## **Exercise Set 5.5**

In each of **Problems 1 – 12**, use the trapezoidal rule to approximate the given integral:

1. 
$$\int_{1}^{4} \frac{dx}{x} = \ln 4$$
 with  $n = 3$  2.  $\int_{0}^{\pi/3} \cos x \ dx = \sqrt{3}/2$  with  $n = 4$ 

3. 
$$\int_{0}^{2} e^{-x^{2}} dx$$
 with  $n = 4$  4.  $\int_{0}^{4} x^{2} dx$  with  $n = 8$ 

5. 
$$\int_{0}^{\pi} \sin x \, dx$$
 with  $n = 6$  6.  $\int_{0}^{2} \frac{dx}{1 + x^3}$  with  $n = 4$ 

7. 
$$\int_{0}^{1} \frac{dx}{\sqrt{4-x^2}}$$
 with  $n=4$  8. 
$$\int_{-2}^{2} (2x^2+1) dx$$
 with  $n=4$ 

9. 
$$\int_{1}^{5} \frac{dx}{x^2}$$
 with  $n = 4$  10.  $\int_{0}^{1} e^{-x} dx$  with  $n = 6$ 

11. 
$$\int_{1}^{2} \ln x \, dx$$
 with  $n = 4$  12.  $\int_{0}^{2} \frac{1}{\sqrt{1+x^2}} \, dx$  with  $n = 4$ 

13. Use Simpson's rule to approximate the integrals of Problems 3, 4, 10, 11 and 12.

Find a bound on the error in approximating the given integral using (i) the trapezoidal rule (ii) Simpson's rule. (Problems 14 - 16):

14. 
$$\int_{-1}^{2} x^5 dx$$
 with  $n = 10$ 

15. 
$$\int_{1}^{3} \frac{dx}{x}$$
 with  $n = 10$ 

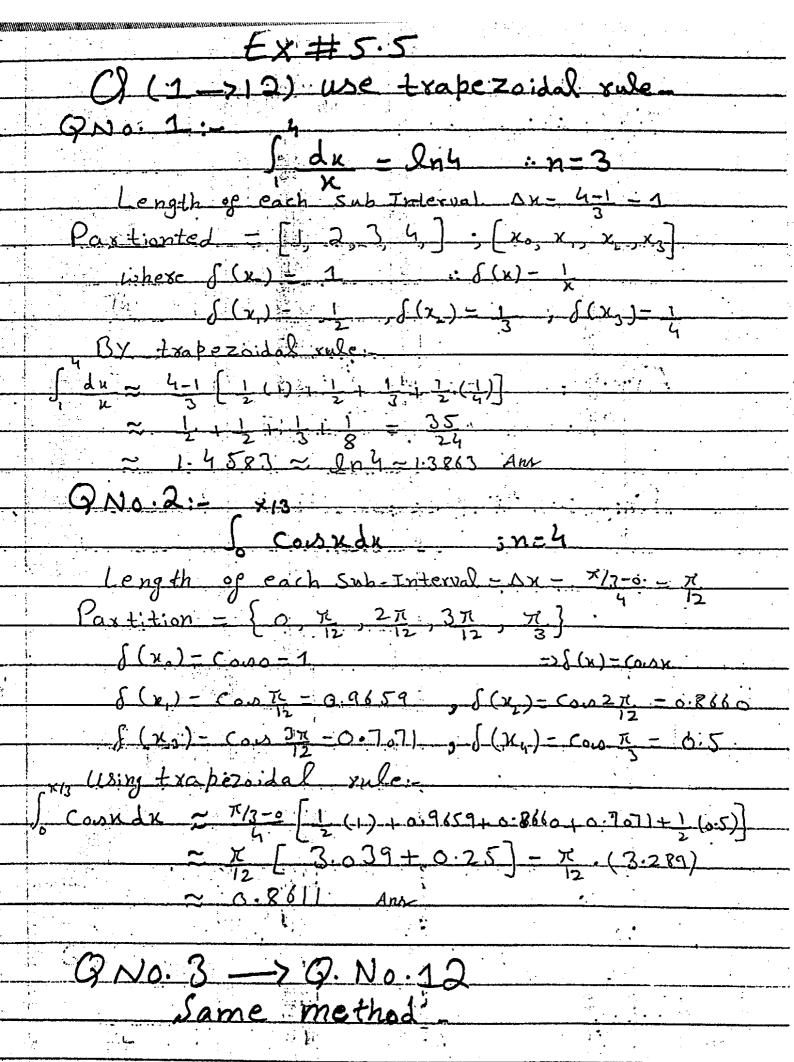
16. 
$$\int_{0}^{2} \frac{dx}{\sqrt{1+x}} \quad \text{with } n = 8$$

