CS 223 – Digital Logic and Design

Lecture 6
Subtraction with Complements

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Subtraction with r's Complements

The direct method of subtraction taught in elementary schools uses the borrow concept. In this method, we borrow a 1 from a higher significant position when the minuend digit is smaller than the subtrahend digit. The method works well when people perform subtraction with paper and pencil. However, when subtraction is implemented with digital hardware, the method is less efficient than the method that uses complements.

The subtraction of two n-digit unsigned numbers $M - N$ in base r can be done as follows:

1. Add the minuend M to the r's complement of the subtrahend N. Mathematically, $M + (r^n - N) = M - N + r^n$.
2. If $M \geq N$, the sum will produce an end carry r^n, which can be discarded; what is left is the result $M - N$.
3. If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the r's complement of $(N - M)$. To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front.

Subtraction with r's Complements (Explained)

- The subtraction of two *n-digit unsigned* numbers ***M – N*** in base ***r*** can be done as follows:

1. Add the *minuend* ***M*** to the ***r's*** *complement* of the *subtrahend* ***N***. Mathematically, $M + (r^n - N)$
2. If ***M >= N***, the sum will produce an end carry, which will be discarded to produce the result ***M – N***.
3. If ***M < N***, the sum does not produce an end carry, take the ***r's*** complement of the sum and place a negative sign in front.

Example 1.5

EXAMPLE 1.5

Using 10's complement, subtract $72532 - 3250$.

$$
\begin{aligned}
M &= \phantom{+ {}} 72532 \\
\text{10's complement of } N &= + \underline{96750} \\
\text{Sum} &= \phantom{+ {}} 169282 \\
\text{Discard end carry } 10^5 &= -100000 \\
\textit{Answer} &= \phantom{+ {}} 69282
\end{aligned}
$$

Note that M has five digits and N has only four digits. Both numbers must have the same number of digits, so we write N as 03250. Taking the 10's complement of N produces a 9 in the most significant position. The occurrence of the end carry signifies that $M \geq N$ and that the result is therefore positive.

Example 1.6

EXAMPLE 1.6

Using 10's complement, subtract $3250 - 72532$.

$$\begin{aligned} M &= 03250 \\ \text{10's complement of } N &= +\ \underline{27468} \\ \text{Sum} &= 30718 \end{aligned}$$

There is no end carry. Therefore, the answer is $-$(10's complement of 30718) $= -69282$.

Note that since $3250 < 72532$, the result is negative. Because we are dealing with unsigned numbers, there is really no way to get an unsigned result for this case. When subtracting with complements, we recognize the negative answer from the absence of the end carry and the complemented result. When working with paper and pencil, we can change the answer to a signed negative number in order to put it in a familiar form.

Example 1.7

EXAMPLE 1.7

Given the two binary numbers $X = 1010100$ and $Y = 1000011$, perform the subtraction **(a)** $X - Y$ and **(b)** $Y - X$ by using 2's complements.

$$
\begin{array}{lrl}
\text{(a)} & X = & \ 1010100 \\
 & \text{2's complement of } Y = & +\ 0111101 \\
 & \text{Sum} = & \ 10010001 \\
 & \text{Discard end carry } 2^7 = & -\ \underline{10000000} \\
 & \textit{Answer: } X - Y = & \ 0010001 \\
\text{(b)} & Y = & \ 1000011 \\
 & \text{2's complement of } X = & +\ 0101100 \\
 & \text{Sum} = & \ 1101111
\end{array}
$$

There is no end carry. Therefore, the answer is $Y - X = -(\text{2's complement of } 1101111) = -0010001$.

Subtraction by using (r – 1)'s Complement

- Subtraction of unsigned numbers can also be done by means of the *(r - 1)'s complement.*
- Remember that the *(r - 1)'s complement is one less than the r's complement.*
- Because of this, the result of adding the *minuend M* to the complement of the *subtrahend N* produces a sum that is one less than the correct difference when an end carry occurs.
- Removing the end carry and adding 1 to the sum is referred to as an *end-around carry.*

Subtraction by using (r – 1)’s Complement (Contd.)

EXAMPLE 1.8

Repeat Example 1.7, but this time using 1’s complement.

(a) $X - Y = 1010100 - 1000011$

$$\begin{array}{rr}
X = & 1010100 \\
\text{1's complement of } Y = & +\ 0111100 \\
\text{Sum} = & 10010000 \\
\text{End-around carry} = & +\ \underline{\qquad\quad 1} \\
\textit{Answer: } X - Y = & 0010001
\end{array}$$

(b) $Y - X = 1000011 - 1010100$

$$\begin{array}{rr}
Y = & 1000011 \\
\text{1's complement of } X = & +\ \underline{0101011} \\
\text{Sum} = & 1101110
\end{array}$$

There is no end carry. Therefore, the answer is $Y - X = -$(1’s complement of 1101110) $=$ -0010001.

Example 1.8 (Contd.)

- Note that the negative result is obtained by taking the *1's complement* of the sum, since this is the type of complement used.
- The procedure with end-around carry is also applicable to subtracting unsigned decimal numbers with 9's complement.

- That’s end of the presentation ! ☺