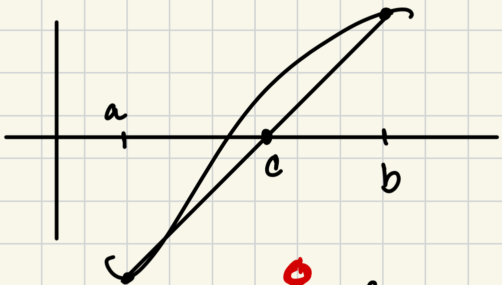
Lecture 6: November 29, 2024

No Class

Friday, December 6, 2024

Tuesday, December 10, 2024

Ex: Find the root of $0.325x^3 + 5x^2 - x + 1.275 = 0$



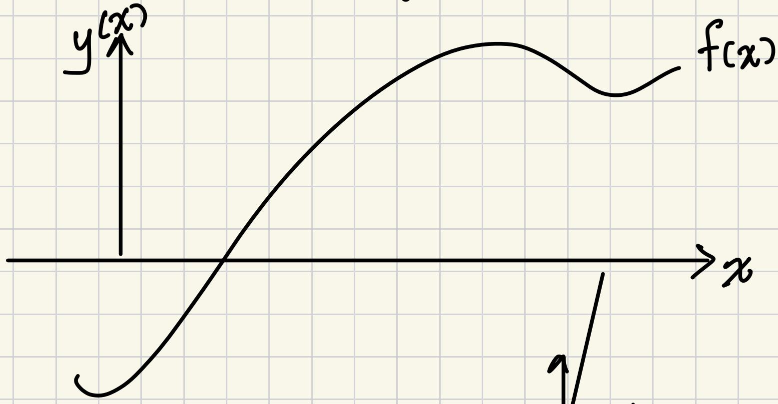
$f(a) \cdot f(b) < 0$

Method of false position

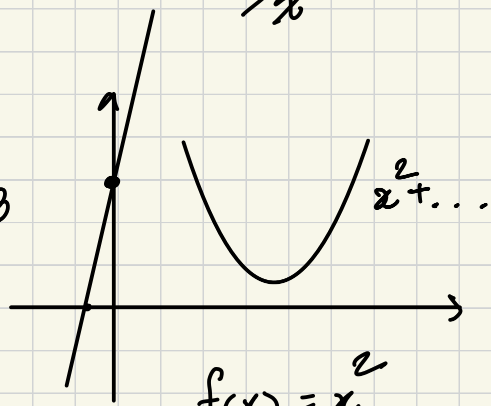
$$\frac{\overset{0}{\cancel{y}} - f(a)}{\underset{c}{\cancel{x}} - a} = \frac{f(b) - f(a)}{b - a}$$

$$\boxed{c = a - \frac{f(a)(b-a)}{f(b) - f(a)}}$$

System of nonlinear equation



$y = 2x + 3$



$f(x) = x^2$

↑ non linear equation

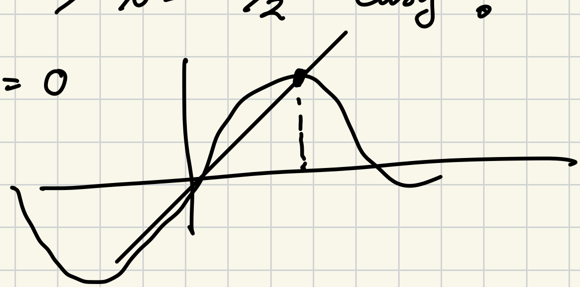
$f(x) = 2\sin(3x)$

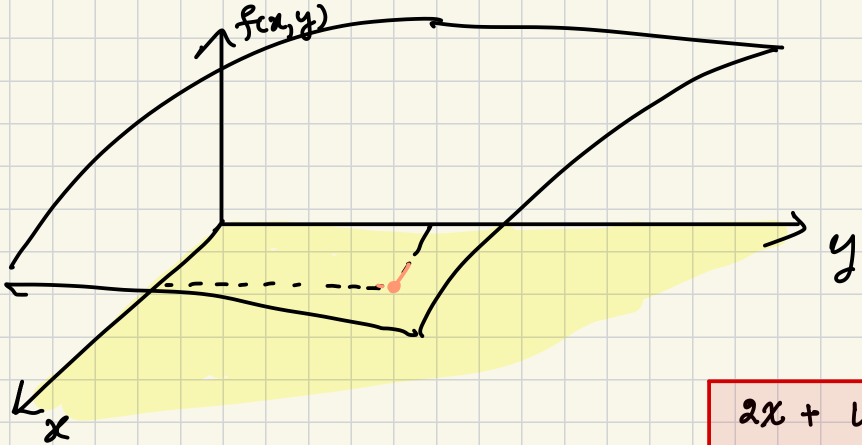
$g(x) = e^{x}\cos(3x)$

$f(x) = 0$

$f(x) = 2x + 3 = 0 \Rightarrow x = -3/2$ easy !

$f(x) = x - \sin x = 0$





$2x + y$	$= 0$
$3x - y$	$= 4$

System of linear equation

① — $2x + y = 0 \Rightarrow y = -2x$

② — $3x - y = 4 \Rightarrow y = 3x - 4$

③ = ① + ② $5x = 4$ Elimination method

$x = 4/5$

$y = -2\left(4/5\right) = -8/5$

Matrix

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5 & -4 \end{bmatrix}$$

2×2

row × column

$$B = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 6 & 3 \end{bmatrix}_{2\times 3}$$

$b_{1,3} = 4$; $b_{13} = 4$

$$A_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$$

$$kA = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

$$k = 2 \Rightarrow 2A = \begin{bmatrix} 2 & 4 \\ -6 & 8 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A + 3B = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ 0 & 7 \end{bmatrix}$$

Matrix multiplication

$$\underset{n\times \boxed{p}}{A} \cdot \underset{\boxed{p}\times m}{B} = \underset{n\times m}{C}$$

Ex: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$ $B = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}_{3\times 1}$

$\underset{2\times 3}{A} \cdot \underset{3\times 1}{B} = \underset{2\times 1}{C}$, $\underset{3\times 1 \neq 2\times 3}{B \cdot A}$ X

Ex: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$ $B = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}_{3\times 1}$

$$\underset{2\times\boxed{3 \quad 3}\times 1}{A \cdot B} = \underset{2\times 1}{C} = \begin{bmatrix} C_{1,1} \\ C_{2,1} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} C_{1,1} \checkmark \\ C_{2,1} \end{bmatrix}$$

1^{st} row A $\quad$ 1^{st} column B

$$C_{1,1} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= 1.1 + (2)(-1) + (3)(0)$$

$$C_{2,1} = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= (4)(1) + (5)(-1) + (6)(0)$$

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}_{3\times 3}$$

$$\underset{2\times 3}{C} = \underset{2\times \boxed{3 \quad 3} \times 3}{A \cdot B} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

$$c_{2,2} = \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= (2)(0) + (1)(0) + (0)(1) = 0$$

$$c_{13} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = (1)(1) + (1)(-1) + (-1)(2)$$

$$2x + y = 0$$

$$3x - y = 4$$

$$\underbrace{\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{1,1} \\ 2,1 \end{bmatrix} = \begin{bmatrix} 2x + 1y \\ 3x - y \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$