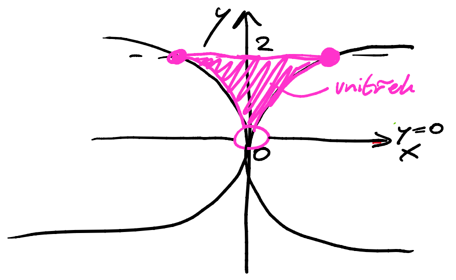
C2 vnitřek, hranice uzávěr

pondělí 26. září 2022 12:50

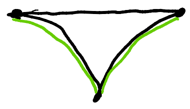
Př: $M: 0 < y \leq 2 \ \wedge \ -y^2 \leq x \leq y^2$

Určete vnitřek, hranice a uzávěr

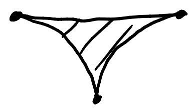


vnitřek M: $M^\circ: 0 < y < 2 \wedge -y^2 < x < y^2$

hranice M: $\partial M: \Big(0 \leq y \leq 2 \ \wedge \ (x = y^2 \vee x = -y^2)\Big) \vee \left(y = 2 \wedge -y^2 \leq x \leq y^2\right)$

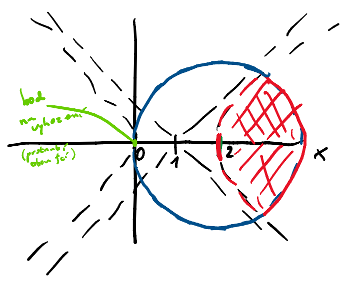


uzávěr M: $\overline{M} = M \cup \{(0,0)\}$

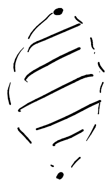


$\overline{M}: 0 \leq y \leq 2 \ \wedge \ -y^2 \leq x \leq y^2$

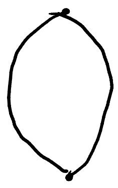
Př: $M: (x-1)^2 - y^2 > 1 \ \wedge \ \underline{(x-2)^2 + y^2 \leq 4}$



Vnitřek M: $M^\circ: (x-1)^2 - y^2 > 1 \ \wedge \ (x-2)^2 + y^2 < 4$

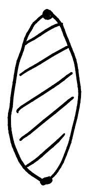


Hranice M: $\partial M: \Big(\underbrace{\left((x-1)^2 - y^2 = 1 \ \wedge \ (x-2)^2 + y^2 < 4\right)}_{\emptyset} \ \vee \ \underbrace{\left((x^2-2)^2 + y^2 = 4 \ \wedge \ (x-1)^2 - y^2 \geq 1\right)}_{\emptyset}\Big) \setminus \{(0,0)\}$

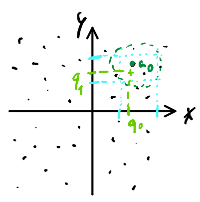


vyhazujeme

Uzávěr M: $\overline{M}: \left((x-1)^2 - y^2 \geq 1 \ \wedge \ (x-2)^2 + y^2 \leq 4\right) \setminus \{(0,0)\}$



Př: $M = \mathbb{Q}^2$ v $\mathbb{R}^2$

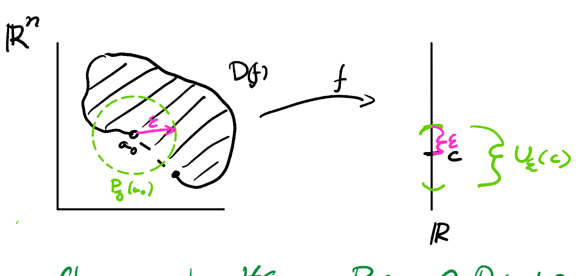


$M^\circ = \emptyset$

$\partial M = \mathbb{R}^2$

$(q_0, q_1) \in U_\varepsilon(a_0) \cap \mathbb{Q}^2$

LIMITY

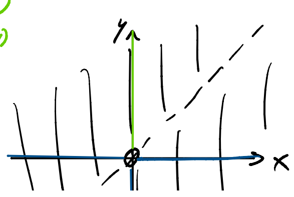


Chceme, aby $\forall \varepsilon > 0 \ \ P_\varepsilon(a_0) \cap D(f) \neq \emptyset$ (tj. a_0 je hromadný bod množiny $D(f)$)

$\lim\limits_{a \to a_0} f(a) = c \iff \forall \varepsilon > 0 \ \exists \delta > 0$

$\forall a \in D(f) \quad a \in P_\delta(a) \Rightarrow f(a) \in U_\varepsilon(c)$

Př: $\lim\limits_{(x,y) \to (0,0)} \underbrace{\frac{x+y}{x-y}}_{f(x,y)}$ $\quad D(f): x \neq y$



Proti příklad

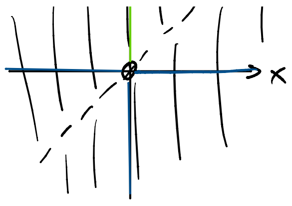
Zkusíme přiblížení po ose x $(y=0)$:

Vezmeme $g(x) = f(x,0) = \frac{x+0}{x-0} = 1$, $x \neq 0$ $\Rightarrow \lim\limits_{x \to 0} g(x) = 1$

Osa y:

$h(y) = f(0,y) = \frac{0+y}{0-y} = -1$, $y \neq 0$ $\rightarrow \lim\limits_{y \to 0} h(y) = -1$

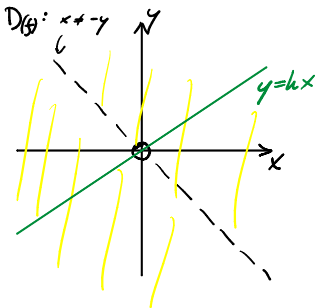
} dvě různé limity $\Rightarrow$ původní limita neexistuje



osa y: $h(y) = f(0,y) = \frac{0+y}{0-y} = -1 \longrightarrow \lim_{y\to 0} h(y) = -1$

$y \neq 0$

Př: $\lim\limits_{(x,y)\to(0,0)} \underbrace{\frac{(x-y)^2}{x+y}}_{f(x,y)}$



Zkusme přiblížení po $\overbrace{\text{přímce } y=kx}^{\text{parametrizace přímky}}$, $k \neq -1$

$$f(x,kx) = \frac{(x-kx)^2}{x+kx} = \frac{x^{\cancel{2}}(1-k)^2}{\cancel{x}(1+k)} = x \cdot \frac{(1-k)^2}{1+k} \xrightarrow[x\to 0]{} 0$$

Zvolíme M: $x=-y \wedge (x,y) \neq (0,0) \overset{\text{kritérium}}{\Longrightarrow}$ limita neexistuje

Kritérium neexistence (konečné) limity

Nechť $M \subseteq \mathbb{R}^n$ je taková množina, že: (vyšetřujeme $\lim\limits_{a\to a_0} \frac{h(a)}{g(a)}$)

- $\forall a \in M \;\; h(a) \neq 0 \wedge g(a) \neq 0$
- fce h a g jsou spojité na nějakém okolí $U_\varepsilon(a_0)$ bodu a_0
- $a_0 \in \overline{M}$

Pak konečná $\lim\limits_{a\to a_0} \frac{h(a)}{g(a)}$ neexistuje