(AB)D

$$AB = \begin{bmatrix} 10 & -6 \\ 14 & -6 \end{bmatrix}$$

$$(AB)D = \begin{bmatrix} 10 & -6 \\ 14 & -6 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 20+6 & 30+12 \\ 28+6 & 42+12 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 42 \\ 34 & 54 \end{bmatrix}$$

possible

A(C+E)

$$C+E = \begin{bmatrix} 2 & 3 & 1 \\ 3 & -4 & 5 \\ 1 & -1 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 5 \\ 3 & 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & -2 \\ 1 & -3 & 10 \\ 4 & +3 & 0 \end{bmatrix}$$

2) find $a \cdot b$

a:- $a = [2 \quad -1]$, $b = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$a \cdot b = [(2)(3) + (-1)(2)]$

$a \cdot b = [6 - 2] \Rightarrow [4]$

b:- $a = [1 \quad -1]$, $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$a \cdot b = [(1)(1) + (-1)(1)]$

$= [1 - 1] = [0]$

c:- $a = [1 \quad 2 \quad 3]$, $b = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

$a \cdot b = [(1)(-2) + (2)(0) + (3)(1)]$

$a \cdot b = [-2 + 0 + 3] \Rightarrow [1]$

d:- $a = [1 \quad 0 \quad 0]$, $b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$a \cdot b = [(1)(1) + (0)(0) + (0)(0)]$

$a \cdot b = [1]$

3) Let $a = [-3 \quad 2 \quad x]$, $b = \begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix}$

if $a \cdot b = 17$ find x

$a \cdot b = [-3 \quad 2 \quad x] \begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix}$

$17 = [+9 + 4 + x^2]$

$= [13 + x^2]$

$17 = 13 + x^2$

$17 - 13 = x^2 \Rightarrow 4 = x^2$

$\boxed{x = \pm 2}$

$$12+0+2 \qquad 4+0-1$$

$$= \begin{bmatrix} 10 & -6 \\ 14 & -6 \end{bmatrix}$$

Possible

b) BA

$$BA = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -1 & 5 \end{bmatrix}_{3\times 2} \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix}_{2\times 3}$$

$$= \begin{bmatrix} 3+4 & 6+0 & -9-2 \\ 2+16 & 4+0 & -6-8 \\ -1+20 & -2+0 & 3-10 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 6 & -11 \\ 18 & 4 & -14 \\ 19 & -2 & -7 \end{bmatrix}$$

15)

Let $A = \begin{bmatrix} 2 & -3 & 4 \\ -1 & 2 & 3 \\ 5 & -1 & -2 \end{bmatrix}$ and $c \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$

Express Ac as a linear combi of the column A

$$Ac = \begin{bmatrix} 2 & -3 & 4 \\ -1 & 2 & 3 \\ 5 & -1 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 3 + 16 \\ -2 + 2 + 12 \\ 10 - 4 - 8 \end{bmatrix}$$

$$Ac = \begin{bmatrix} 17 \\ 12 \\ 1 \end{bmatrix}$$

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te: ______

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix}$$

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AO = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix}_{2\times 3} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 \end{bmatrix}$$

4) Let $W = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$, compute ...

$W \cdot W = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$

$W \cdot W = \left[(\sin\theta)(\sin\theta) + (\cos\theta)(\cos\theta) \right]$

$= \left[\sin^2\theta + \cos^2\theta \right]$

$W \cdot W \quad [1]$

5) find all value of x that $v \cdot v = 1$, where $v = \begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix}$

$v \cdot v = \begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix} \begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix}$

$1 = \left[(1/2)(1/2) + (-1/2)(-1/2) + x^2 \right]$

$1 = \left[1/4 + 1/4 + x^2 \right]$

$1 = \left[\frac{2}{4} + x^2 \right]$

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9) Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 3 \end{bmatrix}$ and $B =$

AB

a) The (1, 2) Entry

$$AB = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6+3 & -2+6 & 6+13 \\ -3+4 & 1+8 & -3+16 \\ 0+3 & 0+6 & 0+12 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 & 9 \\ 1 & 9 & 13 \\ 3 & 6 & 12 \end{bmatrix}$$

(1, 2) Entry is 4

b) The (2, 3) Entry

13

c) The (3, 1) Entry

3

d) The (3, 3) Entry

12.

