LINEAR TRANSFORMATION :-

Let, u and v be two vector spaces over the same field F and let $T: u \rightarrow v$ be a function. Then, T is called a linear transformation if the following conditions are satisfied :-

i) $T(u_1+u_2) = T(u_1) + T(u_2) \quad \forall\, u_1, u_2 \in u$

ii) $T(\alpha u_1) = \alpha T(u_1) \quad \forall\, \alpha \in F$ and $u_1 \in u$

EXAMPLE :-

Let, $T: R^2 \rightarrow R^2$ is defined by $T(x_1, x_2) = (-x_2, x_1)$. Is T linear?

Solution :-

i) Let, $x = (x_1, x_2)$ and $y = (y_1, y_2)$ belongs to R^2, then $T(x) = T(x_1, x_2) = (-x_2, x_1)$

also $T(y) = T(y_1, y_2) = (-y_2, y_1)$

To show that,

$$T(x+y) = T(x) + T(y)$$

Now,

$$x+y = (x_1, x_2) + (y_1, y_2)$$
$$= (x_1+y_1,\ x_2+y_2)$$
$$T(x+y) = T(x_1+y_1,\ x_2+y_2)$$
$$= (-(x_2+y_2),\ x_1+y_1)$$
$$= (-x_2-y_2,\ x_1+y_1)$$
$$= (-x_2, x_1) + (-y_2, y_1)$$
$$T(x+y) = T(x) + T(y)$$

ii) Let, $x = (x_1, x_2)$ and $\alpha \in F$.

To show that,

$$T(\alpha x) = \alpha T(x)$$

Now,

Day: Date: / /

$$\alpha x = \alpha(x_1, x_2)$$
$$= (\alpha x_1, \alpha x_2)$$
$$T(\alpha x) = T(\alpha x_1, \alpha x_2)$$
$$= (-\alpha x_2, \alpha x_1)$$
$$= \alpha(-x_2, x_1)$$
$$T(\alpha x) = \alpha T(x)$$

Since both conditions are satisfied, so T is linear.

EXAMPLE:-

Let, $T : R^2 \to R^2$ be given by $T(x_1, x_2) = (x_1 + 1, x_2$
then T is not linear.

Solution:-

If T is linear then it should map $(0,0) \in R^2$ onto $(0,0) \in R^2$. But $T(0,0) = (1,2) \neq (0$
So, T is not linear.

EXAMPLE:-

Let, $T : R^2 \to R^2$ be given by $T(x_1, x_2) = (|x$

Solution:-

$$T(0,0) = (|0|, 0)$$
$$= (0, 0)$$

EXAMPLE:-

Let, $T : R^2 \to R^2$ be given by $T(x_1, x_2) = \Big(\frac{1}{x_1}$

Solution:-

$$T(0,0) = \left(\frac{1}{0}, 0\right)$$
$$= (\infty, 0)$$

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EXAMPLE :-

Let, $T : R^2 \to R^2$ be given by $T(x_1, x_2) = (x_1, x_2^2)$.

Solution :-

$$T(0,0) = (0, 0^2)$$
$$= (0,0)$$

EXAMPLE :-

Let, $T : R^2 \to R^2$ be given by $T(x_1, x_2) = (x_1 x_2,\ x_2 x_1)$ if $x_1 = -1$ and $x_2 = 2$.

Solution :-

$$T(-1,2) = ((-1)(2),\ (2)(-1))$$
$$= (-2, -2)$$