CS 223 – Digital Logic and Design

Lecture 5 – Complements

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Lecture 5

Complements

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- Complements are used in digital computers to **simplify the subtraction operation** and for **logical manipulation**.
- Simplifying operations leads to simpler, less expensive circuits to implement the operations.
- There are two types of complements for each **base-r** *system:*
- **The radix complement** and **the diminished radix complement**.
- The first is referred to as the ***r's complement*** *and the second as the* ***(r – 1)'s complement****.*
- When the value of the base r is substituted in the name, the two types are referred to as the 2's complement and 1's complement for binary numbers and the 10's complement and 9's complement for decimal numbers.

Diminished Radix Complement OR (r – 1)'s Complement

- Given a number ***N*** *in base* ***r*** *having* ***n*** *digits,*
  the ***(r – 1)'s*** *complement of* ***N*** , is defined as $(r^n - 1) - N$.
- For decimal numbers, $r = 10$ and $r - 1 = 9$, *so the* ***9's*** *complement of* ***N*** *is* $(10^n - 1) - N$.
- In this case, 10^n represents a number that consists of a single ***1*** followed by ***n 0's***.
- $10^n - 1$ is a number represented by ***n 9's***.
- Example:
- N = 546700, r =10, n = 6
  9's complement of N = $(10^6 - 1) - 546700$
  = 999999 – 546700
  = 453299
- The 9's complement of 546700 is 999999 – 546700 = 453299.
- The 9's complement of 012398 is 999999 – 012398 = 987601.

(r – 1)'s Complement (Contd.)

- For binary numbers, **$r = 2$** and **$r - 1 = 1$**, so the **1's** complement of **N** is **$(2^n - 1) - N$**.
- Again, **2^n** is represented by a binary number that consists of a 1 followed by **n 0's**.
- **$2^n - 1$** is a binary number represented by **n *1's***.
- For example, if **$n = 4$**, we have **$2^4 = (10000)_2$**
- and **$2^4 - 1 = (1111)_2$**.
- Thus, the **1's complement** of a binary number is obtained by
- subtracting each digit from **1**.
- However, when subtracting binary digits from **1**, we can have either **$1 - 0 = 1$** or **$1 - 1 = 0$**, which causes the bit to change from **0** to **1** or from **1** to **0**, respectively.
- Therefore, **the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's**.

(r – 1)'s Complement (Contd.)

- Examples:
- The **1's** complement of **1011000** is **0100111**.
- The **1's** complement of **0101101** is **1010010**.
- The **(r – 1)'s** *complement of octal or hexadecimal numbers is obtained by subtracting* each digit from **7** or **F** (decimal **15**), respectively.

Radix Complement OR r’s Complement

- The ***r*’s** complement of an ***n*-digit number *N*** in base ***r*** is defined as

  $r^n - N$ for $N \neq 0$ and as

  0 for $N = 0$.

- Comparing with the ***(r - 1)’*s complement**, we note that the ***r’*s** complement is obtained by adding **1** to the **(*r* - 1)’s complement**, since $r^n - N = [(r^n - 1) - N] + 1$

- Thus, the **10’s complement** of decimal **2389** is **7610 + 1 = 7611** and is obtained by adding**1** to the **9’s complement** value.

- The **2’s** complement of binary **101100** is **010011 + 1 = 010100** and is obtained by adding 1 to the 1’s-complement value.

r's Complement / 10's Complement (Contd.)

- **$10^n - N$** , which is the 10's complement of *N*, can be formed also by leaving all least significant 0's unchanged, subtracting the first nonzero least significant digit from 10, and subtracting all higher significant digits from 9.
- Thus, the 10's complement of 012398 is 987602 and the 10's complement of 246700 is 753300
- The 10's complement of the first number is obtained by subtracting 8 from 10 in the least significant position and subtracting all other digits from 9. The 10's complement of the second number is obtained by leaving the two least significant 0's unchanged, subtracting 7 from 10, and subtracting the other three digits from 9.

r's Complement / 2's Complement (Contd.)

- Similarly, the 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged and replacing 1's with 0's and 0's with 1's in all other higher significant digits.
- For example, the 2's complement of 1101100 is 0010100 and the 2's complement of 0110111 is 1001001
- The 2's complement of the first number is obtained by leaving the two least significant
- 0's and the first 1 unchanged and then replacing 1's with 0's and 0's with 1's in the other four most significant digits. The 2's complement of the second number is obtained by leaving the least significant 1 unchanged and complementing all other digits.
- It is also worth mentioning that **the complement of the complement restores the number to its original value.**

- That’s end of the presentation ! ☺