

Exercise 1.1

Method of elimination

1)

$$x + 2y = 8$$

$$3x - 4y = 4$$

$$\begin{array}{r} 3x + 6y = 24 \\ \hline \end{array}$$

$$\begin{array}{r} 3x - 4y = 4 \\ \hline \end{array}$$

$$0 = -5$$

$$0 \neq -5$$

NO solution.

$$\begin{aligned} 6) \quad & x + y - 2z = 5 \\ & 2x + 3y + 4z = 2 \end{aligned}$$

$$\begin{array}{r} -2x - 2y + 4z = -10 \\ 2x + 3y + 4z = 2 \\ \hline 4 + 8y = -8 \end{array}$$

$$x + y - 2z = 5$$

$$y + 8z = -8$$

$$\boxed{1y = -8 - 8z}$$

$$x - 8 - 8z - 2z = 5$$

$$x - 10z - 8 = 5$$

$$x - 10z - 8 - 5 = 0$$

$$x - 10z - 13 = 0$$

$$x - 10z = 13$$

$$\boxed{x = 13 + 10z}$$

Hence

$$\left. \begin{array}{l} x = 13 + 10z \\ y = -8 - 8z \\ z = z \end{array} \right\} \begin{array}{l} z \text{ is any} \\ \text{real number} \end{array}$$

$$x = 13 + 10r$$

$$y = -8 - 8r$$

$$z = r$$

$$8) \quad 3x + 4y - z = 8$$

$$6x + 8y - 2z = 3$$

$$-6x + 8y + 2z = -16$$

$$6x + 8y - 2z = 3$$

$$14y = -13$$

$$\begin{aligned} d) \quad x &= -3 \\ y &= 3 \\ z &= 3 \end{aligned}$$

$$\begin{aligned} 2(-3) + 3(3) - 3 &= 0 \\ -6 + 9 - 3 &= 0 \\ 0 &= 0 \end{aligned}$$

$$\begin{aligned} -3 - 4(3) + 5(3) &= 0 \\ -3 - 12 + 15 &= 0 \\ 0 &= 0 \end{aligned}$$

15)

$$\begin{aligned} 2x - y &= 5 \\ 4x - 2y &= t \end{aligned}$$

a) determine a value of t so that the system has a solution

b) determine a value of t so that the system has no solution

c) How many different values of t can be selected in part (b)

$$\begin{aligned} 2x - y &= 5 \\ 4x - 2y &= t \end{aligned}$$

19)

Is there a value of x so that $x=1, y=2, z=x$ is the solution to the following linear system? If there is, find it

$$2x + 3y - z = 11$$

$$x - y + 2z = -7$$

$$4x + y - 2z = 12$$

$x=1, y=2$ in eq (1)

$$2(1) + 3(2) - z = 11$$

$$2 + 6 - z = 11$$

$$8 - z = 11$$

$$-z = 11 - 8 \Rightarrow$$

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a) Put $x_1 = 1, y_1 = -1, z_1 = 1$
 $2x + 3y - z = 0$
 $2(1) + 3(-1) - (-1) = 0$
 $2 - 3 + 1 = 0$
 $0 = 0$

$$x - 4y + 5z = 0$$
$$1 - 4(-1) + 5(-1) = 0$$
$$1 + 4 - 5 = 0$$
$$5 - 5 = 0$$
$$0 = 0$$

b) Put $x_2 = -2, y_2 = 2, z_2 = 2$
 $2x + 3y - z = 0$
 $2(-2) + 3(2) - 2 = 0$
 $-4 + 6 - 2 = 0$
 $0 = 0$

$$x - 4y + 5z = 0$$
$$-2 - 4(2) + 5(2) = 0$$
$$-2 - 8 + 10 = 0$$
$$-10 + 10 = 0$$
$$0 = 0$$

c) $x = x_1 + x_2 = -1$

$$y = y_1 + y_2 = 1$$

$$z = z_1 + z_2 = 1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

unique solution

16)

Given the linear system

$$2x + 3y - z = 0$$

$$x - 4y + 5z = 0$$

a) verify that $x_1 = 1, y_1 = -1, z_1 = 1$ is a solution

b) verify that $x_2 = -2, y_2 = 2, z_2 = 2$ is solution

c) is $x = x_1 + x_2 = -1, y = y_1 + y_2 = 1$ and $z = z_1 + z_2 = 1$ solution of linear system?

d) is $3x, 3y, 3z$, where x, y, z are as in part (c), a solution to the linear system

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Unique Solution.

$$\begin{aligned}
 13) \quad x + 3y &= -4 \\
 2x + 5y &= -8 \\
 x + 3y &= -5
 \end{aligned}$$

$$\begin{array}{r}
 -2x - 6y = +8 \\
 2x + 5y = -8 \\
 \hline
 -y = 0 \\
 \boxed{y = 0}
 \end{array}$$

$$x + 3(0) = -4$$

$$\boxed{x = -4}$$

Unique Solution.

