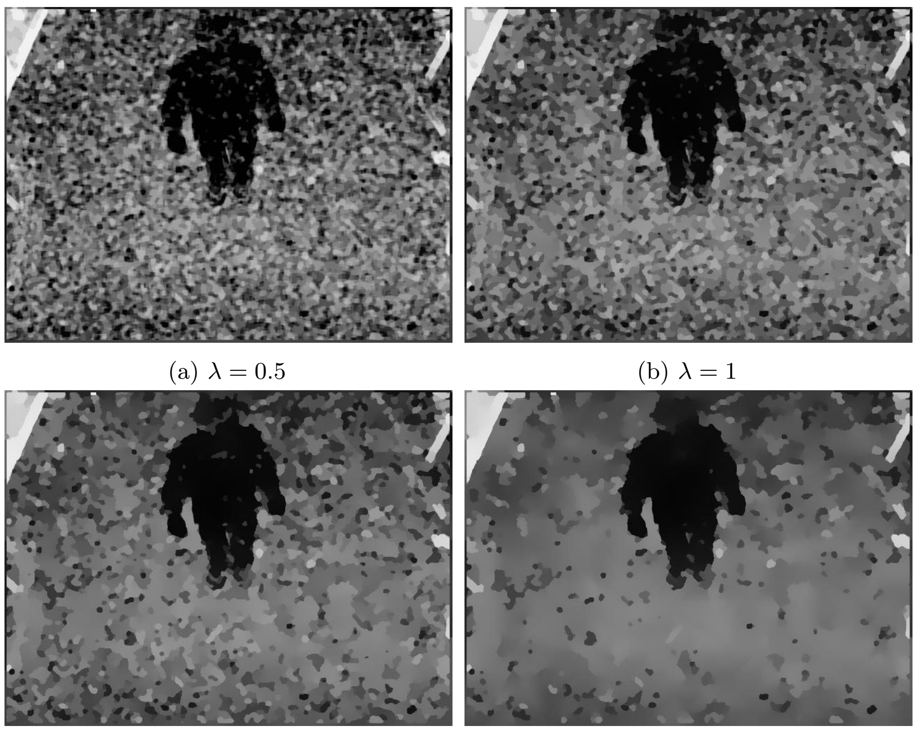
Intelligente Sehsysteme - Übungsblatt 4

Hendrik Walther, Jan Konrad, Berkan Kaya

3 ImageToolBox: Diffusionsfilter



(a) $\lambda = 0.5$ (b) $\lambda = 1$

(c) $\lambda = 1.5$ (d) $\lambda = 2$

Abb. 1: Isotropes inhomogenes Diffusionsfilter mit $\epsilon_0 = 1$ und 500 Iterationen

4 Tensorberechnung fur anisotropes inhomogenes Diffusionsfilter

A. $$\begin{pmatrix} e_{1,1} \\ e_{1,2} \end{pmatrix} = \frac{\nabla u}{\|\nabla u\|}$$

$$\begin{pmatrix} e_{2,1} \\ e_{2,2} \end{pmatrix} = \begin{pmatrix} e_{1,2} \\ -e_{1,1} \end{pmatrix}$$

$$\lambda_1 = \epsilon_0 \frac{\lambda^2}{\|\nabla u\|^2 + \lambda^2}$$

$$\lambda_2 = 1$$

$$\nabla u(x,y) \approx \begin{pmatrix} I(x+1,y) - I(x-1,y) \\ I(x,y+1) - I(x,y-1) \end{pmatrix}$$

Für $x, y = 3$ gilt:

$$\nabla u(3,3) \approx \begin{pmatrix} I(4,3) - I(2,3) \\ I(3,4) - I(3,2) \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$$

$$\|\nabla u(3,3)\| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$$

$$\begin{pmatrix} e_{1,1} \\ e_{1,2} \end{pmatrix} = \frac{1}{10\sqrt{2}} \begin{pmatrix} 10 \\ 10 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} e_{2,1} \\ e_{2,2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\lambda_1 = \epsilon_0 \frac{\lambda^2}{200 + \lambda^2}$$

$$\text{mit } \epsilon_0 = \lambda = 1$$

$$\lambda_1 = \frac{1}{201}$$

$$\lambda_2 = 1$$

B. $$\mathbf{D} = \begin{pmatrix} e_{1,1} & e_{2,1} \\ e_{1,2} & e_{2,2} \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} e_{1,1} & e_{1,2} \\ e_{2,1} & e_{2,2} \end{pmatrix}$$

$$= \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right) \begin{pmatrix} \frac{1}{201} & 0 \\ 0 & 1 \end{pmatrix} \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right)$$

$$= \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right) \left(\frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{201} & 0 \\ 0 & -1 \end{pmatrix} \right)$$

$$= \begin{pmatrix} \frac{1}{402} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

C. $\mathbf{D}$ ist positiv definit, da für alle Eigenvektoren $\lambda_1, \lambda_2 > 0$ gilt.