Linear Algebra

→ Matrix

1: Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & -5 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix}$ and

$C = \begin{bmatrix} 7 & 3 & 2 \\ -4 & 3 & 5 \\ 6 & 1 & -1 \end{bmatrix}$

(a) What is a_{12}, a_{22}, a_{23}

$a_{12} = 3$

$a_{22} = -5$

$a_{23} = 4$

(b) What is b_{11}, b_{31}

$b_{11} = 4$

$b_{31} = 5$

(c) What is c_{13}, c_{31}, c_{33}

$c_{13} = 2$

$c_{31} = 6$

$c_{33} = -1$

→ Question #02

if $\begin{bmatrix} a+b & c+d \\ c-d & a-b \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix}$

$a+b = 4 \longrightarrow (i)$

$a-b = 2 \longrightarrow (ii)$

$c+d = 6 \longrightarrow (iii)$

$c-d = 10 \longrightarrow (iv)$

from (i) & (ii) then add

$$\begin{array}{r} a+b = 4 \\ a-b = 2 \\ \hline 2a = 6 \end{array}$$

$$a = \frac{6}{2}$$

$$\boxed{a = 3}$$

Put in (i)

$3+b = 4$

$b = 4-3$

$$\boxed{b = 1}$$

from (iii) & (iv) then add

$c + d = 6$

$c - d = 10$

$2c = 16$

$c = \frac{16}{2}$

$\boxed{c = 8}$

Put the value of c in eq(iii)

$8 + d = 6$

$d = 6 - 8$

$\boxed{d = -2}$

$a = 3, b = 1, c = 8, d = -2$

Question NO #03

$$\text{if } \begin{bmatrix} a+2b & 2a-b \\ 2c+d & c-2d \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -3 \end{bmatrix}$$

$a + 2b = 4 \rightarrow (i)$

$2a - b = -2 \rightarrow (ii)$

$2c + d = 4 \rightarrow (iii)$

$c - 2d = -3 \rightarrow (iv)$

from (i) and (ii) then add

$a + 2b = 4 \rightarrow (1)$

$2a - b = -2 \rightarrow (2)$

Multiply eq (1) with 2

$2(a + 2b) = 4$

$2a + 4b = 8$

$2a - b = -2$

$3b = 10$

$5b = 10$

$\boxed{b = 2}$

Put in (1)

$a + 2(2) = 4$

$a + 4 = 4$

$a = 4 - 4$

$\boxed{a = 0}$

From (3) & (4)

$2c + d = 4 \rightarrow (3)$

$c - 2d = -3 \rightarrow (4)$

$2(2c + d) = 4$

$4c + 2d = 8$

$$4c + 2d = 8$$
$$c - 2d = -3$$
$$5c = 5$$
$$c = 1$$

Put in (3)

$$2(1) + d = 4$$
$$2 + d = 4$$
$$d = 4 - 2$$
$$\boxed{d = 2}$$

$a = 0,\ b = 2,\ c = 1, d = 2$

→ Question #04

Compute the indicate linear combination

C+E and E+C

$$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, E \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

$$C+E = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C+E = \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}$$

$$E+C = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}$$

⇒ linear combination

(b) A+B

$$A+B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

Impossible there is not linear combination because order of matrices are not same

(c) D-F

$$D-F=\begin{bmatrix}3 & -2\\ 2 & 4\end{bmatrix}-\begin{bmatrix}-4 & 5\\ 2 & 3\end{bmatrix}$$

$$D-F=\begin{bmatrix}7 & -7\\ 0 & 1\end{bmatrix}$$

(d) $-3C+5O$

$$-3\begin{bmatrix}3 & -1 & 3\\ 4 & 1 & 5\\ 2 & 1 & 3\end{bmatrix}+5\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}-9 & 3 & -9\\ -12 & -3 & -15\\ -6 & -3 & 9\end{bmatrix}$$

linear combination

(e) $2C - 3E$

$$2\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} - 3\begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -2 & 6 \\ 8 & 2 & 10 \\ 4 & 2 & 6 \end{bmatrix} - \begin{bmatrix} 6 & -12 & 15 \\ 0 & 3 & 12 \\ 9 & 6 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 10 & -9 \\ 8 & -1 & -2 \\ -5 & -4 & 3 \end{bmatrix}$$

linear combination

$2B + F$

$$2\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

Impossible: There is not linear combination because order of matrices are not same

Question # 05

(i) $3D + 2F$

$$3\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} + 2\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -6 \\ 6 & 12 \end{bmatrix} + \begin{bmatrix} -8 & 10 \\ 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 10 & 18 \end{bmatrix} \Rightarrow \text{linear combination}$$

