Determine all value of 's' and 't' for which following system will have

(i) No solution

(ii) a unique solution

(iii) Infinite many solution

$$3x - y + 5z = 1$$
$$x + 3y + 2z = -t$$
$$x - 2y + 2z = 4$$

Step I:

Augmented matrix

$$= \begin{bmatrix} 3 & -1 & 5 & : & 1 \\ 1 & 3 & 2 & : & -t \\ 1 & -2 & 2 & : & 4 \end{bmatrix}$$

$$= \sim \begin{bmatrix} 1 & -2 & 2 & : & 4 \\ 1 & 3 & 2 & : & -t \\ 3 & 1 & 5 & : & 1 \end{bmatrix} \quad R_3 \leftrightarrow R_1$$

$$= \sim \begin{bmatrix} 1 & -2 & 2 & : & 4 \\ 0 & 5 & 0 & : & (-t-4) \\ 0 & 5 & 5-6 & : & -11 \end{bmatrix} \quad \begin{matrix} -R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3 \end{matrix}$$

$$= \sim \left[\begin{array}{ccc|c} 1 & -2 & 2 & 4 \\ 0 & 1 & 0 & \frac{-t+4}{5} \\ 0 & 5 & 5-6 & -11 \end{array}\right] \quad \frac{1}{5}R_2 \rightarrow R_2$$

$$= \sim \left[\begin{array}{ccc|c} 1 & -2 & 2 & 4 \\ 0 & 1 & 0 & \frac{-t+4}{5} \\ 0 & 0 & 5-6 & t-7 \end{array}\right] \quad -5R_2 + R_3 \rightarrow R_3$$

$$x - 2y + 2z = 4 \rightarrow (i)$$

$$y = \frac{-t-4}{5} \rightarrow (ii)$$

$$(5-6)z = (t-7) \rightarrow (iii)$$

**Case I:**

The system has no solution for $5 = 6$ and $t \neq 7$

$$5 - 6 \Rightarrow 5 = 6$$

$$t - 7 \rightarrow t \neq 7$$

**Case II**

The system has infinite many solutions if s and t are any real number

let say

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$s = r_1$ and $t = r_2$ where $s \neq 6$

$$z = \frac{(t-7)}{(s-6)}$$

$$z = \frac{r_2 - 7}{r_1 - 6}$$

$$y = \frac{-r_2 - 4}{5} \rightarrow (iv)$$

$$x = 4 + 2y - 2z$$

$$x = 4 + 2\left(-\left(\frac{r_2+4}{5}\right)\right) - 2\left(\frac{r_2-7}{r_1-6}\right)$$

$$= 4 - 2\left(\frac{r_2+4}{5}\right) - 2\left(\frac{r_2-7}{r_1-6}\right)$$

$$= \frac{20(r_1-6) - 2(r_2+4)(r_1-6) - 10(r_2-7)}{5(r_1-6)}$$

$$= \frac{20r_1 - 120 - 2(r_2r_1 - 6r_2 + 4r_1 - 24) - 10r_2 + 70}{5(r_1-6)}$$

$$= \frac{20r_1 - 120 - 2r_2r_1 - 12r_2 - 8r_1 + 48 - 10r_2 + 7}{5(r_1-6)}$$

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$$= \frac{12r_1 - 2r_1r_2 + 2r_2 + 2}{5r_1 - 30}$$

## Case III

For single value of $r_1$ and $r_2$ let say

$r_1 = 0$ , $r_2 = 1$

$$x = 4 + 2y - 2z$$

$$y = \frac{-r_2 - 4}{5}$$

$$z = \frac{r_2 - 7}{r_1 - r}$$

$$z = \frac{1-7}{0-6} = \frac{-6}{-6} = 1 \quad \boxed{z = 1}$$

$$y = \frac{-(1+4)}{5} = \frac{-5}{5} = -1 = \boxed{y = 1}$$

$$x = 4 + 2y - 2z$$

$$= 4 + 2(-1) - 2(1)$$

$$= 4 - 2 - 2$$

$$x = 0 = \boxed{x = 0}$$

Checking:

(i)

$$x + 3y + 2z = -1$$

$$0 + 3(-1) + 2(1) = -1$$

$$-3 + 2 = -1$$

$$-1 = -1$$

(ii)

$$x - 2y + 2z = 4$$

$$0 - 2(-1) + 2(1) = 4$$

$$2 + 2 = 4$$

$$4 = 4$$