Theory of Automata

Lecture #17-18-19

Transition Graph (T.G)

- Like an NFA except for start states and transitions
- Transitions
  - Go from ***some*** states to some other states.
  - Labeled by ***strings*** of letters from the alphabet.
  - The labels may include the empty word Λ.
- Start states
  - Maybe ***more*** than one.

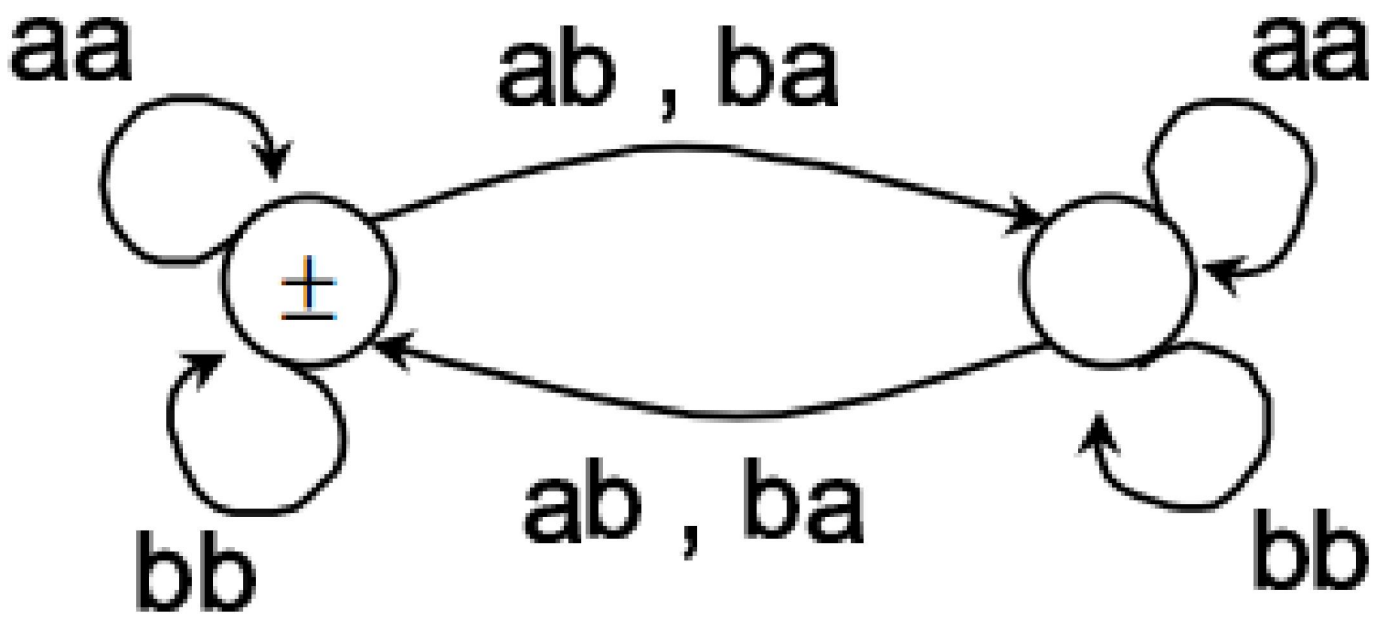
Transition Graph (T.G)

- A finite set of states **at least one** of which is designated as the **start state** (-) and **some** (maybe none) of which are **designated as final states** (+).
- An alphabet Σ of possible input letters from which input strings are formed.
- A **finite set of transitions** that show how to go from one state to another based on reading specified substrings of input letters (possibly even the null string λ).

Transition Graph (T.G)

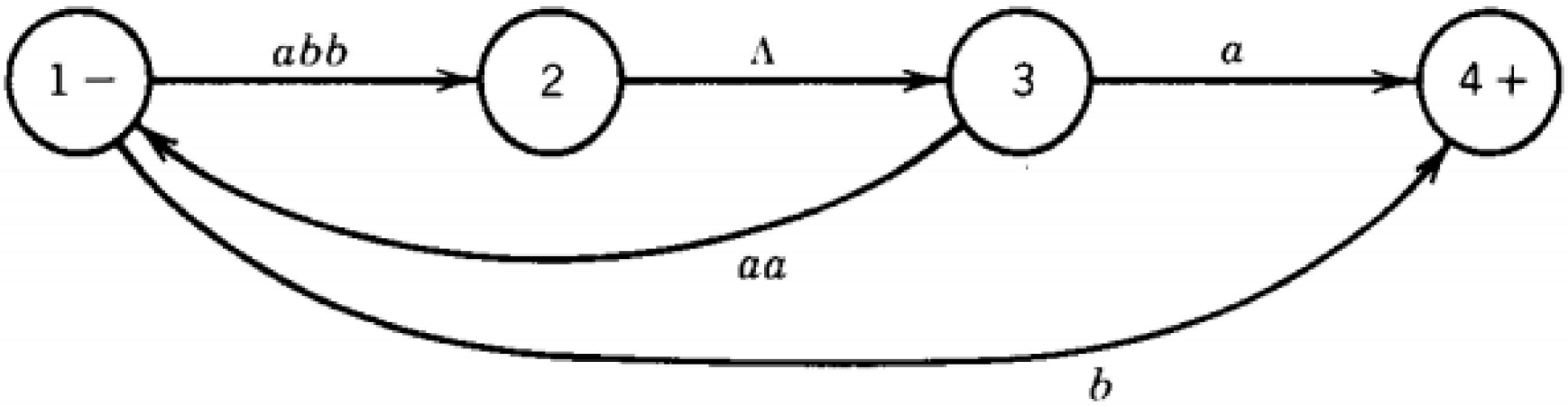
- A **successful path** through a transition graph is a series of edges forming a path beginning at some start state **(there may be several)** and ending at a final state. If we concatenate in order the strings of letters that label each edge in the path, we produce a word that is accepted by this machine.

Transition Graph (T.G)-Even Even



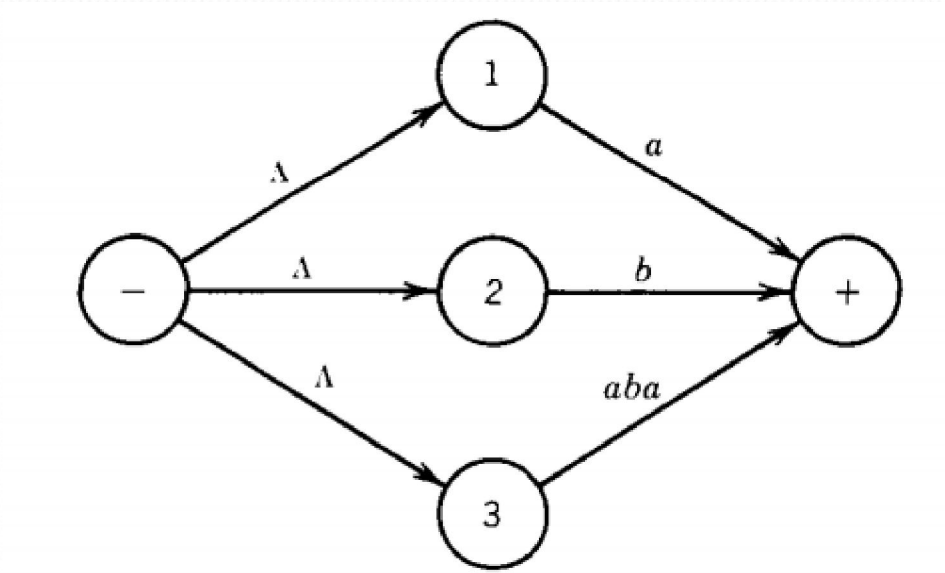
Transition Graph (T.G)

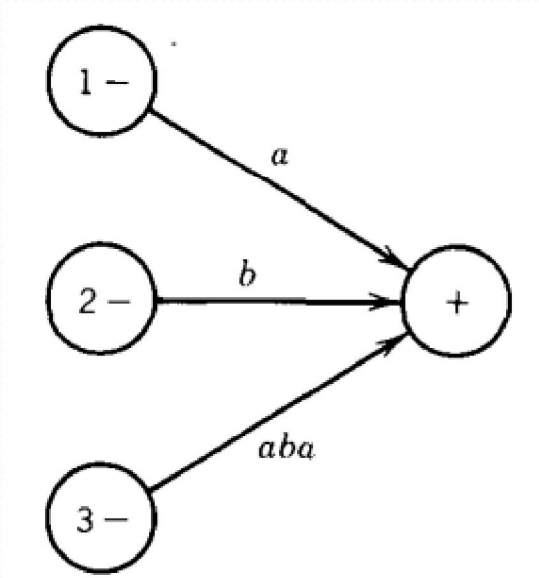
- For example, consider the following TG



- Some accepted words are **abba**, **abbaaabba**, and **b etc**

Transition Graph (T.G)





Both are Same

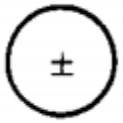
Transition Graph (T.G)

- It is extremely important for us to understand that every FA is also a TG.
- This means that any picture that represents an FA can be interpreted as a picture of a TG. Of course, not every TG satisfies the definition of an FA.
- Let us consider some more examples of TG's.

Transition Graph (T.G)

- The picture above represents a TG that accepts nothing, not even the null string Λ. To be able to accept anything, it must have a final state.

