Therefore
$$\lim_{n \to \infty} \frac{\int_{0}^{\pi/2} \sin^{2n} x \ dx}{\int_{0}^{\pi/2} \sin^{2n+1} x \ dx} = 1,$$
 (7)

by the Sandwiching Theorem 1.32 (v).

Taking limits of both sides of (5) as $n \to \infty$ we have

$$\frac{\pi}{2} = \lim_{n \to \infty} \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdots (2n)(2n)}{1 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdots (2n-1)(2n+1)} \times \lim_{n \to \infty} \frac{0}{\pi/2} \sin^{2n} x \ dx$$

$$= \lim_{n \to \infty} \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdots \frac{2n}{2n-1} \cdot \frac{2n}{2n+1}, \text{ using (7)}.$$

This is known as Wallis' Product Formula for $\frac{\pi}{2}$.

Exercise Set 5.4

Evaluate (Problems 1 – 21):			
1.	$\int \frac{\sec^4 x}{\tan^5 x} dx$	$2. \qquad \int \sin^2 x dx$	$\cos^4 x \ dx$
3.	$\int \sin^6 x \cos^2 x \ dx$	$4. \qquad \int \sin^{1/2} x$	$\cos^3 x \ dx$
5.	$\int \sec^2 x \csc^3 x \ dx$	6. $\int \tan^3 x$	$\sec^5 x \ dx$
7.	$\int \cot^5 x \csc^4 x \ dx$	8. $\int \frac{\sin^2 x}{\cos^5 x}$	dx
9.	$\int_{0}^{\pi/2} \cot^4 x \ dx$ $\pi/4$	$10. \int_{\pi/4}^{\pi/2} \cot^3 x$	$\csc^3 x \ dx$
11.	$\int_{0}^{\pi/2} \tan^{5}\left(\frac{x}{2}\right)^{x} dx$	12. $\int_{0}^{a} (a^2 - x^2)$	
13.	$\int_{0}^{\pi} \frac{\sin^4 x}{(1+\cos x)^2} dx$	14. $\int_{0}^{\pi/4} \sin^4 2$	x dx

$$15. \int_{0}^{\pi/2} \sin^6 3x \ dx$$

16.
$$\int_{0}^{\pi/s} \sin^{5} 4x \cos^{4} 4x \ dx$$

$$17. \int_{0}^{\pi/4} \cos^2 2x \ dx$$

$$18. \int_{0}^{\pi/6} \cos^3 3x \ dx$$

19.
$$\int_{0}^{\pi/2} \sin^2 6x \cos^4 3x \ dx$$

$$20. \int_{\pi/6}^{\pi/2} \frac{\cos^2 x}{\sin x} \ dx$$

21.
$$\int_{0}^{1} \frac{x^{6} dx}{\sqrt{1-x^{2}}}.$$

22. Show that

$$\int \sec^{2n+1} x \ dx = \frac{\sec^{2n-1} x \tan x}{2n} + \left(1 - \frac{1}{2n}\right) \int \sec^{2n-1} x \ dx.$$

- Obtain a reduction formula for $\int \frac{dx}{(a^2 + x^2)^n}$, where *n* is an integer. Show that $\int \frac{dx}{(1+x^2)^5} = \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \frac{\pi}{2}$.
- 24. If I_n denotes $\int_0^1 x^p (1-x^q)^n dx$, where p, q and n are positive, prove that $(qn+p+1) I_n = qn I_{n-1}$. Evaluate I_n when n is a positive integer.
- 25. Obtain a reduction formula for $\int \frac{x^n}{\sqrt{1-x^2}} dx$ and hence evaluate $\int \frac{x^3}{\sqrt{1-x^2}} dx$.
- 26. Calculate the value of $\int_{0}^{2a} x^{n} \sqrt{2ax x^{2}} dx$, n being a positive integer. Hence or otherwise calculate the values of

(i)
$$\int_{0}^{2a} x \sqrt{2ax-x^2} \ dx$$
 (ii) $\int_{0}^{2a} x^4 \sqrt{2ax-x^2} \ dx$

27. If
$$I_n = \int x^n (a^2 - x^2)^{1/2} dx$$
, prove that
$$I_n = -\frac{x^{n-1} (a^2 - x^2)^{3/2}}{n+2} + \frac{n-1}{n+2} a^2 I_{n-2}$$
Hence evaluate $\int_0^a x^4 \sqrt{a^2 - x^2} dx$.

28. Prove that

$$\int x^m (\ln x)^n dx = \frac{x^{m+1} (\ln x)^n}{m+1} - \frac{n}{m+1} \int x^m (\ln x)^{n-1} dx.$$

Hence calculate

(i)
$$\int x^m (\ln x)^3 dx$$
 (ii)
$$\int_0^1 x^m (\ln x)^n dx$$

29. Prove that

$$\int_{0}^{\pi/2} \cos^{m} x \sin nx \ dx = \frac{1}{m+n} + \frac{m}{m+n} \int_{0}^{\pi/2} \cos^{m-1} x \sin (n-1) x \ dx$$

