Lecture 33

Q1 There are 15 girls students and 25 boys Students in a Class. How many Students are there in total?

Solution:

Let G is the no of girl Students and B be the set of boy Students.

Then

$n(G) = 15;$

$n(B) = 25$

and $n(G \cup B) = ?$

since set of girls and boys are disjoint here total number of students are

$$n(G \cup B) = n(G) + n(B)$$
$$= 15 + 25$$
$$= 40$$

$\because n(A \cup B) = n(A) + n(B)$

Q2 Among 200 People, 150 either Swim or Jog or both. If 85 Swim and 60 swim and Jog how many Jog?

let U is the Set of total no of People Considered S be the Set of People Who Swim and J be Set of People who Jog.

$n(U) = 200$
$n(S) = 85$
$n(S \cap J) = 60$
$n(S \cup J) = 150$
$n(J) =$

By inclusive and exclusive Principle

$n(S \cup J) = n(S) + n(J) - n(S \cap J)$

$150 = 85 + n(J) - 60$

$n(J) = 150 - 85 + 60$

$= 125$

Q3 Let A and B are Subset of U with $n(U) = 100$
$n(A) = 50$ $n(B) = 60$ and $n((A \cup B)') = 20$
Fin $n(A \cap B)$

Since $(A \cup B)' = U / (A \cup B)$
$n((A \cup B)') = n(U) - n(A \cup B)$
$20 = 100 - n(A \cup B)$

$n(A \cup B) = 80$

Now by inclusive and exclusive Principle

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$80 = 50 + 60 - n(A \cap B)$$
$$n(A \cap B) = 50 + 60 - 80 =$$
$$n(A \cap B) = 30$$

24 Fifty People are interviwed about their food Preferences. 20 of them like Chinese food. 32 like fast food, and 12 Like neither chinese nor fast food. How many like chines but not fast food

$n(U) = 50$ $\quad$ $n(C) = 20$

$n(F) = 32,$ $\quad$ $n((C \cup F)') = 12$

U is total no of people interviwed. n(c) is no of people like Chinese. n(F) no of People like fast food.

Find $n(C \cap F') = n(C/F)$

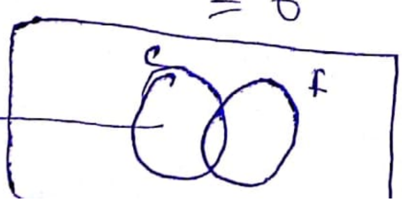
Since $n((C \cup F)') = n(U) - n(C \cup F)$

$$12 = 150 - n(C \cup F)$$
$$n(C \cup F) = 50 - 12$$
$$= 38$$

Next $n(C \cup F) = n(C/F) + n(F)$

$$38 = n(C/F) + 32$$
$$n(C/F) = 38 - 32$$
$$= 6$$

C/F



Q5 Let A and B be subset of U with $n(A) = 10$, $n(B) = 1$
$n(A') = 12$ and $n(A \cap B) = 8$. Find $n(A \cup B)$

$A \cup B' = U / (B/A)$

$n(A \cup B') = n(U / B/A))$

$= n(U) - n(B/A)$ — (i)

Now $U = A \cup A'$ where A & A' are disjoint sets

$n(U) = n(A) + n(A')$

$= 10 + 12$

$n(U) = 22$

Also

$n(B/A) = n(B) - n(A \cap B)$

$= 15 - 8$

$= 7$

Subsituting values in (i) we get

$n(A \cup B') = n(U) - n(B/A)$

$= 22 - 7$

$= 15$ Ans

Lecture 34

Q1 (a) How many integers from 1 through 1000 are multiple of 3 or multiple of 5?

