Newton's form of polynomial

n+1 data

$$P_n(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$
$$+ \dots + f[x_0,x_1,x_2,\dots,x_n](x-x_0)(x-x_1)\dots(x-x_{n-1})$$

Ex: $\overset{x_0,y_0}{(0,0)}$, $\overset{x_1,y_1}{(1,1)}$, $\overset{x_2,y_2}{(2,8)}$, $\overset{x_3\ y_3}{(3,27)}$, $\overset{x_4\ y_4}{(4,64)}$

$$P_4(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$
$$+ \dots + f[x_0,x_1,x_2,x_3,x_4](x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

0	0				
		$\frac{1-0}{1-0}=1$			
1	1		$\frac{7-1}{2-0}=3$		
		$\frac{8-1}{2-1}=7$		$\frac{6-3}{3-0}=1$	
2	8		$\frac{19-7}{3-1}=6$		$\frac{1-1}{4-0}=0$
		$\frac{27-8}{3-1}=19$		$\frac{9-6}{4-1}=1$	
3	27		$\frac{37-19}{4-2}=9$		
		$\frac{64-27}{4-3}=37$			
4	64				

$$P_4(x) = \cancel{0} + 1(x-0) + 3(x-0)(x-1) + 1(x-0)(x-1)(x-2)$$
$$+ 0(x-0)(x-1)(x-2)(x-3)$$

$$= x + 3x(x-1) + (x)(x-1)(x-2)$$

Lagrange form of polynomial.

(x_0, y_0) , (x_1, y_1)

Lagrange polynomial

$$P_1(x) = y_0 \boxed{l_0(x)} + y_1 \boxed{l_1(x)}$$

Ex: $(1, 2)$, $(3, 5)$ Polynomial degree 1. x^1

$$P_1(x) = 2\, l_0(x) + 5\, l_1(x)$$

$$P_1(1) = 2\, \overset{1}{l_0(1)} + 5\, \overset{0}{l_1(1)} = 2$$

$$P_1(3) = 2\, \underset{0}{l_0(3)} + 5\, \underset{1}{l_1(3)} = 5$$

$$l_0(x) = \frac{x-3}{1-3}, \quad l_1(x) = \frac{x-1}{3-1}$$

$$\boxed{P_1(x) = 2\left[\frac{x-3}{-2}\right] + 5\left[\frac{x-1}{2}\right]}$$

$$= -(x-3) + \frac{5}{2}(x-1)$$

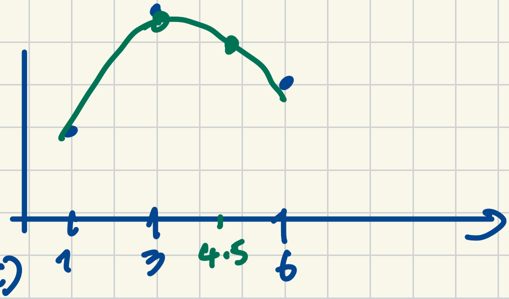
$$\boxed{= \frac{3}{2}x + \frac{1}{2}}$$

$$3 - \frac{5}{2}$$

$$\frac{6-5}{2}$$

Ex2: $(1,2), (3,5), (6,4)$

$$\boxed{P_2(x) = \sum_{i=0}^{2} y_i \, l_i(x)}$$



$$P_2(x) = y_0 l_0(x) + y_1 l_1(x) + y_2 l_2(x)$$
$$= 2\, l_0(x) + 5\, l_1(x) + 4\, l_2(x)$$

$$2\, \overset{1}{l_0(1)} + 5\, \overset{0}{l_1(1)} + 4\, \overset{0}{l_2(1)} = 2$$
$$2\, \overset{0}{l_0(3)} + 5\, \overset{1}{l_1(3)} + 4\, \overset{0}{l_2(3)} = 5$$
$$2\, \overset{0}{l_0(6)} + 5\, \overset{0}{l_1(6)} + 4\, \overset{1}{l_2(6)} = 4$$

$$l_0(x) = \frac{(x-3)(x-6)}{\underset{(-2)(-5)}{(1-3)(1-6)}} = \frac{1}{10}(x-3)(x-6)$$

$$l_1(x) = \frac{(x-1)(x-6)}{(3-1)(3-6)} = -\frac{1}{6}(x-1)(x-6)$$

$$l_2(x) = \frac{(x-1)(x-3)}{(6-1)(6-3)} = \frac{1}{15}(x-1)(x-3)$$

$$P_2(x) = \frac{2}{10}(x-3)(x-6) - \frac{5}{6}(x-1)(x-6) + \frac{4}{15}(x-1)(x-3)$$

$$f(4.5) = \frac{2}{10}(4.5-3)(4.5-6) - \frac{5}{6}(4.5-1)(4.5-6) + \frac{4}{15}(4.5-1)(4.5-3)$$