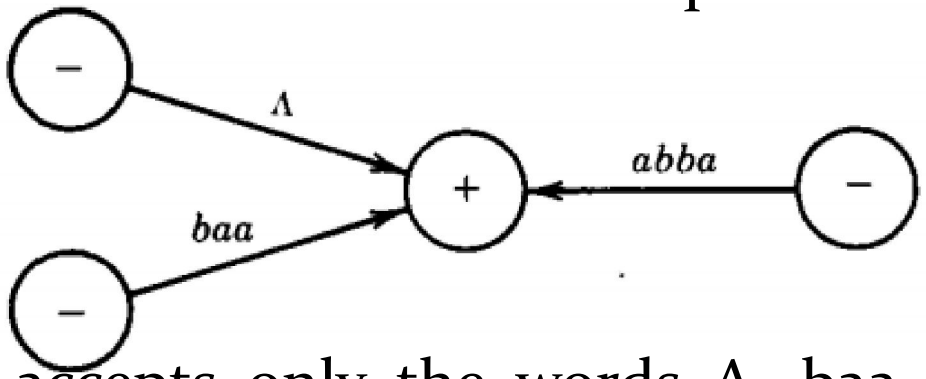
- The machine



accepts only the string λ

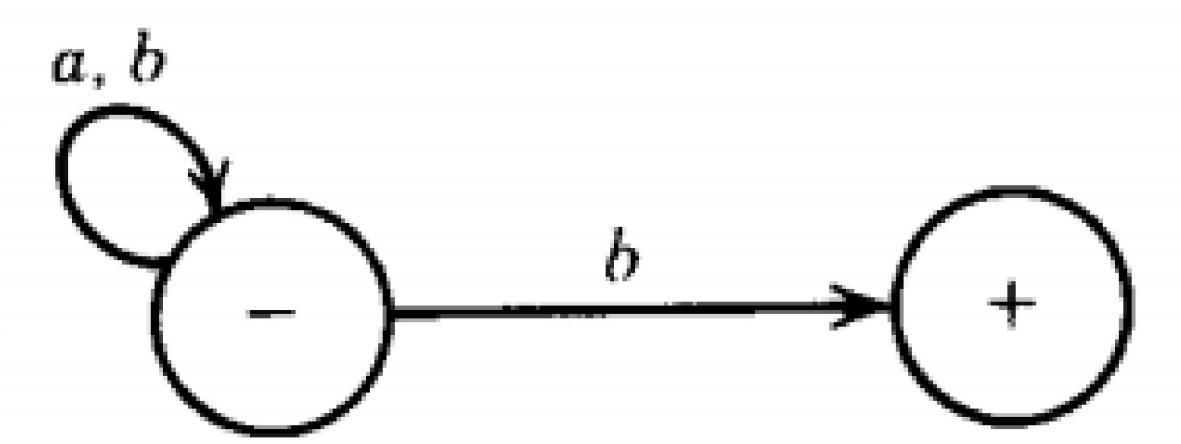
Transition Graph (T.G)

- Any TG in which some start state is also a final state will always accept the string λ; this is also true of FA's
- There are some other TG's that accept the word λ, for example:



- This machine accepts only the words A, baa, and abba. Anything read while in the + state will cause a crash, since the + state has no outgoing edges

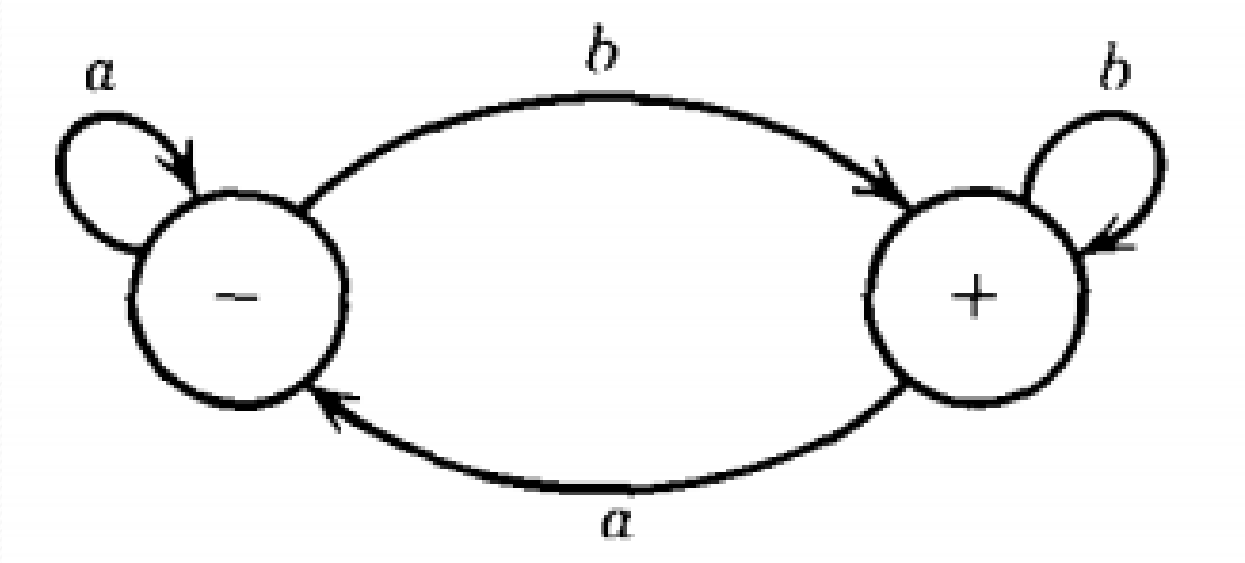
Transition Graph (T.G)



- We can read all the input letters one at a time and stay in the left-side state. When we read a b in the - state there are two possible edges we can follow. If the very last letter is a b, we can use it to go to the + state

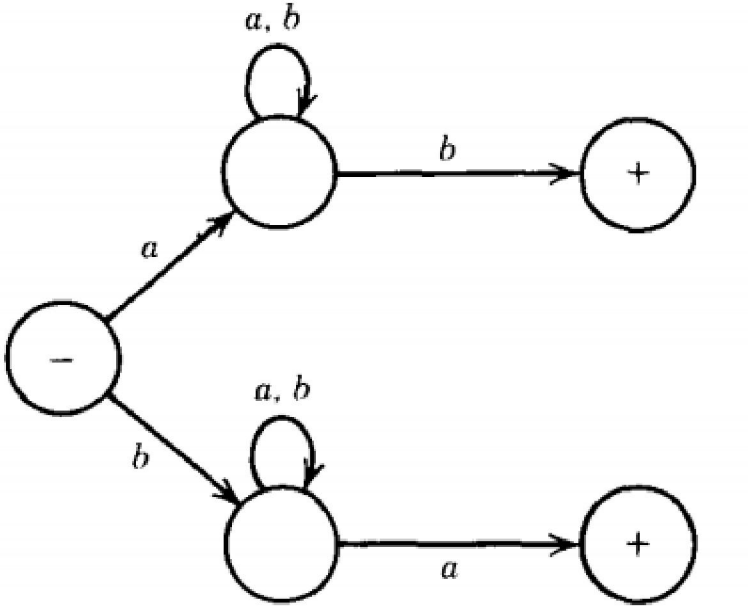
Transition Graph (T.G)

- The language accepted by this TG is all words ending in b. One regular expression for this language is (a + b)*b and an FA that accepts the same language is



Transition Graph (T.G)

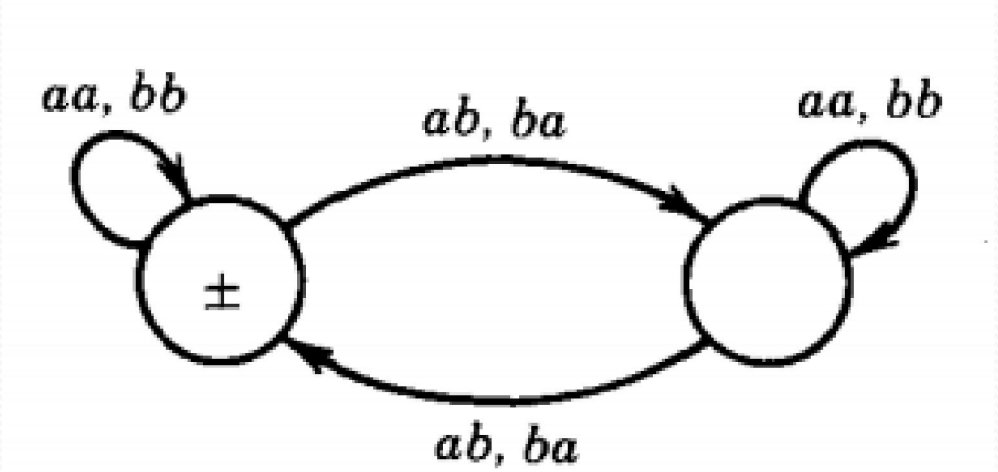
- A TG



- accepts the language of all words that begin and end with different letters

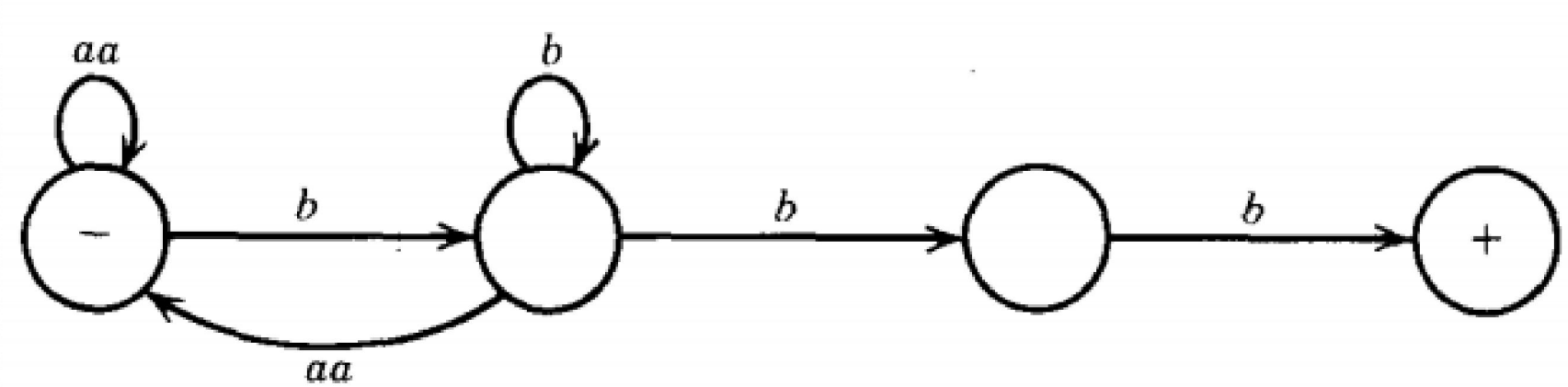
Transition Graph (T.G)

- Consider the following TG:



