CS 223 – Digital Logic and Design

Lecture 3 – Number Systems

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Number Systems

BINARY NUMBERS

A decimal number such as 7,392 represents a quantity equal to 7 thousands, plus 3 hundreds, plus 9 tens, plus 2 units. The thousands, hundreds, etc., are powers of 10 implied by the position of the coefficients (symbols) in the number. To be more exact, 7,392 is a shorthand notation for what should be written as

$$7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

However, the convention is to write only the numeric coefficients and, from their position, deduce the necessary powers of 10 with powers increasing from right to left. In general, a number with a decimal point is represented by a series of coefficients:

$$a_5a_4a_3a_2a_1a_0.\ a_{-1}a_{-2}a_{-3}$$

The coefficients a_j are any of the 10 digits $(0, 1, 2, \ldots, 9)$, and the subscript value j gives the place value and, hence, the power of 10 by which the coefficient must be multiplied. Thus, the preceding decimal number can be expressed as

$$10^5a_5 + 10^4a_4 + 10^3a_3 + 10^2a_2 + 10^1a_1 + 10^0a_0 + 10^{-1}a_{-1} + 10^{-2}a_{-2} + 10^{-3}a_{-3}$$

with $a_3 = 7$, $a_2 = 3$, $a_1 = 9$, and $a_0 = 2$.

Number Systems (Contd.)

- The decimal equivalent of the binary number **11010.11** is **26.75**, as shown from the multiplication of the coefficients by powers of **2**:

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 26.75$$

There are many different number systems. In general, a number expressed in a base-r system has coefficients multiplied by powers of r:

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \cdots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \cdots + a_{-m} \cdot r^{-m}$$

The coefficients a_j range in value from 0 to $r - 1$. To distinguish between numbers of different bases, we enclose the coefficients in parentheses and write a subscript equal to the base used (except sometimes for decimal numbers, where the content makes it obvious that the base is decimal). An example of a base-5 number is

$$(4021.2)_5 = 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1} = (511.4)_{10}$$

Number Systems (Contd.)

- **The letters of the alphabet are used to supplement the 10 decimal digits when the base of the number is greater than 10.**
- For example, in the hexadecimal (base-16) number system, the first 10 digits are borrowed

from the decimal system. The letters A, B, C, D, E, and F are used for the digits 10, 11, 12, 13, 14, and 15, respectively. An example of a hexadecimal number is

$$(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0 = (46{,}687)_{10}$$

The hexadecimal system is used commonly by designers to represent long strings of bits in the addresses, instructions, and data in digital systems. For example, B65F is used to represent 1011011001010000.

Number Systems (Contd.)

As noted before, the digits in a binary number are called *bits*. When a bit is equal to 0, it does not contribute to the sum during the conversion. Therefore, the conversion from binary to decimal can be obtained by adding only the numbers with powers of two corresponding to the bits that are equal to 1. For example,

$$(110101)_2 = 32 + 16 + 4 + 1 = (53)_{10}$$

There are four 1's in the binary number. The corresponding decimal number is the sum of the four powers of two. Zero and the first 24 numbers obtained from 2 to the power of n are listed in Table 1.1. In computer work, 2^{10} is referred to as K (kilo), 2^{20} as M (mega), 2^{30} as G (giga), and 2^{40} as T (tera). Thus, $4\text{K} = 2^{12} = 4{,}096$ and $16\text{M} = 2^{24} = 16{,}777{,}216$. Computer capacity is usually given in bytes. A *byte* is equal to eight bits and can accommodate (i.e., represent the code of) one keyboard character. A computer hard disk with four gigabytes of storage has a capacity of $4\text{G} = 2^{32}$ bytes (approximately 4 billion bytes). A terabyte is 1024 gigabytes, approximately 1 trillion bytes.

Table 1.1 Powers of Two

Table 1.1
Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024 (1K)	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096 (4K)	20	1,048,576 (1M)
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

Arithmetic Operations

Arithmetic operations with numbers in base *r* follow the same rules as for decimal numbers. When a base other than the familiar base 10 is used, one must be careful to use only the *r*-allowable digits. Examples of addition, subtraction, and multiplication of two binary numbers are as follows:

$$
\begin{array}{lr}
\text{augend:} & 101101 \\
\text{addend:} & \underline{+100111} \\
\text{sum:} & 1010100
\end{array}
\qquad
\begin{array}{lr}
\text{minuend:} & 101101 \\
\text{subtrahend:} & \underline{-100111} \\
\text{difference:} & 000110
\end{array}
\qquad
\begin{array}{lr}
\text{multiplicand:} & 1011 \\
\text{multiplier:} & \underline{\times\ 101} \\
 & 1011 \\
 & 0000 \\
\text{partial product:} & \underline{1011} \\
\text{product:} & 110111
\end{array}
$$

- That’s end of the presentation ! ☺