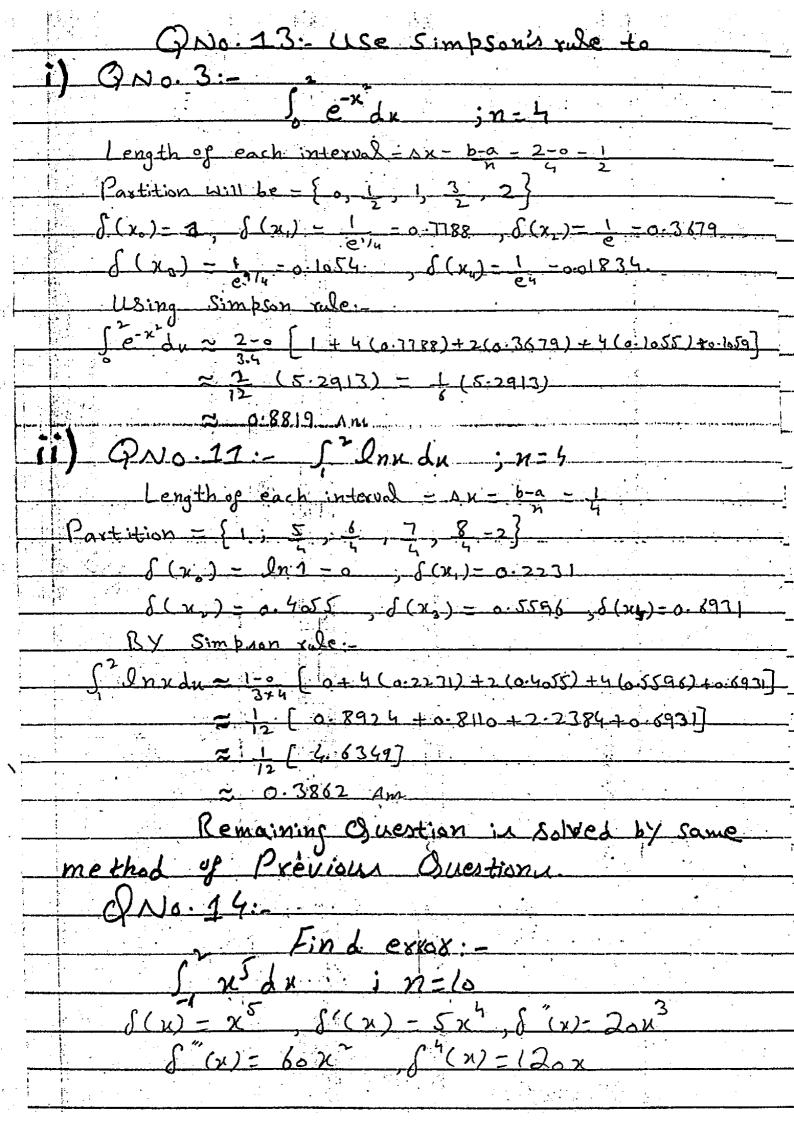
- 17. With n = 8, find the area under the semicircle  $y = \sqrt{4 x^2}$  and above the x-axis by (i) the trapezoidal rule (ii) Simpson's rule.
- 18. A reading of the velocity of a ship was made every quarter hour as shown below:

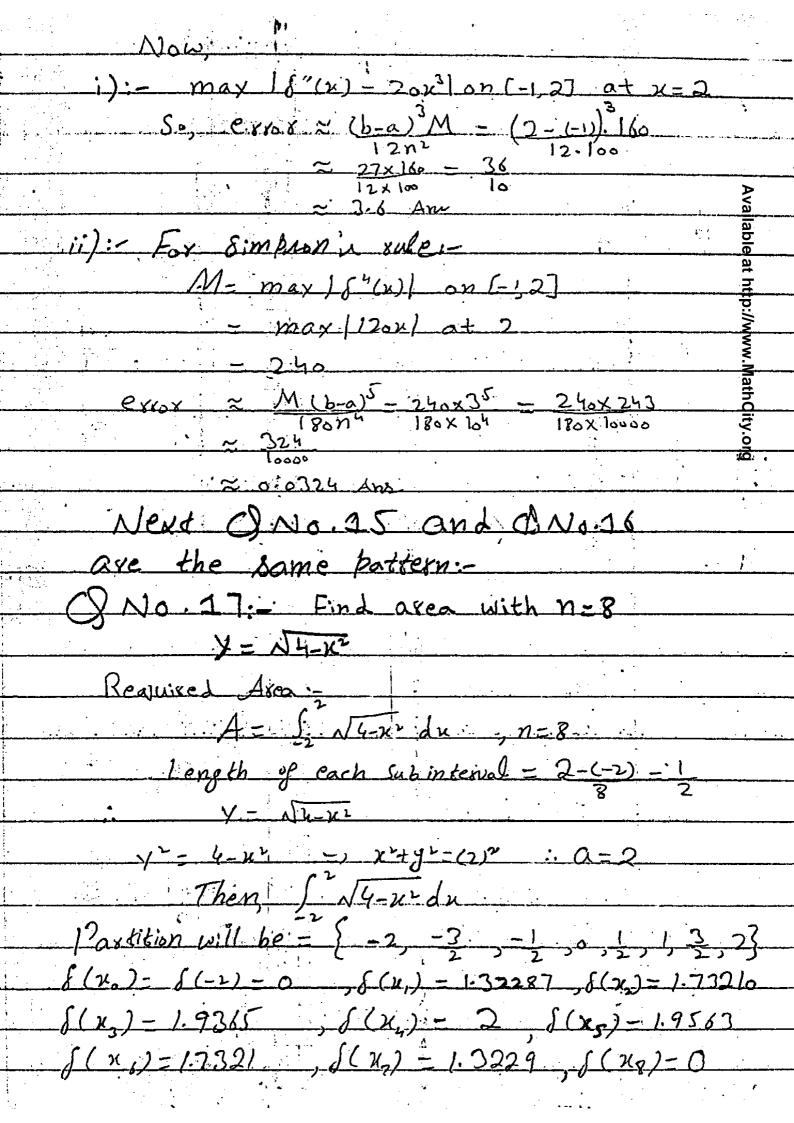
Time t (hours) =	0	1/4	1 2	34	1
Velocity $v(t)$ (mph) =	19.5	24.3	34.2	40.5	38.4
:::::::::::::::::::::::::::::::::::::	5/4	3/2	7/4	2	
v(t) =	26.2	18	16	8	