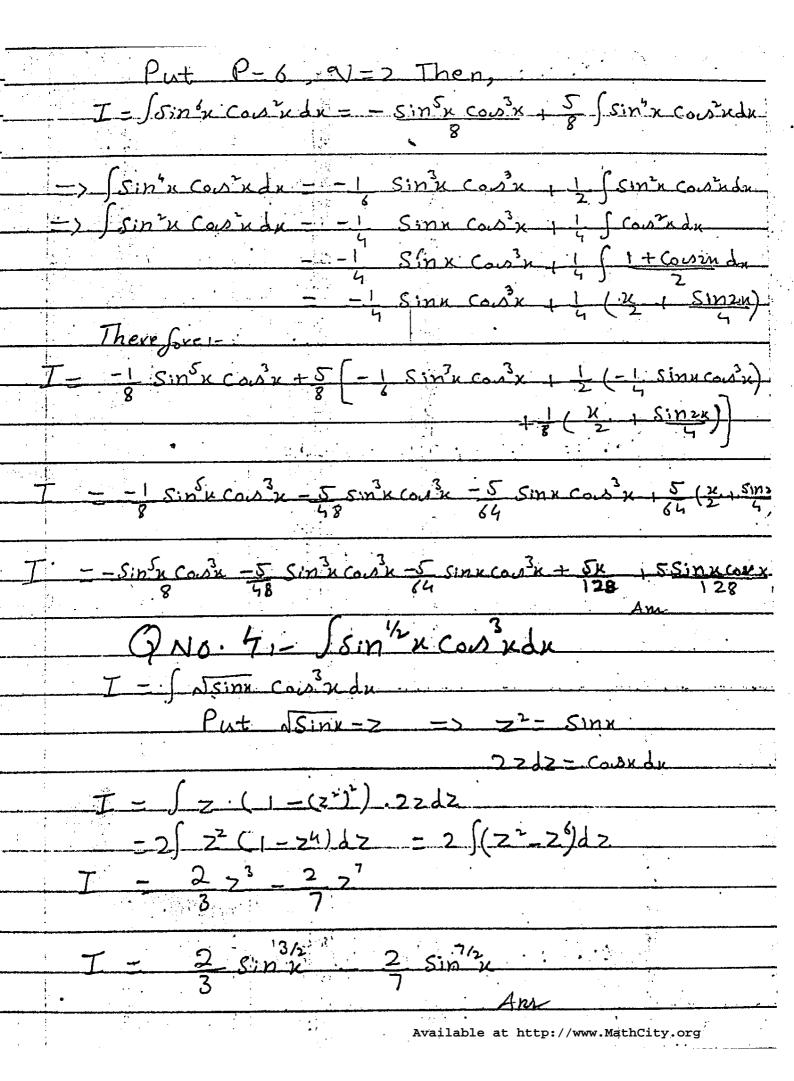
Hence evaluate
$$\int_{0}^{\pi/2} \cos^6 x \sin 3x \ dx.$$

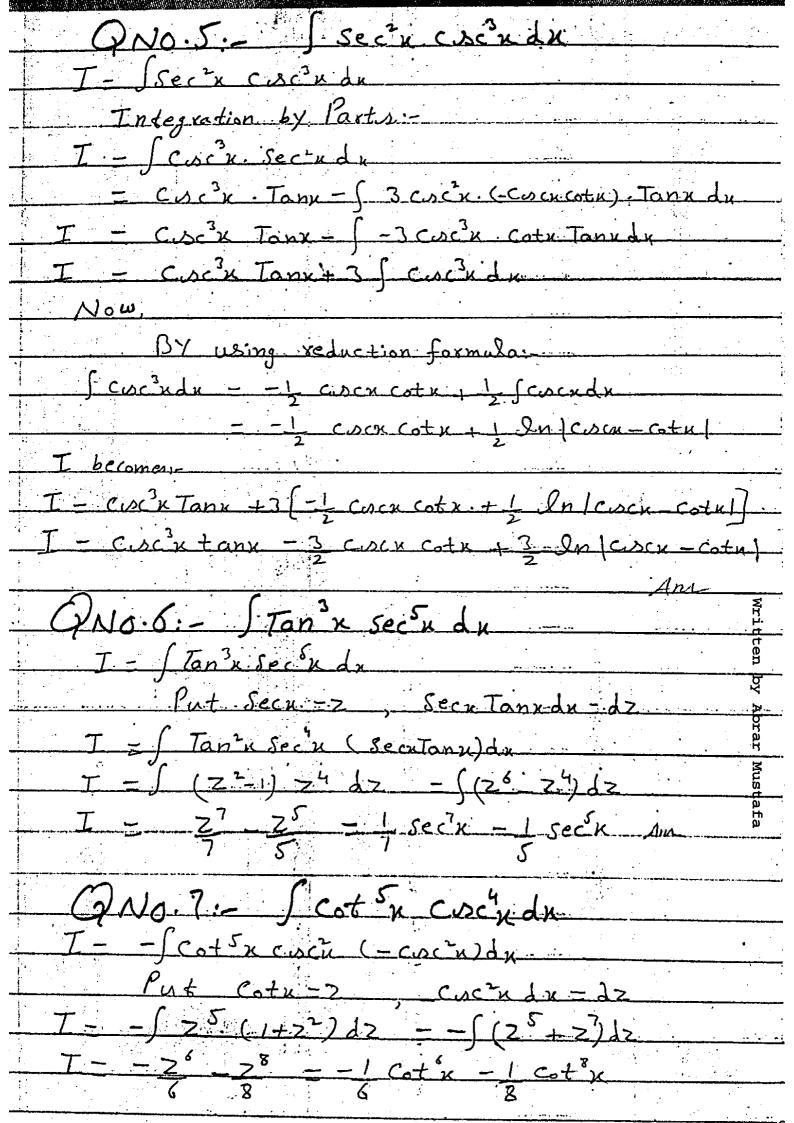
30. Find a reduction formula for
$$\int \frac{x^n}{\sqrt{ax^2 + 2bx + c}} dx$$
.

Numerical Integration

In Chapter 4 we studied various methods of evaluating antiderivatives of certain functions. In order to evaluate definite integrals, the Fundamental Theorem of Integral Calculus (5.3) is a basic tool. But this theorem fails to deliver if the antiderivative of the integral cannot be found in terms of elementary functions (i.e., functions that can be expressed as a finite combination of algebraic and transcendental functions). For such cases Riemann sums provide an approximation of a definite integral when the number of points in partition is large. In practice this method is seldom used since there are better techniques and formulas which give a more efficient way to approximate such integrals. The methods of approximating definite integrals are called numerical integration. In this section, we discuss two such methods.

	Ex#5.4 M.Sc. Mathematics (P.U) Mob. # 0345-7845311
1	QNO. 1: - Sect K dk
	Tansx
	Sectudi - Dut Tanx - 7, Sectudi - dz
	Tansk
	Sectular -
	tanin Tanin
	$-\int \left(\frac{1}{2^{5}} + \frac{1}{2^{3}}\right) dz = -\frac{1}{52^{3}} - \frac{1}{22^{3}}$
	Jeck dx 1 - 1 Ans Tansu Granx 2 Tanx
	GNO.2:- J Sinzx cosxxdx
	We Know that,
• •	Sink contatx - Sinx Cont + a-1 Sink contatx
	-> Sin2x consudx - Sin3x consu + 1 Sin2x consudx Now
	Now
	Sint Court dr= Sin'i Court + + Sin'i Courdn
	- Sin3x Cour 1 [1-Cousex 12
·	$= -Sin \times Coox + \frac{\chi}{\chi} = Sin 2 \chi$
,	$= \frac{\sin^3 x \cos x}{4} + \frac{1}{8} \int \frac{1 - \cos 2x}{4x} dx$ $= \frac{\sin^3 x \cos x}{4} + \frac{x}{8} - \frac{\sin^2 x}{8}$ We have, $\int \sin^3 x \cos^3 x dx - \sin^3 x \cos x + \frac{x}{8} - \frac{\sin^3 x \cos x}{16}$ $\int \sin^3 x \cos x dx - \frac{x}{8} - \frac{\sin^3 x \cos x}{16} + \frac{\sin^3 x \cos x}{16}$
	Sinzucous x du = Sinx cousu + Sinzucousu + K = Sinx cour
	8 16 16 Ann
	QNO.3: - J Sin'x Contadx
	we have reduction formula:
S	QNO.3:- JS:n'x Con'x dx We have reduction formula:- in'x coundx = -Sin'x con'x', P-1 Ssn' con xdx P+0 Sn' con xdx
7	Pta Pta J





