Round of error

$x \equiv$ real value

$fl(x) \equiv$ floating number

$|x - fl(x)|$ = absolute error
$\equiv$ "round off error"

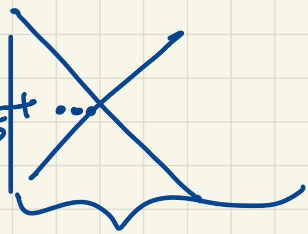
relative error $= \dfrac{|x - fl(x)|}{x}$

↑ don't know.

Error bound

<u>Truncation error</u>

$$\sum_{n=0}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$



Truncation error.

Taylor's series expansion

$f(x)$ continuous and differentiable
n+1 time

$f(x) = \sin x$

$f'(x) = \cos(x)$

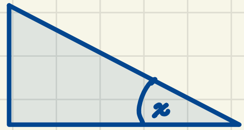
$f''(x) = -\sin(x)$

$f'''(x) = -\cos(x)$

$f^{(4)}(x) = +\sin(x)$

$f^{(5)}(x) = \cos(x)$

$\vdots$



Taylor's series $f(x)$ near point x_0

$$f(x) = f(x_0) + \frac{(x-x_0)f'(x_0)}{1!} + \frac{(x-x_0)^2 f''(x_0)}{2!} + \frac{(x-x_0)^3 f^{(3)}(x_0)}{3!} + \dots + \frac{(x-x_0)^n}{n!} f^{(n)}(x_0) + \dots$$

infinite series

truncation error

$$\boxed{f(x) = f(x_0) + \frac{(x-x_0)f'(x_0)}{1!}} + \frac{(x-x_0)^2 f''(x_0)}{2!} + \frac{(x-x_0)^3 f^{(3)}(x_0)}{3!} + \boxed{\frac{(x-x_0)^4}{4!} f^{(4)}(\xi)}$$

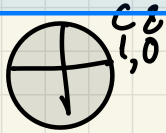
xi

$\boxed{x \le \xi\ x_0}$

error bound

$f(x) = \cos x$; $\boxed{x_0 = 0}$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f^{(3)}(0) + \frac{x^4}{4!} f^{(4)}(0) + \dots$$



$f(x) = \cos x$	$f(0) = \cos(0) = 1$
$f'(x) = -\sin x$	$f'(0) = -\sin(0) = 0$
$f''(x) = -\cos x$	$f''(0) = -\cos(0) = -1$
$f^{(3)}(x) = +\sin x$	$f^{(3)}(0) = +\sin(0) = 0$
$f^{(4)}(x) = \cos x$	$f^{(4)}(0) = \cos(0) = 1$
$f^{(5)}(x) = -\sin x$	$f^{(5)}(0) = -\sin(0) = 0$
$f^{(6)}(x) = -\cos x$	$f^{(6)}(0) = -\cos(0) = -1$

$\vdots$

$$f(x) = \cos x$$
$$= 1 - \frac{1}{2} x^2 + \frac{1}{4!} x^4 - \frac{1}{6!} x^6 + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

" when θ small : $\sin\theta \approx \theta$ "

Root finding $\boxed{f(x)=0}$

zero of the solution

(1) $x+2=0 \qquad x=-2$

$-2+2=\checkmark 0 \qquad \boxed{y=x+2} \qquad f(x)=x+2$

(2) $\underbrace{x^2+3x-4}_{f(x)}=0 \qquad f(x)=x^2+3x-4$

$4x-1x=3x$

$(x-1)(x+4)=0 \qquad (-1)(4)=-4$

x^2

$x-1=0 \Rightarrow x=1$

$x+4=0 \Rightarrow x=-4$

(3) $x^2+3.25x-9.1=0$

$ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-3.25\pm\sqrt{(3.25)^2-4(1)(-9.1)}}{2(1)}$$