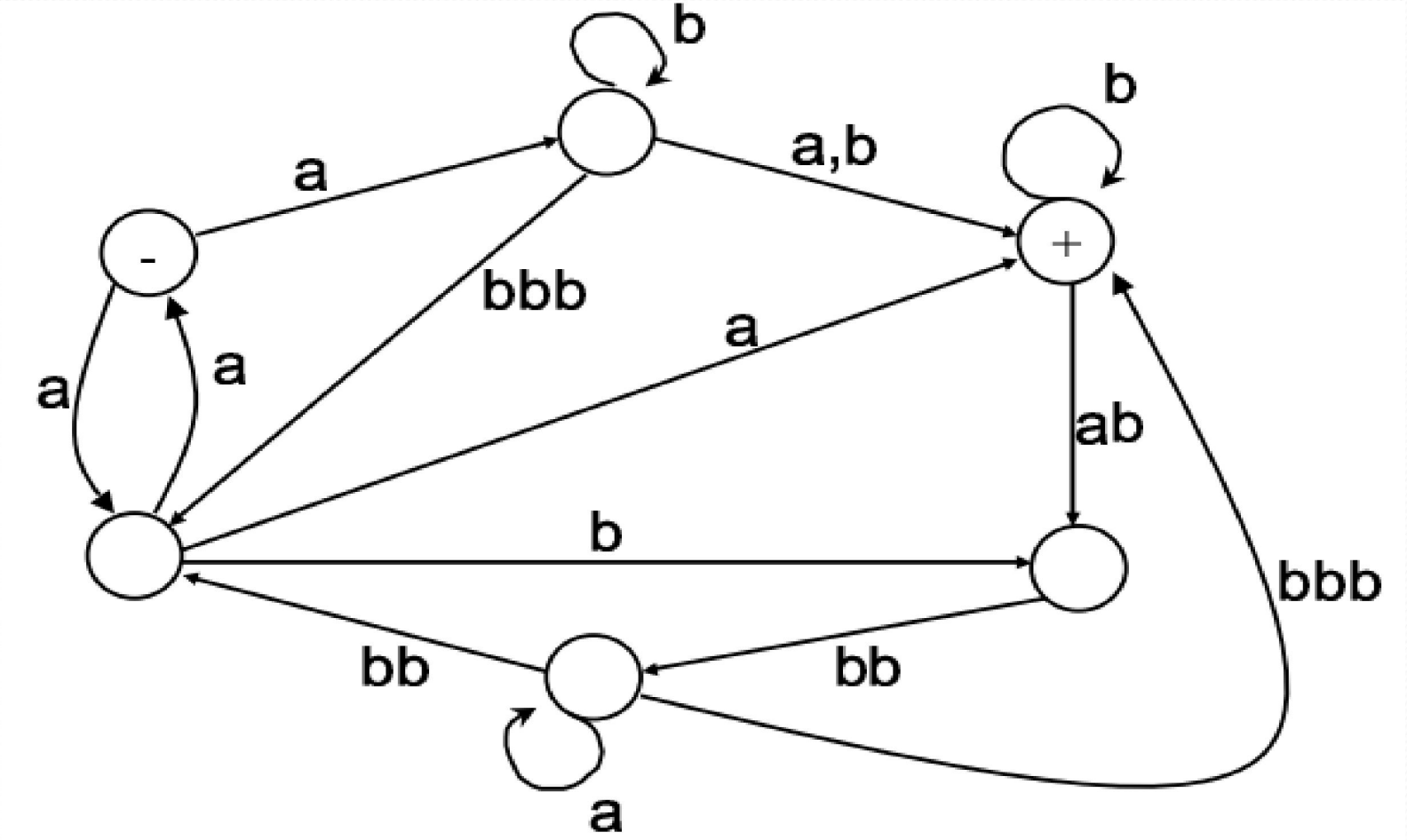
- in this TG every edge is labeled with a pair of letters. This means that for the string to be accepted it must have an **even number of letters** that are read in and processed in groups of two's.

Transition Graph (T.G)



- accepts the language of all words in which the **a's occur only in even clumps** and that end in **three or more b's**.

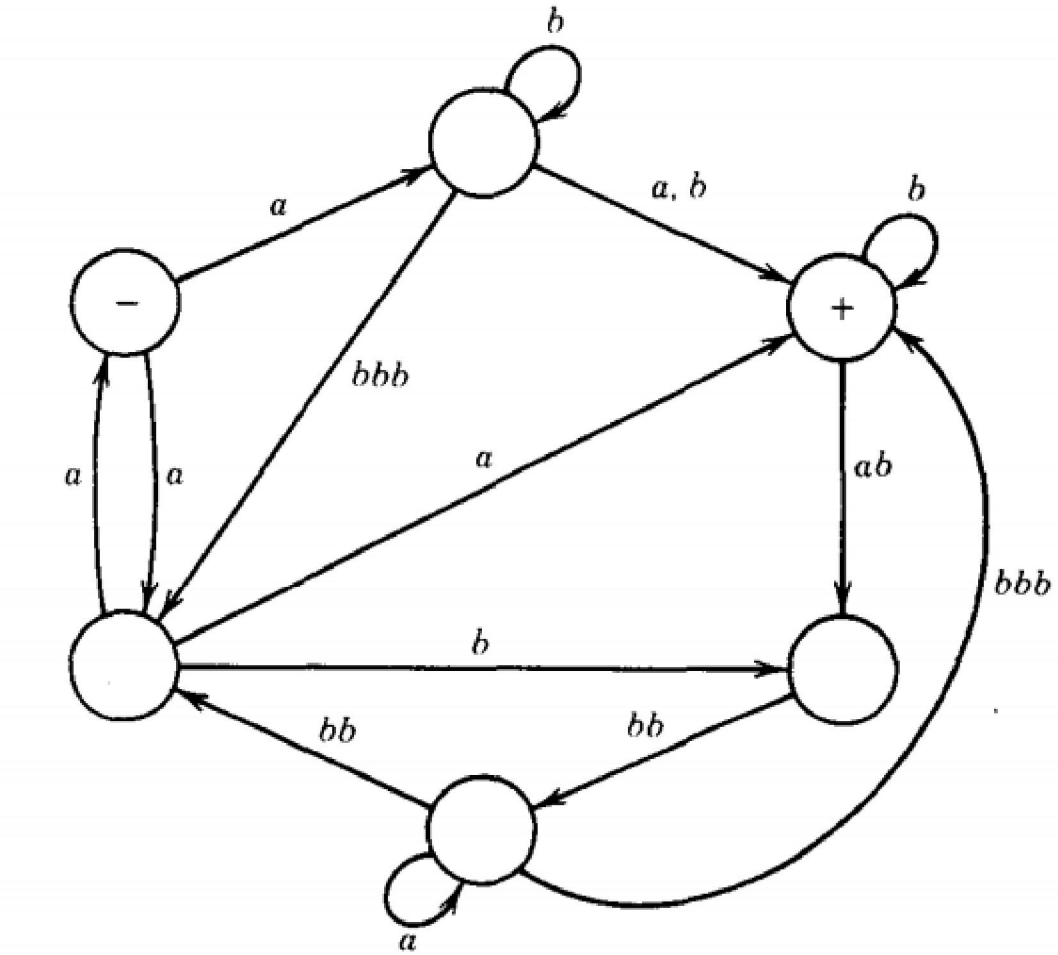
Transition Graph (T.G)-Even Even



is **abbbabbbabba** is accepted?

Transition Graph (T.G)

- Consider this TG

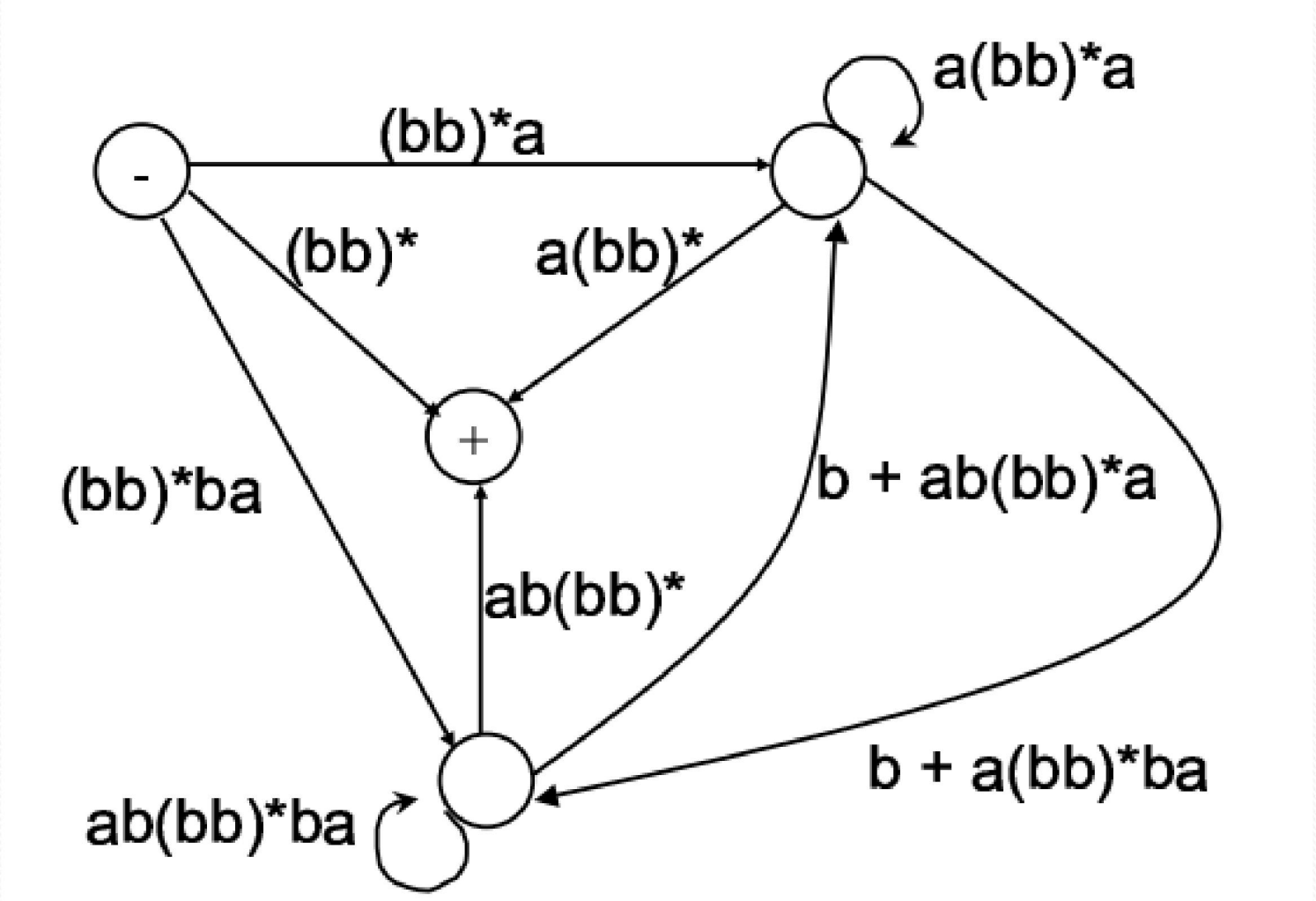


Is the word *abbbabbbabba* accepted by this machine? (Yes, in two ways.)

