Linear Algebra

(Week 09-13)

Lecture 1

Determinants (Chapter No. 5)

Mathematical Method

5

Consider the simultaneous eqs.

$$a_1x + b_1 = 0 \quad \text{———— (1)}$$
$$a_2x + b_2 = 0 \quad \text{———— (2)}$$

Let us eliminate x from these two eqs.

From (1) $x = -\frac{b_1}{a_1}$

Put in (2)

$$a_2\left(-\frac{b_1}{a_1}\right) + b_2 = 0$$
$$-a_2b_1 + a_1b_2 = 0$$
or $$a_1b_2 - a_2b_1 = 0$$

The expression on the left i.e., $a_1b_2 - a_2b_1$ is symbolically written as

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$$ & is called a determinant

As this determinant has two rows & two columns, so it is said to be a determinant of order 2.

Again

Consider the simultaneous eqs.

$$a_1x + b_1y + c_1 = 0 \quad \text{———— (1)}$$
$$a_2x + b_2y + c_2 = 0 \quad \text{———— (2)}$$
$$a_3x + b_3y + c_3 = 0 \quad \text{———— (3)}$$

Let us eliminate x & y from these three eqs.

from (2) & (3)

$$\frac{x}{b_2c_3 - b_3c_2} = \frac{-y}{a_2c_3 - a_3c_2} = \frac{1}{a_2b_3 - a_3b_2}$$

$$\Rightarrow x = \frac{b_2c_3 - b_3c_2}{a_2b_3 - a_3b_2} \quad \& \quad y = -\frac{a_2c_3 - a_3c_2}{a_2b_3 - a_3b_2}$$

Put Values in ①

$$a_1\left(\frac{b_2c_3-b_3c_2}{a_2b_3-a_3b_2}\right)+b_1\left(-\frac{a_2c_3-a_3c_2}{a_2b_3-a_3b_2}\right)+c_1 = 0$$

or

$$a_1(b_2c_3-b_3c_2)-b_1(a_2c_3-a_3c_2)+c_1(a_2b_3-a_3b_2)=0$$

or

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

As it Consists of three row & three Columns, so it is said to be a determinant of order 3.

<u>Properties of determinants:</u>

Following are some important properties of determinants.

(i) The value of a determinant is the same as the value of its transpose.

(ii) The interchange of two adjacent rows or Columns changes the sign of the determinant.

(iii) If a row or Column of a determinant is passed over m rows or Columns then its value is multiplied by $(-1)^m$.

(iv) If any two rows or Columns of a determinant are identical then value of determinant is zero.

(v) If all the elements in a row or Column of a determinant are zero then value of the determinant is zero.

(vi) If a non zero scalar is multiplied by a determinant then this scalar will be multiplied by any one of the rows or Columns of that det.

(vii) If each element in a row or Column of a determinant is the sum of two elements then this determinant will be written as the sum of two determinants as

$$\begin{vmatrix} a_1 & b_1+t_1 & c_1 \\ a_2 & b_2+t_2 & c_2 \\ a_3 & b_3+t_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \begin{vmatrix} a_1 & t_1 & c_1 \\ a_2 & t_2 & c_2 \\ a_3 & t_3 & c_3 \end{vmatrix}$$

(viii) Addition of some scalar multiple of a row or Column to any other row or Column does not Change the value of that determinant.

Minors & Cofactors:

Let $\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$

be a given determinant of order n.

The minor of an element a_{ij} of Δ is the det. M_{ij} obtained by deleting the rows & Columns in which a_{ij} lies. Clearly M_{ij} is a determinant of order $n-1$.

The Cofactor A_{ij} of an element a_{ij} of Δ is

$$A_{ij} = (-1)^{i+j} \cdot M_{ij}$$

Note

(i) $\Delta = a_{i1}A_{i1} + a_{i2}A_{i2} + \cdots\cdots + a_{in}A_{in}$ for any i

(ii) $\Delta = a_{1j}A_{1j} + a_{2j}A_{2j} + \cdots\cdots + a_{nj}A_{nj}$ for any j

(iii) If the elements of a line are multiplied by the cofactors of the corresponding elements of any other parallel line & the results so obtained are added the answer will be zero.