17) Without using the method of elimination, solve the linear system

$$2x + y - 2z = -5$$

$$3y + z = 7$$

$$z = 4$$

$$3y + 4 = 7$$

$$3y = 7 - 4$$

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$$\begin{aligned}
 3y &= 3 \\
 \boxed{y} &= \boxed{1} \\
 \boxed{z} &= \boxed{4}
 \end{aligned}$$

$$2x + 1 - 2(4) = -5$$

$$2x + 1 - 8 = -5$$

$$2x - 7 = -5$$

$$2x = -5 + 7$$

$$2x = 2$$

$$\boxed{x = 1}$$

unique solution.

18)

Without using The method of elimination, solve linear system

$$4x = 8$$

$$-2x + 3y = -1$$

$$3x + 5y - 2z = 11$$

$$\boxed{x = 2}$$

$$-2(2) + 3y = -1$$

$$-4 + 3y = -1$$

Date

$$\begin{aligned}
 3y &= -4 + 1 \\
 3y &= -3 \\
 \boxed{y} &= \boxed{-1}
 \end{aligned}$$

16) Give

a) 1

is

b)

22

c)

an

line

d) is

as

to

$x=1, y=2$ in eq (1)

$$1 - 2 + 2z = -7$$

$$-1 + 2z = -7 \Rightarrow 2z = -7 + 1$$

$$2z = -6 \Rightarrow \boxed{z = -3}$$

$x=1, y=2$ in eq (2)

$$4(1) + 2 - 2z = 12$$

$$6 - 2z = 12$$

$$-2z = 12 - 6$$

$$-2z = 6 \Rightarrow \boxed{z = -3}$$

Is there a value of r so that $x=r, y=2, z=1$ is a solution of the following linear system? If there is find it.

$$3x - 2z = 4$$

$$x - 4y + z = -5$$

$$-2x + 3y + 2z = 9$$

$x=r, y=2, z=1$ in eq (1)

$$3x - 2(1) = 4$$

$$3x - 2 = 4$$

$$3x = 4 + 2 \Rightarrow 3x$$

$$\boxed{x = 2}$$

$$\therefore y = \frac{4-z}{24} \quad \dots \dots \dots$$

$$\therefore x \pm 16z \dots$$

$$z = z$$

Where z is any real no

$$\begin{aligned} \text{ii)} \quad 2x + 3y &= 13 \\ x - 2y &= 3 \\ 5x + 2y &= 27 \end{aligned}$$

$$\begin{aligned} 2x + 3y &= 13 \\ - 7x + 4y &= -6 \end{aligned}$$

$$[y = -1]$$

$$x - 5(-1) = 6$$

$$x - 5 = 6 \Rightarrow [x = 11]$$

unique solution

$$2) \quad \begin{aligned} x + 4y - z &= 12 \\ 3x + 8y - 2z &= 4 \end{aligned}$$

$$\begin{array}{r} -3x - 12y + 3z = -36 \\ 3x + 8y - 2z = 4 \\ \hline -4y + z = -32 \end{array}$$

$$\begin{aligned} x + 4y - z &= 12 \\ -4y + z &= 32 \\ -4y &= -z \\ y &= \frac{4-z}{-4} \Rightarrow \end{aligned}$$

$$x + 4\left(\frac{4-z}{-4}\right) - z = 12$$

$$x - (4 - z) - z = 12$$

$$x - 4 + z - z = 12$$

$$\begin{aligned} x - 4 &= 12 \\ x &= 12 + 4 = \boxed{x = 16} \end{aligned}$$

No Solution

$$\begin{aligned} 10) \quad & x + y = 1 \\ & 2x - y = 5 \\ & 3x + 4y = 2 \end{aligned}$$

$$-2x - 2y = -2$$

$$2x - 3y = 5$$

$$-3y = 3$$

$$\boxed{y = -1}$$

$$x + (-1) = 1$$

$$x - 1 = 1$$

$$x = 1 + 1$$

$$\boxed{x = 2}$$

Unique Solution

12)

$$x - 5y = 6$$

$$3x + 2y = 1$$

$$5x + 2y = 1$$

$$-2x - 15y = 10$$

$$3x + 2y = 1$$

in eq (3) $-2x + 6t = 9 \Rightarrow -2x + 8 = -9 \Rightarrow$
 $-2x = -17 \Rightarrow x = \frac{17}{2}$
 there is no such the
 value of x .

