

Techniques of Integration

EXERCISE 4.1

Q No. 1 $\int 0 \, dx = c$

Q No. 2 $\int \sqrt{x} \, dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{2}{3} x^{\frac{3}{2}} + c$

Q No. 3 $\int \frac{1+x}{x} \, dx = \int \left(\frac{1}{x} + \frac{x}{x} \right) dx = \int \frac{dx}{x} + \int dx$
 $= \ln(x) + x + c$

Q No. 4 $\int \frac{x^2-1}{x^2+1} \, dx$
 $= \int \left(1 - \frac{2}{x^2+1} \right) dx$ by long division
 $= \int dx - 2 \int \frac{dx}{x^2+1}$
 $= x - 2 \tan^{-1} x + c$

Q No. 5 $\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx$
 $= \int \sec^2 x \, dx - \int dx$
 $= \tan x - x + c$

Q No. 6 $\int \cot^2 x \, dx = \int (\operatorname{cosec}^2 x - 1) \, dx$
 $= \int \operatorname{cosec}^2 x \, dx - \int dx$
 $= -\cot x - x + c$

Q No. 7 $\int \cos^2 x \, dx = \int \left(\frac{1+\cos 2x}{2} \right) dx$
 $= \frac{1}{2} \int dx + \frac{1}{2} \int \cos 2x \, dx$
 $= \frac{x}{2} + \frac{1}{2} \left(\frac{\sin 2x}{2} \right) + c$

Q No. 8 $\int \sin^2 x \, dx = \int \left(\frac{1-\cos 2x}{2} \right) dx$
 $= \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x \, dx$
 $= \frac{x}{2} - \frac{1}{2} \left(\frac{\sin 2x}{2} \right) + c$

Q No. 9 $\int \sqrt{1-\cos x} \, dx = \int \left(\sqrt{2} \sin \frac{x}{2} \right) dx$
 $= \sqrt{2} \int \sin \frac{x}{2} \, dx$
 $= \sqrt{2} \left(-\frac{\cos \frac{x}{2}}{\frac{1}{2}} \right) + c$
 $= -2\sqrt{2} \cos \frac{x}{2} + c$

Q No. 10 $\int \sqrt{4-x^2} \, dx = \frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \frac{x}{2}$

Q No. 11 $\int \sqrt{4+x^2} \, dx = \frac{x}{2} \sqrt{4+x^2} + 2 \sinh^{-1} \frac{x}{2}$

Q No. 12 $\int \sqrt{x^2-4} \, dx = \frac{x}{2} \sqrt{x^2-4} - 2 \cosh^{-1} \frac{x}{2}$