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FA17-BCS-090-B

Question # 01

Part a

(i) Construct the CFG

$$L = \{ 0^i 1^j 2^k \mid j > i + k \}$$

$S \to ABC$

$A \to 0A1 \mid \varepsilon$

$B \to 1B \mid 1$

$C \to 1C2 \mid \varepsilon$

(ii)

$S \to XY$

$X \to Zb \mid b$

$Y \to bw \mid b$

$Z \to AB \mid A \mid B$

$w \to Z$

$A \to aA \mid bA \mid a \mid b$

$B \to Ba \mid Bb \mid a \mid b$

Part b

Converting into GNF

$S \to XA \mid BB$

$B \to b \mid SB$

$X \to b$

$A \to a$

Step 1

$$S \to XA \mid BB$$
$$B \to b \mid XAB \mid BBB$$
$$X \to b$$
$$A \to a$$

Step 2

$$S \to bA \mid BB$$
$$B \to b \mid bAB \mid BBB$$
$$X \to b$$
$$A \to a$$

Step 3 : Removing left recursion

$$S \to bA \mid BB$$
$$B \to bC \mid bABC$$
$$C \to BBC \mid \epsilon$$
$$X \to b$$
$$A \to a$$

Step 4: Removing null production

$$S \to bA \mid BB$$
$$B \to bC \mid bABC \mid b \mid bAB$$
$$C \to BBC \mid BB$$
$$X \to b$$
$$A \to a$$

Step 5:   Substituting B →

S → bA| bcB| bABCB| bB| bABB

B → bC| bABC| b| bAB

C → BBC| BB

X → b

A → a

Step 6:

S → bA| bcB| bABCB| bB| bABB

B → bC| bABC| b| bAB

C → BBC

C → bCB| bABCB| bB| bABB

X → b

A → a

Step 7:

S → bA| bCB| bABCB| bB| bABB

B → bC| bABC| b| bAB

C → bCBC| bABCBC| bBC| bABBC

C → bCB| bABCB| bB| bABB

X → b

A → a

PART c

Converting into CNF

$S \rightarrow ASA \mid aB$

$A \rightarrow B \mid S$

$B \rightarrow b \mid \varepsilon$

Step 1: Adding new start symbol

$S_0 \rightarrow S$

$S \rightarrow ASA \mid aB$

$A \rightarrow B \mid S$

$B \rightarrow b \mid \varepsilon$

Step 2: Removing ε from B

$S_0 \rightarrow S$

$S \rightarrow ASA \mid aB \mid a$

$A \rightarrow B \mid S \mid \varepsilon$

$B \rightarrow b$

Step 3: Removing ε from A

$S_0 \rightarrow S$

$S \rightarrow ASA \mid aB \mid a \mid AS \mid SA \mid S$

$A \rightarrow B \mid S$

$B \rightarrow b$

Step 4: Removing unit rule

$S_0 \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$S \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$A \rightarrow B \mid S$

$B \rightarrow b$

Step 5: Remove the rule for $A \rightarrow B$

$S_0 \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$S \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$A \rightarrow S \mid b$

$B \rightarrow b$

Step 6: Remove the rule for $A \rightarrow S$

$S_0 \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$S \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$A \rightarrow b \mid ASA \mid aB \mid a \mid AS \mid SA$

$B \rightarrow b$

Step 7:

$S_0 \rightarrow AA_1 \mid A_2 B \mid a \mid AS \mid SA$

$A_1 \rightarrow SA$

$A_2 \rightarrow a$

$S \rightarrow AA_1 \mid A_2 B \mid a \mid AS \mid SA$

$A \rightarrow b \mid AA_1 \mid A_2 B \mid a \mid AS \mid SA$

$B \rightarrow b$

Question # 02

PART a

Moore Machine

Input: $\{a, b\}$

Output: $\{P, Q\}$
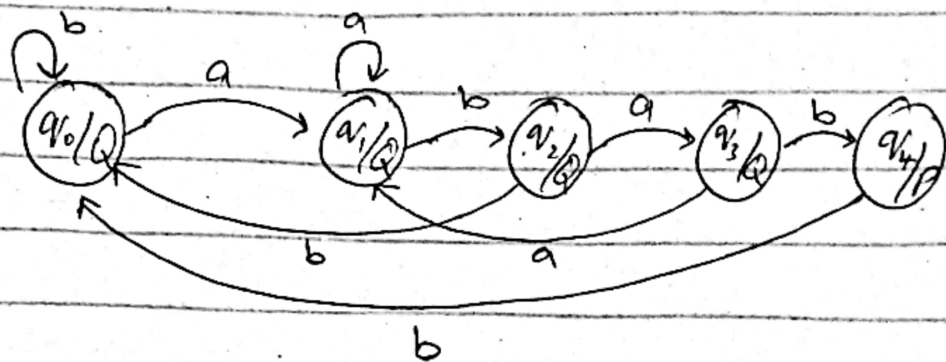
PART b

Moore to Mely Machine

State	0	1
→ v_0	$v_1, 1$	$v_0, 0$
v_1	$v_2, 0$	$v_3, 1$
v_2	$v_3, 1$	$v_1, 0$
v_3	$v_0, 0$	$v_1, 1$

Mely Machine
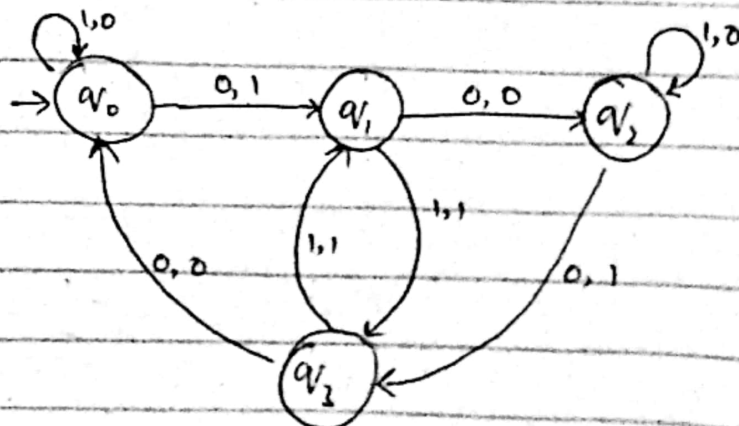
Question # 3

	a	g	f	e	d	c	b
g	✔						
f		✔					
e		✔	✓				
d	✓	✔	✓	✓			
c	✔		✓	✓	✔		
b	✓	✔		✓		✔	
a		✓	✓	✓	✓	✔	✓

$\delta(a,0) = b$ $\delta(d,0) = g$ (b,g) _marked_
$\delta(a,1) = g$, $\delta(g,1) = e$ (g,e) $\frac{a,b}{marked}$
$\frac{}{d,f}$