$$4) \begin{aligned} 2x + 3y - z &= 6 \\ 2x - y + 2z &= -8 \\ 3x - y + z &= -7 \end{aligned}$$

$$\begin{aligned} 2x + 3y - z &= 6 \\ \pm \quad 2x - y + 2z &= -8 \\ \hline \end{aligned}$$

$$\begin{array}{r}
 6x - 3y + 6z = -24 \\
 \underline{+ 6x + 2y + 2z = -14} \\
 -y + 4z = -10
 \end{array}$$

$$\begin{array}{r}
 2x + 3y - z = 6 \\
 4y - 3z = 14 \\
 -y + 4z = -10
 \end{array}$$

$$\begin{array}{r}
 2x + 3y - z = 6 \\
 4y - 3z = 14 \\
 -y + 4z = -10
 \end{array}$$

$$\begin{array}{r}
 4y - 3z = 14 \\
 \underline{-4y + 16z = -40} \\
 13z = -26
 \end{array}$$

$$2x + 3(2) - (-2) = 6$$

$$2x + 6 + 2 = 6$$

$$2x + 8 = 6$$

$$2x = 6 - 8 \Rightarrow$$

$$2(-1) + 3(2) - (-2) = 6$$

$$-2 + 6 + 2 = 6$$

$$6 = 6 \text{ Satisfy.}$$

9) $x + y + 3z = 12$

$$2x + 2y + 6z = 6$$

$$-2x - 2y - 6z = -24$$

$$\underline{2x + 2y + 6z = 6}$$

$$-17y - 22z = 61$$

$$2x + 4y + 6z = -12$$

$$7y + 10z = -27$$

$$-17y - 22z = 61$$

$$119y + 170z = -459$$

$$-119y - 154z = 127$$

$$16z = -32$$

$$z = \frac{-32}{16}$$

$$\boxed{z = -2}$$

$$7y + 10(-2) = -27$$

$$7y + (-20) = -27$$

$$7y = -27 + 20$$

$$7y = -7$$

$$\boxed{y = -1}$$

$$2x + 4(-1) + 6(-2) =$$

$$2x - 4 - 12 = -12$$

$$2x - 16 = -12$$

$$2x = -12 + 16 \Rightarrow 2x = 4 \Rightarrow$$

$$\begin{aligned}
 3) \quad & 3x + 2y + z = 2 \\
 & 4x + 2y + 2z = 8 \\
 & x - y + z = 4
 \end{aligned}$$

$$\begin{array}{r}
 3x + 2y + z = 2 \\
 + 4x + 2y + 2z = 8 \\
 \hline
 -x - z = -6
 \end{array}$$

$$\begin{array}{r}
 4x + 2y + 2z = 8 \\
 2x - 2y + 2z = 8 \\
 \hline
 6x + 4z = 0
 \end{array}$$

$$\begin{aligned}
 & 3x + 2y + z = 2 \\
 & -x - z = -6 \\
 & 6x + 4z = 0
 \end{aligned}$$

$$\begin{aligned}
 & 6x + 4z = 0 \\
 & -4x - 4z = -24
 \end{aligned}$$

$$-(-12) - z = -6$$

$$12 - z = -6$$

$$-z = -6 - 12$$

$$-z = -18$$

$$\boxed{z = 18}$$

$$3x + 2(-12) + 18 = 2$$

$$3x - 24 + 18 = 2$$

$$3x - 6 = 2$$

$$3x = 2 + 6$$

$$3x = 8$$

$$\boxed{x = 8/3}$$

$$3\left(\frac{8}{3}\right) + 2(-12) + 18 = 2$$

$$8 - 24 + 18 = 2$$

$$8 - 6 = 2$$

$$2 = 2$$

Satisfy.

$$\begin{array}{l} 2) \quad 2x - 3y + 4z = -12 \\ \quad \quad x - 2y + z = -5 \\ \quad \quad 3x + y + 2z = 1 \end{array}$$

from eq 1 and 2

$$\begin{array}{r} (2x - 3y + 4z = -12) \\ - 2(x - 2y + z = -5) \\ \hline \end{array}$$

$$\begin{array}{r} 2x - 3y + 4z = -12 \\ + 2x + 4y - 2z = 10 \\ \hline y + 2z = -2 \quad \text{--- A} \end{array}$$

from eq (2) and (3)

$$\begin{array}{r} (3x + y + 2z = 1) \\ - 3(x - 2y + z = -5) \\ \hline \end{array}$$

$$\begin{array}{r} 3x + y + 2z = 1 \\ - 3x + 6y - 3z = 15 \\ \hline 7y - z = 16 \quad \text{--- B} \end{array}$$

from A and B

$$\begin{array}{r} (y + 2z = -2) \\ + 2(7y - z = 16) \\ \hline \end{array}$$

$$\begin{array}{r} y + 2z = -2 \\ + 14y - 2z = 32 \\ \hline 15y = 30 \\ y = 2 \end{array}$$

$$\boxed{y=2}$$

put in A

$$2+2z=-2$$

$$2z=-2-2$$

$$2z=-4$$

$$\boxed{z=-2} \text{ put in (1)}$$

$$2(x)-3(z)+4(-2)=-12$$

$$2x-6-8=-12$$

$$2x=-12+6+8$$

$$2x=-12+14$$

$$2x=2$$

$$\boxed{x=1}$$