Find LU decomposition using Crout's method of Matrix ...

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Solution:
Crout's method for LU decomposition
Let $A = LU$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \times \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

This implies

$l_{11} = 1$

$l_{11}u_{12} = 2 \Rightarrow 1 \times u_{12} = 2 \Rightarrow u_{12} = 2$

$l_{11}u_{13} = 3 \Rightarrow 1 \times u_{13} = 3 \Rightarrow u_{13} = 3$

$l_{21} = 4$

$l_{21}u_{12} + l_{22} = 5 \Rightarrow 4 \times 2 + l_{22} = 5 \Rightarrow l_{22} = -3$

$l_{21}u_{13} + l_{22}u_{23} = 6 \Rightarrow 4 \times 3 + (-3) \times u_{23} = 6 \Rightarrow u_{23} = 2$

$l_{31} = 7$

$l_{31}u_{12} + l_{32} = 8 \Rightarrow 7 \times 2 + l_{32} = 8 \Rightarrow l_{32} = -6$

$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 9 \Rightarrow 7 \times 3 + (-6) \times 2 + l_{33} = 9 \Rightarrow l_{33} = 0$

$\therefore A = L \times U = LU$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & -3 & 0 \\ 7 & -6 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Find LU decomposition using Crout's method of Matrix ...

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & 5 & 3 \\ 7 & 2 & -1 \end{bmatrix}$$

Solution:
Crout's method for LU decomposition
Let $A = LU$

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & 5 & 3 \\ 7 & 2 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \times \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & 5 & 3 \\ 7 & 2 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

This implies
$l_{11} = 3$

$l_{11}u_{12} = 4 \Rightarrow 3 \times u_{12} = 4 \Rightarrow u_{12} = 1.333$

$l_{11}u_{13} = 7 \Rightarrow 3 \times u_{13} = 7 \Rightarrow u_{13} = 2.333$

$l_{21} = 2$

$l_{21}u_{12} + l_{22} = 5 \Rightarrow 2 \times 1.333 + l_{22} = 5 \Rightarrow l_{22} = 2.333$

$l_{21}u_{13} + l_{22}u_{23} = 3 \Rightarrow 2 \times 2.333 + 2.333 \times u_{23} = 3 \Rightarrow u_{23} = -0.714$

$l_{31} = 7$

$l_{31}u_{12} + l_{32} = 2 \Rightarrow 7 \times 1.333 + l_{32} = 2 \Rightarrow l_{32} = -7.333$

$l_{31}u_{13} + l_{32}u_{23} + l_{33} = -1 \Rightarrow 7 \times 2.333 + (-7.333) \times (-0.714) + l_{33} = -1 \Rightarrow l_{33} = -22.571$

$\therefore A = L \times U = LU$

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & 5 & 3 \\ 7 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 2.333 & 0 \\ 7 & -7.333 & -22.571 \end{bmatrix} \times \begin{bmatrix} 1 & 1.333 & 2.333 \\ 0 & 1 & -0.714 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 5 & 3 \\ 7 & 2 & -1 \end{bmatrix}$$