Generalized Transition Graph(GTG)

- Its is similar to transition graph except for transitions
- Transitions
  - Goto from some states to some other states
  - Labeled by **regular expressions** formed from letters from the alphabets

Generalized Transition Graph(GTG)



Types of Automata

- **FA**
  - For each state and letter there is a **unique transition.**
- **NFA**
  - For **some** states and letters there is a transition.
- **NFA-Λ**
  - For **some** states and letters and **Λ** is a transition.

Types of Transition Graphs

- **TG**
  - For some states and **strings** there is a transition.
  - Maybe **more than one start state.**
- **GTG**
  - For some states and **regular expressions** there is a transition.
  - Maybe **more than one start state.**

Assignment

- Solve question # 4 of book on page 103

Kleene Theorem-PART-I

- Part I: Every FA is a TG

FA	TG
Single Start State and multiple end states	Multiple Start States and multiple end states
Finite set of input symbols	Same
Finite set of transitions	Same
Deterministic	Non-Deterministic
Distinguishing Rule	No such rule

Kleene Theorem-PART-II

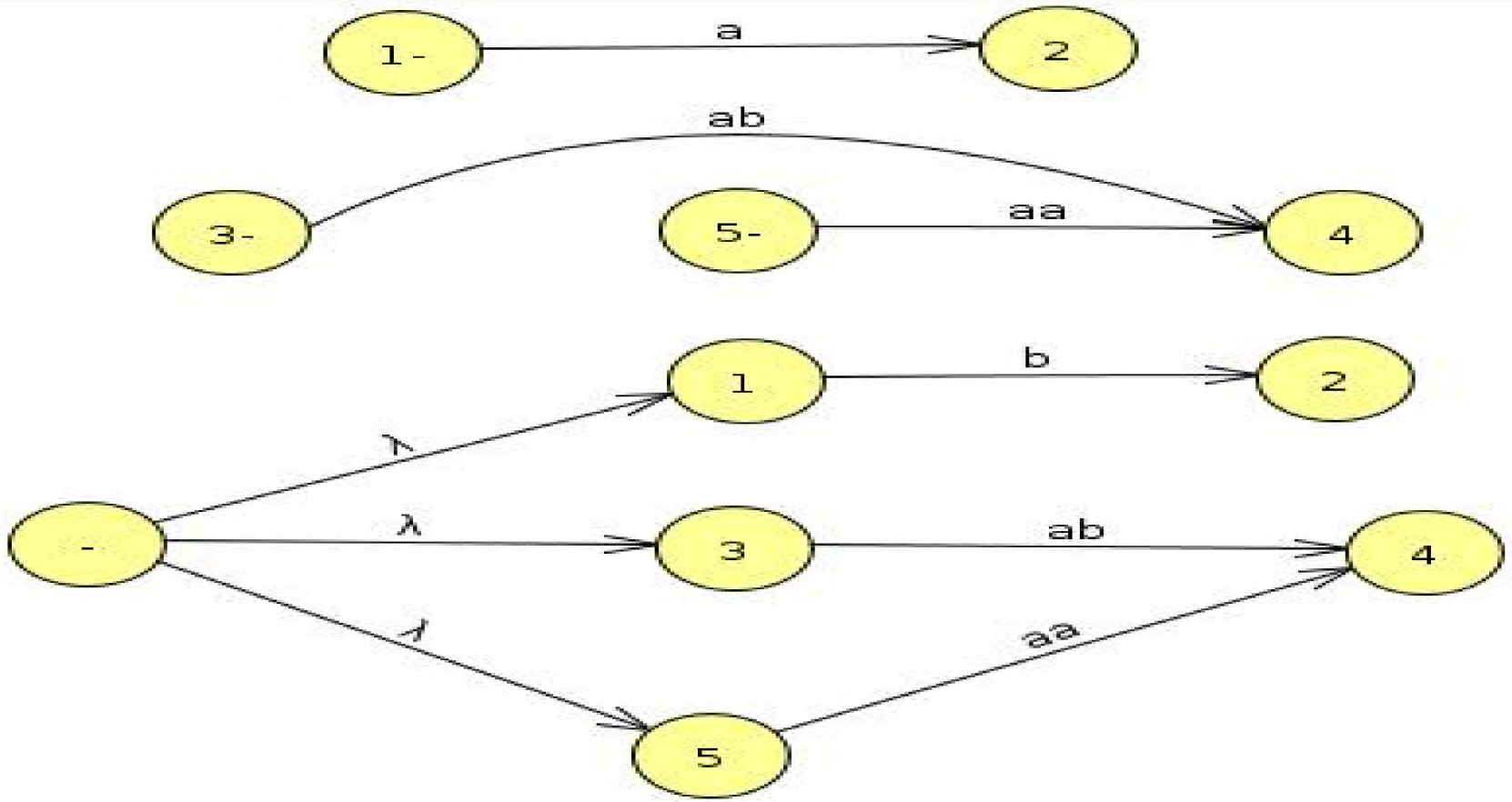
- Part I: Every language that can be defined by a finite automaton can also be defined by a transition graph. (Every FA is a TG)
- This is the easiest part.
- Every finite automaton is itself a transition graph.
- Therefore, any language that has been defined by a finite automaton has already been defined by a transition graph.
- Done

Kleene Theorem-PART-II

- Part II : Every language that can be defined by a transition graph can also be defined by a regular expression
- The prove of this Part requires a systematic algorithm through which a **TG can be converted to a GTG, in which all transitions are actually regular expressions**. **Thus, we need to transform a TG to a GTG and eliminate its various states and convert it to a single state or single transition GTG**, so that we can get the regular expression associated with the TG.
- Steps involved in TG to GTG conversion Examples taken from Daniel I Cohen Book

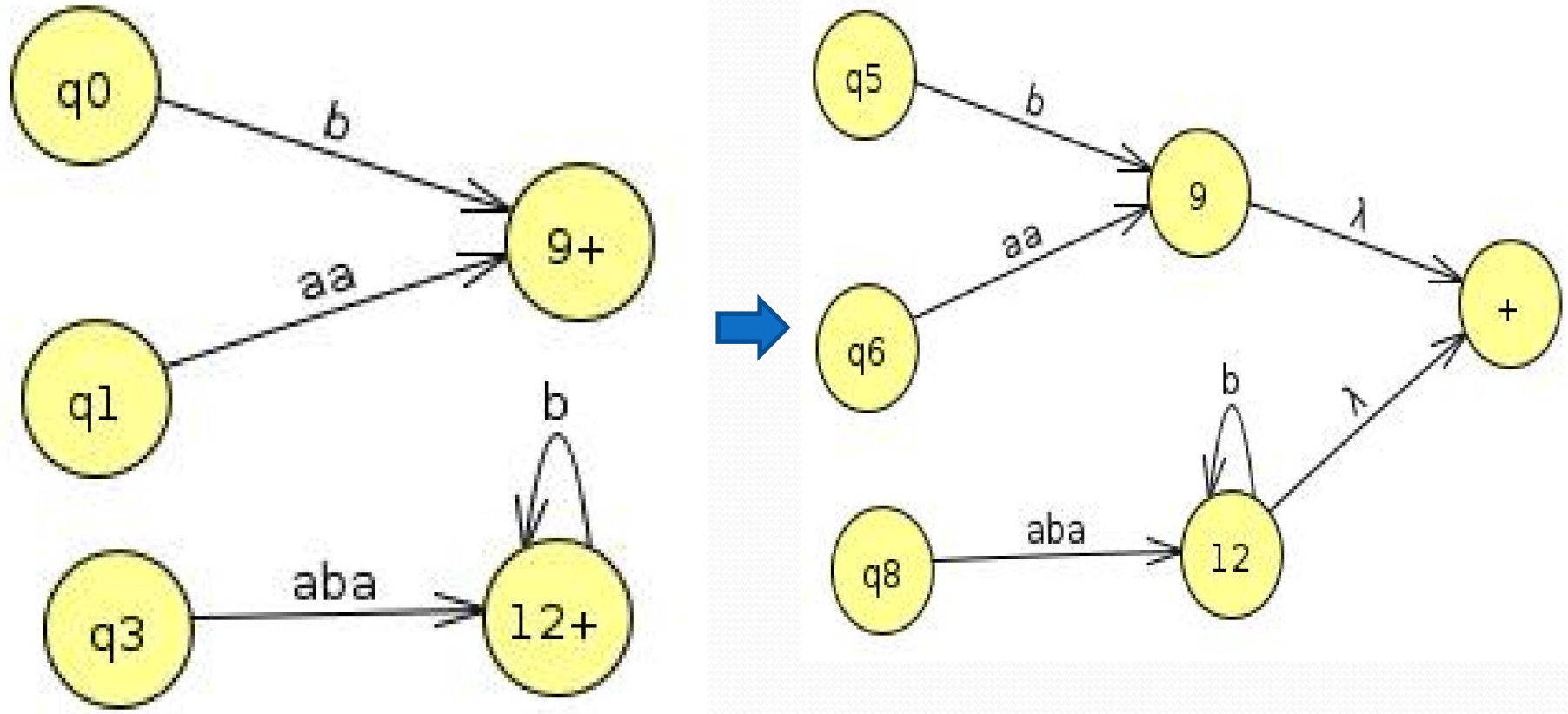
Kleene Theorem-PART-II

- **Step I: More than one Initial states conversion:**



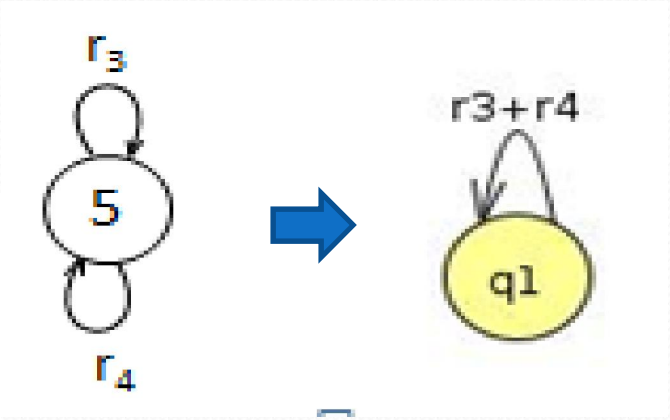
Kleene Theorem-PART-II

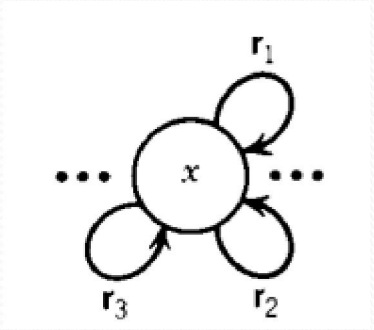
- **Step II: More than one Final states conversion:**

