Homework #1F

Solve the following system of linear equations

$$3x_1 - 5x_2 - 4x_3 - x_4 = 28,$$
$$-4x_2 + 3x_3 - 4x_4 = 41,$$
$$2x_1 + 3x_2 + 3x_3 - 3x_4 = 11,$$
$$-2x_1 + 2x_2 - 5x_3 - 4x_4 = -21.$$

a) Using Naïve Gaussian elimination method.
b) Using Gaussian elimination method with partial pivoting.

a). We know that correspond to the augmented matrix:

$$\begin{pmatrix} 3 & -5 & -4 & -1 & | & 28 \\ 0 & -4 & 3 & -4 & | & 41 \\ 2 & 3 & 3 & -3 & | & 11 \\ -2 & 2 & -5 & -4 & | & -21 \end{pmatrix} \xrightarrow[R_4 - \frac{2}{3}R_1]{R_3 - \frac{2}{3}R_1} \begin{pmatrix} 3 & -5 & -4 & -1 & | & 28 \\ 0 & -4 & 3 & -4 & | & 41 \\ 0 & -\frac{1}{3} & \frac{5}{3} & -\frac{7}{3} & | & -\frac{35}{3} \\ 0 & -\frac{4}{3} & -\frac{17}{3} & -\frac{10}{3} & | & -\frac{35}{3} \end{pmatrix}$$

$$\xrightarrow[R_4 - \frac{-\frac{4}{3}}{-4}R_2]{R_3 - \frac{-\frac{1}{3}}{-4}R_2} \begin{pmatrix} 3 & -5 & -4 & -1 & | & 28 \\ 0 & -4 & 3 & -4 & | & 41 \\ 0 & 0 & \frac{52}{12} & -\frac{43}{12} & | & -\frac{434}{12} \\ 0 & 0 & -\frac{49}{12} & -\frac{94}{12} & | & -\frac{668}{12} \end{pmatrix} \xrightarrow{R_4 - \frac{-\frac{49}{12}}{\frac{52}{12}}R_3} \begin{pmatrix} 3 & -5 & -4 & -1 & | & 28 \\ 0 & -4 & 3 & -4 & | & 41 \\ 0 & 0 & \frac{52}{12} & -\frac{43}{12} & | & -\frac{434}{12} \\ 0 & 0 & 0 & -\frac{168}{52} & | & -\frac{624}{52} \end{pmatrix}$$

$$\therefore X_4 = \frac{-\frac{624}{52}}{-\frac{168}{52}} = -3, \quad \frac{52}{12}X_3 - \frac{43}{12}(-3) = -\frac{434}{12} \Rightarrow X_3 = 3, \quad -4X_2 + 3\times3 - 4\times(-3) = 41 \Rightarrow X_2 = -5,$$

$$3X_1 - 5\times(-5) - 4\times3 - (-3) = 28 \Rightarrow X_1 = 4$$

b) Use row 1 as the pivot, still
$$\begin{pmatrix} 3 & -5 & -4 & -1 & | & 28 \\ 0 & -4 & 3 & -4 & | & 41 \\ 0 & -\frac{1}{3} & \frac{5}{3} & -\frac{7}{3} & | & -\frac{35}{3} \\ 0 & -\frac{4}{3} & -\frac{17}{3} & -\frac{10}{3} & | & -\frac{35}{3} \end{pmatrix} \xrightarrow[R_4 - \frac{2}{3}R_1]{R_3 - \frac{2}{3}R_1} \begin{pmatrix} 3 & -5 & -4 & -1 & | & 28 \\ 0 & -4 & 3 & -4 & | & 41 \\ 0 & -\frac{1}{3} & \frac{5}{3} & -\frac{7}{3} & | & -\frac{35}{3} \\ 0 & -\frac{4}{3} & -\frac{17}{3} & -\frac{10}{3} & | & -\frac{35}{3} \end{pmatrix}$$

$$\xrightarrow[R_4 - \frac{-\frac{4}{3}}{-4}R_2]{R_3 - \frac{-\frac{1}{3}}{-4}R_2} \begin{pmatrix} 3 & -5 & -4 & -1 & | & 28 \\ 0 & -4 & 3 & -4 & | & 41 \\ 0 & 0 & \frac{52}{12} & -\frac{43}{12} & | & -\frac{434}{12} \\ 0 & 0 & -\frac{49}{12} & -\frac{94}{12} & | & -\frac{668}{12} \end{pmatrix} \xrightarrow{R_4 - \frac{-\frac{49}{12}}{\frac{52}{12}}R_3} \begin{pmatrix} 3 & -5 & -4 & -1 & | & 28 \\ 0 & -4 & 3 & -4 & | & 41 \\ 0 & 0 & \frac{52}{12} & -\frac{43}{12} & | & -\frac{434}{12} \\ 0 & 0 & 0 & -\frac{168}{52} & | & -\frac{624}{52} \end{pmatrix} \text{, the same result as method 1}$$

$$\therefore X_4 = -3, \; X_3 = 3, \; X_2 = -5, \; X_1 = 4.$$