) The fourth column

$$A\,\text{Col}_4 B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4+2 \\ 6+8+4 \\ 8-8+3 \\ 4+4+5 \end{bmatrix}$$

$$A\,\text{Col}_4 B = \begin{bmatrix} 0 \\ 18 \\ 3 \\ 13 \end{bmatrix}$$

) Using the method in Example 12, compute the following columns of AB

a) the first column

$$A\,\text{col}_1 B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

8) if possible, compute

a) $A(BD)$

$$BD = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$$BD = \begin{bmatrix} 6-1 & 9-2 \\ 4-4 & 6-8 \\ -2-5 & -3-10 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 7 \\ 0 & -2 \\ -7 & -13 \end{bmatrix}$$

$$A(BD) = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} \begin{bmatrix} 5 & 7 \\ 0 & -2 \\ -7 & -13 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+21 & 7-4+39 \\ 20+0+14 & 28-0+26 \end{bmatrix}$$

$$A(BD) = \begin{bmatrix} 26 & 42 \\ 34 & 54 \end{bmatrix}$$

possible.

$$\begin{bmatrix} -7 \\ 16 \\ -11 \end{bmatrix} = \begin{bmatrix} -7 \\ 18 \\ -11 \end{bmatrix}$$

yes, linear Combination.

1.4) using the method in Example 12, compute the following columns of AB

a) The second column.

$$AB = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix}_{4\times 3} \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 3 & -3 & 4 \\ 4 & 2 & 5 & 1 \end{bmatrix}_{3\times 4}$$

$$\text{col}_2 B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0-3+4 \\ 0+6+8 \\ 0-6+6 \\ 0+3+10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 14 \\ 0 \\ 13 \end{bmatrix}$$

c) CB + D

$$CB = \begin{bmatrix} 2 & 3 & 1 \\ 3 & -4 & 5 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6+6-1 & 2+12+5 \\ 9-8-5 & 3-16+25 \\ 3-2+2 & 1-4-10 \end{bmatrix}$$

$$CB = \begin{bmatrix} 11 & 19 \\ -4 & 12 \\ 3 & -13 \end{bmatrix}$$

$$CB + D = \begin{bmatrix} 11 & 19 \\ -4 & 12 \\ 3 & -13 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

Not Possible.

d) AB + DF

$$AB = \begin{bmatrix} 10 & -6 \\ 14 & -6 \end{bmatrix}$$

$$DF = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$$

$$DF = \begin{bmatrix} 4+12 & -6+3 \\ -2-8 & -3-2 \end{bmatrix}$$

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$$= \begin{bmatrix} 1-3+8 \\ 3+6+16 \\ 4-6+1.2 \\ 2+3+20 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 \\ 25 \\ 10 \\ 25 \end{bmatrix}$$

b) the Third column

$$A\,col_3 B \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1+3+10 \\ -3-6+20 \\ -4+6+15 \\ -2-3+25 \end{bmatrix} \Rightarrow \begin{bmatrix} 12 \\ 11 \\ 18 \\ 20 \end{bmatrix}$$

12) if A is the matrix in Example 4 and O is the 3×2 matrix every one of whose entries is zero compute AO

$$AC + AE = \begin{bmatrix} 5 & -2 & 17 \\ 6 & 14 & 8 \end{bmatrix} + \begin{bmatrix} \cdots \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -12 & 18 \\ 4 & 6 & -8 \end{bmatrix}$$

$(D+F)A$

$$D+F = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$$

$$D+F = \begin{bmatrix} 4 & 0 \\ 3 & -1 \end{bmatrix}$$

$$(D+F)A = \begin{bmatrix} 4 & 0 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 8+0 & -12- \\ 3-4 & 6-0 & -9+ \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 & -12 \end{bmatrix}$$

$1 = \frac{1}{2} + x$

$1 - \frac{1}{2} = x \Rightarrow \frac{2-1}{2} = x \Rightarrow x = \frac{1}{2}$

6) Let $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$. if $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ find x & y

$AB = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

$= \begin{bmatrix} y + 2x + x \\ 3y - x + 2 \end{bmatrix} \begin{matrix} 6 \\ 8 \end{matrix}$

$y + 3x = 6$

$3y - x = 6$

$3y + 9y = 18$

$\pm 3y \mp x = \pm 6$

$10x = 12 \Rightarrow \boxed{x = 6/5}$

$y + 3(6/5) = 6$

$y + \frac{18}{5} = 6$

$y = 6 - \frac{18}{5}$

$\boxed{y = \frac{12}{5}}$

Ex # 1.3

find a.b

1)

a:- $a = \begin{bmatrix} 1 & 2 \end{bmatrix}$, $b \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

$a \cdot b = \begin{bmatrix} (1)(4) + (2)(-1) \end{bmatrix}$

$a \cdot b = \begin{bmatrix} 4 - 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \end{bmatrix}$

b:- $a = \begin{bmatrix} -3 & -2 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$a \cdot b = \begin{bmatrix} (-3)(1) + (-2)(-2) \end{bmatrix}$