GNO.8: - Sin'x dx I = Sinzu du - Sinzu Conzu Conzu I - S Tan'n Sec'ndu - S(Sec'n-1) Sec'nda I = Sectudn - Secondn Now, By reduction formulea

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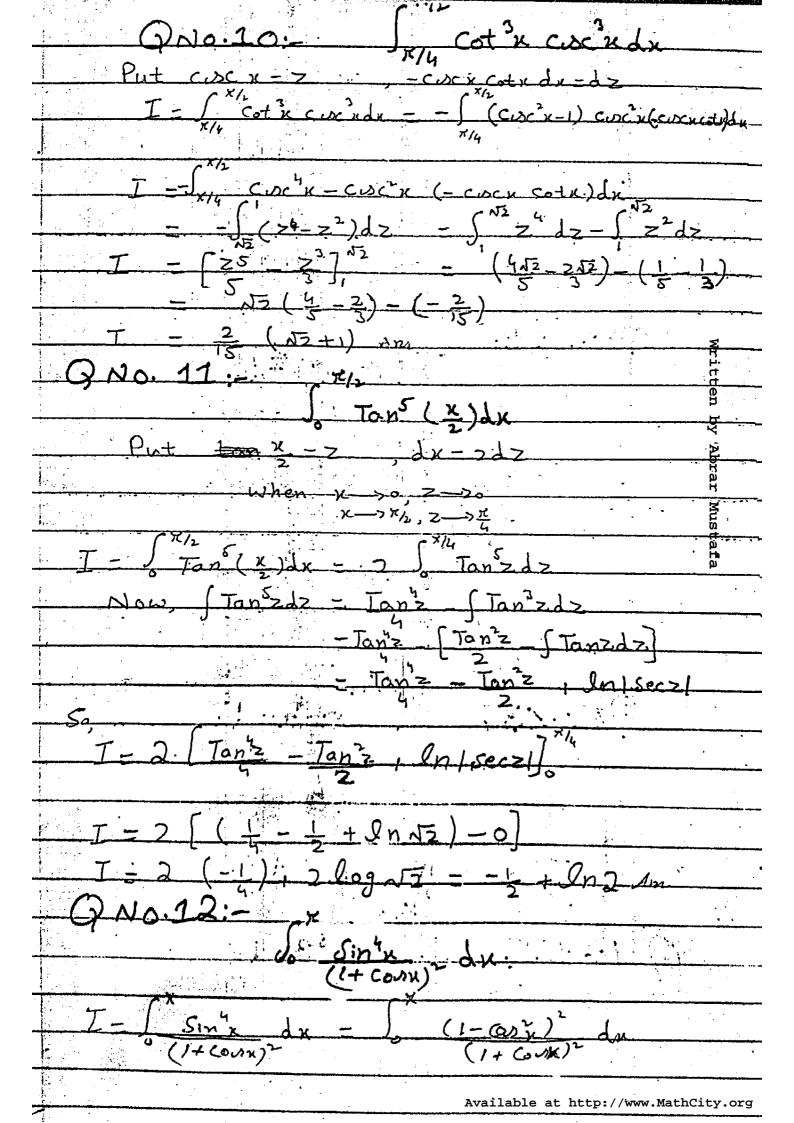
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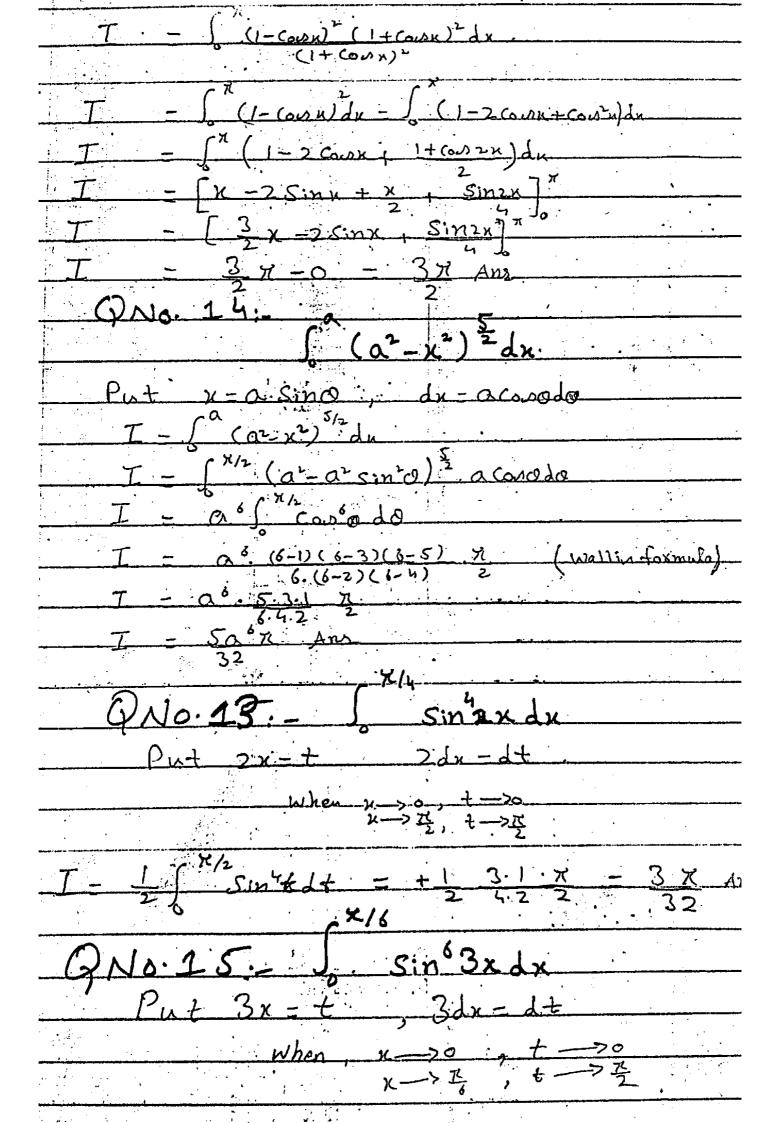
Seconda So,

T - Sec3x Tanu + 3 Sec3x du - Sec3x dx - Secutar - 1 Seculu Again, Seconda = Secutaria 1 & Secudar - Secritary 1.1 In/secretary J = Sec3x Tanx - 1 Secx Tanx - 1 In | Secx | Tanx |

No. 9: - Sely Cot u.d.

