

Question #3.23

The following table shows the age distribution of 1,143 horses

| Age (Year) | Number of horses (f_i) | Average age (\bar{x}_i) |
|-------------|----------------------------|-----------------------------|
| 1-4 | 12 | 2.7 |
| 5-9 | 223 | 7.6 |
| 10-14 | 435 | 12.0 |
| 15-19 | 272 | 16.3 |
| 20-24 | 161 | 20.8 |
| 25-29 | 34 | 25.8 |
| 30 and Over | 6 | 31.8 |

Solution:

| Age (year) | f | \bar{x} | x | $D = x - a$ | fD | $\bar{x}_i w$ |
|------------|-----------------|-----------|------|-------------|-------|---------------|
| 1-4 | 12 | 2.7 | 2.5 | -9.5 | -114 | 32.4 |
| 5-9 | 223 | 7.6 | 7.0 | -5 | -1115 | 1694.8 |
| 10-14 | 435 | 12.0 | 12.0 | 0 | 0 | 5220.0 |
| 15-19 | 272 | 16.3 | 17.0 | 5 | 1360 | 4433.6 |
| 20-24 | 161 | 20.8 | 22.0 | 10 | 1610 | 3348.8 |
| 25-29 | 34 | 25.8 | 27.0 | 15 | 510 | 877.2 |
| 30-34 | 6 | 32.0 | 32.0 | 20 | 120 | 190.8 |
| | $\sum f = 1143$ | | | | 2371 | 1579.6 |

$$\text{Average age} = a + \frac{\sum f D}{n}$$

$$= 12.0 + \frac{2371}{1143}$$

$$= 14.07 \text{ year}$$

b) Average age = $\frac{\sum \bar{x}_i w_i}{\sum w_i}$

$$= \frac{1579.6}{1143} \Rightarrow 13.82 \text{ year}$$

The weighted average age is more nearly the real average age because the mean age of each and every value age group has been multiplied by the corresponding frequency.

Question #3.24

Find the arithmetic and geometric means of the series $1, 2, 4, 8, 16, \dots, 2^n$. Find also the harmonic mean.

Solution:

$$1, 2, 4, 8, 16, \dots, 2^n$$

Sum of the Value of G.P is Obtain by the formula

$$(a=1, r=2)$$

$$S = \frac{a(r^n - 1)}{r-1}, \text{ Where } r > 1$$

$$= \frac{a(1-r^n)}{1-r}, \text{ Where } r < 1$$

$$\text{Sum of Values} = \frac{a(r^n - 1)}{r-1}$$

$$= \frac{1(2^{n+1} - 1)}{2-1}$$

$$= \frac{2^{n+1} - 1}{1}$$

$$\begin{aligned} \text{Product of the Value} &= 1 \times 2 \times 4 \times 8 \times 6 \\ &= 2^0 \times 2^1 \times 2^2 \times 2^3 \times 2^4 \dots \dots 2^n \end{aligned}$$

$$= 2^0 + 1 + 2 + 3 + 4 - n$$

$$= 2^n + 1(n+1)/2$$

$$\text{Sum of reciprocal} = \frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \dots + \frac{1}{2^n}$$

$$= 1 \left(1 - \frac{1}{2^n}\right)$$

$$= \frac{a(1-r^n)}{1-r} \Rightarrow \frac{1 \left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} \quad \because a=1, r=\frac{1}{2}$$

$$= 2 \left(1 - \frac{1}{2^n}\right)$$

$$\text{Hence } \bar{x} = \frac{\text{Sum of Values}}{\text{No. of Values}} \Rightarrow \frac{2^{n+1} - 1}{n + 1}$$

$$\begin{aligned} \text{G.M.} &= (\text{Product of Value})^{\frac{1}{n+1}} \\ &= [2^n(n+1)/2]^1/(n+1) \\ &= 3^n/2 \end{aligned}$$

$$\text{H.M.} = \frac{\text{Number of Values}}{\text{Sum of their reciprocals}}$$

$$\text{H.M.} = \frac{n+1}{2\left(1 - \frac{1}{2^{n+1}}\right)}$$

Question # 3.25

Find ① arithmetic mean ② geometric mean ③ harmonic mean of the series 1, 3, 9, 27, 81, ..., 3^n .

