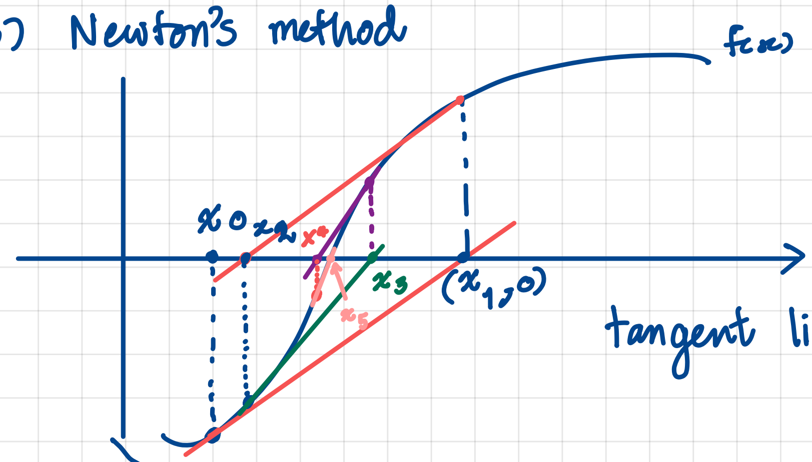
Root finding

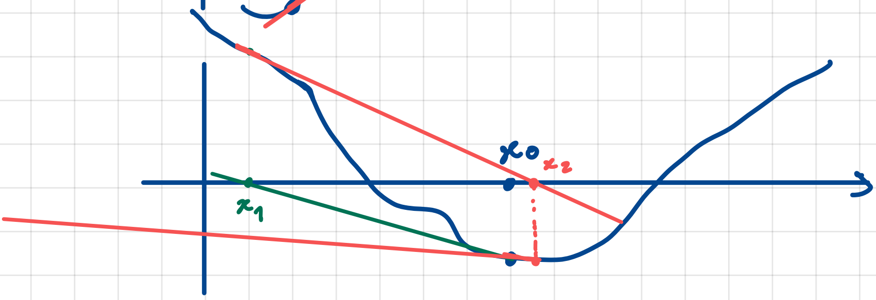
$\boxed{f(x) = 0}$

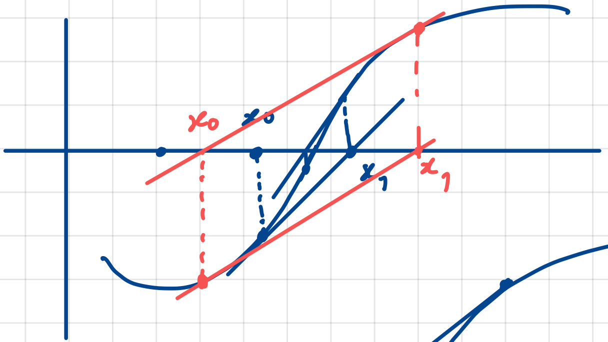
1) Bisection method
2) Method of false position
3) Newton's method

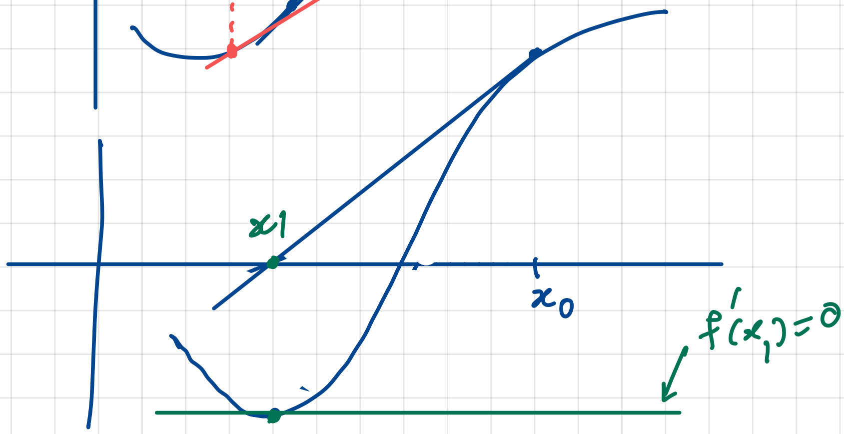
$$\left.\begin{array}{l}\text{1) Bisection method}\\ \text{2) Method of false position}\end{array}\right\} \quad f(a)\cdot f(b) < 0$$