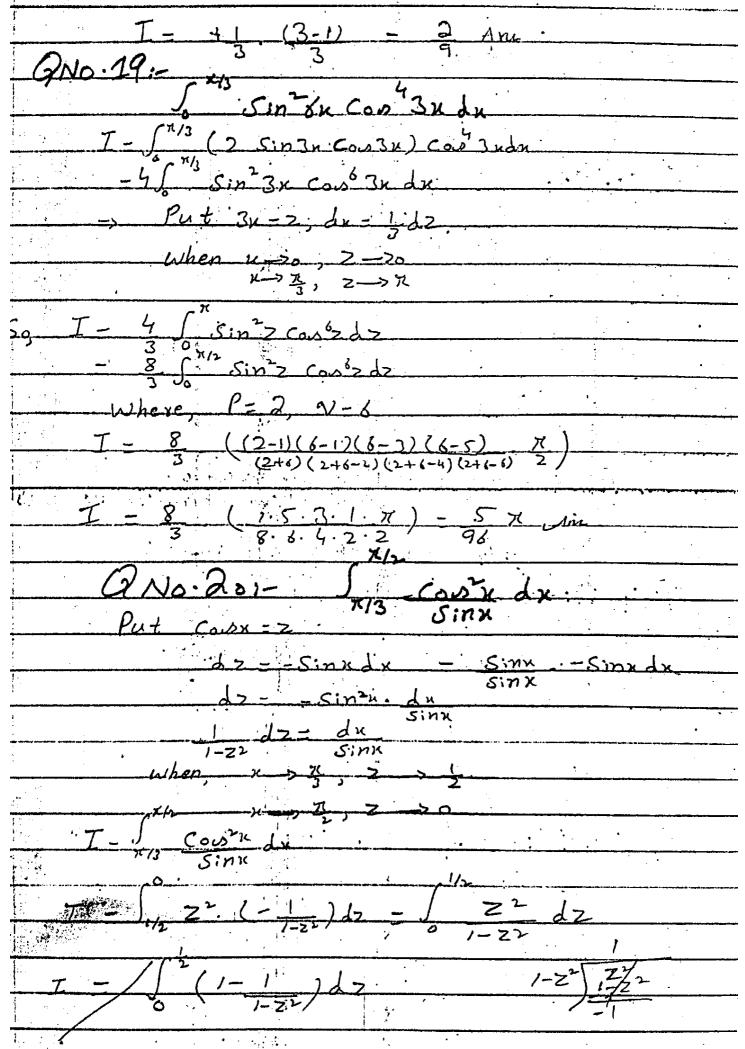
By reduction formula: Jostyndn - - Cot3x - Scot2xda -- cot n S (coch - 1)dn $= -\cot^{3}x \quad (-\cot x) + x$ $= -\cot^{3}x \quad + \cot x + x$ $=-\cot^{3}x \left(-\cot x\right)+x$ T = 7 -[-1+(+3] = 3-2 Am



