Assignment #2

Question # 1:

Show that if $W = F(s)$ is any differentiable Function of s and if $s = y + 5x$, then $\frac{\partial w}{\partial x} - 5\frac{\partial w}{\partial y} = 0$

Solve:

$$\frac{\partial w}{\partial x} - 5\frac{\partial w}{\partial y} = 0$$

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}(y+5x)$$

$$= 0 + 5(1)$$

$$= 5$$

$$\frac{\partial w}{\partial y} = \frac{\partial}{\partial y}(y+5x)$$

$$= \frac{\partial y}{\partial y} + 0$$

$$\frac{\partial w}{\partial y} = 1$$

$$\frac{\partial w}{\partial x} - 5\frac{\partial w}{\partial y} = 0$$

$$5 - 5(1) = 0$$

$$5 - 5 = 0$$

$$0 = 0$$

Hence prove

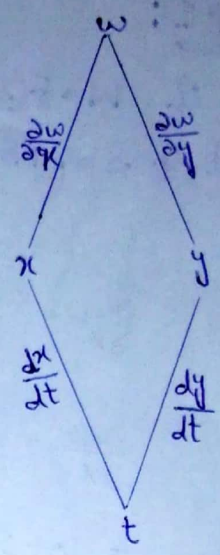
Question #2 :

Find $\frac{dw}{dt}$ at $t=0$ if $w=\sin(xy+\pi)$, $x=e^t$, $y=\ln(t+1)$

Solve :

Apply Chain rule

$$\frac{dw}{dt}=\frac{\partial w}{\partial x}\left(\frac{dx}{dt}\right)+\frac{\partial w}{\partial y}\left(\frac{dy}{dt}\right)$$



$$\frac{\partial w}{\partial x}=\frac{\partial}{\partial x}\left(\sin(xy+\pi)\right)$$

$$=\cos(xy+\pi)\cdot\frac{\partial}{\partial x}(xy+\pi)$$

$$=\cos(xy+\pi)(y+0)$$

$$\frac{\partial w}{\partial x}=y\cos(xy+\pi)$$

$$\frac{\partial w}{\partial y}=\frac{\partial}{\partial y}\left(\sin(xy+\pi)\right)$$

$$=\cos(xy+\pi)(x+0)$$

$$=x\cos(xy+\pi)$$

$\because\ x=e^t$

$$\frac{dx}{dt}=\frac{d}{dt}(e^t)$$

$$=e^t\left(\frac{d}{dt}(t)\right)$$

$$=e^t$$

$\because\ y=\ln(t+1)$

$$\frac{dy}{dt}=\frac{d}{dt}\left(\ln(t+1)\right)$$

$$= \frac{1}{t+1} \frac{d}{dt}(t+1)$$

$$= \frac{1}{t+1}(1+0)$$

$$\frac{dy}{dt} = \frac{1}{t+1}$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\left(\frac{dx}{dt}\right) + \frac{\partial w}{\partial y}\left(\frac{dy}{dt}\right)$$

$$= y\cos(xy+\pi)(e^t) + x\cos(xy+\pi)$$

$$= y\cos(xy+\pi)\,e^t + x\cos(xy+\pi)\left(\frac{1}{t+1}\right)$$

$$= e^t y\cos(xy+\pi) + x\left(\frac{1}{t+1}\right)\cos(xy+\pi)$$

$$\left.\frac{\partial w}{\partial t}\right|_{t=0} = e^0 y\cos(xy+\pi) + x\left(\frac{1}{0+1}\right)\cos(xy\pi)$$

$$= y\cos(xy+\pi) + x\cos(xy+\pi)$$

Question #3:

Find Linearization of $f(x,y) = (x+y+z)^2$ at $(1,2)$

Solve:

$$L(x,y) = F(x_0,y_0) + F_x|_{(x_0,y_0)}(x-x_0) + F_y|_{(x_0,y_0)}(y-y_0)$$

$$F(1,2) = (1+2+2)^2$$

$$= (5)^2$$

$$= 25$$

$$F_x \Rightarrow \frac{\partial F}{\partial x} = \frac{\partial}{\partial x}(x+y+2)^2$$

$$= 2(x+y+2)(1)$$

$$= 2(x+y+2)$$

$$F_x\Big|_{(1,2)} = 2(1+2+2)$$
$$= 2(5)$$
$$= 10$$

$$F_y \Rightarrow \frac{\partial F}{\partial y} = \frac{\partial}{\partial y}(x+y+2)^2$$
$$= 2(x+y+2)(1)$$
$$= 2(x+y+2)$$

$$F_y\Big|_{(1,2)} = 2(1+2+2)$$
$$= 2(5)$$
$$= 10$$

$$L(x,y) = F(x_0,y_0) + F_x\Big|_{(x_0,y_0)}(x-x_0) + F_y\Big|_{(x_0,y_0)}(y-y_0)$$

$$= 25 + 10(x-1) + 10(y-2)$$

$$= 25 + 10x - 10 + 10y - 20$$

$$= 10x + 10y - 5$$