Adjoint of a square matrix:

Let $A = [a_{ij}]_{n\times n}$ be a square matrix of order n. Denoting the co-factors by A_{ij} of the elements a_{ij} of A, we define

$$\text{Adj}\,A = [A_{ij}]^t_{n\times n} = [A_{ji}]_{n\times n}$$

Inverse of a square matrix:

Let A be a non singular square matrix of order n then inverse of A is defined as

$$A^{-1} = \frac{\text{Adj}\,A}{|A|}$$

Note If A & B are square matrices of order n then

(i) $\det(AB) = \det(A).\det(B)$

(ii) $\det(BA) = \det(B).\det(A)$

(iii) $\det(A^{-1}) = (\det(A))^{-1}$ if A is non singular

(iv) $\det(A^t) = \det(A)$

(v) $\det(A^n) = [\det(A)]^n$ where $n \in Z^+$

(vi) $\det(KA) = K^n.\det(A)$

Exercise No. 5.1

5 Mathematical Method

Q1 Let M_2 be the set of all 2×2 matrices. Set up the transformation $A \longrightarrow \det(A)$, $A \in M_2$. What is the range of this mapping? Is the mapping one-to-one?

Sol.

Let $f : A \longrightarrow \det(A)$; $A \in M_2$
be defined by

$$f(A) = \det(A)$$

Suppose the field for all $A \in M_2$ be the set of Complex no's. C, then the range of f is C. But if the field is taken as the set of real no's. R then range of f is also R

This mapping f is not one-to-one as shown by the following example

Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 6 & 0 \\ 9 & 1 \end{bmatrix}$

then clearly $A \neq B$

Now

$$\det(A) = \begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} = 8-2 = 6$$

& $$\det(B) = \begin{vmatrix} 6 & 0 \\ 9 & 1 \end{vmatrix} = 6-0 = 6$$

Hence $\det(A) = \det(B)$

So we have proved that

$$A \neq B \Rightarrow \det(A) = \det(B)$$

Hence by def., f is not one-to-one.

Q2 For 2×2 matrices A & B which of the following equations hold?

(i) $\det(A+B) = \det(A) + \det(B)$

Sol.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ & $B = \begin{bmatrix} f & g \\ h & k \end{bmatrix}$

then

$$A+B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} f & g \\ h & k \end{bmatrix}$$

or

$$A+B = \begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}$$

$$\Rightarrow \det(A+B) = \begin{vmatrix} a+f & b+g \\ c+h & d+k \end{vmatrix}$$

$$= (a+f)(d+k) - (b+g)(c+h)$$

$$= ad + ak + fd + fk - bc - bh - gc - gh \qquad (1)$$

Now

$$\det A + \det B = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} f & g \\ h & k \end{vmatrix}$$

$$= ad - bc + fk - gh \qquad (2)$$

from (1) & (2)

$$\det(A+B) \neq \det A + \det B$$

(ii) $\det(A+B)^2 = [\det(A+B)]^2$

Sol.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ & $B = \begin{bmatrix} f & g \\ h & k \end{bmatrix}$

then

$$A+B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} f & g \\ h & k \end{bmatrix}$$

or $$A+B = \begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}$$

Now

$$(A+B)^2 = \begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}\begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}$$

$$\Rightarrow (A+B)^2 = \begin{bmatrix} (a+f)^2+(b+g)(c+h) & (a+f)(b+g)+(b+g)(d+k) \\ (c+h)(a+f)+(d+k)(c+h) & (c+h)(b+g)+(d+k)^2 \end{bmatrix}$$

So

$$\det(A+B)^2 = \begin{vmatrix} (a+f)^2+(b+g)(c+h) & (a+f)(b+g)+(b+g)(d+k) \\ (c+h)(a+f)+(d+k)(c+h) & (c+h)(b+g)+(d+k)^2 \end{vmatrix} \quad \text{———} \ (1)$$