(b) How many integers from 1 through 1000 are neither a multiple of 3 nor multiple of 5?

a) $n(T) = \{1, 2, \ldots\ldots 1000\}$

$n(A) \overset{\text{Divisible by 3}}{=} \{3, 6, 9, \ldots\ldots\ldots 999\}$

$n(A) = 333$

$n(B) \overset{\text{Divisible by 5}}{=} \{5, 10, 15, \ldots\ldots\ldots 1000\}$

$n(B) = 200$

$n(A \cap B) = 66$

$\because (A \cap B)$ = is element which are common in both multiple of 3 and 5

$\therefore n(A \cap B) = \left[\frac{1000}{15}\right] = 66.$

Hence by inclusive - exclusive Principle

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 333 + 200 - 66$

$= 467.$

b) The set $(A \cup B)$ are either ~~divisible~~ multiple of 5 or 3.

Now

$$n((A \cup B)') = n(U) - n(A \cup B)$$

$$= 1000 - 467$$

$$= 533$$

$\because n(U) = 1000$

$\because n(A \cup B) = 467$

2. How many integers from first 100 integers which is divisble by 6 or 8.

$n(T) = \{1, 2, 3 \dots\dots 100\}$

A = divisible by 6

B = divisible by 8

$A = \{6, 12, 18, 24 \dots 96\}$

$B = \{8, 16, 24, \dots\dots 96\}$

$n(B) = 12$

$n(A) = 16$

$n(A \cap B) = 4$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 16 + 12 - 4$$

$$= 28 - 4$$

$$= 24$$

Q3 What is the minimum number of Students in a class to be sure that two of them are born in the Same Month.

There are $n=12$ month in a year. The Pigeon Principle Shows that among any $13 (=n+1)$ or more Studerd there must be at least two Students Who are born in Same month.

$$13 = 12+1$$
$$= 13$$

Q4 Given any Set of seven integers must there be two that have the same remainder when divided by 6?

The set of Possible remainders that Can be obtained when an integer is divided By Six is $\{0, 1, 2, 3, 4, 5\}$.

The Set has 6 elements. By Peigonhole Principle

$$\cancel{N} = k+1$$
$$7 = 6+1$$
$$\cancel{7=7}$$

integers are each divided by six, then at least two of them have Same remainder.

QNo5) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

The Total no of digit divisible by 5 from 1 through 100 is 20. Hence 80 digits are not divisble by 5

$n(T) = 100$

$n(D) = 20$

~~n(B) = 80~~

$n(B) = 100 - 20$

$= 80$

Thus by Pigeon hole Principle $81 = 80 + 1$ integers from 1 through 100 must be Picked in order to be Sure of getting ~~out~~ one that is divisibly by 5.

QNo6) Let $A = \{1, 2, 3 \ldots 10\}$. Suppose six integers are chosen from A. Must there be two integers whose sum is 11.

$A = \{1, 2, 3, \ldots 10\}$

Set A Can be Partioned into five Subsets

$\{1, 10\}$, $\{2, 9\}$, $\{3, 8\}$, $\{4, 7\}$ and $\{5, 6\}$

each Consisting of two integers whose Sum is 11

The 5 Subsets are Considerd as 5 Pigeon holes

If $6 = 5+1$ integers selected from A, then by Pigeonhole Principle at least two must be from one of the five subsets. But then the sum of these two integers is 11.

Q7 Compute $\lfloor x \rfloor$ and $\lceil x \rceil$ for each of the following values of x

a) $25/4$

$\lfloor 25/4 \rfloor = \lfloor 6 + \frac{1}{4} \rfloor = 6$

$\lceil 25/4 \rceil = \lceil 6 + \frac{1}{4} \rceil = 7$

b) $\lfloor 0.999 \rfloor = \lfloor 0 + 0.999 \rfloor = 0$

$\lceil 0.999 \rceil = \lceil 0 + 0.999 \rceil = 1$

c) $\lfloor -2.01 \rfloor = \lfloor -3 + 0.99 \rfloor = -3$

$\lceil -2.01 \rceil = \lceil -3 + 0.99 \rceil = -2$

Q8 What is the smallest integer N such that

a) $\lceil N/7 \rceil = 5$

$N = 7 \cdot (5-1) + 1$

$= 7 \cdot 4 + 1$

$= 28 + 1$

$= 29$

b) $\lceil N/9 \rceil = 6$

$N = 9 \cdot (6-1) + 1$

$= 9 \cdot 5 + 1$

$= 45 + 1$

$= 46$