Solution:

Sum of Values in G.P is Obtain the formula.

$$\text{Sum} = \frac{a(r^n - 1)}{r - 1}, \text{ Where } r > 1$$

$$= \frac{a(1 - r^n)}{1 - r}, \text{ Where } r < 1$$

$$a=1 \text{ and } r=3 \text{ (common ratio)}$$

$$\text{Sum of Value} = \frac{1}{3} (3^{n+1} - 1) \Rightarrow \frac{1}{2} (3^{n+1} - 1)$$

$$\text{Product of Values} = 1 \times 3 \times 9 \times 27 \times 81 \dots \times 3^n$$

$$\begin{aligned} &= 3^{0+1+2+3+4+\dots+n} \\ &= 3^{(0+n)(n+1)/2} \Rightarrow 3^{n(n+1)/2} \end{aligned}$$

$$\text{Sum of Reciprocals} = \frac{1}{1} + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} \dots + \frac{1}{3^n}$$

$$= \frac{1}{3^{n+1}} \left(1 - \frac{1}{3^{n+1}}\right) \quad (\because a=1, r=1/3)$$

$$= \bar{X} = \frac{1}{2} \left(3^{n+1} - 1\right)$$

$$\text{G.M.} = (\text{Product of Value})^{1/n+1} \Rightarrow [3^{n(n+1)/2}]^{1/(n+1)} \Rightarrow 3^n/2$$

$$\text{H.M.} = \frac{n+1}{3/2 \left(1 - \frac{1}{3^{n+1}}\right)}$$

Question # 3.30

Find the Geometric mean and the harmonic mean of the following frequency distribution:

| Weekly Income (Rs) | 35-39 | 40-44 | 45-49 | 50-54 | 55-59 | 60-64 | 65-69 |
|--------------------|-------|-------|-------|-------|-------|-------|-------|
| No. of Workers | 15 | 13 | 17 | 29 | 11 | 10 | 5 |

Solution:

| (Rs) | No of Workers | x_i | $f \times (1/x)$ | $\log x_i$ | $f \log i$ |
|-------|----------------|-------|---------------------|------------|---------------------|
| 35-39 | 15 | 37 | 0.40540 | 1.56820 | 23.5230 |
| 40-44 | 13 | 42 | 0.309523 | 1.623249 | 21.10224 |
| 45-49 | 17 | 47 | 0.361702 | 1.672097 | 28.4256 |
| 50-54 | 29 | 52 | 0.551692 | 1.7160033 | 49.7640 |
| 55-59 | 11 | 57 | 0.192982 | 1.755874 | 19.31462 |
| 60-64 | 10 | 62 | 0.1612903 | 1.79239 | 17.9239 |
| 65-69 | 5 | 67 | 0.074626 | 1.82607 | 9.13037 |
| | $\Sigma = 100$ | | $\Sigma = 2.063216$ | | $\Sigma = 169.1637$ |

We Know that

$$\text{Harmonic Mean} = \frac{n}{\sum f \times (1/x)}$$

$$= \frac{100}{2.063216}$$

$$H.M = 48.468$$

$$H.M = 48.47 \text{ Rs}$$

We Know that

$$G.M = \text{Antilog } \frac{1}{n} \sum f \log$$

$$= \text{Antilog } \frac{469.1837}{100}$$

$$= \text{Antilog } (1.691837)$$

$$G.M = 49.1855$$

$$\boxed{G.M = 49.19 \text{ R}}$$

Question #3.31

Calculate the Geometric and the harmonic means for the distribution given below:

| Variable | 0-5 | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 | 30-35 | 35-40 |
|-----------|-----|------|-------|-------|-------|-------|-------|-------|
| Frequency | 2 | 5 | 7 | 13 | 21 | 16 | 8 | 3 |