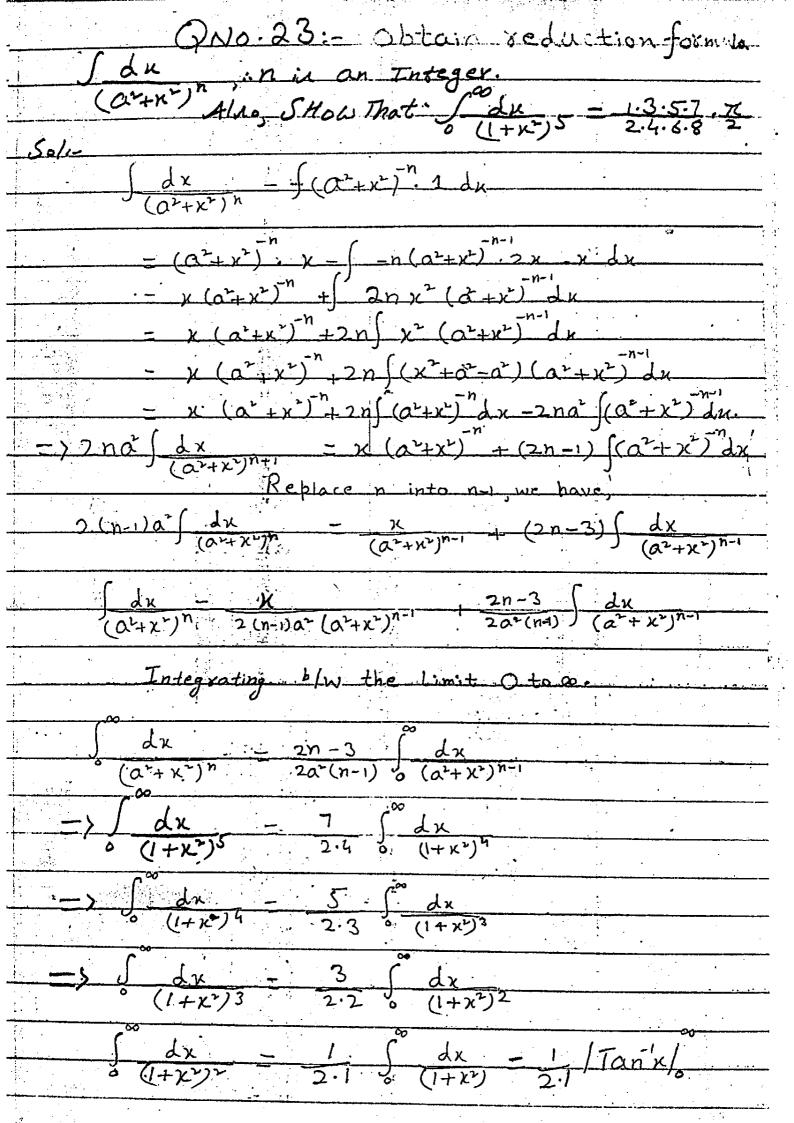


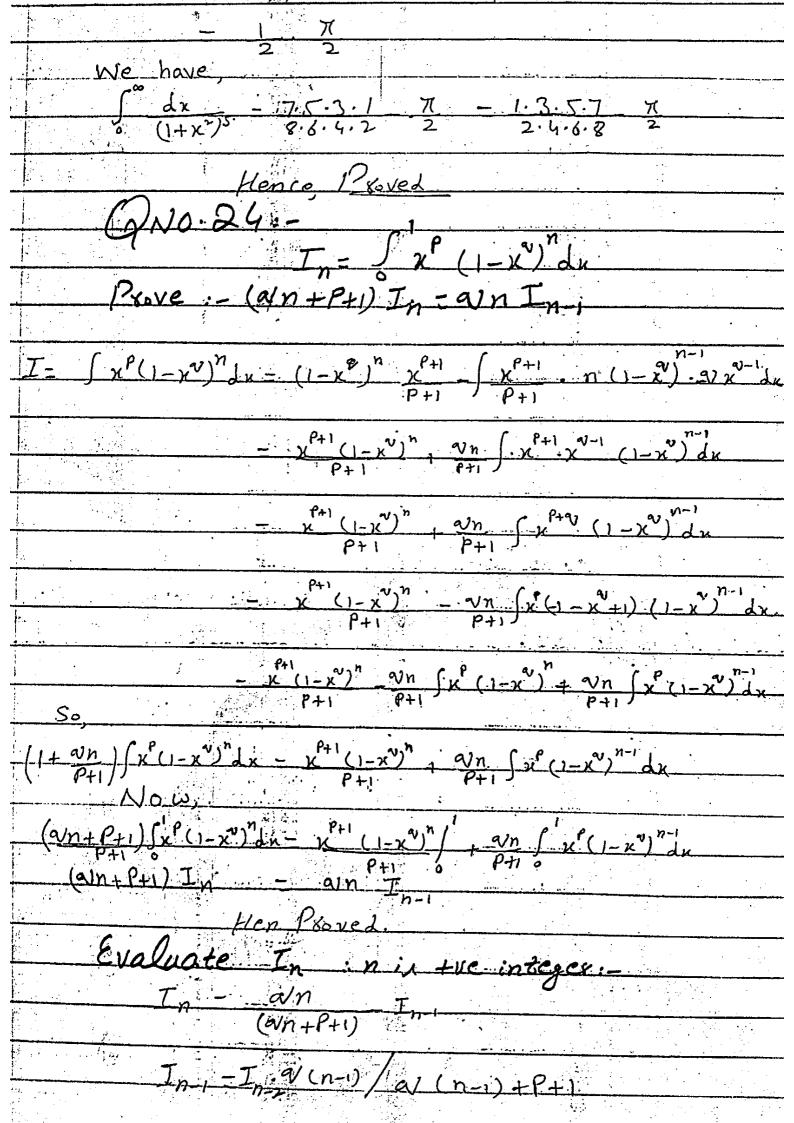
T - Sin 3x dx = (Sin 4) - 1 1t $\frac{T-1}{3}\int_{0}^{\pi/2}\sin^{6}t\,dt$ $\frac{T-1}{3} = \frac{(6-1)(6-3)(6-5)}{6\cdot(6-2)(6-4)} = \frac{7}{2}$ QNO. 16:- 57. Am. QNO. 16:- 5/8 Sin 54x con 4xdx Put 4x-2, dx-112 So, $T = \int_{0}^{\pi/8} \frac{4\pi}{5} \frac{1}{5} \frac{dz}{4}$ T - 1 5 " Sin 5 . cos 2 2 22' Hence Pindd and alinenen so,

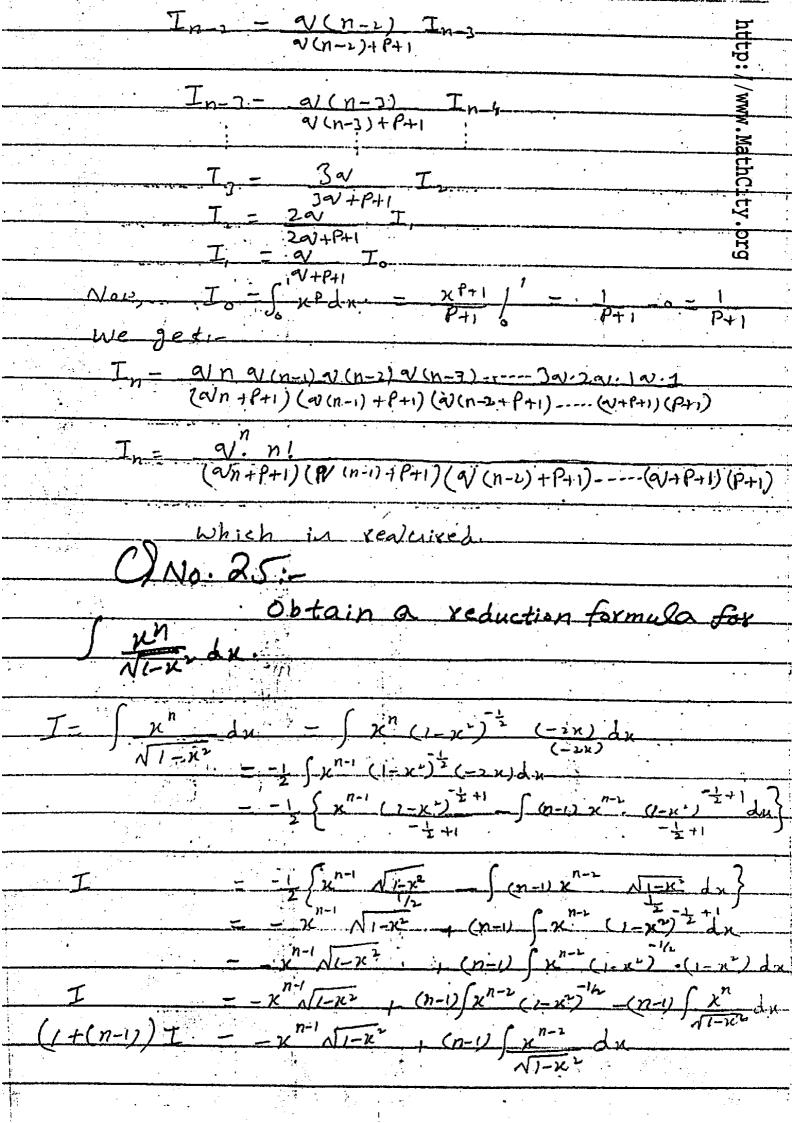
1 \int \(\sin^2 \) \(\sin QNO. 17:- 5 CONZX 1x (4.2.3.1) - 2 Am Put 2x=t , dx= 1dt So, $T - \int_{0}^{x/4} \frac{1}{2} dx = \int_{0}^{x/2} \frac{1}{2} dt = \int_{0}^{x/2} \frac{1}{2} dt$ $L = \frac{1}{2} \left(\frac{2-1}{2}, \frac{\chi}{2} \right) = \frac{\pi}{8} Am$ QNO.18:- 5 Cos33xdx Put 3x-+, dx= 1dt when, $x \rightarrow 0$, $t \rightarrow 0$. $x \rightarrow x$, $t \rightarrow x$. $T = \int_{0}^{\infty} Cox^{3} 3x dx - \int_{0}^{\infty} Cox^{3} t dt - \frac{1}{3} \int_{0}^{\infty} Cox^{3} t dt$.