Now

$$A+B = \begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}$$

then

$$\det(A+B) = \begin{vmatrix} a+f & b+g \\ c+h & d+k \end{vmatrix}$$

&

$$[\det(A+B)]^2 = \begin{vmatrix} a+f & b+g \\ c+h & d+k \end{vmatrix}\begin{vmatrix} a+f & b+g \\ c+h & d+k \end{vmatrix}$$

$$\Rightarrow [\det(A+B)]^2 = \begin{vmatrix} (a+f)^2+(b+g)(c+h) & (a+f)(b+g)+(b+g)(d+k) \\ (c+h)(a+f)+(d+k)(c+h) & (c+h)(b+g)+(d+k)^2 \end{vmatrix} \quad \text{———} \ (2)$$

from (1) & (2)

$$\det(A+B)^2 = [\det(A+B)]^2$$

(iii) $\det(A+B)^2 = \det(A^2+B^2)$

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ & $B = \begin{bmatrix} f & g \\ h & k \end{bmatrix}$

then

$$A+B = \begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}$$

Now

$$(A+B)^2 = \begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}\begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} (a+f)^2+(b+g)(c+h) & (a+f)(b+g)+(b+g)(d+k) \\ (c+h)(a+f)+(d+k)(c+h) & (c+h)(b+g)+(d+k)^2 \end{bmatrix}$$

So

$$\det(A+B)^2 = \begin{vmatrix} (a+f)^2+(b+g)(c+h) & (a+f)(b+g)+(b+g)(d+k) \\ (c+h)(a+f)+(d+k)(c+h) & (c+h)(b+g)+(d+k)^2 \end{vmatrix} \quad \text{——— (1)}$$

Now

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

&

$$B^2 = \begin{bmatrix} f & g \\ h & k \end{bmatrix}\begin{bmatrix} f & g \\ h & k \end{bmatrix}$$

$$= \begin{bmatrix} f^2+gh & fg+gk \\ hf+kh & gh+k^2 \end{bmatrix}$$

So

$$A^2+B^2 = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} + \begin{bmatrix} f^2+gh & fg+gk \\ hf+kh & gh+k^2 \end{bmatrix}$$

$$A^2+B^2 = \begin{bmatrix} a^2+f^2+bc+gh & ab+bd+fg+gk \\ ac+cd+hf+kh & d^2+k^2+bc+gh \end{bmatrix}$$

So

$$\det(A^2+B^2) = \begin{vmatrix} a^2+f^2+bc+gh & ab+bd+fg+gk \\ ac+cd+hf+kh & d^2+k^2+bc+gh \end{vmatrix} \quad \text{——— (2)}$$

from ① & ②

$$\det(A+B)^2 \neq \det(A^2+B^2)$$

(iv) $\det(A+B)^2 = \det(A^2+2AB+B^2)$

Sol.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ & $B = \begin{bmatrix} f & g \\ h & k \end{bmatrix}$

then

$$A+B = \begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}$$

Now

$$(A+B)^2 = \begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}\begin{bmatrix} a+f & b+g \\ c+h & d+k \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} (a+f)^2+(b+g)(c+h) & (a+f)(b+g)+(b+g)(d+k) \\ (c+h)(a+f)+(d+k)(c+h) & (c+h)(b+g)+(d+k)^2 \end{bmatrix}$$

So

$$\det(A+B)^2 = \begin{vmatrix} (a+f)^2+(b+g)(c+h) & (a+f)(b+g)+(b+g)(d+k) \\ (c+h)(a+f)+(d+k)(c+h) & (c+h)(b+g)+(d+k)^2 \end{vmatrix} \quad \text{——— (1)}$$

Now

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} f & g \\ h & k \end{bmatrix}\begin{bmatrix} f & g \\ h & k \end{bmatrix} = \begin{bmatrix} f^2+gh & fg+gk \\ hf+kh & gh+k^2 \end{bmatrix}$$

&

$$2AB = 2\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} f & g \\ h & k \end{bmatrix}$$

$$= 2\begin{bmatrix} af+bh & ag+bk \\ cf+dh & gc+dk \end{bmatrix}$$

$$= \begin{bmatrix} 2af+2bh & 2ag+2bk \\ 2cf+2dh & 2gc+2dk \end{bmatrix}$$

So

$$A^2+2AB+B^2 = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} + \begin{bmatrix} 2af+2bh & 2ag+2bk \\ 2cf+2dh & 2gc+2dk \end{bmatrix} + \begin{bmatrix} f^2+gh & fg+gk \\ hf+kh & gh+k^2 \end{bmatrix}$$

$$A^2+2AB+B^2 = \begin{bmatrix} a^2+bc+2af+2bh+f^2+gh & ab+bd+2ag+2bk+fg+gk \\ ac+cd+2cf+2dh+hf+kh & bc+d^2+2gc+2dk+gh+k^2 \end{bmatrix}$$

So

$$\det(A^2+2AB+B^2) = \begin{vmatrix} a^2+bc+2af+2bh+f^2+gh & ab+bd+2ag+2bk+fg+gk \\ ac+cd+2cf+2dh+hf+kh & bc+d^2+2gc+2dk+gh+k^2 \end{vmatrix} \quad — (2)$$

from (1) & (2)

$$\det(A+B)^2 \neq \det(A^2+2AB+B^2)$$

Q3 Find the Value of each of the following determinants:

(i) $\begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix}$.

