EXAMPLE:-

Let $v_1 = (1,1)$ and $v_2 = (1,0)$ are the basis of R^2. Find the formula of linear transformation $T: R^2 \to R^3$ for which

$$T(v_1) = (1,2,1) \text{ and } T(v_2) = (-1,0,2)$$

i.e $T(1,1) = (1,2,1)$ and $T(1,0) = (-1,0,2)$

Solution:-

Let $x = (x_1, x_2) \in R^2$ then x can be written as a linear combination of v_1 and v_2.

By definition of linear combination:-

$$x = av_1 + bv_2 \quad \to ①$$

$$(x_1, x_2) = a(1,1) + b(1,0)$$

$$= (a,a) + (b,0)$$

$$= (a+b, a) \quad \to ②$$

From eq. ②:-

$$a+b = x_1 \quad \to (i)$$

$$a = x_2 \quad \to (ii)$$

Putting the values of a into eq. (i):-

$$x_2 + b = x_1$$

$$b = x_1 - x_2$$

Using the values of both a and b into eq. ①

$$x = x_2 v_1 + (x_1 - x_2) v_2$$

$$T(x) = T(x_2 v_1) + T((x_1 - x_2) v_2)$$

$$T(x_1, x_2) = x_2 T(v_1) + (x_1 - x_2) T(v_2)$$

$$= x_2(1,2,1) + (x_1 - x_2)(-1,0,2)$$

$$= (x_2, 2x_2, x_2) + (-x_1 + x_2, 0, 2x_1 - 2x_2)$$

$$= (x_2 - x_1 + x_2, 2x_2, x_2 + 2x_1 - 2x_2)$$

$$= (2x_2 - x_1, 2x_2, 2x_1 - x_2)$$

Day: Date: / /

EXAMPLE :-

Let, $v_1=(1,1,1)$, $v_2=(1,1,0)$ and $v_3=(1,0,0) \in R^3$. Find a linear transformation $T: R^3 \to R^2$ such that $T(v_1)=(1,0)$, $T(v_2)=(2,-1)$, $T(v_3)=(4,3)$.

Solution:-

Let $x=(x_1,x_2,x_3) \in R^3$ then x can be written as a linear combination of v_1, v_2, v_3.

By definition of linear combination:-

$$x = av_1 + bv_2 + cv_3 \quad \to (1)$$

$$\begin{aligned}(x_1,x_2,x_3) &= a(1,1,1)+b(1,1,0)+c(1,0,0)\\ &= (a,a,a)+(b,b,0)+(c,0,0)\\ &= (a+b+c,\ a+b,\ a) \quad \to (2)\end{aligned}$$

From eq. (2) :-

$$a+b+c = x_1 \quad \to (i)$$

$$a+b = x_2 \quad \to (ii)$$

$$a = x_3 \quad \to (iii)$$

Putting the values of a into eq. (ii) :-

$$x_3 + b = x_2$$

$$b = x_2 - x_3$$

Using the values of both a and b into eq. (i):-

$$\cancel{x_3} + x_2 - \cancel{x_3} + c = x_1$$

$$x_2 + c = x_1$$

$$c = x_1 - x_2$$

Using the values of both a and b into eq. (1):-

$$x = x_3 v_1 + (x_2-x_3)v_2 + (x_1-x_2)v_3$$

$$T(x) = x_3 T(v_1) + (x_2-x_3)T(v_2) + (x_1-x_2)T(v_3)$$

$$\begin{aligned}T(x_1,x_2,x_3) &= x_3(1,0)+(x_2-x_3)(2,-1)+(x_1-x_2)(4,3)\\ &= (x_3,0)+(2x_2-2x_3,\ -x_2+x_3)\\ &\quad +(4x_1-4x_2,\ 3x_1-3x_2)\end{aligned}$$

Day: Date: / /

$$= (x_3 + 2x_2 - 2x_3 + 4x_1 - 4x_2, -x_2 + x_3 + 3x_1 - 3x_2)$$
$$= (-x_3 - 2x_2 + 4x_1, -4x_2 + x_3 + 3x_1)$$

EXAMPLE:-

Let, $T: R^2 \to R$ be the linear transformat for which $T(1,1) = 3$ and $T(0,1) = -2$. Find $T(x_1, x_2)$.

Solution:-

Let, $x = (x_1, x_2) \in R^2$ then x can be written as a linear combination of v_1, v_2.

By definition of linear combination:-

$$x = av_1 + bv_2 \quad \to ①$$
$$(x_1, x_2) = a(1,1) + b(0,1)$$
$$= (a, a) + (0, b)$$
$$= (a, a+b) \quad \to ②$$

From eq ②:-

$$a = x_1 \quad \to (i)$$
$$a + b = x_2 \quad \to (ii)$$

Putting the values of a into eq. (ii):-

$$x_1 + b = x_2$$
$$b = x_2 - x_1$$

Using the values of a and b into eq. ①:-

$$x = x_1 v_1 + (x_2 - x_1) v_2$$
$$T(x) = x_1 T(v_1) + (x_2 - x_1) T(v_2)$$
$$= x_1(3) + (x_2 - x_1)(-2)$$
$$= 3x_1 + 2x_1 - 2x_2$$
$$= 5x_1 - 2x_2$$