(5.2) $\max\{\sum_{i=1}^n c_i x_i ;\ -1 \le x_i \le 1\}$

a) pro $c_i > 0$, $x_i = 1$
pro $c_i < 0$, $x_i = -1$
pro $c_i = 0$, x_i libovolné
$\rightarrow = \sum_{i=1}^n |c_i|$

b) $\max c^T x \rightarrow \min 1^T y - 1^T z \quad , y \ge 0 \ , z \ge 0$

$$\begin{bmatrix} \cdots & 0 \\ 1 & \\ & \ddots \\ 0 & 1 \\ 1 & 0 \\ & \ddots \\ 0 & \cdots \end{bmatrix} x \le \begin{bmatrix} 1 \\ \vdots \\ 1 \\ -1 \\ \vdots \\ -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \cdots & 0 & 1 & \cdots & 0 \\ & \ddots & \vdots & \vdots & \ddots & \\ 0 & \cdots & 1 & 0 & \cdots & \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} = c$$

$\min\{x_i - z_i ;\ x_i + z_i = c_i , x_i \ge 0 , z_i \ge 0\} \quad i = 1, 2, \dots, n$
$\rightarrow$ pro $c_i > 0$: $x_i + 0 = c_i$ neboť $0 + z_i = c_i$ ($z_i \le 0$)
pro $c_i < 0$: $0 + z_i = c_i \rightarrow x_i + z_i = |c_i|$

$\rightarrow \sum |c_i|$.

c) podmínky: $x_i = 1$ nebo $y_i = 0$; $x_i = -1$ nebo $z_i = 0$

d) $n = 3$, $c = \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix}$
$x = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \rightarrow y = \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}, z = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

(5.3) a) $\min 2x_1 - 3x_2 + x_4$ $\rightarrow$ $\max 5y_2 + 6y_3$

$x_1 - x_2 - x_3 \ge 0$
$-x_1 + 2x_2 - x_3 \le 5$
$2x_1 - x_2 - x_3 + 2x_4 = 6$
$x_1, x_2, x_3, x_4 \ge 0$

$y_1 - y_2 + 2y_3 \le 2$
$-y_1 + 2y_2 - y_3 \le 0$
$-y_1 - y_2 - y_3 \le -3$
$2y_3 \le 1$
$y_1 \ge 0$
$y_2 \le 0$

$$\begin{pmatrix} 1 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ 2 & -1 & -1 & 2 \end{pmatrix}$$

podmínky: $y_1(x_1 - x_2 - x_3) = 0$
$y_2(-x_1 + 2x_2 - x_3 - 5) = 0$

$x_1(y_1 - y_2 + 2y_3 - 2) = 0$
$x_2(-y_1 + 2y_2 - y_3) = 0$
$x_3(-y_1 - y_2 - y_3 + 3) = 0$
$x_4(2y_3 - 1) = 0$

b) $\min_{x \in \mathbb{R}} \max_{i=1,\dots,n} |a_i - x|$

$\min\{z : z \in \mathbb{R}, x \in \mathbb{R}, z \ge a_i - x, z \ge x - a_i\}$

$\max\{a^T u - a^T v ;\ u \ge 0, v \ge 0\}$

$$\begin{bmatrix} -1 & \cdots & -1 & 1 & \cdots & 1 \\ 1 & \cdots & 1 & -1 & \cdots & -1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\min z$

$$\begin{bmatrix} 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \\ -1 & 1 \\ \vdots & \vdots \\ -1 & 1 \end{bmatrix} \begin{bmatrix} z \\ x \end{bmatrix} \ge \begin{bmatrix} a_1 \\ \vdots \\ a_n \\ -a_1 \\ \vdots \\ -a_n \end{bmatrix}$$