

Tiling With Dominoes

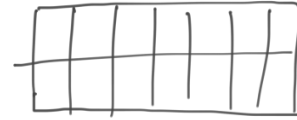
Our "tiles" : 



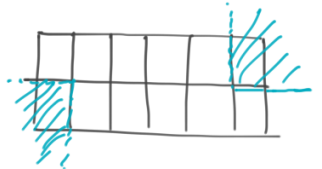
If you are in Breakout
Room n , go to page
 $5n + 1$

Qn : Can a $1 \times n$ grid be completely tiled using dominoes ?

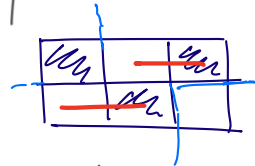
Qn: What about a $2 \times n$ grid?



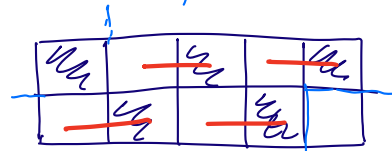
Qn: A $2 \times n$ grid with opposite corners removed?



Ans: If n odd, can do:



If n even, can't, because of the colouring like a checkerboard argument.



It's quite possible here that they'll figure out you can't do it for even n using a case-by-case analysis / argument.

In this case, ask if one can tile an $n \times n$ board with dominoes, for n even.

Qn: A $2 \times n$ grid with its opposite corners removed
can be tiled when n is odd.
How many different ways are there to tile it?

(Only one: working our way up from ~~the~~ a
corner square, we're forced to place each tile

Hint: Think about the tile
covering the first square horizontally)

Qn: A $2 \times n$ grid can be tiled with dominoes.
How many different ways are there to tile it?

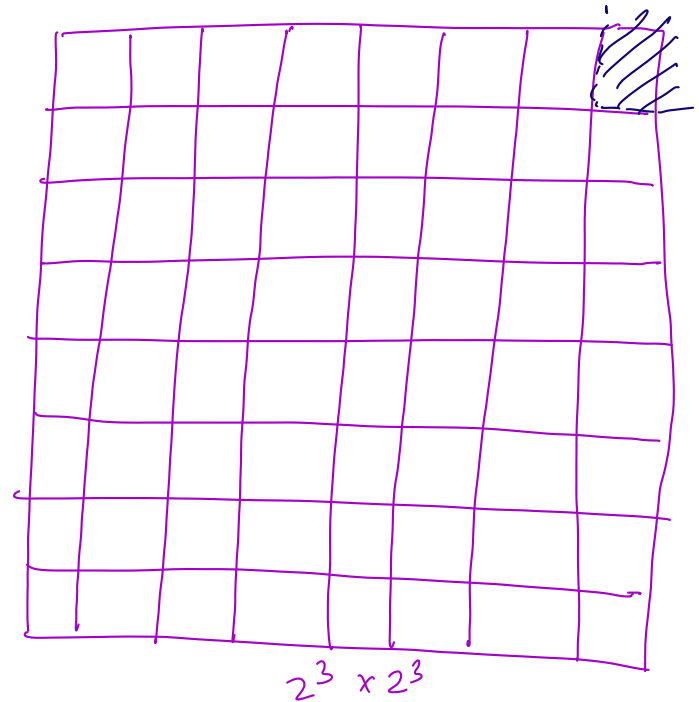
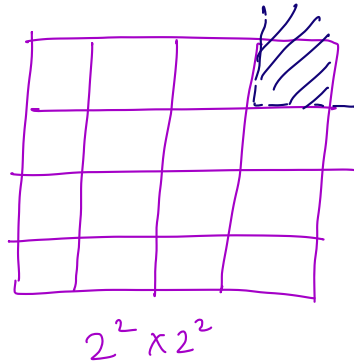
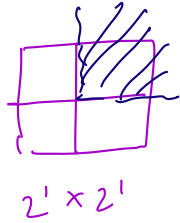
(Fibonacci nos :

first tile could be
horizontal or vertical;
that decides the # ways
for the rest of the board)

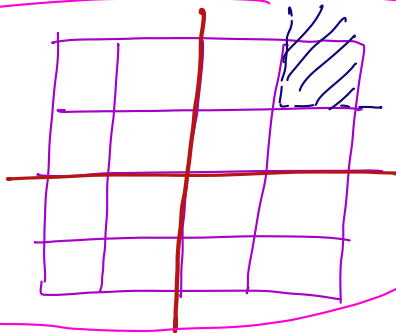
Hint: Think about the tile
covering the first square

Qn : New tiles : 

$2^n \times 2^n$ grid with a corner removed



Hint :

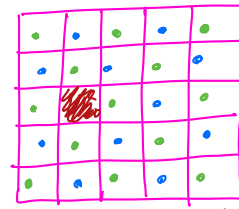
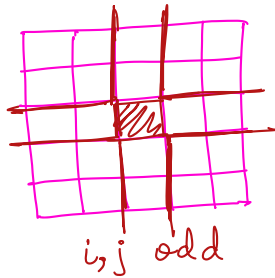
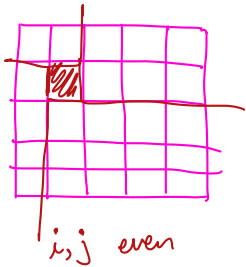


Follow-up Qn : Does it matter which 1×1 square we remove?
What if it isn't a corner? Can we still tile it?

If there's still time, or some leftover time after the domino problems but not enough for the domino example, ask the following:

- In general, if n is odd, is it possible to tile an $n \times n$ board with 2×1 tiles if one square is covered with a 1×1 tile? Does it matter which square is covered? *Yes it matters; use a checkerboard colouring argument - see below*
- In general, if n is even, is it possible to tile an $n \times n$ board with 2×1 tiles if two squares are removed? Does it matter which two squares are removed? *Yes it matters; checkerboard colouring argument*

Say (i, j) th square has been removed



$(i$ odd, j even case similar)

Can't do it - we took away a blue square and so now there's an imbalance; we have 13 green squares and 11 blue ones