1. Find the integral of the function, $f(x) = x^3 + x$ using Monte Carlo integration where $x \in [0, 2]$

$$I \approx \int_0^2 dx \frac{1}{N} \sum_{i=1}^{N} f(x_i)$$

use the above formula to calculate the integral. Calculate the integral say (n) times and take the average. Now plot the absolute error as a function of n.

2. Find the integral

$$\int_0^2 \left(3x^2 - 5x + 7\right) dx$$

using Monte Carlo integration in both normal (using for loop) and vectorized code. Compare the computation time and the solution, to find which method is faster and accurate.

3. Solving the problem of particle in a 1-D box we get the wavefunction as

$$\psi = A \sin\left(\frac{n\pi x}{L}\right)$$

Where the normalizing constant A can be calculated from the integral for $n = 1$

$$A^2 \int_0^L \sin^2\left(\frac{\pi x}{L}\right) dx = 1$$

Use Monte Carlo integration for a box of length $L = 1$ to find A and compare it with analytic solution, $A = \sqrt{2}$.

4. Radioactive decay of a material is given as

$$\frac{dN}{dt} = -\lambda N$$

where N is the amount of the substance, λ and t is the decay constant and time respectively. Take the initial amount of the substance as 10 and step size as 0.01. Calculate N for different values of t using Euler's method and plot it along with the exact solution. Compare the results and plot the absolute error.

5. Consider the differential equation

$$\frac{dx}{dt} = -3x \cos t$$

with initial condition $x(0) = 1$. Solve for different values of t. Compare the computation time from Euler's method and Euler's method in vectorized code. Also, compare the results with exact solution.

6. Consider an LR series circuit which is excited by a battery of emf V. The differential equation for the growth of current in the circuit is given as

$$\frac{di}{dt} = \frac{V}{L} - \frac{Ri}{L}$$

where R, V and L are resistance, voltage and inductance. Calculate current i for different instant of time using modified Euler's method (take initial i as 0) and plot the result along with the exact solution. Compare the results with inbuilt python function in scipy 'odeint' by plotting the absolute error.

7. Consider a simple harmonic oscillator with differential equation

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

where x and t is position and time respectively and ω is the angular frequency of oscillation. Solve for x and v, at $t = 0$ $x = 0$ and $v = 1$. Plot x vs t and v vs t, using RK 2^{nd} order method.

8. Van der Pol's equation given by 2^{nd} order differential equation is given by

$$\frac{d^2x}{dt^2} - \mu(1-x^2)\frac{dx}{dt} + x = 0$$

Given initial conditions are $x = 0.5$, $\frac{dx}{dt} = 0$ at $t = 0$. Using RK 4^{th} order method plot x vs $\frac{dx}{dt}$ for $t = [0, 2]$ taking step size 0.1 and $\mu = 1$.

9. Write a vectorized code for RK 2^{nd} order method and solve $Q.4$ with it. Compare the computation time for the two methods to find which method is faster.

10. A mass-spring-damper system is a second order equation with constant coefficients:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

where m, c and k are the mass, damping and stiffness coefficients. Also $F(t)$ is the forcing function of the system. Take $m = 1, k = 10$ for a time period $t \in [0, 20]$ for different damping (say $c = 0, 1, 10$) and calculate the displacement using RK 4^{th} order method and plot it.