| Variable | F | x_i | $\log x_i$ | $f \log x_i$ | $f \times \left(\frac{1}{n}\right)$ |
|----------|---------------|-------|------------|----------------------|-------------------------------------|
| 0-5 | 2 | 2.5 | 0.39794 | 0.795880 | 0.8 |
| 5-10 | 5 | 7.5 | 0.875061 | 4.375306 | 0.6666 |
| 10-15 | 7 | 12.5 | 1.096910 | 7.6783700 | 0.56 |
| 15-20 | 13 | 17.5 | 1.24303 | 16.159494 | 0.74285 |
| 20-25 | 21 | 22.5 | 1.35218 | 28.395832 | 0.93333 |
| 25-30 | 16 | 27.5 | 1.43933 | 23.0293 | 0.581818 |
| 30-35 | 8 | 32.5 | 1.51188 | 12.095066 | 0.246158 |
| 35-40 | 3 | 37.5 | 1.57403 | 4.722093 | 0.08 |
| | $\Sigma = 75$ | | | $\Sigma = 97.251341$ | $\Sigma = 4.610751$ |

We Know that

$$G.M = \text{Antilog } \frac{1}{n} \sum f \log x$$

$$= \text{Antilog } \frac{1}{75} (97.251341)$$

$$G.M = \text{Antilog } (1.296684)$$

$$G.M = 19.800$$

$$\boxed{G.M = 19.80}$$

We Know that

$$H.M = \frac{n}{\sum F\left(\frac{1}{n}\right)}$$

$$= \frac{75}{4.610751}$$

$$H.M = 16.26633$$

$$\boxed{H.M = 16.27}$$

Question # 3-33

The following data relate to Sizes of Shoes Sold at a Store during a given Week. Find the median of the Shoes. Also Calculate the Quartiles, the 7th decile and the 64th Percentile.

| | | | | | | | | | | |
|---------------|---|----|----|----|----|----|----|----|---|----|
| Size of Shoes | 5 | 5½ | 6 | 6½ | 7 | 7½ | 8 | 8½ | 9 | 9½ |
| No. of Pairs | 2 | 5 | 15 | 30 | 60 | 40 | 23 | 11 | 4 | 1 |

Solution:

| Size of Shoes | No of Pairs | C.P | C.B |
|---------------|-------------|-----|-------------|
| 5 | 2 | 2 | 4.75 - 5.25 |
| 5½ | 5 | 7 | 5.25 - 5.75 |
| 6 | 15 | 22 | 5.75 - 6.25 |
| 6½ | 30 | 52 | 6.25 - 6.75 |
| 7 | 60 | 112 | 6.75 - 7.25 |
| 7½ | 40 | 152 | 7.25 - 7.75 |
| 8 | 23 | 175 | 7.75 - 8.25 |
| 8½ | 11 | 186 | 8.25 - 8.75 |
| 9 | 4 | 190 | 8.75 - 9.25 |
| 9½ | 1 | 191 | 9.25 - 9.75 |

Median:

$$\begin{aligned}\text{Median} &= l + \frac{h}{f} \left(\frac{n}{2} - c \right) \\ &= l + \frac{h}{f} \left(\frac{191}{2} - c \right) \\ &= 6.75 + \frac{0.5}{60} (95.5 - 52) \\ &= 6.75 + \frac{0.5}{60} (43.5) \\ &= 6.75 + \frac{(21.75)}{60} \\ &= 6.75 + 0.3625\end{aligned}$$

$$\boxed{\text{Median} = Q_2 = 7.11}$$

Quartiles:

$$Q_1 = l + \frac{h}{f} \left(\frac{n}{4} - c \right)$$

$$\frac{n}{4} = \frac{191}{4} \Rightarrow 47.75$$

$$l = 6.25, h = 0.5, C = 22, f = 30$$

$$Q_1 = 6.25 + \frac{0.5}{30} (47.75 - 22)$$

$$Q_1 = 6.25 + \frac{0.5}{30} (25.75)$$

$$Q_1 = 6.25 + \frac{12.875}{30}$$

$$Q_1 = 6.25 + 0.4291666$$

$$Q_1 = 6.679166$$

$$Q_1 = 6.68 \Rightarrow \boxed{Q_1 = 6.5}$$

$$Q_3 = l + \frac{h}{f} \left(\frac{3n}{4} - c \right)$$

$$\frac{3n}{4} = 3 \left(\frac{191}{4} \right)$$

$$\frac{3n}{4} = 143.25, l = 7.25, h = 0.5, f = 40, C = 112$$

$$Q_3 = 7.25 + \frac{0.5}{40} (143.25 - 112)$$

$$Q_3 = 7.25 + \frac{0.5}{40} (31.25)$$

$$= 7.25 + \frac{15.625}{40}$$

$$= 7.25 + 0.390625$$

$$Q_3 = 7.640625$$

$$Q_1 = 6.5$$

$$Q_2 = 7.11$$

$$Q_3 = 7.640625$$

7th decile:

$$D_7 = l + \frac{h}{f} \left(\frac{7n}{10} - c \right)$$

$$\frac{7n}{10} \Rightarrow \frac{7(191)}{10} \Rightarrow 133.7$$

$$l = 7.25, h = 0.5, f = 40, C = 112$$

$$D_7 = 7.25 + \frac{0.5}{40} (133.7 - 112)$$

$$D_7 = 7.25 + \frac{0.5}{40} (21.7)$$

$$D_7 = 7.25 + \frac{10.85}{40}$$

$$= 7.25 + 0.27125$$

$$D_7 = 7.52125 \Rightarrow D_7 = 7.5$$

64th Percentile:

$$P_{64} = l + \frac{h}{f} \left(\frac{64n}{100} - c \right)$$

$$\frac{64(n)}{100} \Rightarrow \frac{64(191)}{100} \Rightarrow 122.24$$

$$l = 7.25, h = 0.5, C = 112, f = 40$$

$$P_{64} = 7.25 + \frac{0.5}{40} (122.24 - 112)$$

$$P_{64} = 7.25 + \frac{0.5}{40} (10.24)$$

$$= 7.25 + \frac{5.12}{40}$$

$$= 7.25 + 0.128$$

$$P_{64} = 7.378 \Rightarrow P_{64} = 7.5$$

Question #3.34

Calculate the Mean, Median and Modal numbers of Persons Per house from the data:

| | | | | | | | | | | |
|-------------------------|----|-----|-----|----|----|----|----|----|---|----|
| No of Persons Per house | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| No of houses | 26 | 113 | 120 | 95 | 60 | 42 | 21 | 14 | 5 | 4 |

Solution:

| No of Persons Per house | No of houses | C.F | C.B | $f_x i$ |
|-------------------------|--------------|-----|-----------|---------|
| 1 | 26 | 26 | 0.5 - 1.5 | 26 |
| 2 | 113 | 139 | 1.5 - 2.5 | 226 |
| 3 | 120 | 259 | 2.5 - 3.5 | 360 |
| 4 | 95 | 354 | 3.5 - 4.5 | 380 |
| 5 | 60 | 414 | 4.5 - 5.5 | 300 |

| | | | | |
|----|----|-----|------------|---------------------|
| 6 | 42 | 456 | 5.5 - 6.5 | 252 |
| 7 | 21 | 477 | 6.5 - 7.5 | 147 |
| 8 | 14 | 491 | 7.5 - 8.5 | 112 |
| 9 | 5 | 496 | 8.5 - 9.5 | 45 |
| 10 | 4 | 500 | 9.5 - 10.5 | 40 |
| | | | | $\sum f_x i = 1888$ |

Median:

$$l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

$$\frac{n}{2} = \frac{500}{2}$$

$$\frac{n}{2} = 250, f = 120, h = 1, C = 139, l = 2.5$$

$$\begin{aligned} &= l + \frac{h}{f} \left(\frac{n}{2} - c \right) \\ &= 2.5 + \frac{1}{120} (250 - 139) \end{aligned}$$

$$\frac{1}{120} (111)$$

$$\text{Median} = 2.5 + 0.925$$

$$\boxed{\text{Median} = 3.425}$$

Mean:

we know that

$$\text{Mean} = \frac{\sum f_x i}{n}$$

$$\text{Mean} = \frac{1888}{500}$$

$$\text{Mean} = 3.776 \Rightarrow \boxed{\text{Mean} = 3.78}$$

Mode:

Question #3.37

Compute the median and quartiles of the following distribution of Heights and Check the results on a graph.

| | |
|------------------|-------------------------------------------------------|
| Heights (inches) | 57.5-, 60.0-, 62.5-, 65.0-, 67.5-, 70.0-, 72.5- |
| Number | 6 26 190 281 412 127 38 |