Sol:

Let $\Delta = \begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix}$

Expanding from R_1

$$= 1\begin{vmatrix} 4 & 5 \\ 6 & 7 \end{vmatrix} - 0\begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} + 2\begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix}$$

$= 1(28-30) - 0 + 2(18-20)$

$= -2 + 2(-2)$

$= -2-4$

$\Delta = -6$

(ii) $\begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & 4 \\ -1 & 0 & 3 \end{vmatrix}$

Sol.

Let $\Delta = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & 4 \\ -1 & 0 & 3 \end{vmatrix}$

Expanding from R_1

$= 2\begin{vmatrix} 2 & 4 \\ 0 & 3 \end{vmatrix} + 1\begin{vmatrix} 3 & 4 \\ -1 & 3 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ -1 & 0 \end{vmatrix}$

$= 2(6-0) + (9+4) + (0+2)$

$= 2(6) + 13 + 2$

$= 12 + 15$

$\Delta = 27$

(iii) $\begin{vmatrix} 6 & -6 & 6 \\ 2 & 4 & -6 \\ 15 & -5 & 5 \end{vmatrix}$

Sol.

Let $\Delta = \begin{vmatrix} 6 & -6 & 6 \\ 2 & 4 & -6 \\ 15 & -5 & 5 \end{vmatrix}$

Expanding from R_1

$= 6\begin{vmatrix} 4 & -6 \\ -5 & 5 \end{vmatrix} + 6\begin{vmatrix} 2 & -6 \\ 15 & 5 \end{vmatrix} + 6\begin{vmatrix} 2 & 4 \\ 15 & -5 \end{vmatrix}$

$= 6(20-30) + 6(10+90) + 6(-10-60)$

$= 6(-10) + 6(100) + 6(-70)$

$= -60 + 600 - 420$

$\Delta = -60 + 180 = 120$

Q4 Evaluate

(i) $\begin{vmatrix} 2 & 3 & -2 & 4 \\ 7 & 4 & -3 & 10 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{vmatrix}$

Sol.

Let $\Delta = \begin{vmatrix} 2 & 3 & -2 & 4 \\ 7 & 4 & -3 & 10 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{vmatrix}$

$= \begin{vmatrix} 2 & 3 & -2 & 4 \\ 7 & 4 & -3 & 10 \\ 1 & -1 & 5 & 0 \\ -2 & 4 & 0 & 5 \end{vmatrix}$ $\quad R_3 - R_1$

$= \begin{vmatrix} 0 & 5 & -12 & 4 \\ 0 & 11 & -38 & 10 \\ 1 & -1 & 5 & 0 \\ 0 & 2 & 10 & 5 \end{vmatrix}$ $\quad R_1 - 2R_3$, $R_2 - 7R_3$, $R_4 + 2R_3$

Expanding from C_1

$= 0 - 0 + \begin{vmatrix} 5 & -12 & 4 \\ 11 & -38 & 10 \\ 2 & 10 & 5 \end{vmatrix} - 0$

$= \begin{vmatrix} 5 & -12 & 4 \\ 11 & -38 & 10 \\ 2 & 10 & 5 \end{vmatrix}$

Expanding from R_1

$= 5\begin{vmatrix} -38 & 10 \\ 10 & 5 \end{vmatrix} + 12\begin{vmatrix} 11 & 10 \\ 2 & 5 \end{vmatrix} + 4\begin{vmatrix} 11 & -38 \\ 2 & 10 \end{vmatrix}$