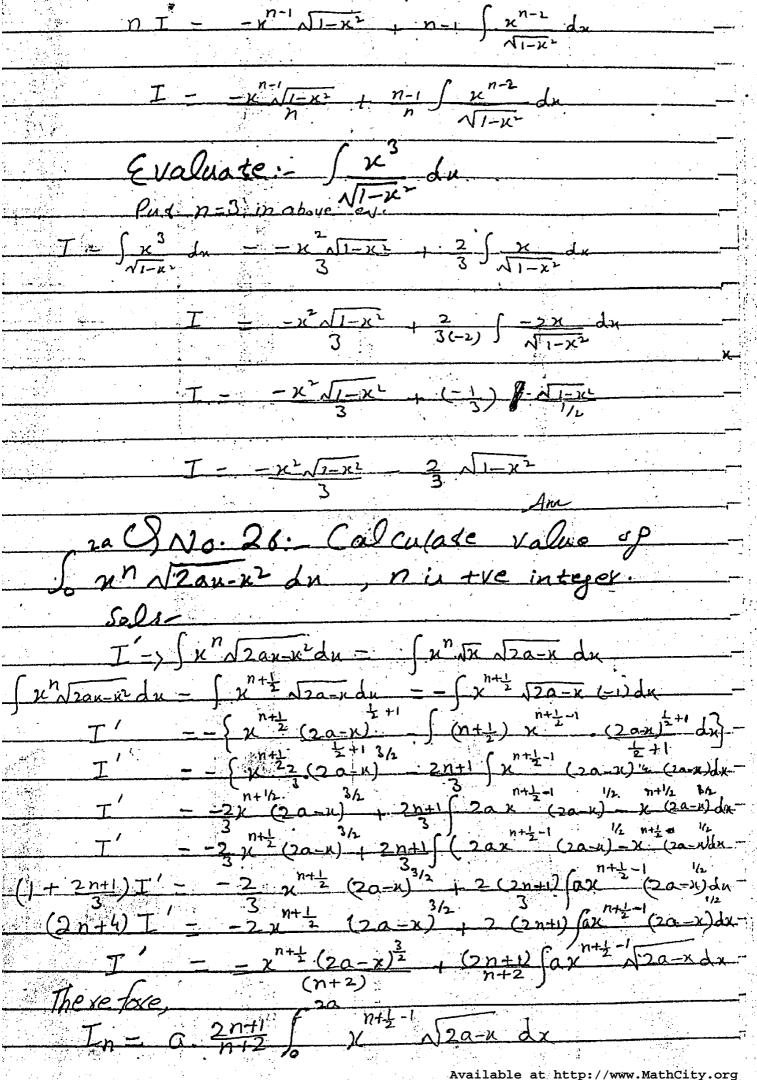


1/2 By Partial Fraction.
I =] [1- + (+++=)] d=
$T = \left[\frac{2}{2}\right]^{1/2} - \frac{1}{2}\left[\frac{1+2}{1-2}\right]^{1/2}$
$I = (o-1)^{2} - \frac{1}{2} \left[2n + 2n \left(\frac{3/2}{1/2} \right) \right]$
$T = -\frac{1}{2} - (+\frac{1}{2} \ln 3)\frac{1}{2} + \ln \sqrt{3}$
$T = -1$ $2n\sqrt{3}$ Am
QNO:21:- 1
$\int_{0}^{\infty} \frac{\chi^{\delta} dx}{x^{\delta}}$
$\sqrt{1-\lambda^2}$
Put x - Sino, dx = conodo
When $\chi \rightarrow 0$ $Q \rightarrow 0$
77 6
Therefore J K dx - Sinfo concodo NI-NI CONO
- Sintodo
= (6-1)(8-3)(6-5) <u>\tau</u>
6. (6-2)(8-4) 2 - 5.3.1. T.
6.4.2 2 - ST Am
GNO.22:-
Sec 2 dx = Sec x tanx + (1-1) Sec xdx- 2n
T- Sec xdx = Sec x. Sec xdx
I - Sec 2. Tank - STank (2n-1) Sec x Secutaria
I = Sec x tanx - (2n-1) \ Sec2n-1 Tan nda
I = Sec 2 tanu - (2n-1) \ Sec 2n-1 (Sec 2 u - 1) du
$I = Sec^{2n-1} + tanx - (2n-1) \int Sec^{2n+1} dx + (2n-1) \int Sec^{2n-1} dx$
(2n-1) I+I- Seen danx+ (2n-1) Sec2n-1 du
$T - \operatorname{Sec}^{2n-1} \operatorname{tanu} + (1-1) \operatorname{Sec}^{2n-1} \operatorname{du}$
2n 2n Hence Proved