Solution

| Heights | Number | C.F |
|-----------|--------|------|
| 57.5-60.0 | 6 | 6 |
| 60.0-62.5 | 26 | 32 |
| 62.5-65.0 | 190 | 222 |
| 65.0-67.5 | 281 | 503 |
| 67.5-70.0 | 412 | 915 |
| 70.0-72.5 | 127 | 1042 |
| 72.5-75.0 | 38 | 1080 |

We Know that

$$\text{Median} = l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

$$\frac{n}{2} \Rightarrow \frac{1080}{2} \Rightarrow 540$$

$$l = 67.5, C = 503, F = 412, h = 2.5$$

$$\text{Median} = 67.5 + \frac{2.5}{412} (540 - 503)$$

$$= 67.5 + \frac{2.5}{412} (37)$$

$$= 67.5 + 0.2245$$

$$\boxed{\text{Median} = 67.72 \text{ inches}}$$

$$Q_1 = l + \frac{h}{f} \left(\frac{n}{4} - c \right)$$

$$\frac{n}{4} \Rightarrow \frac{1080}{4} = 270$$

$$l = 65.0, h = 2.5, F = 281, C = 22.2$$

$$Q_1 = 65.0 + \frac{2.5}{281} (270 - 222)$$

$$= 65.0 + \frac{2.5}{281} (48)$$

$$= 65.0 + \frac{120}{281}$$

$$\boxed{Q_1 = 65.43 \text{ inches}}$$

$$Q_3 = l + \frac{h}{f} \left(\frac{3n}{4} - c \right)$$

$$\frac{3n}{4} \Rightarrow 3\left(\frac{1080}{4}\right) \Rightarrow 810$$

$$l = 67.5, F = 412, n = 2.5, C = 503$$

$$Q_3 = 67.5 + \frac{2.5}{412} (810 - 503)$$

$$= 67.5 + \frac{2.5}{412} (307)$$

$$= 67.5 + 1.86286$$

$$\boxed{Q_3 = 69.3628}$$

$$Q_1 = 68.43 \text{ inches}$$

$$Q_2 = 67.72 \text{ inches}$$

$$Q_3 = 69.36 \text{ inches}$$

Question # 3.39

The Frequency distribution of a group of Persons according to age is given below:

| Age in Years | <1 | 1-4 | 5-9 | 10-19 | 20-29 | 30-39 | 40-59 | 60-79 |
|---------------|----|-----|-----|-------|-------|-------|-------|-------|
| No of Persons | 5 | 10 | 11 | 12 | 22 | 18 | 8 | 7 |

Calculate the Mean and Median ages of the distribution.

Solution:

| Age in Year | No of Person | C.F | x | fx | C.B |
|-------------|--------------|-----|------|-----------------|-------------|
| <1 | 5 | 5 | 0.5 | 2.5 | 4pt0 - 05 |
| 1-4 | 10 | 15 | 2.5 | 25 | 0.5 - 4.5 |
| 5-9 | 11 | 26 | 7 | 77 | 4.5 - 9.5 |
| 10-19 | 12 | 38 | 14.5 | 174 | 9.5 - 19.5 |
| 20-29 | 22 | 60 | 24.5 | 539 | 19.5 - 29.5 |
| 30-39 | 18 | 78 | 34.5 | 621.0 | 29.5 - 39.5 |
| 40-59 | 8 | 86 | 49.5 | 396 | 39.5 - 59.5 |
| 60-69 | 7 | 93 | 69.5 | 486.5 | 59.5 - 79.5 |
| | | | | $\Sigma = 2321$ | |

We know that

$$\begin{aligned} \text{Mean} &= \frac{\sum fx}{n} \\ &= \frac{2321}{93} \end{aligned}$$

$$\boxed{\text{Mean} = 24.96 \text{ Year}}$$

$$\text{Median} = l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

$$\frac{n}{2} \Rightarrow \frac{93}{2} \Rightarrow 46.5$$

$$l = 19.5, h = 10, f = 22, C = 38$$

$$= 19.5 + \frac{10}{22} (46.5 - 38)$$

$$= 19.5 + \frac{85}{22}$$

$$\boxed{\text{Median} = 23.36 \text{ Year}}$$

Question # 3.40

- Describe the merits and demerits of mean and median
- Calculate the median, the upper and lower quartiles from the following data. Also draw a box plot.