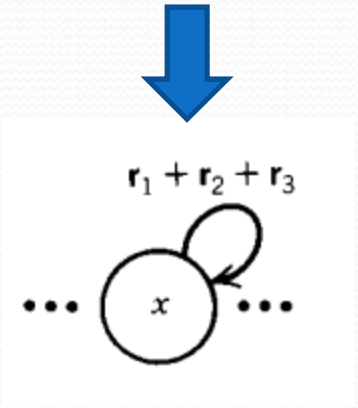


Kleene Theorem-PART-II

- Step III: Multiple Loops conversion:



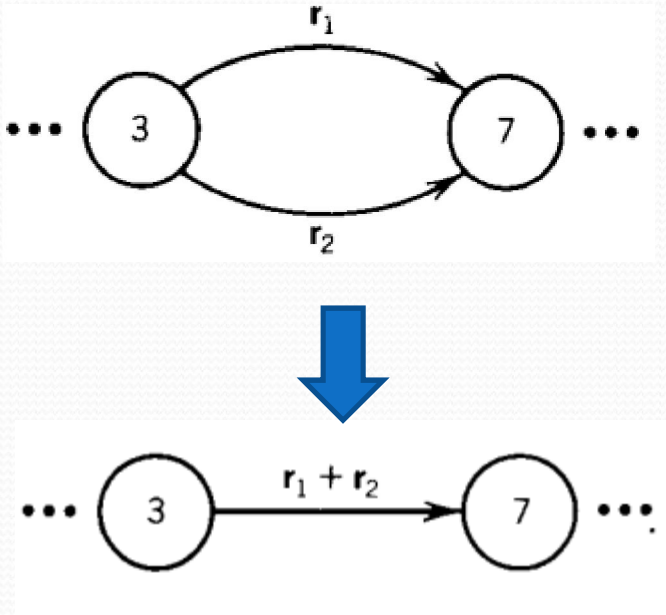




Kleene Theorem-PART-II

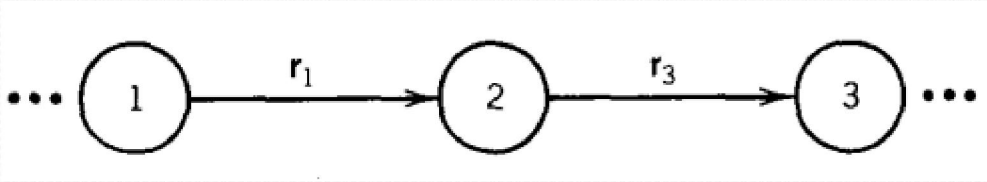
- Step 4: State Merging

(Alternative)

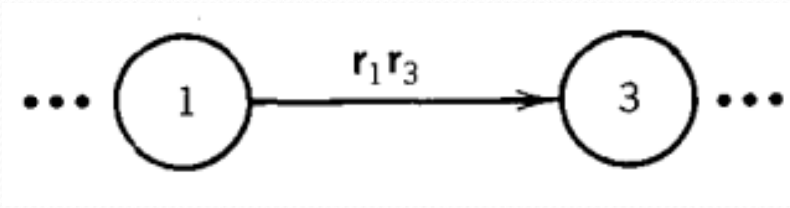


Kleene Theorem-PART-II

- State Merging

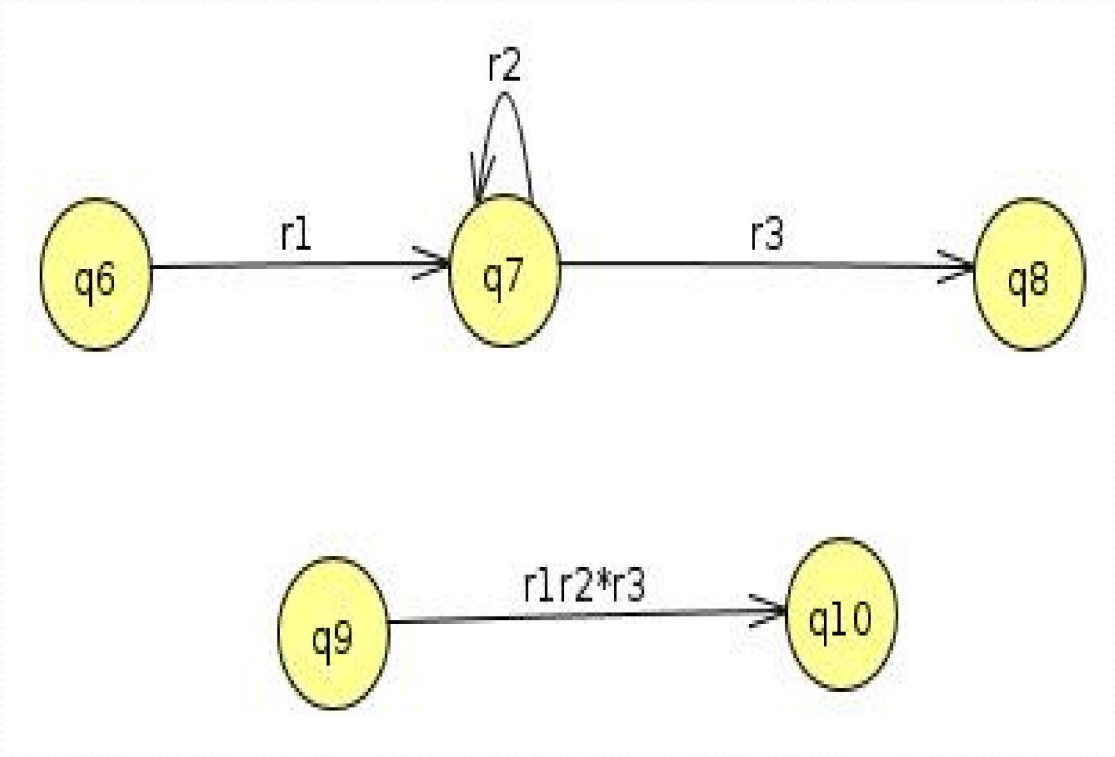


(Concatenation)



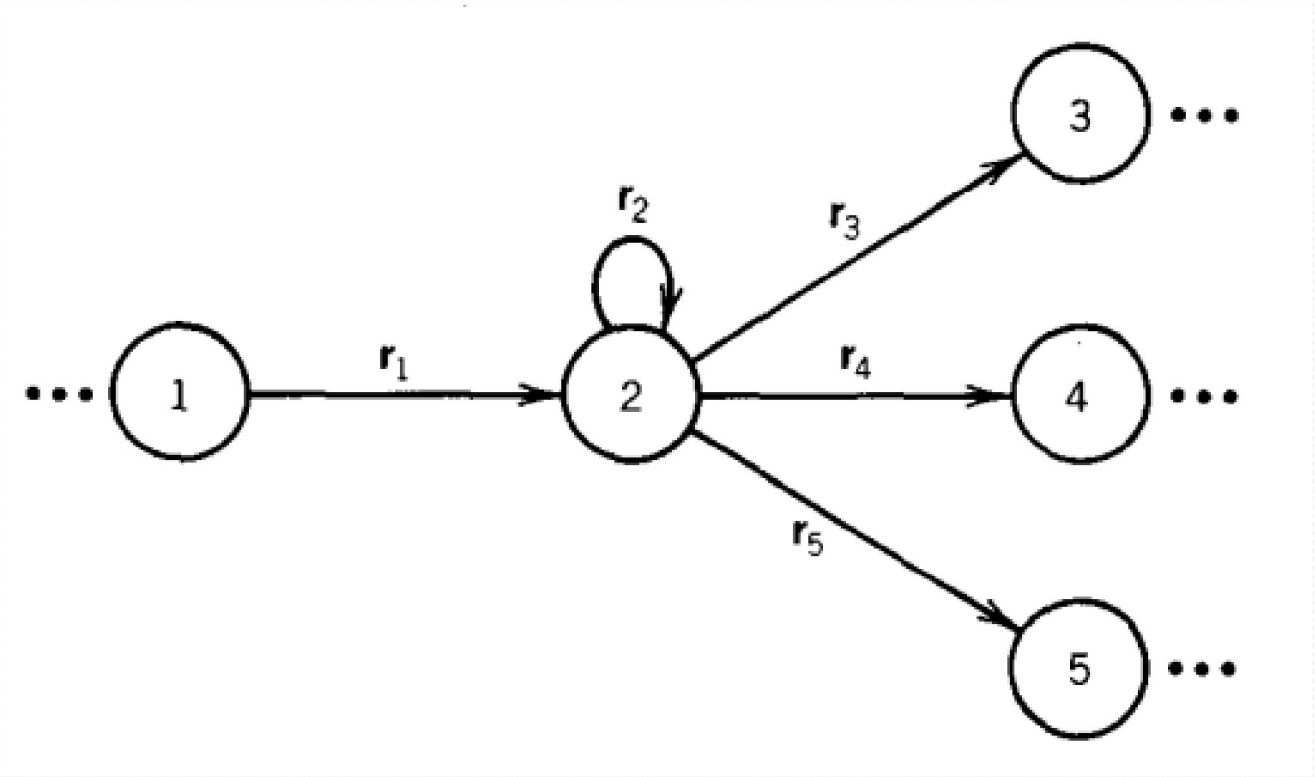
Kleene Theorem-PART-II

- State Merging (Remove self state Loops)



