① $p(x,y) = x^2 - 6xy - 18x + y^2 + 22y + 49 =$

a)

$$= (x\ y)\underbrace{\begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}}_{D=A}\begin{pmatrix} x \\ y \end{pmatrix} + \underbrace{(-18\ 22)}_{b^T}\begin{pmatrix} x \\ y \end{pmatrix} + \underbrace{49}_{c}$$

$$b^* = -2Dx_0 \Rightarrow \begin{pmatrix} -18 \\ 22 \end{pmatrix} = -2\begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}x_0 = \begin{pmatrix} -2 & 6 \\ 6 & -2 \end{pmatrix}x_0$$

$$\left(\begin{array}{cc|c} -2 & 6 & -18 \\ 6 & -2 & 22 \end{array}\right) \sim \begin{matrix} \frac{1}{2}① \\ \frac{1}{2}② \end{matrix} \left(\begin{array}{cc|c} -1 & 3 & -9 \\ 3 & -1 & 11 \end{array}\right) \sim$$

$$\sim \begin{matrix} \\ ②+3\cdot① \end{matrix} \left(\begin{array}{cc|c} -1 & 3 & -9 \\ 0 & 8 & -16 \end{array}\right) \sim \begin{matrix} \\ \frac{1}{8}② \end{matrix} \left(\begin{array}{cc|c} -1 & 3 & -9 \\ 0 & 1 & -2 \end{array}\right)$$

$$x_0 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$c = x_0^T D x_0 + y_0$$

$$49 = (3\ -2)\begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix} + y_0 = (3+6\ -9-2)\begin{pmatrix} 3 \\ -2 \end{pmatrix} + y_0 =$$

$$= (9\ -11)\begin{pmatrix} 3 \\ -2 \end{pmatrix} + y_0 = 27 + 22 + y_0 = 49$$

$$49 + y_0 = 49 \Rightarrow y_0 = 0$$