Ex: $(1,2), (3,5), (6,4), (7,11)$

$$P_3(x) = 2l_0(x) + 5l_1(x) + 4l_2(x) + 11l_3(x)$$

$$2\overset{1}{l_0(1)} + 5\overset{0}{l_1(1)} + 4\overset{0}{l_2(1)} + 11\overset{0}{l_3(1)} = 2$$

$$2\overset{0}{l_0(3)} + 5\overset{1}{l_1(3)} + 4\overset{0}{l_2(3)} + 11\overset{0}{l_3(3)} = 5$$

$$2\overset{0}{l_0(6)} + 5\overset{0}{l_1(6)} + 4\overset{1}{l_2(6)} + 11\overset{0}{l_3(6)} = 4$$

$$2\overset{0}{l_0(7)} + 5\overset{0}{l_1(7)} + 4\overset{0}{l_2(7)} + 11\underset{1}{l_3(7)} = 11$$

$$l_0(x) = \frac{(x-3)(x-6)(x-7)}{\underset{(-2)(-5)(-6)}{(1-3)(1-6)(1-7)}} = -\frac{1}{60}(x-3)(x-6)(x-7)$$

$$l_1(x) = \frac{(x-1)(x-6)(x-7)}{(3-1)(3-6)(3-7)} =$$

$$l_2(x) = \frac{(x-1)(x-3)(x-7)}{(6-1)(6-3)(6-7)}$$

$$l_3(x) = \frac{(x-1)(x-3)(x-6)}{(7-1)(7-3)(7-6)}$$

$$P_3(x) = 2l_0(x) + 5l_1(x) + 4l_2(x) + 11l_3(x)$$

Ex: $(-1, 2), (0, 1), (1, 5), (2, 7), (3, 2)$

$$P_4(x) = 2\,l_0(x) + 1\,l_1(x) + 5\,l_2(x) + 7\,l_3(x) + 2\,l_4(x)$$

$$l_0(x) = \frac{(x-0)(x-1)(x-2)(x-3)}{\underbrace{(-1-0)(-1-1)(-1-2)(-1-3)}_{(-1)(-2)(-3)(-4)}} = \frac{1}{24}(x)(x-1)(x-2)(x-3)$$

$$l_1(x) = \frac{(x+1)(x-1)(x-2)(x-3)}{\underbrace{(0+1)(0-1)(0-2)(0-3)}_{(1)(-1)(-2)(-3)}} = -\frac{1}{6}(x+1)(x-1)(x-2)(x-3)$$

$$l_2(x) = \frac{(x+1)(x-0)(x-2)(x-3)}{\underbrace{(1+1)(1-0)(1-2)(1-3)}_{(2)(1)(-1)(-2)}} = \frac{1}{4}(x+1)(x)(x-2)(x-3)$$

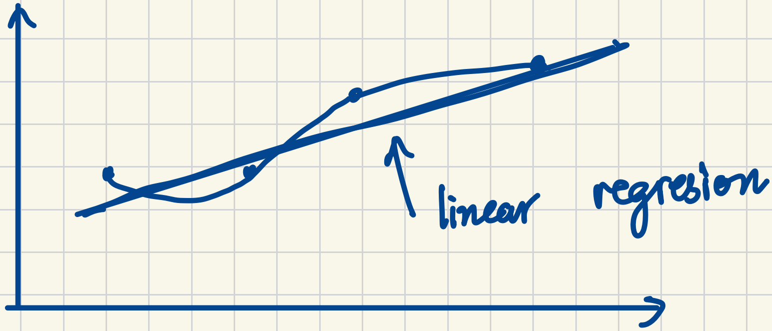
$$l_3(x) = \frac{(x+1)(x-0)(x-1)(x-3)}{\underbrace{(2+1)(2-0)(2-1)(2-3)}_{(3)(2)(1)(-1)}} = -\frac{1}{6}(x+1)(x)(x-1)(x-3)$$

$$l_4(x) = \frac{(x+1)(x-0)(x-1)(x-2)}{\underbrace{(3+1)(3-0)(3-1)(3-2)}_{(4)(3)(2)(1)}} = \frac{1}{24}(x+1)(x)(x-1)(x-2)$$

$$P_4(x) = \frac{1}{12}(x)(x-1)(x-2)(x-3) - \frac{1}{6}(x+1)(x-1)(x-2)(x-3) + \frac{5}{4}(x+1)(x)(x-2)(x-3) - \frac{7}{6}(x+1)(x)(x-1)(x-3) + \frac{1}{12}(x+1)(x)(x-1)(x-2)$$

$$P_4(x) = a + bx + cx^2 + dx^3 + ex^4$$

$$P_4(x) = a + b(x-x_0) + c(x-x_0)(x-x_1) + \dots + e(x-x_0)(x-x_1)(x-x_2)(x-x_3)$$



linear polynomial curve fitting.