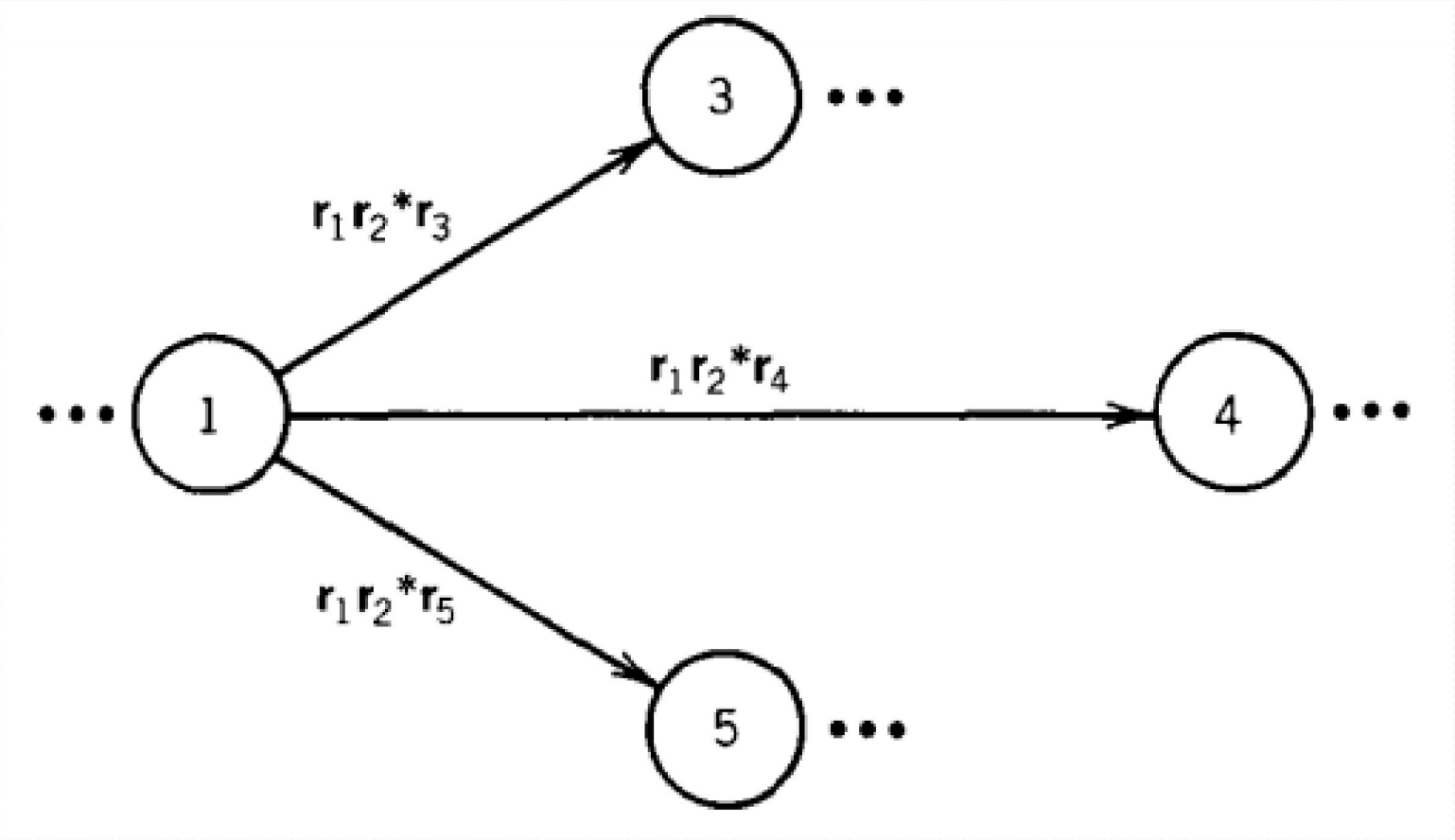
Kleene Theorem-PART-II

- State Merging

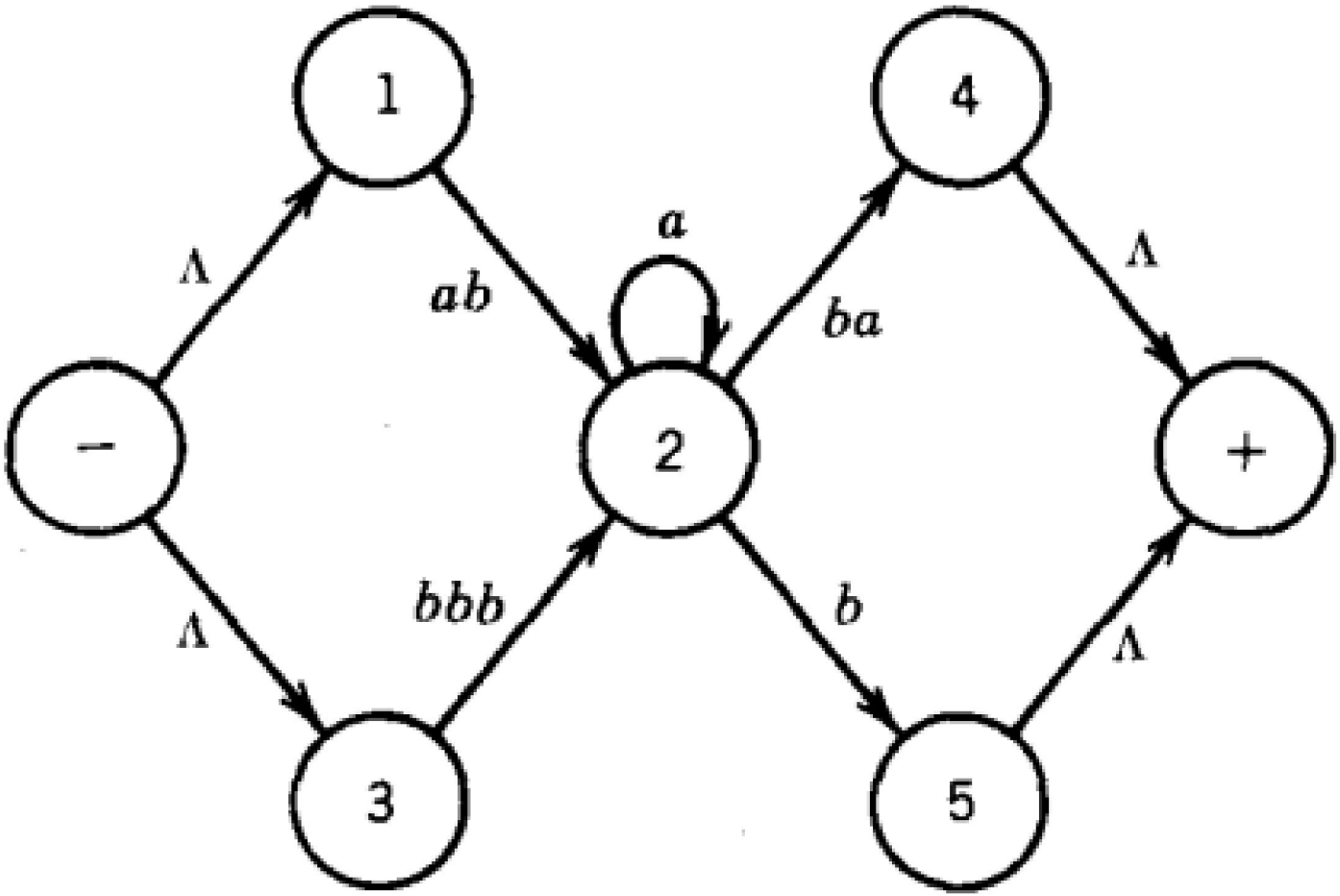


Kleene Theorem-PART-II

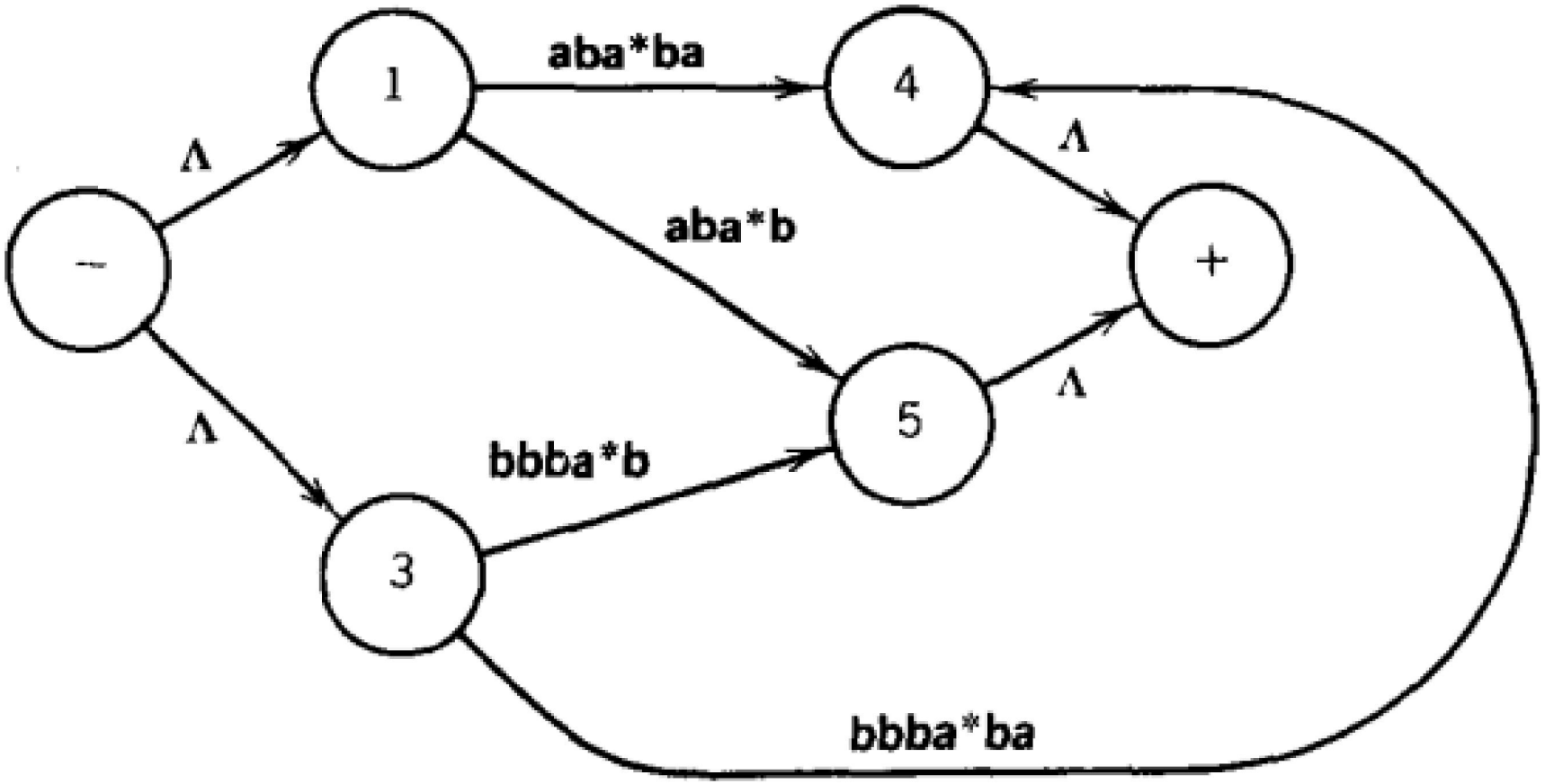
- State Merging



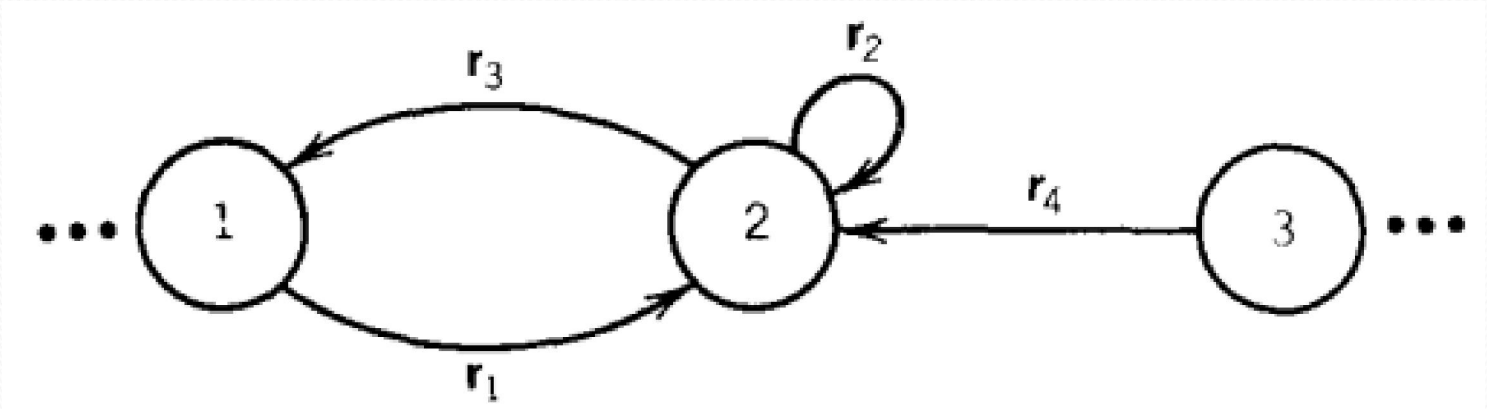
State elimination in Kleene Theorem

