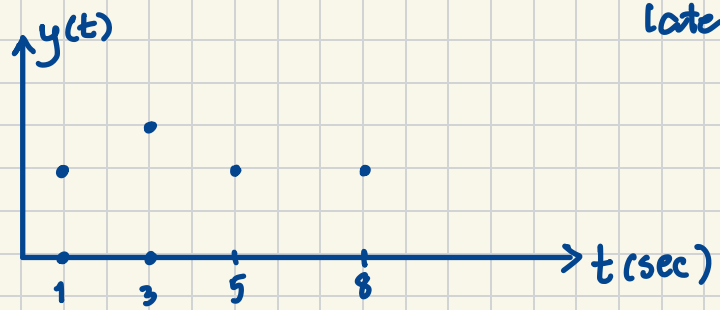
Interpolation

Curve fitting later.

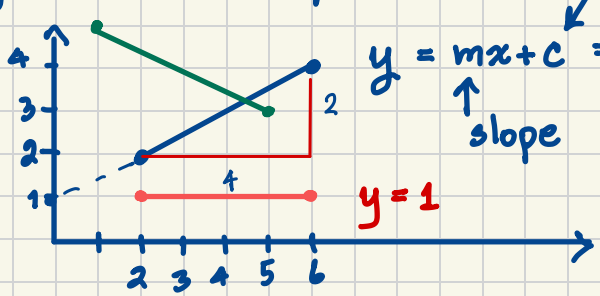


$$\text{velocity} = \frac{dy}{dt}$$

Find continuous function y(t) that pass through the given points.

$$f(x) = 2x^3 + 3x$$

Polynomial interpolation



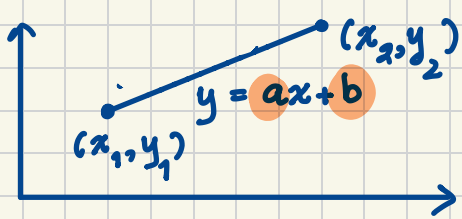
y-intercept

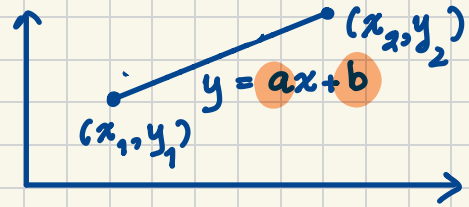
$y = mx + c = \frac{1}{2}x + 1$

slope

$m = \frac{2}{4} = \frac{1}{2}$

$c = 1$





$$f(x) = y = ax + b$$

$(x_1, y_1) \Rightarrow \quad y_1 = ax_1 + b$

$(x_2, y_2) \Rightarrow \quad y_2 = ax_2 + b$

$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$a = \frac{\begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix}}, \quad b = \frac{\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}}{\begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix}}$$

Ex: $(1,2)$, $(3,5)$ $\quad y = a + bx$

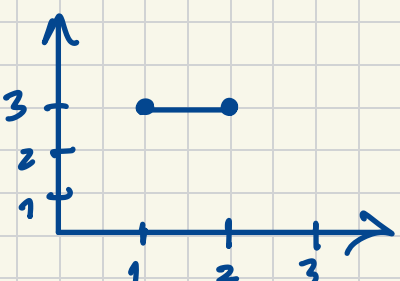
$$\left.\begin{array}{l} (1,2) \Rightarrow \quad 2 = a + b(1) \\ (3,5) \Rightarrow \quad 5 = a + b(3) \end{array}\right\} \quad \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$a = \frac{\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix}} = \frac{6-3}{3-1} = \frac{3}{2}$$

$$b = \frac{\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix}} = \frac{3-2}{2} = \frac{1}{2}$$

$$\boxed{y = \frac{3}{2} + \frac{1}{2}x}$$

Ex: $(1,3)$, $(2,3)$ $\quad y = 3 + 0x$ $\quad y = a + bx$



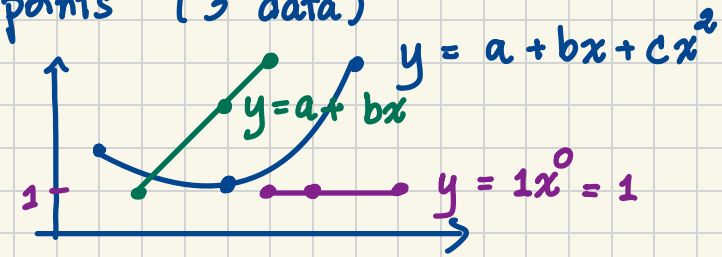
$$\boxed{y = 3}$$

$$\left.\begin{array}{l} 3 = a + b(1) \\ 3 = a + b(2) \end{array}\right\} \quad \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$a = \frac{\begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}} = \frac{6-3}{2-1} = 3$$

$$b = \frac{\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}} = \frac{3-3}{2-1} = 0$$

3 points (3 data)



Ex: $(1,2), (3,5), (5,4)$

$$y = a + bx + cx^2$$

$$a + b(1) + c(1)^2 = 2$$

$$a + b(3) + c(3)^2 = 5$$

$$a + b(5) + c(5)^2 = 4$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 5 & 25 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}$$

$a = -1$

$b = \frac{7}{2}$

$c = -\frac{1}{2}$

$$y = -1 + 3.5x - 0.5x^2$$

Newton form of polynomial