| Class-Interval | Frequency |
|----------------|-----------|
| under 25 | 222 |
| 25-29 | 405 |
| 30-34 | 508 |
| 35-39 | 520 |
| 40-44 | 525 |
| 45-49 | 490 |
| 50-54 | 457 |
| 55-59 | 416 |
| 60 and Over | 166 |

Solution:

| Class Interval | f | C.f | C.B |
|----------------|-----|------|-------------|
| Under 25 | 222 | 222 | Up to 24.5 |
| 25 - 29 | 405 | 627 | 24.5 - 29.5 |
| 30 - 34 | 508 | 1135 | 29.5 - 34.5 |
| 35 - 39 | 520 | 1655 | 34.5 - 39.5 |
| 40 - 44 | 525 | 2180 | 39.5 - 44.5 |
| 45 - 49 | 490 | 2670 | 44.5 - 49.5 |
| 50 - 54 | 457 | 3127 | 49.5 - 54.5 |
| 55 - 59 | 416 | 3543 | 54.5 - 59.5 |
| 60 and Over | 166 | 3709 | 59.5 + Over |

$$\text{Median} = l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

$$\frac{n}{2} \Rightarrow \frac{3709}{2} \Rightarrow 1854.4$$

$$l = 39.5, h = 5, f = 525, c = 1655$$

$$= 39.5 + \frac{5}{525} (1854.5 - 1655)$$

$$= 39.5 + \frac{5}{525} (199.5)$$

$$= 39.5 + 1.9$$

$$\boxed{\text{Median} = 41.4}$$

$$\boxed{Q_2 = 41.4}$$

$$Q_1 = l + \frac{h}{f} \left(\frac{n}{4} - c \right)$$

$$\frac{n}{4} \Rightarrow \frac{3709}{4} \Rightarrow 927.25$$

$$l = 29.5, h = 5, C = 627, F = 508$$

$$= 29.5 + \frac{5}{508} (927.25 - 627)$$

$$= 29.5 + \frac{5}{508} (300.25)$$

$$= 29.5 + 2.95521$$

$$Q_1 = 32.46$$

$$Q_3 = l + \frac{h}{F} \left(\frac{3n}{2} - c \right)$$

$$3n = 3 \left(\frac{3709}{4} \right) \Rightarrow 2781.75$$

$$l = 49.5, h = 5, F = 457, C = 2670$$

$$= 49.5 + \frac{5}{457} (2781.75 - 2670)$$

$$Q^3 = 111.75$$

Question # 3.41

The following distribution shows Kilowatt-Hours of Electricity used in one month by 75 residential Consumers in a certain Locality of Lahore.

| Consumers in Kilowatt hour | 5-24 | 25-44 | 45-64 | 65-84 | 85-104 | 105-124 | 125-144 | 145-164 |
|----------------------------|------|-------|-------|-------|--------|---------|---------|---------|
| No of Consumers | 4 | 6 | 14 | 22 | 14 | 5 | 7 | 3 |

Solution

| Consumption in kilowatt hour | No of Consumer | C.F | XI | C.B | FxI |
|------------------------------|----------------|-----|-------|-------------|-------|
| 5-24 | 4 | 4 | 14.5 | 4.5-24.5 | 58 |
| 25-44 | 6 | 10 | 34.5 | 24.5-44.5 | 207 |
| 45-64 | 14 | 24 | 54.5 | 44.5-64.5 | 763 |
| 65-84 | 22 | 46 | 74.5 | 64.5-84.5 | 1639 |
| 85-104 | 14 | 60 | 94.5 | 84.5-104.5 | 1323 |
| 105-124 | 5 | 65 | 114.5 | 104.5-124.5 | 572.5 |
| 125-144 | 7 | 72 | 134.5 | 124.5-144.5 | 941.5 |

| | | | | | |
|---------|---------------|----|-------|-------------|--------|
| 145-164 | 3 | 75 | 154.5 | 144.5-164.5 | 463.5 |
| | $\Sigma = 75$ | | | | 5967.5 |

