

$$f(x,y) = (x-3; y+2) \begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} x-3 \\ y+2 \end{pmatrix} + 0$$


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$$b) \det \begin{pmatrix} 1-\lambda & -3 \\ -3 & 1-\lambda \end{pmatrix} = (1-\lambda)^2 - 9 = 1 - 2\lambda + \lambda^2 - 9 =$$

$$= \lambda^2 - 2\lambda - 8 = (\lambda - 4)(\lambda + 2)$$

$$\underline{\lambda_1 = 4}$$

$$\underline{\lambda_2 = -2}$$

$$\underline{\lambda = 4}: \begin{pmatrix} -3 & -3 \\ -3 & -3 \end{pmatrix}$$

$$\begin{pmatrix} -3 & -3 & | & 0 \\ -3 & -3 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \quad v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\underline{\lambda = -2}: \begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -3 & | & 0 \\ -3 & 3 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \quad w = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\underline{\begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}}$$


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$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}^{-1} \Rightarrow \begin{pmatrix} 1 & 1 & | & 1 & 0 \\ -1 & 1 & | & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 1 & 0 \\ 0 & 2 & | & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 1 & 0 \\ 0 & 1 & | & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & | & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & | & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$