x}\,x - H\|\} \Rightarrow \min\{\| \quad - a^T x - b\|\}$?   vzdálenost nadroviny H od počátku

0

z.p. $a \neq 0$

b)

c) $\min |(1-x)^2+(1-y)^2| \; | y=2x-3$ Ale jak jste našel tento bod (1,1) ?

0

$(1-x)^2+(1-2x+3)^2 = (1-x)^2+(4-2x)^2$

$f = 1-2x+x^2+16-16x+4x^2$

$\frac{\partial f}{\partial x} = -2+2x-16+8x$

$10x = 18$

$x = 1{,}8 \Rightarrow y = 0{,}6 \Rightarrow \sqrt{(1-1{,}8)^2+(1-0{,}6)^2} = \frac{2\sqrt{5}}{5}$

8)

Sillinger

$\min \{ b^T x \mid x \in \mathbb{R}^n, 0 \le Ax \le 1 \}$

$Ax \ge 0 : c_1$

$Ax \le 1 : c_2$

$\min b^T x = \min \sum b_i x_i$

z.p. $Ax \ge 0$ z.p. ....

$Ax \le 1 \;/\cdot(-1)$

$-Ax \ge -1$

$c = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \\ \vdots \\ -1 \end{pmatrix}$

a)

Duální úloha

2

$\max$ $c^T y$

z.p. $y \ge 0$

$A^T y \le b$

$[A^T \; -A^T] y = b$

... ale maly je

min $b^T x$		max $c^T y$
z.p. $Ax \ge b$		$y \ge 0$
$x \ge 0$	2	$A^T y \le c$

b) A mít plnou hodnost LN řádky a sloupce ?

0 $b \ne 0$

Úloha by mohla být neomezená nebo

$\sum_{j \in J} a_{ij} x_j = b$ nebo $y_i = 0$ $\forall i \in I$

$x_j = 0$ nebo $\sum_{i \in I} a_{ij} y_i = c_i$ $\forall j \in J$