Mean:

$$\bar{x} = \frac{\sum f_i x_i}{n}$$

$$= \frac{5967.5}{75}$$

$$\bar{x} = 79.566 \Rightarrow \boxed{\bar{x} = 79.57}$$

Median:

$$= l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

$$\frac{75}{2} = 37.5, C = 24, f = 22, L = 64.5, h = 20$$

$$= 64.5 + \frac{20}{22} (37.5 - 24)$$

$$= 64.5 + \frac{20}{22} (13.5)$$

$$= 64.5 + \frac{270}{22}$$

$$= 64.5 + 12.727$$

$$\boxed{\text{Median} = 76.772}$$

$$\boxed{Q_2 = 76.772}$$

$$Q_1 = l + \frac{h}{f} \left(\frac{n}{4} - c \right)$$

$$\frac{n}{4} = \frac{75}{4} \Rightarrow 18.75, C = 10, f = 14, l = 44.5, h = 20$$

$$= 44.5 + \frac{20}{14} (18.75 - 10)$$

$$= 44.5 + \frac{20}{14} (18.75)$$

$$= 44.5 + \frac{175}{14}$$

$$= 44.5 + 12.5$$

$$\boxed{Q_1 = 57}$$

$$Q_3 = l + \frac{h}{f} \left(\frac{3n}{4} - c \right)$$

$$\frac{3n}{4} = \frac{3(15)}{4} \Rightarrow 56.25, C=46, h=20, l=84.5, f=14$$

$$Q_3 = 84.5 + \frac{20}{14} (56.25 - 46)$$

$$= 84.5 + \frac{20}{14} (10.25)$$

$$= 84.5 + \frac{205}{14}$$

$$= 84.5 + 14.64$$

$$\boxed{Q_3 = 99.142}$$

$$Q_1 = 57$$

$$Q_2 = 76.772$$

$$Q_3 = 99.142$$

Question #3.4.5

Describe the advantages and disadvantages of the mean, the median and the mode. Explain the empirical relation between them.

b) The Weight of the 40 male Students at a University are given in the following frequency table.

| Weight | 118-126 | 127-135 | 136-144 | 145-153 | 154-162 | 163-171 | 172-180 |
|-----------|---------|---------|---------|---------|---------|---------|---------|
| frequency | 3 | 5 | 9 | 12 | 5 | 4 | 3 |

Calculate the mean, median and the mode.

Solution:

| f_xi | Weight | f | C.f | C.B | (MidPoint) x_i |
|---------------|---------|---------------|-----|-------------|------------------|
| 366 | 118-126 | 3 | 3 | 117.5-126.5 | 122 |
| 655 | 127-135 | 5 | 8 | 126.5-135.5 | 131 |
| 1260 | 136-144 | 9 | 17 | 135.5-144.5 | 140 |
| 1788 | 145-153 | 12 | 29 | 144.5-153.5 | 149 |
| 790 | 154-162 | 5 | 34 | 153.5-162.5 | 158 |
| 668 | 163-171 | 4 | 38 | 162.5-171.5 | 167 |
| 352 | 172-180 | 2 | 40 | 171.5-180.5 | 176 |
| $\sum f_xi =$ | | $\sum f = 40$ | | | |
| 5879 | | | | | |

Median:

$$\text{Median} = l + \frac{h}{f} \left(\frac{n}{2} - C.f \right)$$

$$\frac{n}{2} = \frac{40}{2} \Rightarrow 20$$

$$= 144.5 + \frac{9}{12} (20-17)$$

$$= 144.5 + \frac{27}{12}$$

$$\text{Median} = 146.75$$

$\boxed{\text{Median} = 146.75}$

1 Mode:

$$\text{Mode} = l + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 144.5 + \frac{12 - 9}{(12 - 9) + (12 - 5)} \times 9$$

$$\text{Mode} = 144.5 + \frac{3}{3+7} \times 9$$

$$= 144.5 + 0.3 \times 9$$

$$= 144.5 + 2.7$$

$$\boxed{\text{Mode} = 147.2}$$

Mean:

$$\text{Mean} = \frac{\sum F_i x_i}{n}$$

$$= \frac{5879}{40}$$

$$\boxed{\text{Mean} = 146.975}$$

Question #3.46

The following table shows the distribution of the maximum loads in short tons supported by certain cables produced by a company.