Similarly all pointes will be
marked
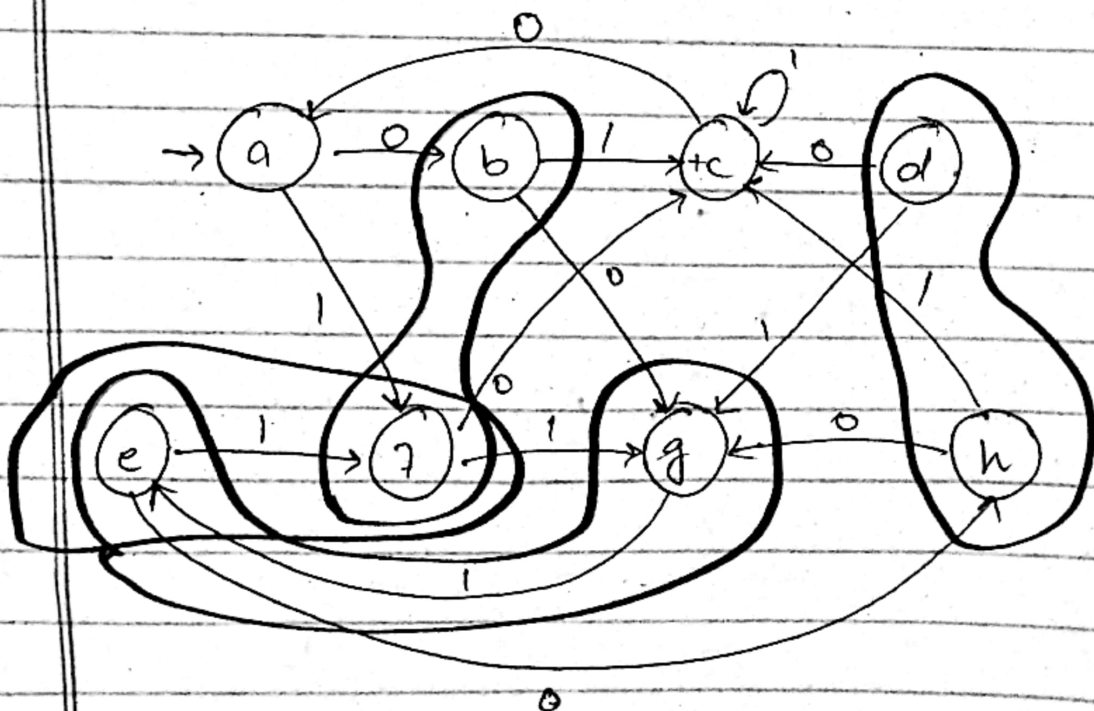
Turing Machine that accepts even & odd
palindromes over $\Sigma = \{a, b\}$
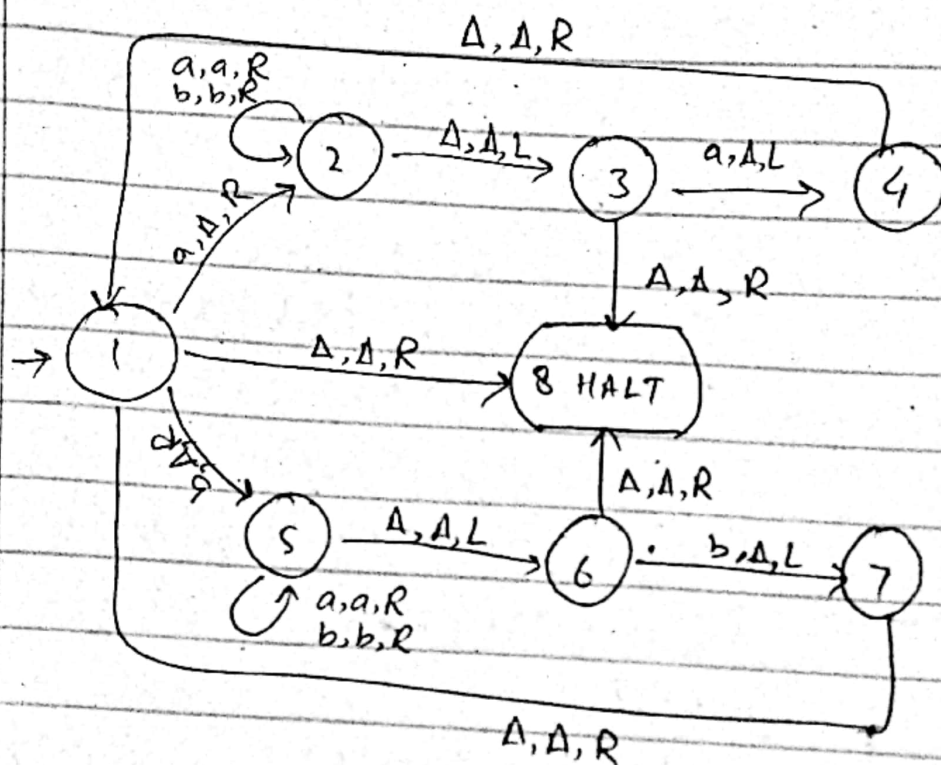
Question # 04

PART a

$$S \to aY \mid Ybb \mid Y$$
$$X \to \wedge \mid a$$
$$Y \to aXY \mid bb \mid XXa$$
$$X \to \wedge$$

$$\left(\begin{array}{l} S \to aY \mid Ybb \mid Y \\ X \to a \\ Y \to aXY \mid aY \mid bb \mid XXa \mid Xala \end{array}\right.$$