Estimate the distance travelled by the ship during the 2-hour period.

Numerical Integration:
Trapezoidal rule:
Let a function f be Continuous on [a,b] and let
[a, b] be partioned into n excial subintervals [x,x].[x,x]
[xn, xn] each of length b-a. Then,
<u>b</u>
$\int_{a} f(x) dx = b - a \left[ \frac{1}{2} S(x_0) \cdot f(x_0) + f(x_0) + \cdots + \frac{1}{2} f(x_0) \right]$
$Error_{i-} \approx \frac{(b-a)M}{12n^2}$ $M = \max \{f''(u)\} \text{ on } [a,b]$
$M = \max f''(x) j$ on $[a,b]$
Simpson's Rule:
=> Lenna 1-
Axea = \frac{h}{3} (S(x) + 4f(x,) + S(x))
Then:
96 fin Continuous on [a,b] and [a,b] is Partitioned
into n even number of carual Subinterval, by the Points,
$Q = X_0, X_1, X_2, \dots, X_{n-1}, X_n = b$
So, Con a local and a local an
$\int_{a} f(x) dx \approx \frac{b-a}{3n} \left[ f(x_{0}) + 4\delta(x_{0}) + 2f(x_{0}) + 4f(x_{0}) +$
$\frac{1}{2\delta(x_{n-2})+1\delta(x_{n-1})+\delta(x_n)}$
Whex Coefficiend Pattern is that:
1, 4, 2, 4, 2, 4, 2, 4, 2, 4, 1
$Exx_{0}x_{0}$ $Exx_{0}x_{0}$ $M(b-a)^{5}$ $= M = Max / f'(x) / (x) / (x)$
180n4







i):- BY Trapezoidal rule:
$A \approx 2^{-(-2)} \left( \frac{1}{2} (6) + 1.3229 + 1.7321 + 1.9365 + 2 + 1.9365 \right)$
1 1.7321+1.3229+16)
≈ 1 [11.9830] = 5-9915
$\approx$ 5.9915 Ans
ii):- By Simpson rule,-
$A \approx 2^{-(-2)} \left[ 0 + 4(1.3229) + 2(1.7321) + 4(1.9365) + \frac{3.8}{2.8} \right]$
3.8 2 (2)+4(1.9765)+2(1.7321)+4(1.7229)+2)
~ [5.2916+3.4642+7.746x+7.4642+B.2916]
≈ <u>31.09</u> 36
~ 6.1673 Am
Available at http://www.MathCity.org
ON0.18:-
So.l:-
The total distance travelled by the ship during
2-hour period in Sv(t)dt ; n=8
=>By trapezoidal rule:-
Length of each Sub-interval = 2-0 - 1
Se 4
$\int_{S} V(t) dt \approx 2 - o \left[ \frac{1}{2} (19.5) + 24.7 + 34.2 + 26.2 \right]$
2. 1. [9.75+24.3+34.2+4a.5+38.4+2 (22)]
≈ - [9.75+24.3+34.2+40.5+38.4+26.24)8
11-1-1
$\simeq \left(20.35\right)$
≈ 52.8375 ≈ 52.84 Ans
M.Sc. Mathematics (P.U).
Mob # 0345-78453111 Proposed by: Abrox
Mustofa

1.