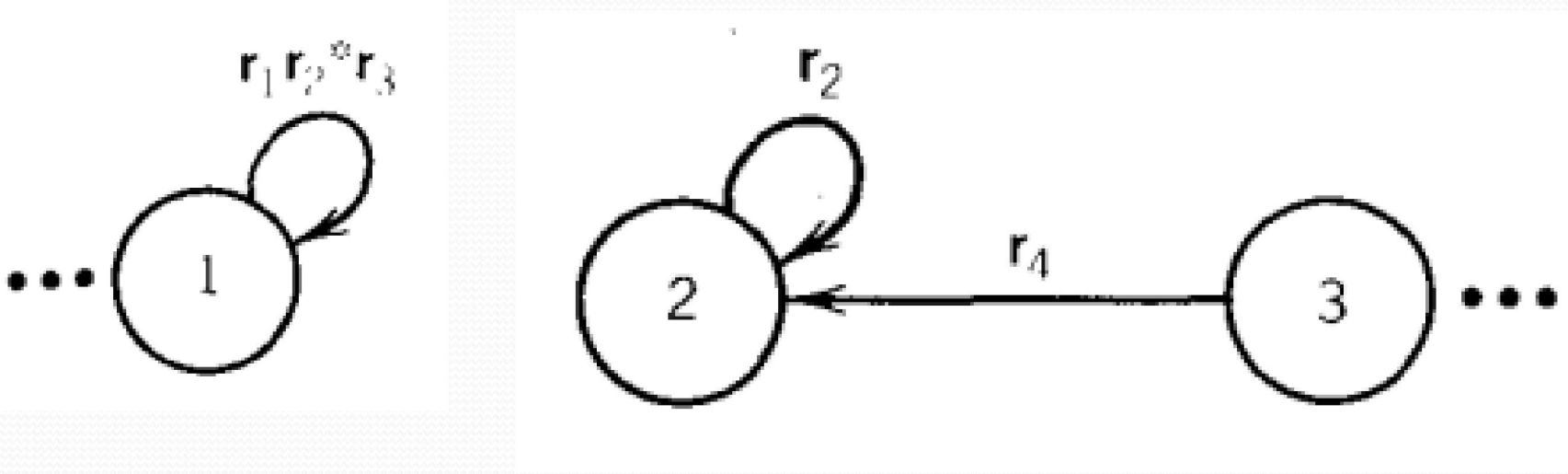


State elimination in Kleene Theorem



State elimination in Kleene Theorem





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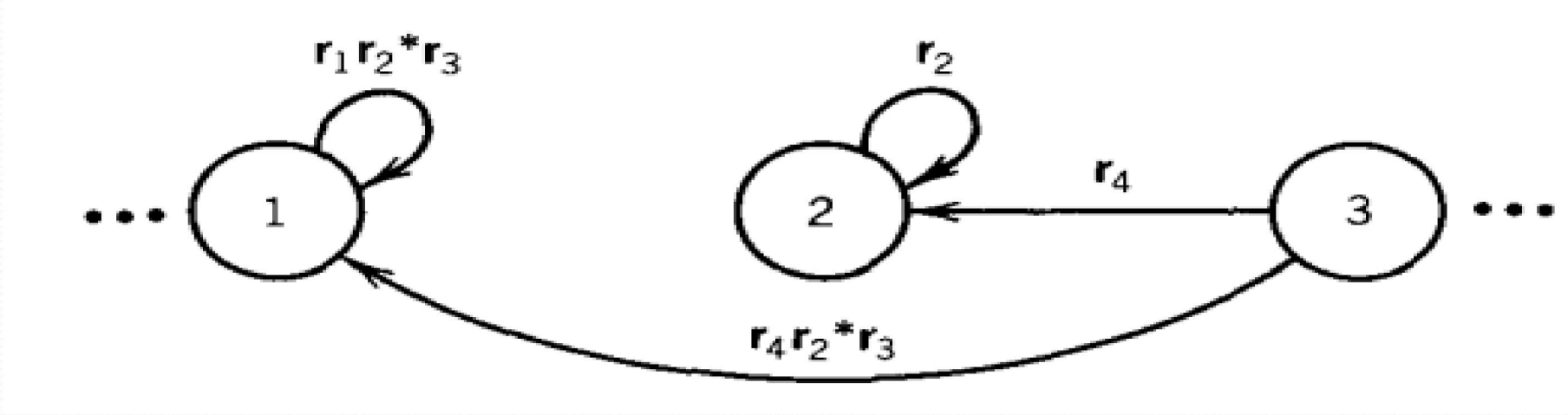
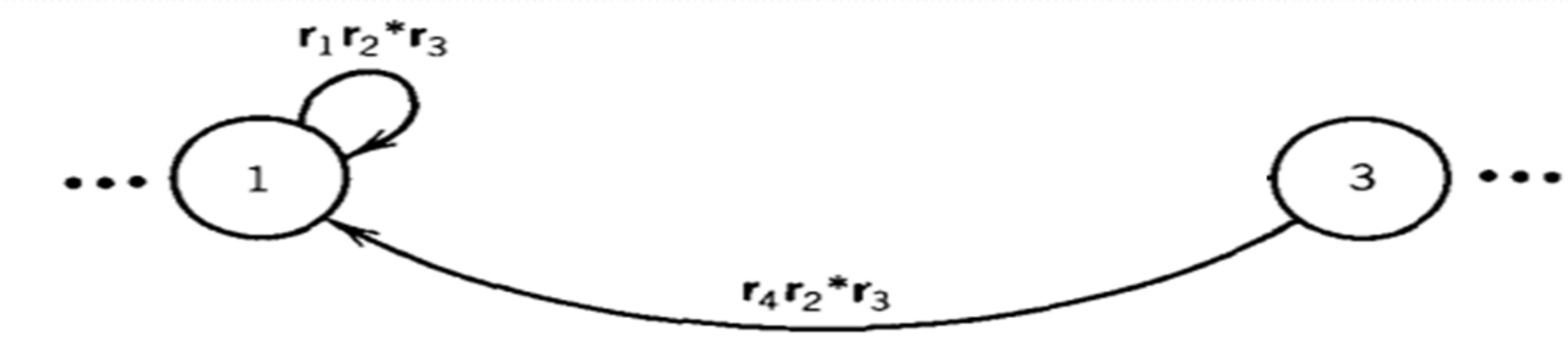