$= 5(-190 - 100) + 12(55 - 20) + 4(110 + 76)$

$$= 5(-290) + 12(35) + 4(186)$$

$$= -1450 + 420 + 744$$

$$= -1450 + 1164$$

$$\Delta = -286$$

(ii) $\begin{vmatrix} 3 & 7 & 5 & 2 \\ 2 & 4 & 1 & 1 \\ -2 & 0 & 0 & 0 \\ 1 & 1 & 3 & 4 \end{vmatrix}$

Sol.

$$\text{Let } \Delta = \begin{vmatrix} 3 & 7 & 5 & 2 \\ 2 & 4 & 1 & 1 \\ -2 & 0 & 0 & 0 \\ 1 & 1 & 3 & 4 \end{vmatrix}$$

Expanding from R_3

$$= -2\begin{vmatrix} 7 & 5 & 2 \\ 4 & 1 & 1 \\ 1 & 3 & 4 \end{vmatrix}$$

Expanding from R_1

$$= -2\left\{7\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} - 5\begin{vmatrix} 4 & 1 \\ 1 & 4 \end{vmatrix} + 2\begin{vmatrix} 4 & 1 \\ 1 & 3 \end{vmatrix}\right\}$$

$$= -2\left\{7(4-3) - 5(16-1) + 2(12-1)\right\}$$

$$= -2\left\{7(1) - 5(15) + 2(11)\right\}$$

$$= -2(7 - 75 + 22)$$

$$= -2(-46)$$

$$= 92$$

(iii) $\begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & -3 \\ 0 & -7 & 3 & 1 \end{vmatrix}$

Sol.

$$\text{Let } \Delta = \begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & -3 \\ 0 & -7 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 0 & 5 & -3 & 1 \\ 0 & -7 & 3 & 1 \end{vmatrix} \qquad R_3 - R_1$$

$$= \begin{vmatrix} 1 & -1 & 1 \\ 5 & -3 & 1 \\ -7 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -4 \\ 0 & -4 & 8 \end{vmatrix} \qquad \begin{matrix} R_2 - 5R_1 \\ R_3 + 7R_1 \end{matrix}$$

Expanding from C_1

$$= 1 \begin{vmatrix} 2 & -4 \\ -4 & 8 \end{vmatrix}$$

$$= 16 - 16$$

$$\Delta = 0$$

(iv) $\begin{vmatrix} 9 & 93 & 12 & -6 \\ 1 & 92 & 84 & -6 \\ 2 & 185 & 100 & -12 \\ 4 & 270 & 196 & -24 \end{vmatrix}$

Sol.

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$$\text{Let } \Delta = \begin{vmatrix} 9 & 93 & 12 & -6 \\ 1 & 92 & 84 & -6 \\ 2 & 185 & 100 & -12 \\ 4 & 270 & 196 & -24 \end{vmatrix}$$

taking -6 Common from C_4

$$= -6 \begin{vmatrix} 9 & 93 & 12 & 1 \\ 1 & 92 & 84 & 1 \\ 2 & 185 & 100 & 2 \\ 4 & 270 & 196 & 4 \end{vmatrix}$$

$$= -6 \begin{vmatrix} 9 & 93 & 12 & 1 \\ -8 & -1 & 72 & 0 \\ -16 & -1 & 76 & 0 \\ -32 & -102 & 148 & 0 \end{vmatrix} \qquad \begin{matrix} R_2 - R_1 \\ R_3 - 2R_1 \\ R_4 - 4R_1 \end{matrix}$$

Expanding from C_4

$$= -6 \begin{vmatrix} -8 & -1 & 72 \\ -16 & -1 & 76 \\ -32 & -102 & 148 \end{vmatrix}$$

taking $-8, -1, 4$ Common from C_1, C_2, C_3

$$= (-6)(-8)(-1)(4) \begin{vmatrix} 1 & 1 & 18 \\ 2 & 1 & 19 \\ 4 & 102 & 37 \end{vmatrix}$$

$$= -192 \begin{vmatrix} 1 & 1 & 18 \\ 2 & 1 & 19 \\ 4 & 102 & 37 \end{vmatrix}$$

$$= -192 \begin{vmatrix} 1 & 1 & 18 \\ 0 & -1 & -17 \\ 0 & 98 & -35 \end{vmatrix} \qquad \begin{matrix} R_2 - 2R_1 \\ R_3 - 4R_1 \end{matrix}$$

Expanding from C_1

$$= -192 \begin{vmatrix} -1 & -17 \\ 98 & -35 \end{vmatrix}$$