| Max. loads (short tons) | 9.8-10.2 | 10.3-10.7 | 10.8-11.2 | 11.3-11.7 | 11.8-12.2 | 12.3-12.7 |
|-------------------------|----------|-----------|-----------|-----------|-----------|-----------|
| Number of Cables | 7 | 12 | 17 | 14 | 6 | 4 |

| Max.loads (Short tons) | No of cables | C.f | C.B | x_i | $f x_i$ |
|---------------------------|-----------------|-----|---------------|--------|--------------------|
| 9.8 - 10.2 | 7 | 7 | 9.75 - 10.5 | 10 | 70 |
| 10.3 - 10.7 | 12 | 19 | 10.25 - 10.75 | 10.5 | 126 |
| 10.8 - 11.2 | 17 | 36 | 10.75 - 11.25 | 11.015 | 187 |
| 11.3 - 11.7 | 14 | 50 | 11.25 - 11.75 | 11.5 | 161 |
| 11.8 - 12.2 | 6 | 56 | 11.75 - 12.25 | 12 | 72 |
| 12.3 - 12.7 | 4 | 60 | 12.25 - 12.75 | 12.5 | 50 |
| | | | | | $\sum f x_i = 666$ |

Mode:

$$= l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 10.75 + \frac{(17-12)}{(17-12) + (17-14)} \times 0.5$$

$$= 10.75 + \frac{5}{(5+3)} \times 0.5$$

$$= 10.75 + 0.3125$$

Mode = 11.06 Short tons

Median:

$$= l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

$$\frac{n}{2} = \frac{60}{2} = 30, C = 19, L = 10.75, h = 0.5, f = 17$$

$$= 10.75 + \frac{0.5}{17} (30-19)$$

$$= 10.75 + \frac{0.5}{17} (11)$$

$$= 10.75 + 0.3235$$

Median = 11.07352

Median = 11.07 Short Tons

Mean:

$$= \frac{\sum f x_i}{\sum f}$$

$$= \frac{666.0}{60}$$

Mean = 11.10 Short tons

Question #3.47

The following is the distribution of Wages per thousand employees in a certain factory.

| | | | | | | | | | | | | |
|------------------------|----|----|----|-----|-----|-----|-----|-----|----|----|----|----|
| Daily Wages (Rs) | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 |
| Number of employees | 3 | 13 | 43 | 102 | 175 | 220 | 204 | 139 | 69 | 25 | 6 | 1 |

Calculate the Modal and Median wages and explain why there is a difference between the two.

Solution:

| Daily Wages | Number of Employees | C.F | C.B |
|-------------|---------------------|------|-------|
| 22 | 3 | 3 | 21-23 |
| 24 | 13 | 16 | 23-25 |
| 26 | 43 | 59 | 25-27 |
| 28 | 102 | 161 | 27-29 |
| 30 | 175 | 336 | 29-31 |
| 32 | 220 | 556 | 31-33 |
| 34 | 204 | 760 | 33-35 |
| 36 | 139 | 899 | 35-37 |
| 38 | 69 | 960 | 37-39 |
| 40 | 25 | 993 | 39-41 |
| 42 | 6 | 999 | 41-43 |
| 44 | 1 | 1000 | 43-45 |

Mode:

$$= l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 31 + \frac{220 - 175}{(220 - 175) + (220 - 204)} \times 2$$

$$= 31 + \frac{45}{45+16} \times 2$$

$$= 31 + \frac{45}{61} \times 2$$

$$= 31 + 0.7377.0 \times 2$$

Mode = 32.48 Rs

Median:-

$$= l + \frac{h}{F} \left(\frac{n}{2} - c \right)$$

$$\frac{n}{2} = \frac{1000}{2} \Rightarrow 500$$

$$C = 336, F = 220, l = 31, h = 2$$

$$= 31 + \frac{2}{220} (500 - 336)$$

$$= 31 + \frac{2}{220} (164)$$

$$= 31 + 1.49090$$

Median = 32.49 Rs