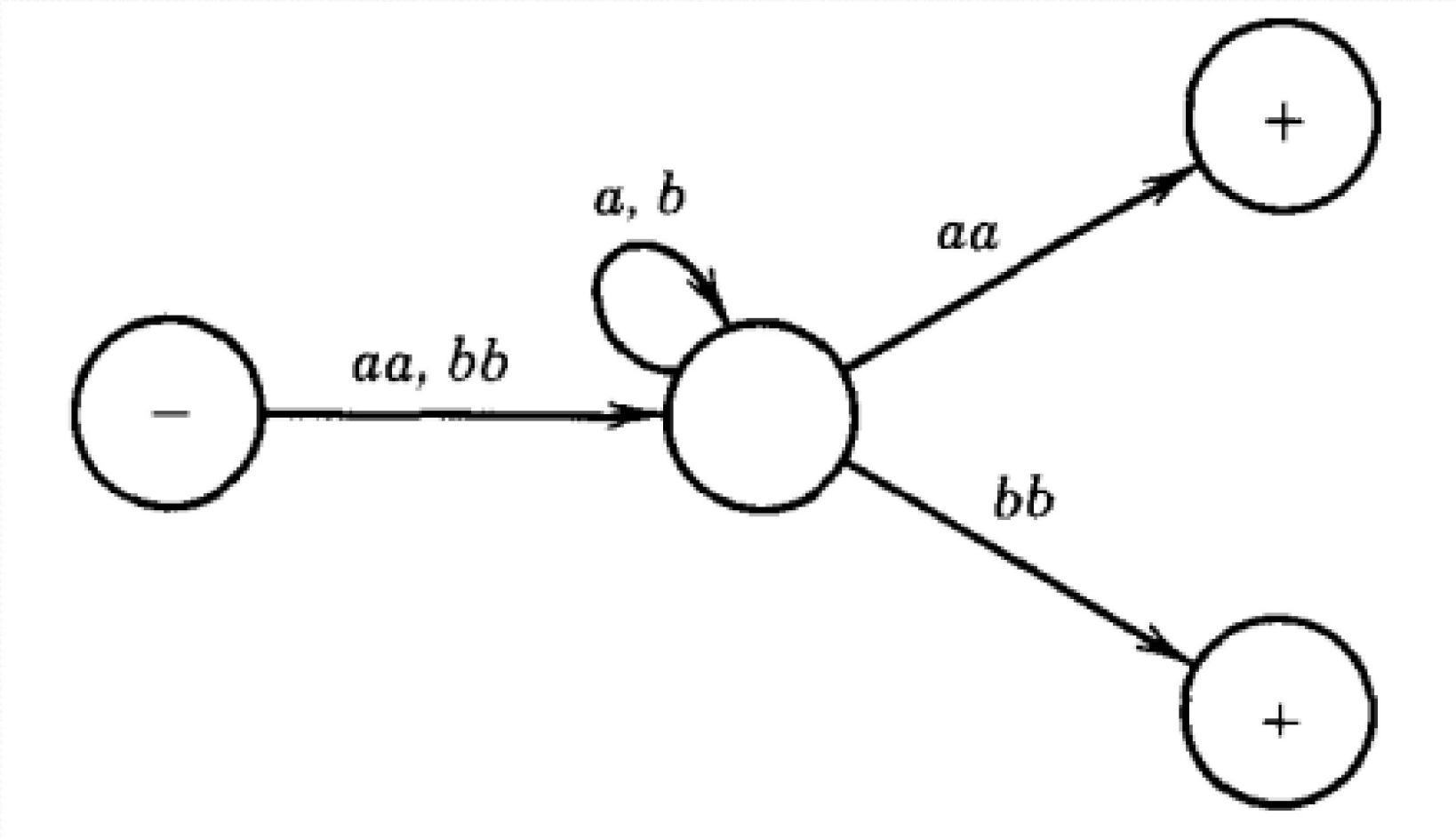
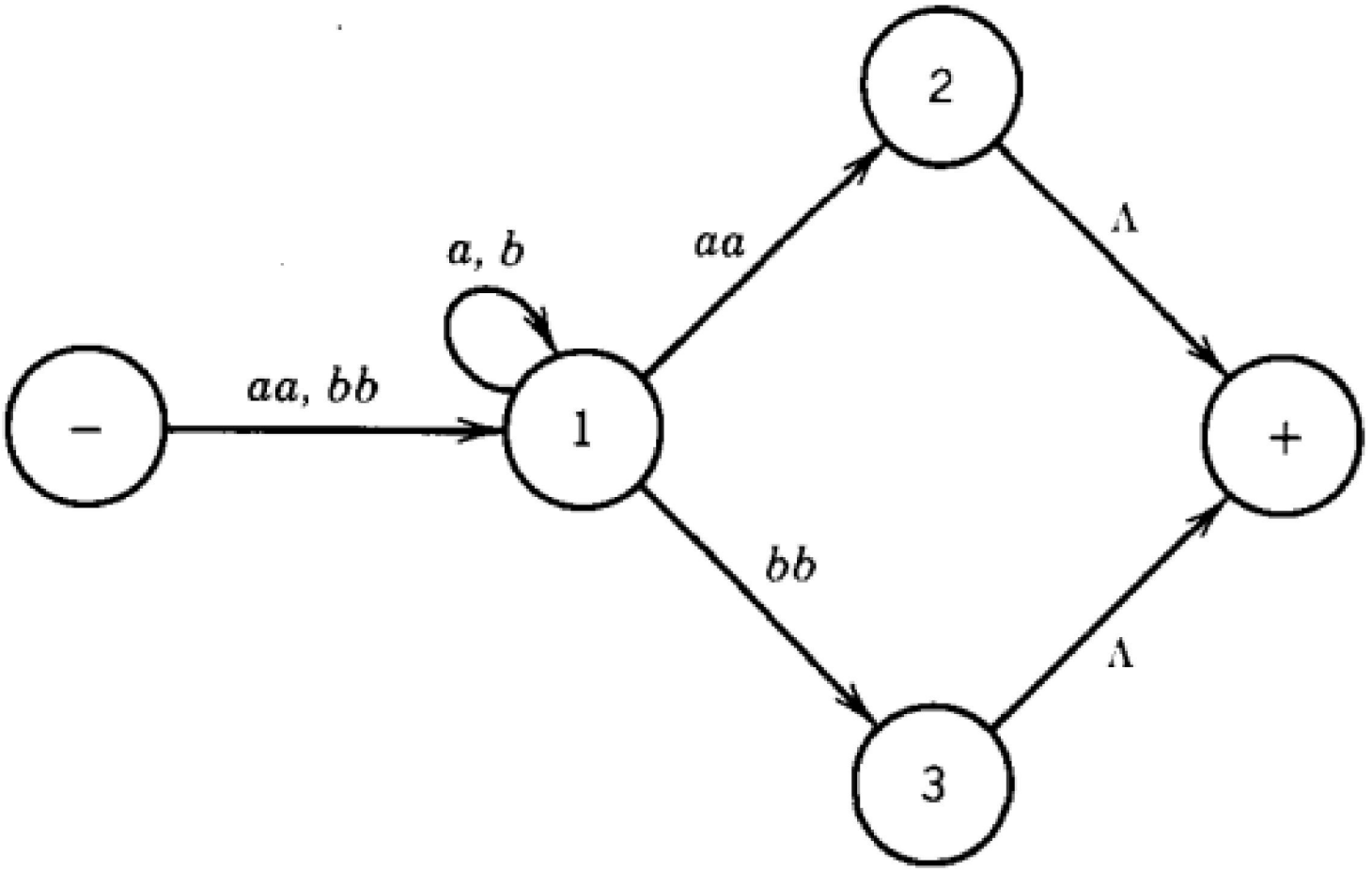




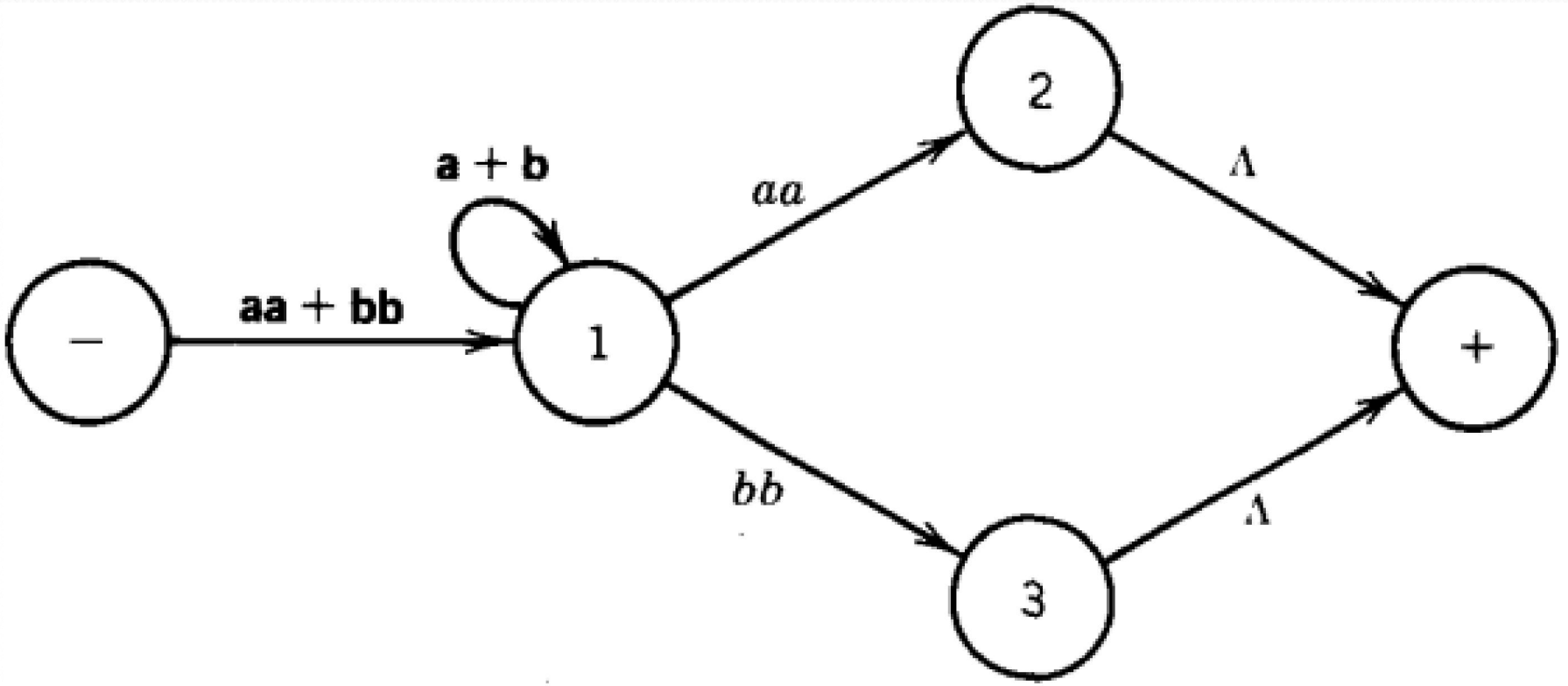
State elimination in Kleene Theorem



State elimination in Kleene Theorem



State elimination in Kleene Theorem



State elimination in Kleene Theorem