

a) $f(x,y) = e^x + |y| - 2$ konvexní?

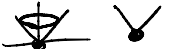
b) $\{(x,y) \in \mathbb{R}^2 \mid -1 \leq x \leq 1; x^2 \leq y \leq 1; |x| \leq y\}$ konvexní polyedr?

c) $f(x,y) = \sqrt{x^2+y^2} + |x| + |y|$ má lok. min, lok. max, žádný, nebo globální?

d) Každý bod $[-1,1]$ je reg. bod $f(x) = (x-1)^{10}$

a)  +  - **ANO**

b) **ANO** uzavřená polopr.

c)  **NE**

d)  **ANO** z obrázku

e) Reg. bod: deriv. 0
 NE \leftarrow $\begin{cases} \text{žádný: } LU \equiv [-1,1] \\ -1 \rightarrow (-2)^{10} \\ 1 \rightarrow 0 \rightarrow f' = 0 \end{cases}$

Matice $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$ rozložte ve tvaru SVD $A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T$:

$A^T A = \begin{bmatrix} -3 & 6 & 6 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix} \rightarrow \det(A - \lambda I) = \det \begin{bmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{bmatrix} = 81 \cdot 9 - 81\lambda - 9\lambda + \lambda^2 - 27^2 = \lambda^2 - 90\lambda$

$v_{\text{sing}} = \frac{1}{\|v\|} \cdot v = \frac{1}{\sqrt{10}} \cdot \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\lambda_2: \begin{bmatrix} -9 & -27 \\ -27 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \xrightarrow{u_2, v_2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda(\lambda - 90) = 0$

$\lambda_1 = 0 \rightarrow \sqrt{0} = 0 \leftarrow \text{nulla nejednotka}$

$\lambda_2 = 90 \rightarrow \sqrt{90} = 3\sqrt{10} = \sigma$

$u = \frac{1}{\sigma_1} \cdot A \cdot v = \frac{1}{3\sqrt{10}} \cdot \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} \cdot \frac{1}{\sqrt{10}} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \frac{1}{30} \begin{bmatrix} -10 \\ 20 \\ 20 \end{bmatrix}$