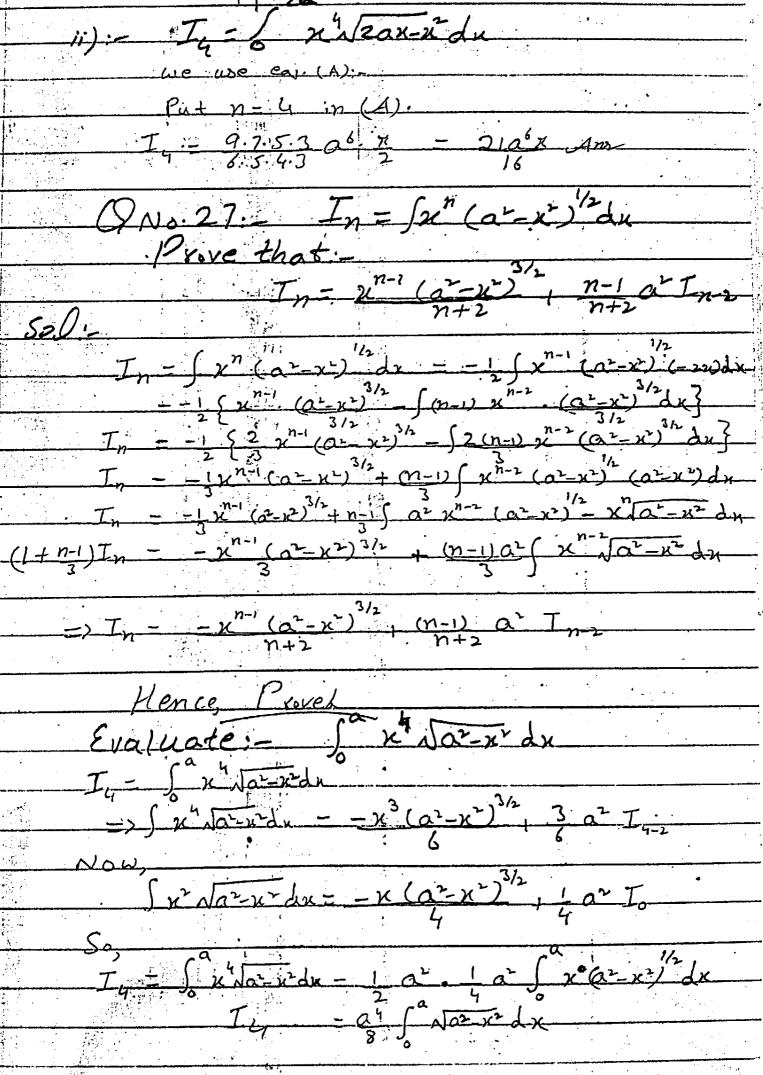


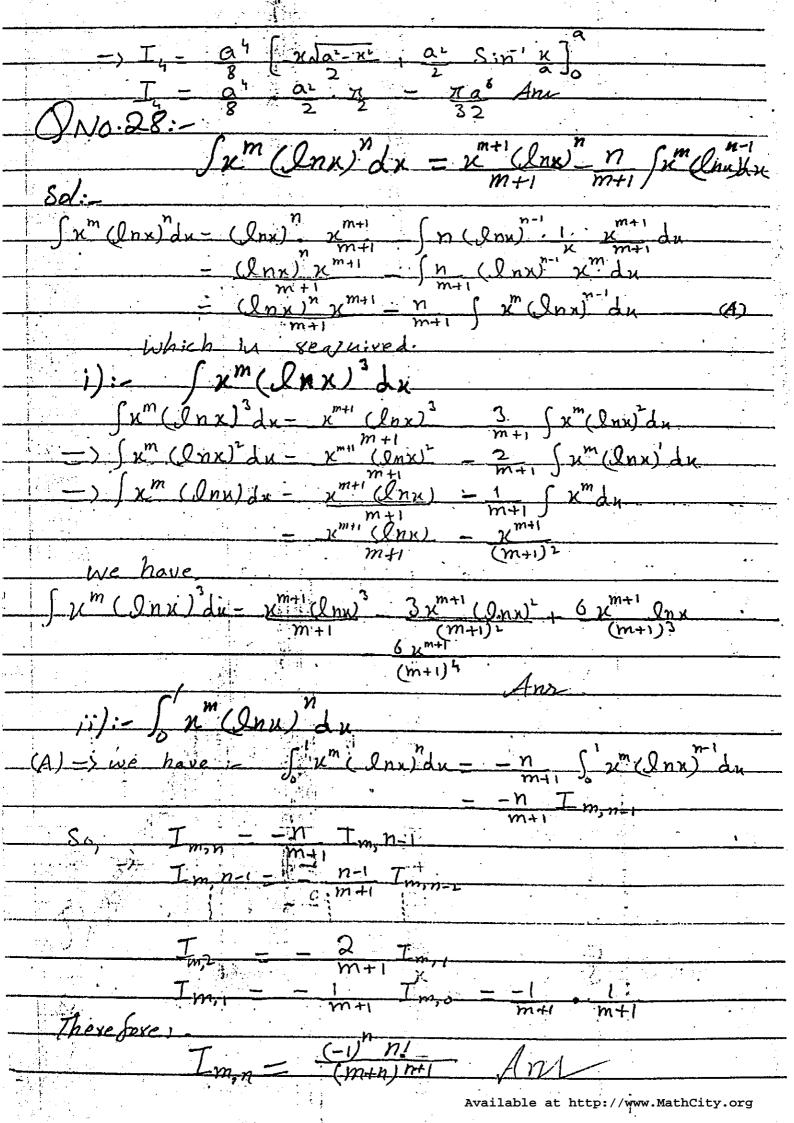


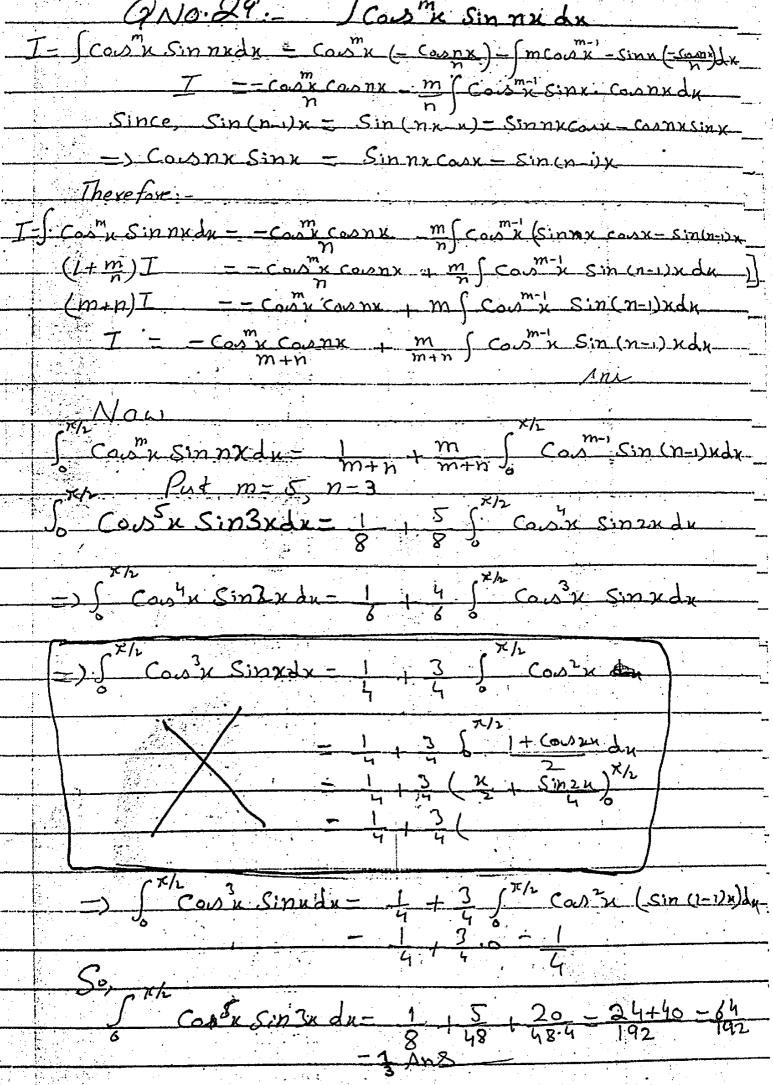




 $\frac{T_{n}-2n+1}{n+2}aT_{n-1}$ 2(n-1)+1aTn2 - $I_{n-2} = (2n-3)\alpha I_{n-1}$ 7a T. 3a I. - 1/2 x 1/2 N 2a-x dx -> Put x-2a Sin2Q; dx-4a Sina conda - Law dx Stra 2 a Sin O Condido 2 a Sino cono Gasinacono da = 802 5 ×12 Sin20 Con2010 $T_0 = 8a^2 - \frac{1 \cdot 1}{4 \cdot 2 \cdot 2} = \frac{1}{2}$ So We have: (2n+1)(2n-1)---5.3 a^n a^{-71} (n+2)(n+1)n---4.3 2 $I_n = (2n+1)(2n-1)---5.3 a^{n+2}$ (n+2) (n+1) --- 4.3 i): Evaluade: I, = 520 x NZa-x2 dx There are two mayn to silve thin I. alua T. (1) Put, x = 2 > (ii) = Put 10 n= 1 in en (A):- $T_1 - 3 \cdot a^3 \pi - a^3 \pi$







GNO. 30: - Find a reduction formula for Sun Non+26x+C
for fxn du
Nak+2bx+C
$\int_{\mathbb{R}^{n}} \int_{\mathbb{R}^{n}} \int_{$
$\int_{0}^{\infty} \frac{x^{n}}{\sqrt{ax^{2}+2bx+c}} dx = \int_{0}^{\infty} \frac{x^{n}(ax^{2}+2bx+c)^{\frac{1}{2}}dx}{-\int_{0}^{\infty} x^{n}(ax^{2}+2bx+c)^{\frac{1}{2}}dx - 2ax + 2b-2b}$
5. T = 1 (xn-1 (ax+2bx+c) = (2ax+2b-2b)dx