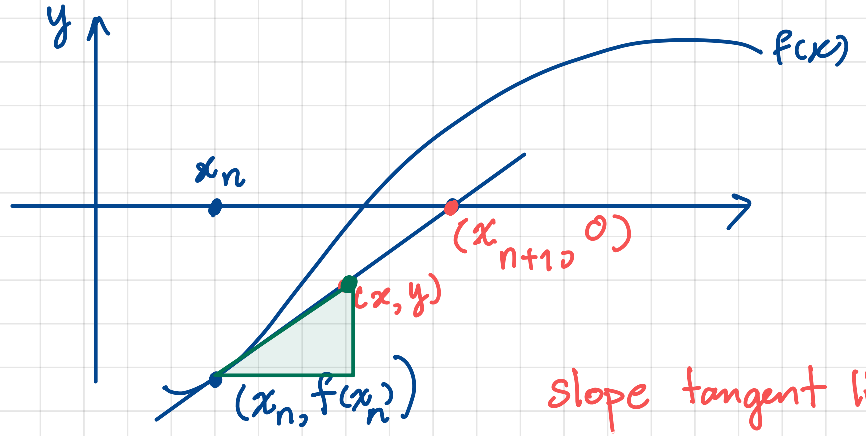


tangent line









slope tangent line $= f'(x_n)$

$$\boxed{\frac{y - f(x_n)}{x - x_n} = f'(x_n)} \leftarrow \text{tangent line}$$

$$\frac{\cancel{y}^{\,0} - f(x_n)}{x_{n+1} - x_n} = f'(x_n)$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}} \quad n = 0, 1, 2 \ldots$$

Ex: $\underbrace{x^3 - x - 1}_{f(x)} = 0$; $x_0 = 1$

$f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

$f(1) = 1 - 1 - 1 = -1$

$f'(1) = 3 - 1 = 2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{(-1)}{2} = \frac{3}{2}$$

$f(x_1) = f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right) - 1 =$

$f'(x_1) = f'\left(\frac{3}{2}\right) =$

Ex: $\sqrt{3.7182} = x$

$$x^2 = 3.7182$$

$$\underbrace{x^2 - 3.7182}_{f(x)} = 0$$

$1^2 = 1$

$2^2 = 4$

$$f(x) = x^2 - 3.7182$$

$$f'(x) = 2x$$

$x_0 = 1$; $\quad f(1) = 1^2 - 3.7182$

$$f'(1) = 2(1)$$

$$x_1 = 1 - \frac{(1 - 3.7182)}{2} =$$

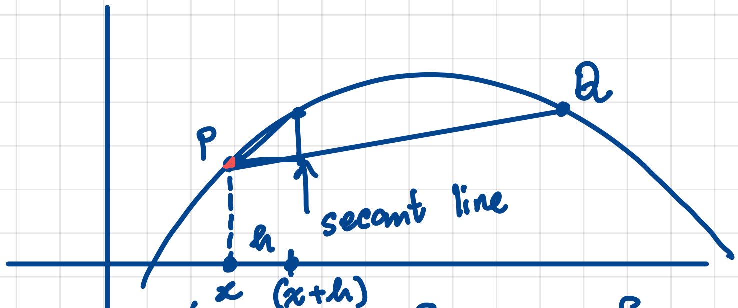
$$\sec(2x+1) = \frac{1}{x+2}$$

$$\underbrace{\sec(2x+1) - \frac{1}{x+2}}_{f(x)} = 0$$

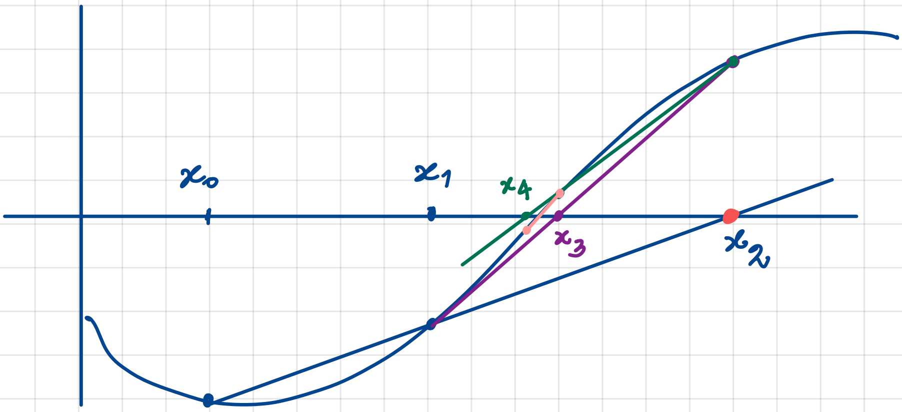
, $f'(x)$

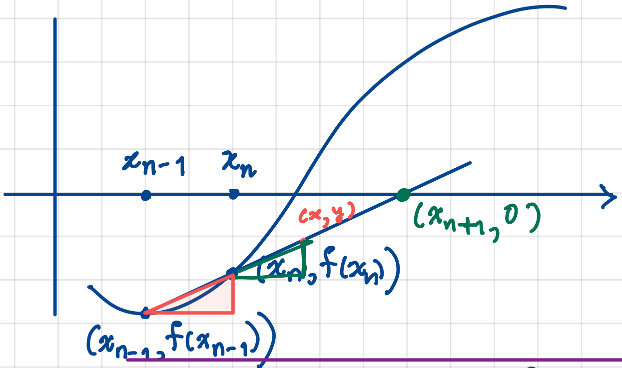
Secant method

Secant line



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$





$$\boxed{\frac{y - f(x_n)}{x - x_n} = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}}$$

$$\frac{\overset{0}{\cancel{y}} - f(x_n)}{x_{n+1} - x_n} = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

$$\boxed{x_{n+1} = x_n - f(x_n) \cdot \frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}}$$

$n = 1, 2, 3, \ldots$