$a \cdot b = \begin{bmatrix} -3 + 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \end{bmatrix}$

c:- $a = \begin{bmatrix} 4 & 2 & -1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$

$a \cdot b = \begin{bmatrix} (4)(1) + (2)(3) + (-1)(6) \end{bmatrix}$

$a \cdot b = \begin{bmatrix} 4 + 6 - 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 \end{bmatrix}$

d:- $a = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$a \cdot b = \begin{bmatrix} 1 + 0 + 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \end{bmatrix}$

$$\begin{bmatrix}17\\12\\1\end{bmatrix} = 2\begin{bmatrix}2\\-1\\5\end{bmatrix} + 1\begin{bmatrix}-3\\2\\-1\end{bmatrix} + 4\begin{bmatrix}4\\3\\-2\end{bmatrix}$$

$$= \begin{bmatrix}4\\-2\\10\end{bmatrix} + \begin{bmatrix}-3\\2\\-1\end{bmatrix} + \begin{bmatrix}16\\12\\-8\end{bmatrix}$$

$$\begin{bmatrix}17\\12\\1\end{bmatrix} = \begin{bmatrix}17\\12\\1\end{bmatrix}$$

Yes, linear combination

16) Let $A = \begin{bmatrix}1 & -2 & -1\\2 & 4 & 3\\3 & 0 & -2\end{bmatrix}$ and $B = \begin{bmatrix}1 & -1\\3 & 2\\2 & 4\end{bmatrix}$

Express The column of AB as a linear combination of column A

$$AB = \begin{bmatrix}1 & -2 & -1\\2 & 4 & 3\\3 & 0 & -2\end{bmatrix}\begin{bmatrix}1 & -1\\3 & 2\\2 & 4\end{bmatrix}$$

$$= \begin{bmatrix}1-6-2 & -1-4-4\\2+12+6 & -2+8+12\\3+0-4 & -3+0-8\end{bmatrix}$$

$$\begin{bmatrix} 16 & -3 \\ -10 & -5 \end{bmatrix}$$

$$AB+DF = \begin{bmatrix} 10 & -6 \\ 14 & -6 \end{bmatrix} + \begin{bmatrix} 16 & -3 \\ -10 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & -9 \\ 4 & -11 \end{bmatrix}$$

possible.

BA+FD

$$BA = \begin{bmatrix} 7 & 6 & -11 \\ 18 & 4 & -14 \\ 19 & -2 & -7 \end{bmatrix}$$

$$FD = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 8-1 & 12-2 \end{bmatrix}$$

$$FD = \begin{bmatrix} 7 & 12 \\ 7 & 10 \end{bmatrix}$$

BA+FD Not possible

Because order of Matrices are Not same.

$$A(C+E) = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} \begin{bmatrix} 3 & 3 & -2 \\ 1 & -3 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+2-12 & 3-6-9 & -2+20+0 \\ 12+0-8 & 12-0-6 & -8+0-0 \end{bmatrix}$$

$$\begin{bmatrix} -7 & -12 & 18 \\ 4 & 6 & -8 \end{bmatrix}$$

possible

d) $AC+AE$

$$AC = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 3 & -4 & 5 \\ 1 & -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+6-3 & 3-8+3 & 1+10+6 \\ 8+0-2 & 12+0+2 & 4+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 & 17 \\ 6 & 14 & 8 \end{bmatrix}$$

$$AE = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ -2 & 1 & 5 \\ 3 & 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4-9 & 0+2-12 & -3+10 \\ 4+0-6 & 0+0-8 & -12+0 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -10 & 1 \\ -2 & -8 & -16 \end{bmatrix}$$

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11) Show $AB \neq BA$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6 & -1+8 \\ 6-6 & -3+8 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 7 \\ 0 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 4-2 \\ -3+12 & -6+8 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ 9 & 2 \end{bmatrix}$$

$AB \neq BA$

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10) Let $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$, compute DI_2 and I_2D

$$DI_2 = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 0+3 \\ -1+0 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$$I_2D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2+0 & 3+0 \\ 0-1 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$DI_2 = I_2D = D$

$$3.\begin{bmatrix} -7 & 1 \\ 20 & 18 \\ -1 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} + c_3\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 9 \\ 8 \\ 11 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} + c_3\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -7 \\ 20 \\ -1 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 3\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} -7 \\ 20 \\ -1 \end{bmatrix} = \begin{bmatrix} -7 \\ 20 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 9 \\ 18 \\ -11 \end{bmatrix} = -1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} + 4\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$