$$=\frac{-3.25\pm\boxed{\sqrt{46.9625}}}{2} \qquad 6.863$$

$$\frac{-3.25+6.853}{2}$$

$$\frac{-3.25-6.853}{2}$$

$$x^3 + x - 2 = 0$$

$$x^3 - x + 2 = 0$$

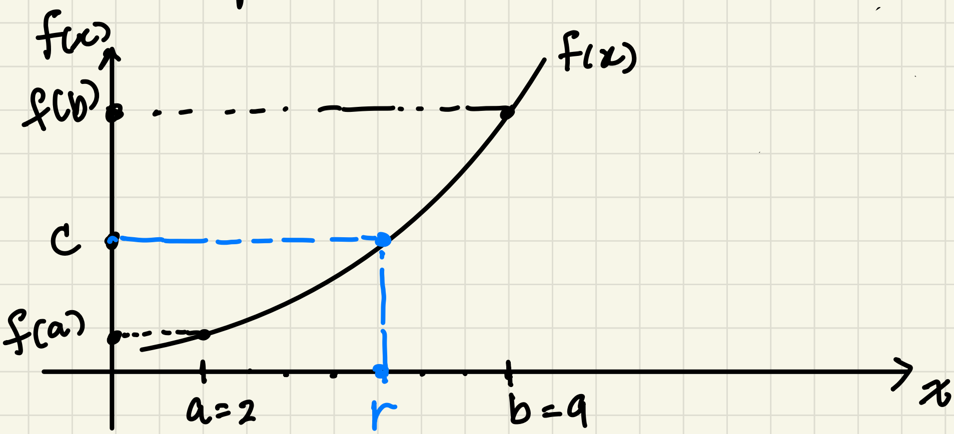
$$0.325x^3 - 1.25x - 1.23 = 0$$

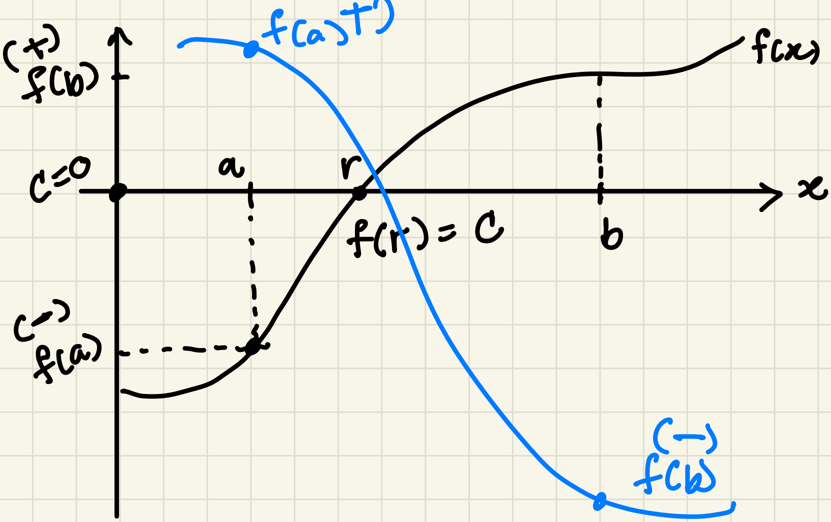
$$\sin(2x) - \frac{x^2}{x+1} + 1.256 = 0$$

Bisection method

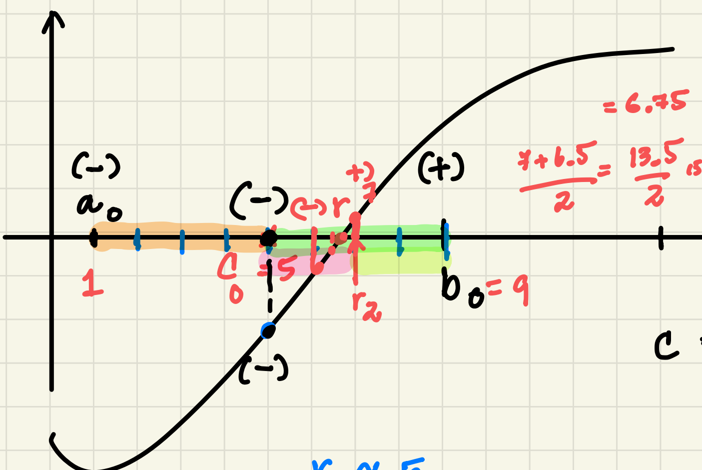
Intermediete value theorem.

$f(x)$ continuous function in $[a,b]$ and $c \in [f(a), f(b)]$, then there exists at least one value of r such that $f(r) = c$





$$\boxed{f(a) \cdot f(b) < 0}$$



[a, b]

$$c = \frac{b+a}{2}$$

$$= \frac{9+1}{2} = 5$$

$r_1 \approx 5$

$$f(5) \cdot f(9) < 0 \Rightarrow r_2 = \frac{9+5}{2} = \frac{14}{2} = 7$$