Module 6: 2D Matrix Transformations

CS 476: Computer Graphics

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Translation Matrix

$$f((x, y)) = (x + a, y + b)$$

$$\mathbb{R}^2 \to \mathbb{R}^2$$

$$\begin{bmatrix} - & - \\ - & - \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + a \\ y + b \end{bmatrix}$$

??

Translation Matrix

$$f((x, y)) = (x + a, y + b)$$

$$\mathbb{R}^2 \to \mathbb{R}^2$$

$$\begin{bmatrix} - & - \\ - & - \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + a \\ y + b \end{bmatrix}$$

- This is impossible!

Example:

$$f((0, 0)) = (a, b)$$

but every matrix times $(0, 0)$ gives $(0, 0)$, so this is a contradiction

Homogenous Coordinates

We have to go to 3x3 matrices to accomplish this.
Example: Pure translation with homogenous coordinates

$$\begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix} \\ 1 \end{bmatrix}$$

Homogenous Coordinates

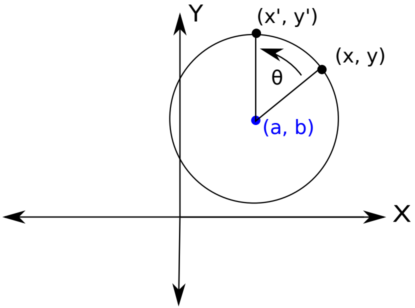
General 2D transformation + translation

$$\begin{bmatrix} a & b & T_x \\ c & d & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix} \\ 1 \end{bmatrix}$$

We have some extra baggage, but we have more freedom now

Group Raffle Point Question

Write down a matrix which rotates a vector around a point



Formulas to help you

$$T_{(x,y)} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix}, R_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$