Lecture # 33

1:- There are 15 girls students and 25 boys Students in a class. How many students are there in total?

Solution

Let G is the no of girl students and B be the set of boy students

Then

$n(G) = 15$

$n(B) = 25$

and $n(G \cup B) = ?$

Since set of girls and boys are disjoint here total number of students are

$$n(G \cup B) = n(G) + n(B)$$

$$15 + 25$$

$\because n(A \cup B) = n(A) + n(B)$ $\quad 40$

(2) Among 200 people, 150 either swing or Jog or both If 85 swim and 60 swim and Jog. How many Jog?

let U is the set of total no of people considered S be the set of people who swim. and J be the set of people who Jog.

$n(U) = 200$
$n(S) = 85$
$n(S \cap J) = 60$
$n(S \cup J) = 150$
$n(J) = ?$

By inclusive and exclusive Principle

$$n(S \cup J) = n(S) + (n(J)) - n(S \cap J)$$
$$150 = 85 + n(J) - 60$$
$$n(J) = 150 - 85 + 60$$
$$= 125$$

(3) Let A and B are subset of U within $(U) = 100$
$n(A) = 50$, $n(B) = 60$ and $n(A \cup B)' = 20$
find $n(A \cap B)$

Since $(A \cup B)' = U / (A \cup B)$

$$n(A \cup B') = n(U) - n(A \cup B)$$
$$20 = 100 - n(A \cup B)$$
$$n(A \cup B) = 80$$

Now by inclusive and exclusive principle

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$80 = 50 + 60 - n(A \cap B)$$
$$n(A \cap B) = 50 + 60 - 80$$
$$n(A \cap B) = 30$$

(4) Fifty people are interviewed about their food preferences. 20 of them like Chineese food 32 like fast food. 12 like neither chineese nor fast food. How many like chinese but not fast food

$n(U)=50$ $\quad$ $n(C)=20$

$n(F)=32$ $\quad$ $n((C\cup F)')=12$

U is total no of people interviewed = $n(C)$ is name of people like Chineese. $n(F)$ no of people like fast food.

find $n(C\cup F') = n(C/F)$

Since

$$n((C\cup F')) = n(0) - n(C\cup F)$$

$$12 = 150 - n(C\cup F)$$

$$n(C\cup F) = 50-12$$

$$= 38$$

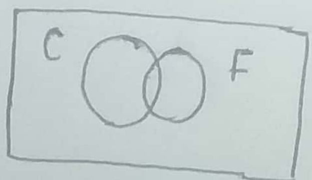
Next $n(C\cup F) = n(C/F) + n(F)$

$$38 = n(C/F) + 32$$

$$n(C/F) = 38-32$$

$$6$$

C/F



5: Let A and B be subset of U with $n(A)=10$, $n(B)=15$, $n(A')=12$ and $n(A \cap B)=8$. find $n(A \cup B)$

$A \cup B' = U \setminus (B \setminus A)$

$n(A \cup B') = n(U \setminus B \setminus A)$

$n(U) - n(B \setminus A)) \longrightarrow (i)$

Now $U = A \cup A'$ where A & A' are disjoint sets

$n(U) = n(A) + n(A')$

$10 + 12$

$n(U) = 22$

Also

$n(B \setminus A) = n(B) - n(A \cap B)$

$15 - 8$

7

Substituting values in i) we get

$n(A \cup B') = n(U) - n(B \setminus A)$

$22 - 7$

15 Ans

Lecture #34

(a) How many integers from 1 through 1000 are multiple of 3 or multiplies of 5.

Solve:

$n(T) = \{1, 2, 3 \cdots 1000\}$

A = divisible by 3

B = divisible by 5

$A = \{3, 6, 9 \cdots 999\}$

$B = \{5, 10, 15 \cdots 1000$

$n(A) = 333$

$n(B) = 200$

$n(A \cap B) = 66$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$333 + 200 - 60$

467

(b) How many integers from 1 through 1000 are neither multiplies of nor multiplies of 5?

$n(A \cup B)' = n(U) - n(A \cup B)$

$1000 - 467$

533 Ans

(c) How many integers from first 100 integer which is divisible by 6 or 8

$n(T) = \{1, 2, 3, 4 \ldots 100\}$

A = divisible by 6

B = divisible by 8

$A = \{6, 12, 18 \ldots 96\}$

$n(A) = 16$

$B = \{8, 16, 24 \ldots 96\}$

$n(B) = 12 \rightarrow (A \cap B) = \{24, 48, 72, 96\}$

$(A \cup B) = n(A) + n(B) - n(A \cap B)$

$16 + 12 - 4$

24 Answer

Example: 3

Solve = The set of possible remainder that can be obtained when an integer is divided by 6 is $\{0, 1, 2, 3, 4, 5\}$ This set has six elements. Thus by pigeonhole Principle $7 = 6 + 1$ integer are each divided by 6 at least two of them must have the same remainder

Example: 4

How many integer through 100. must you pick in order to be sure of getting one that is divisible by 5

Solve:

$n(J) = \{1 \dots 100\}$

There are 20 integer from 1 through 100 divisible by 5. Hence there are eighty integers are not divisible by 5

Example:-

What is the minimum number of students in a class to be sure that two of them are born in same month

Solve:

There are 12 months in a year. The Pigeonhole principle shows that among any

$13 = n + 1$

$13 = 12 + 1$

$13 = 13$

Or more students are must be at least two students who are born in the same month

Let A = {1, 2...10}. Suppose six integer are chosen from A. must be two integers whose sum is 11

Solve

The set A can be partitioned into five Subset {1, 10}, {2, 9}, {4, 7}, {3, 8} and {5, 6}

each consisting of two integer whose sum is 11. These 5 subsets can be considered as 5, pigeonholes if 6 = 5+1 integer can be selected from A. Then B the pigeonhole at least two must be from one of the 5 subsets. But then the sum of these two integer is 11