$S \rightarrow aY \mid Ybb \mid a$

$X \rightarrow a$

$Y \rightarrow aXY \mid aY \mid bb \mid XXa \mid Xa \mid a$

$S \rightarrow aY \mid YY \mid a$

$X \rightarrow a$

$Y \rightarrow aXY \mid aY \mid bb \mid XXa \mid Xa \mid a$

$S \rightarrow XY \mid YY \mid a$

$X \rightarrow a$

$Y \rightarrow aXY \mid aY \mid bb \mid XXa \mid Xa \mid b$

$S \rightarrow XY \mid YY \mid a$

$X \rightarrow a$

$Y \rightarrow XXY \mid XY \mid bb \mid XXX \mid XX \mid a$

$S \rightarrow XY \mid YY \mid a$

$X \rightarrow a$

$Y \rightarrow RY \mid XY \mid bb \mid RX \mid R \mid c$

$R \rightarrow XX$

PART b
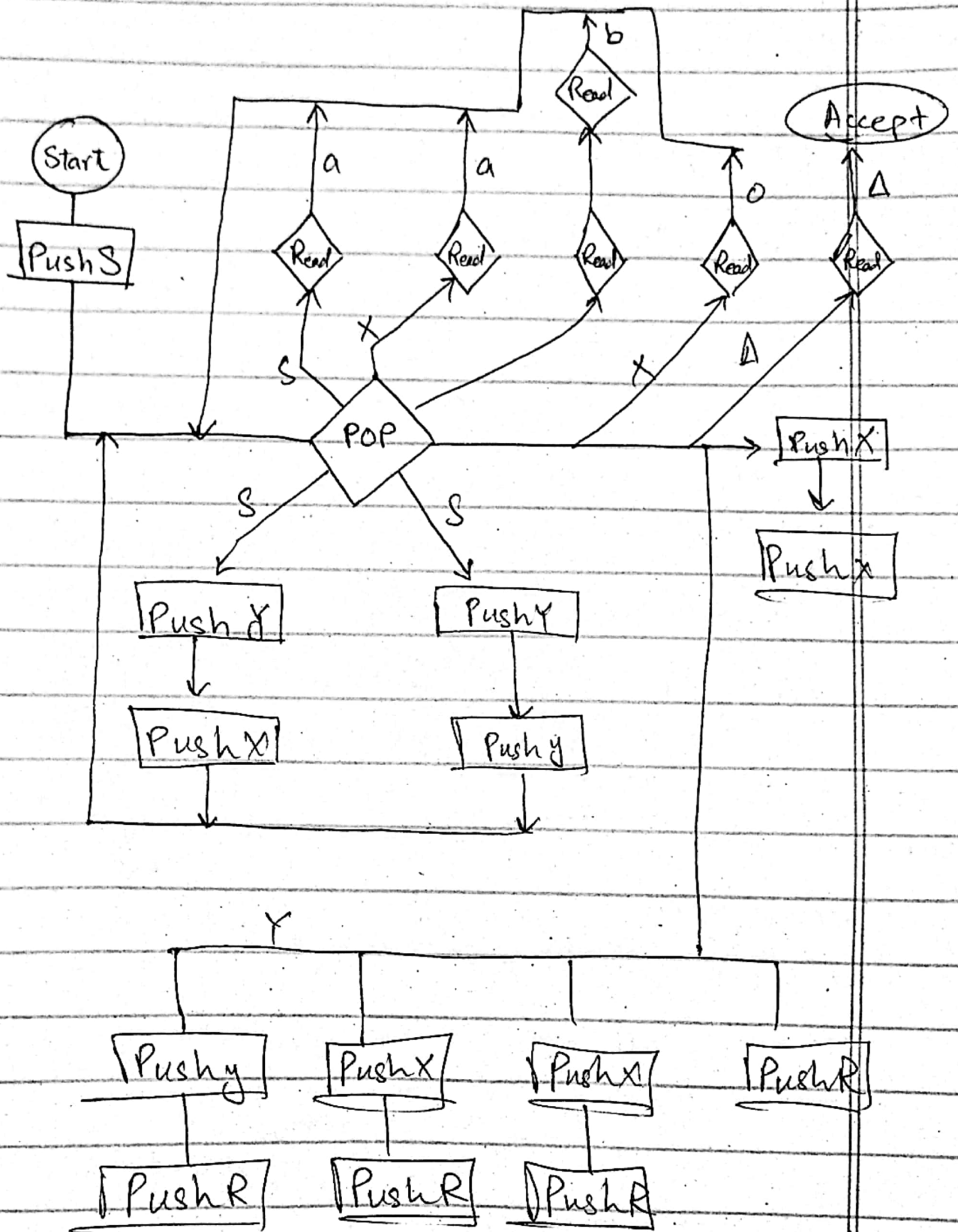
Question # 05

Odd length Palindrome

$L = w c w^r$

Let

$w = abb$

$w^r = bba$

$w c w^r = ab\,bc\,\underbrace{bba}$

Push ↙ ↓ ↘ Pop
Skip

| a | b | b | c | b | b | a | Δ |

→ (q_0) ⟲

$a, z_0 \mid a, z_0$
$a, a \mid a, a$
$a, b \mid a, b$

$b, z_0 \mid b, z_0$
$b, b \mid b, b$
$b, a \mid b, a$

Stack:
| b |
| b |
| a |
| z_0 |

$c, z_0 \mid z_0$
$c, a \mid a$
$c, b \mid b$

(q_1) ⟲

$b, b \mid \varepsilon$
$a, a \mid \varepsilon$

$\wedge, z_0 \mid z_0$

(q_f)