1. Use third and seventh order polynomial interpolation to interpolate Runges function:

$$f(x) = \frac{1}{1+25x^2}$$

at the $n = 11$ points $x_i = -1.0, -0.8, \ldots, 0.8, 1.0$. Compare the results with the linear interpolation.

2. An object is standing on a plane whose slope varies with constant velocity ω. After t seconds its position is

$$s(t,\omega) = \frac{g}{2\omega^2}[\sinh(\omega t) - \sin(\omega t)],$$

where $g = 9.8$ m/s^2 denotes the gravity acceleration. Write a function script which takes in the values s and t and returns the value of ω calculated using the secant method with a tolerance of 10^{-5}. As a test case, assume that the object has moved by 1 meter in 1 second, compute the corresponding value of ω.

3. The variation of gravitational acceleration g with altitude y is given by:

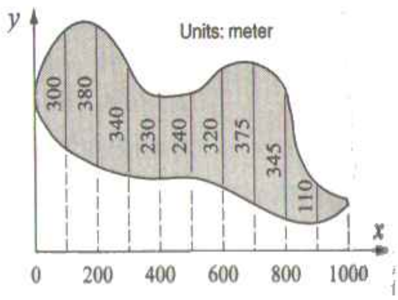
$$g = g_0 \frac{R^2}{(R+y)^2}$$

where $R = 6371$ km is the radius of the earth, and g_0 = 9.81 m/s^2 is the gravitational acceleration at sea level. The change in the gravitational potential energy, ΔU of an object that is raised up from the earth is given by:

$$\Delta U = \int_0^n mg\, dy.$$

Determine the change in the potential energy of a satellite with a mass of 500 kg that is raised from the surface of earth to a height of 800 km.

4. The surface area of a lake is estimated by measuring the width of the lake at intervals of 100 m. The measurements are shown in the figure. Use numerical integration to estimate the area of the lake.



5. The intensity profile in a single-slit diffraction is given by $I(\theta) = I_0 \mathrm{sinc}^2(\frac{d}{\lambda}\sin\theta)$, where $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$ if $x \neq 0$ and $\mathrm{sinc}(0) = 1$. Assuming I_0 and λ as inputs, write a script to write in a text (ascii) file, the table of values of θ and I.

6. The normalized wave-functions of quantum harmonic oscillator can be written as $\psi_n(x) = A_n\, e^{-x^2/2}\, H_n(x)$. $H_n(x)$ are the Hermite polynomials which can be evaluated using the series form

$$H_n(x) = \sum_{s=0}^{n/2} (-1)^s\, (2x)^{n-2s} \frac{n!}{(n-2s)!\, s!}\ .$$

Plot $H_n(x)$ for $n = 1, 2$ and 3 in the range $-5 \leq x \leq 5$.

7. The energy gap E at temperature T is given by the BCS theory as

$$E(T) = 3.52 k_B T_c \sqrt{1 - (T/T_c)},$$

where $k_B = 1.38 \times 10^{-23}$ J/K. Write a script to plot $E(T)$ for given T_c.

8. Write a function definition to transpose a matrix.

9. For $y = \int_{-1}^{1} 3x^2 + 2x\, dx$, compare the results of trapezoidal rule ($n = 20$), Simpson's rule ($n = 20$) and 5-point Gauss quadrature.

10. For $y = \int_{-1}^{1} 3x^5 + 2x^3\, dx$, compare the number of steps (n) required to achieve an accuracy in result up to the second decimal place, for the trapezoidal rule and Simpson's rule.

11. A file named `expt.dat` comprises the values of C and V written as a 10×2 matrix. Write a program which writes the values of C, V, Q and E in a file `ce.out` in a tabular form (10×4 matrix) using the relations $C = Q/V$ and $E = \frac{1}{2}CV^2$.

12. The distance covered by the rocket from $t = t_0$ to $t = t_1$ is

$$x = \int_{t_0}^{t_1} f(t)dt \text{ , with } f(t) = \left[u \ln\left(\frac{m_0}{m_0 - qt}\right) - gt \right].$$

Write a function definition which reads t and returns $f(t)$. You may try all the numerical integration function definitions that you developed, to evaluate x. The values of m_0, u, q, t_0, t_1 and $g = 9.8$ m/s^2, have to be defined as global in the main script. A rocket goes vertically up and expels fuel at a velocity $u = 2000$ m/s at a consumption rate of $q = 2100$ kg/s. Plot the distance travelled by the rocket in first 30 seconds as a function of the initial mass of the rocket which ranges from 100,000 kg to 200,000 kg.