(ii) $3A + 2A$ and $5A$

$$3\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 & 9 \\ 6 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 10 & 15 \\ 10 & 5 & 20 \end{bmatrix}$$

$$5A = \begin{bmatrix} 5 & 10 & 15 \\ 10 & 5 & 20 \end{bmatrix}$$

(b) 3(2A) and 6A

$$3\left[2\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\end{bmatrix}\right]$$

$$3\begin{bmatrix}2 & 4 & 6\\ 4 & 2 & 8\end{bmatrix}$$

$$6A=\begin{bmatrix}6 & 12 & 18\\ 12 & 6 & 24\end{bmatrix}$$ linear combination

(c) 2(D+F) and 2D+2F

$$2\left[\begin{bmatrix}3 & -2\\ 2 & 4\end{bmatrix}+\begin{bmatrix}-4 & 5\\ 2 & 3\end{bmatrix}\right]$$

$$2\begin{bmatrix}-1 & 3\\ 4 & 7\end{bmatrix}$$

$$2D+2F=\begin{bmatrix}-2 & 6\\ 8 & 14\end{bmatrix}$$ = linear combination

(e) $(2+3)D$ and $2D+3D$

$(2+3)D$

$(5)D$

$5D$

$$5\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 15 & -10 \\ 10 & 20 \end{bmatrix} \Rightarrow \text{linear combination}$$

$$2D = 2\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -4 \\ 4 & 8 \end{bmatrix}$$

$$3D = 3\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -6 \\ 6 & 12 \end{bmatrix}$$

$$3D+2D = \begin{bmatrix} 15 & -10 \\ 10 & 20 \end{bmatrix}$$

(f) $3(B+D)$

$B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$

Not linear combination

6 If possible, compute

(a) A^t and $(A^t)^t$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$

$A^t = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$

$(A^t)^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$

(b) $(C+E)^T$ and $C^T + E^T$

$C+E = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

$$\begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}$$

$$(C+E)^T = \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}$$

$$C^T + E^T = \begin{bmatrix} 3 & 4 & 2 \\ -1 & 1 & 1 \\ 3 & 5 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 3 \\ -4 & 1 & 2 \\ 5 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}$$

(c) $(2D+3F)^T$

$$2\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} + 3\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 4 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} -12 & 15 \\ 6 & 9 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 11 \\ 10 & 17 \end{bmatrix} = (2D+3F)^T = \begin{bmatrix} -6 & 10 \\ 11 & 17 \end{bmatrix}$$

d $D - D^T$

$D^T = \begin{bmatrix} 3 & 2 \\ -2 & 4 \end{bmatrix}$

$D - D^t = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -2 & 4 \end{bmatrix}$

$\begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$

(e) $2A^T + B$

$2\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$

$\begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 6 & 8 \end{bmatrix}$

$2A^T + B = \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$

$2A^t + B = \begin{bmatrix} 3 & 4 \\ 6 & 3 \\ 9 & 10 \end{bmatrix}$

$(3D-2F)^T$

$$3\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} - 2\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 6 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} 8 & 10 \\ 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 17 & -16 \\ 2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 17 & 2 \\ -16 & 6 \end{bmatrix}$$

Q # 07

(a) $(2A)^T$

$$2A = 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$$

$$(2A)^T = \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 6 & 8 \end{bmatrix}$$

$(A-B)^T$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 2 & 4 \end{bmatrix}$

NOT possible because order of matrices are not same

$(3B^T - 2A)^T$

$3\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \end{bmatrix}$

$3B - 2A = \begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$

$\begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 3 \\ -4 & 1 & -2 \end{bmatrix}$

$(3B-2A)^t = \begin{bmatrix} 1 & -4 \\ 2 & 1 \\ 3 & -2 \end{bmatrix}$

(d) $(3A^T - 5B^T)^T$

$$3\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 \\ 6 & 3 \\ 9 & 12 \end{bmatrix}$$

$$5B^t = 5\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 10 & 15 \\ 0 & 5 & 10 \end{bmatrix}$$

Not possible

(e) $(-A^T)$ and $(-A^T)^t$

$$-\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}^T$$

$$\begin{bmatrix} -1 & -2 & -3 \\ -2 & -1 & -4 \end{bmatrix}^T$$

$$\begin{bmatrix} -1 & -2 \\ -2 & -1 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -2 & -1 \\ -3 & -4 \end{bmatrix}$$

(d) $(3A^T - 5B^T)^T$

$$3\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 \\ 6 & 3 \\ 9 & 12 \end{bmatrix}$$

$$5B^t = 5\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 10 & 15 \\ 0 & 5 & 10 \end{bmatrix}$$

Not possible

(e) $(-A^T)$ and $(-A^T)^t$

$$-\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}^T$$

$$\begin{bmatrix} -1 & -2 & -3 \\ -2 & -1 & -4 \end{bmatrix}^T$$

$$\begin{bmatrix} -1 & -2 \\ -2 & -1 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -2 & -1 \\ -3 & -4 \end{bmatrix}$$

$(C+E+F)^{T}$

$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$, $E \begin{bmatrix} 2 & -4 & 0 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$,

$F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$ Not possible

Question #08

Is the matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ a linear combination of matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$? justify your answer.

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 1\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ a linear combination