# BioSignal Processing (BM-451)

## Topic: Spectral Analysis

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# Manipulation of Discrete time signals

- **Manipulation of time axis**

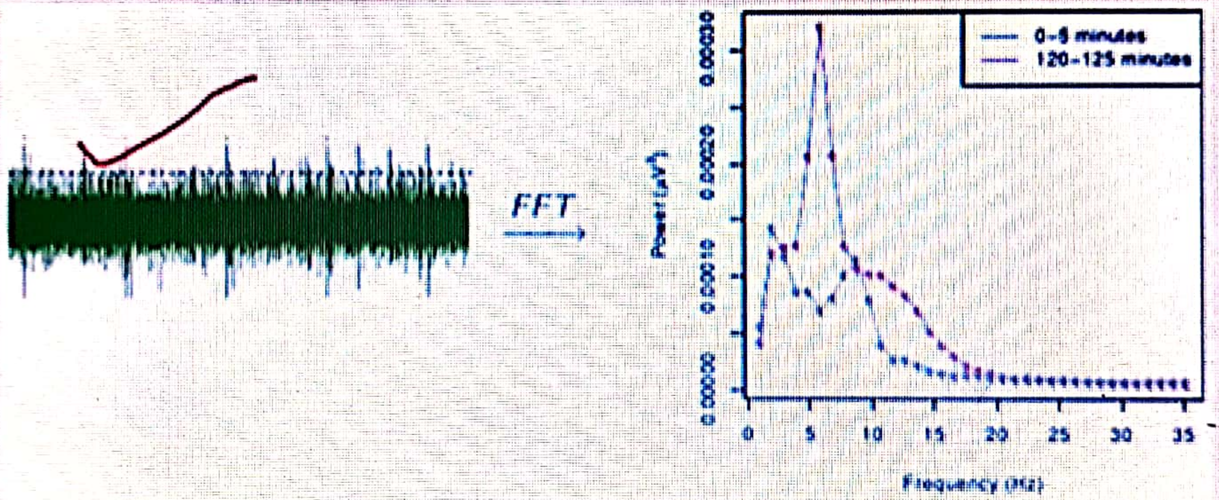
Folding, Time-scaling, Delayed/ Advanced

- **Manipulation of amplitude**

Addition, multiplication, amplitude scaling

# Spectral Analysis

- The spectral analysis represents the signal amplitude in frequency domain instead of time.




- Spectral analysis is concerned with the determination of the energy or power spectrum of a signal

# Spectral Analysis

- Three typical areas of spectral analysis are:

1) Spectral analysis of stationary sinusoidal signals
2) Spectral analysis of nonstationary signals
3) Spectral analysis of random signals

# Applications of Spectral Analysis

- Spectral analysis finds applications in many diverse fields.

- In vibration monitoring, the spectral content of measured signals give information on the wear and other characteristics of mechanical parts under study.

- In economics, meteorology, astronomy and several other fields, the spectral analysis may reveal \hidden periodicities" in the studied data, which are to be associated with cyclic behavior or recurring processes.

# Applications of Spectral Analysis

- In speech analysis, spectral models of voice signals are useful in better understanding the speech production process, and in addition can be used for both speech synthesis (or compression) and speech recognition.

- In radar and sonar systems, the spectral contents of the received signals provide information on the location of the sources (or targets) situated in the field of view.

# Applications of Spectral Analysis

- In seismology, the spectral analysis of the signals recorded prior to and during a seismic event (such as a volcano eruption or an earthquake) gives useful information on the ground movement associated with such events and may help in predicting them.

- Seismic spectral estimation is also used to predict subsurface geologic structure in gas and oil exploration.

# Applications of Spectral Analysis

- In control systems, there is a resurging interest in spectral analysis methods as a means of characterizing the dynamical behaviour of a given system, and ultimately synthesizing a controller for that system.

- In medicine, spectral analysis of various signals measured from a patient, such as electrocardiogram (ECG) or electroencephalogram (EEG) signals, can provide useful material for diagnosis.