$\frac{-\int x (ax+2bx+c) dx - 2ax + 2b-1}{2ax}$ $\frac{\int x^{n-1} (ax^{2}+2bx+c)^{\frac{1}{2}} (2ax+2b-2b) dx}{I - \int x^{n-1} (ax^{2}+2bx+c)^{\frac{1}{2}} (2ax+2b) dx - \frac{2b}{2a} \int x^{n-1} (ax^{2}+2bx+c)^{\frac{1}{2}} x}$ $\frac{\int x^{n-1} (ax^{2}+2bx+c)^{\frac{1}{2}} (2ax+2b) dx - \frac{2b}{2a} \int x^{n-1} (ax^{2}+2bx+c)^{\frac{1}{2}} x}{(2ax^{2}+2bx+c)^{\frac{1}{2}} (2ax^{2}+2bx+c)^{\frac{1}{2}}}$ $\frac{\int x^{n-1} (ax^{2}+2bx+c)^{\frac{1}{2}} (2ax^{2}+2bx+c)^{\frac{1}{2}} (2ax^{2}+2bx+c)^{\frac{1}{2}}}{(2ax^{2}+2bx+c)^{\frac{1}{2}}}$
Dy fixet Integral:
$- \frac{1}{T} = \frac{1}{1} \left(\frac{x^{n-1}(ax^2 + 2bx + C)^{\frac{1}{2}}}{(n-1)x^{n-2}} \frac{(ax^2 + 2bx + C)^{\frac{n}{2}}}{(ax^2 + 2bx + C)^{\frac{n}{2}}} \right) du$
22
$T_{1} - x^{n-1} (ax^{2} + 2bx + c)^{1/2} = n-1 \int x^{n-2} (ax^{n} + 2bx + c)^{2} dx$
$T = x^{n-1} (\alpha x^{2} + 2hx + c)^{\frac{1}{2}} - n - 1 \left(x^{n-1} (\alpha x^{2} + 1hx + c)^{\frac{1}{2}} \alpha x^{2} dx \right)$
$\frac{n-1}{a} + \frac{1}{2} + $
a) / (()) / (
$T = \chi^{n-1} \left[\frac{1}{x^{n-1}} \right] \left[\frac{1}{x^{n$
Nax+2bx+c a Nax+2bx+c
(, n->
(n-1)C K dn A Nax +264+C
We get:
I(1+n-1) = 1 Nax+2bx+c -(2n-2)b / 2 dx (n-1)c
73
x 3 dx b x (ax 12bx +c) dx
Wax + 2 bk + K
T - x n-1 Nax+2bx+(- b (2n-1) / 2n-1)
Nax4-Lbx+c
6-1) C'(-x^-2
on a) Nax2+xbx+c. Am

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