# Methods of Spectral Analysis

- The spectral analysis method broadly falls into three categories:

(i) Non-parametric approach,

(ii) Parametric approach, and

(iii) Sub-space approach.

# Non-parametric Approach for Spectral Analysis

- Non-parametric methods are the classical methods for the spectrum estimation.

- Non-parametric methods are those in which the PSD is estimated directly from the signal.

- The two common approaches are:

(i) Periodogram (direct method, FFT algorithm)

(ii) Correlogram (indirect method, correlation then FFT)

# Non-parametric Approach for Spectral Analysis

- The periodogram and correlogram methods provide reasonably high resolution for sufficiently long data lengths, but are poor spectral estimators because their variance is high.

$$f = \frac{1}{t} = \frac{1}{1} = 1\,Hz$$

$$f = \frac{1}{2} = 0.5\,Hz$$

# Non-parametric Approach for Spectral Analysis: Periodogram computation via FFT algorithm

- In the periodogram method, the spectral analysis often performed using Fast Fourier Transform (FFT) algorithm.

$$P(F) = \frac{1}{N} * |X(F)|^2 = \sum_{n=0}^{N-1} \left| x(n) e^{-j2\pi F n} \right|^2$$

*Where,*

$x(n) = Discrete\ Time\ Signal$

$X(F) = Fourier\ of\ a\ signal\ x(n)$

$P(F) = Power\ of\ a\ signal\ x(n)\ N = Total\ number\ of\ samples$

# Non-parametric Approach for Spectral Analysis: Periodogram computation via FFT algorithm

- The frequency of FFT is calculated using below Equation.

$$F = \frac{F_S}{N}$$

Where, $F =$
$Frequency\ resolution\ of\ a\ signal\ x(n)\ whose\ fft\ is\ calculated$
$F_s = Sampling\ Frequency$
$N = Total\ number\ of\ samples$

- It shows that the frequency resolution is based on length of data epoch extracted for FFT.

# Non-parametric Approach for Spectral Analysis: Correlogram method

- In the correlogram method, the autocorrelation is calculated first then FFT algorithm is applied.

$$P(F) = \sum_{m=-(N-1)}^{N-1} r_{xx}(m) e^{-j2\pi Fm}$$

*Where,*

$r_{xx}(m) = autocorrelation\ of\ a\ Signal$

$P(F) = Power\ of\ a\ signal\ x(n)\ N =$
$Total\ number\ of\ samples$

$m = lag\ parameter$

# Non-parametric Approach for Spectral Analysis: Biased

- The periodogram and correlogram methods for spectral analysis are biased with:

i. Variance
ii. Spectral leakage

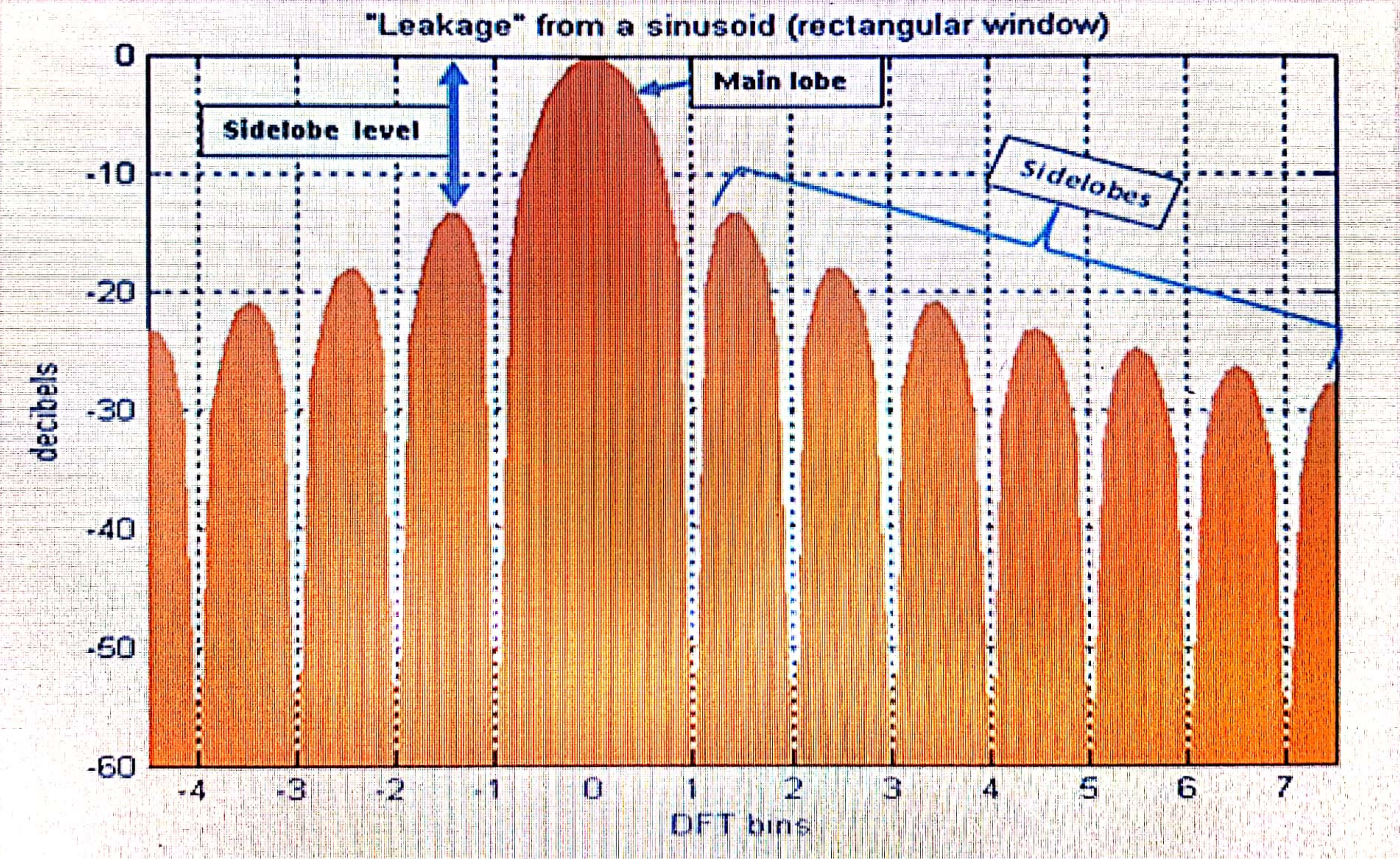
# Biased in Non-parametric Approach for Spectral Analysis: Spectral Leakage

## i. Spectral Leakage:

- The principal effect of the sidelobes on the estimated spectrum consists of transferring power from the frequency bands that concentrate most of the power (main lobe) in the signal to bands that contain less or no power (side lobes). This effect is called leakage.
- The main lobe width is the point where power is half of the peak main lobe power.

# Biased in Non-parametric Approach for Spectral Analysis: Spectral Leakage

- The spectral leakage means resulting



# Non-parametric Approach for Spectral Analysis: Steps for minimization of Variance and Spectral Leakage

## i. Variance minimization

Divide the data sequence into small overlap and or non-overlap segments (K segments). *(will be discussed in the topic: The Bartlett method and welch modified periodogram)*

*Advantage:* Variance is reduced by a factor K

*Disadvantage:* Frequency resolution is reduced by a factor K.

# Non-parametric Approach for Spectral Analysis: Steps for minimization of Variance and Spectral Leakage

## ii. Spectral Leakage

- Multiply sequence x(n) with a window function W(n). *(types of windows will discuss later). (will be discussed in the topic: The Bartlett method and welch modified periodogram)* [https://en.wikipedia.org/wiki/Window_function](https://en.wikipedia.org/wiki/Window_function)

- A window function is a numerical series containing same number of samples as in a single segment.

# Non-parametric Approach for Spectral Analysis: Steps for minimization of Variance and Spectral Leakage

- A non-rectangular window is shaped so that is exactly zero at the end and beginning of epoch and has some non-zero number in between which reduce the height of the side lobes or spectral leakage.

- Non-rectangular windows also broaden the main lobe, which result in a reduction of a resolution.