LAB ASSIGNMENT 02

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Artificial Intelligence

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1. Short Questions

1.1 Informed and Uninformed Searches

1.1.1 Informed Search

Informed search algorithm contains an array of knowledge such as how far we are from the goal, path cost, how to reach to goal node, etc. This knowledge help agents to explore less to the search space and find more efficiently the goal node. The informed search algorithm is more useful for large search space. Informed search algorithm uses the idea of heuristic, so it is also called Heuristic search.

Heuristics function: Heuristic is a function which is used in Informed Search, and it finds the most promising path. It takes the current state of the agent as its input and produces the estimation of how close agent is from the goal. The heuristic method, however, might not always give the best solution, but it guaranteed to find a good solution in reasonable time. Heuristic function estimates how close a state is to the goal. It is represented by h(n), and it calculates the cost of an optimal path between the pair of states. The value of the heuristic function is always positive.

1.1.1.1 Types

1. Pure Heuristic Search
2. Best-first Search Algorithm (Greedy Search)
3. A* Search Algorithm

1.1.2 Uninformed Search

Uninformed search is a class of general-purpose search algorithms which operates in brute force-way. Uninformed search algorithms do not have additional information about state or search space other than how to traverse the tree, so it is also called blind search.

1.1.2.1 Types

1. Breadth-first Search
2. Depth-first Search
3. Depth-limited Search
4. Iterative deepening depth-first search
5. Uniform cost search
6. Bidirectional Search

1.2 Uniform-Cost Search (Dijkstra for large Graphs)

Uniform-Cost Search is a variant of Dijikstra's algorithm. Here, instead of inserting all vertices into a priority queue, we insert only source, then one by one insert when needed. In every step, we check if the item is already in priority queue (using visited array). If yes, we perform decrease key, else we insert it. This variant of Dijsktra is useful for infinite graphs and those graph which are too large to represent in the memory. Uniform-Cost Search is mainly used in Artificial Intelligence.

1.3 Bidirectional Search Algorithm

Bidirectional search is a graph search algorithm that finds a shortest path from an initial vertex to a goal vertex in a directed graph. It runs two simultaneous searches: one forward from the initial state, and one backward from the goal, stopping when the two meet.

1.4 Difference between graph and tree traversal

1. In a tree there exist only one path between any two vertices whereas a graph can have unidirectional and bidirectional paths between the nodes.
2. In the tree, there is exactly one root node, and every child can have only one parent. As against, in a graph, there is no concept of the root node.
3. A tree cannot have loops and self-loops while graph can have loops and self-loops.
4. Graphs are more complicated as it can have loops and self-loops. In contrast, trees are simple as compared to the graph.
5. The tree is traversed using pre-order, in-order and post-order techniques. On the other hand, for graph traversal, we use BFS (Breadth First Search) and DFS (Depth First Search).
6. A tree can have n-1 edges. On the contrary, in the graph, there is no predefined number of edges, and it depends on the graph.
7. A tree has a hierarchical structure whereas graph has a network model.

1.5 Heuristic Search

A heuristic function, also called simply a heuristic, is a function that ranks alternatives in search algorithms at each branching step based on available information to decide which branch to follow. For example, it may approximate the exact solution.

Heuristics function: Heuristic is a function which is used in Informed Search, and it finds the most promising path. It takes the current state of the agent as its input and produces the estimation of how close agent is from the goal. The heuristic method, however, might not always give the best solution, but it guaranteed to find a good solution in reasonable time. Heuristic function estimates how close a state is to the goal. It is represented by h(n), and it calculates the cost of an optimal path between the pair of states. The value of the heuristic function is always positive.

Admissibility of the heuristic function is given as:

$$h(n) <= h^*(n)$$

Here h(n) is heuristic cost, and h*(n) is the estimated cost. Hence heuristic cost should be less than or equal to the estimated cost.

1.6 Adversarial Search

Adversarial search is search when there is an "enemy" or "opponent" changing the state of the problem every step in a direction you do not want. Examples: Chess, business, trading, war. You change state, but then you don't control the next state. Opponent will change the next state in a way: unpredictable.

2. Codes

2.1 Depth-Limited Search

```python
# Python program to print DFS traversal from a given
# given graph
from collections import defaultdict

# This class represents a directed graph using adjacency
# list representation
class Graph:

    def __init__(self,vertices):

        # No. of vertices
        self.V = vertices

        # default dictionary to store graph
        self.graph = defaultdict(list)

    # function to add an edge to graph
    def addEdge(self,u,v):
        self.graph[u].append(v)

    # A function to perform a Depth-Limited search
    # from given source 'src'
    def DLS(self,src,target,maxDepth):

        if src == target : return True
        # If reached the maximum depth, stop recursing.
        if maxDepth <= 0 : return False

        # Recur for all the vertices adjacent to this vertex
        for i in self.graph[src]:
                if(self.DLS(i,target,maxDepth-1)):
                    return True
        return False

    # IDDFS to search if target is reachable from v.
    # It uses recursive DLS()
    def IDDFS(self,src, target, maxDepth):

        # Repeatedly depth-limit search till the
        # maximum depth
        for i in range(maxDepth):
            if (self.DLS(src, target, i)):
                return True
        return False

# Create a graph given in the above diagram
g = Graph (7);
g.addEdge(0, 1)
g.addEdge(0, 2)
g.addEdge(1, 3)
g.addEdge(1, 4)
g.addEdge(2, 5)
g.addEdge(2, 6)

target = 6; maxDepth = 3; src = 0

if g.IDDFS(src, target, maxDepth) == True:
    print ("Target is reachable from source " +
        "within max depth")
else :
    print ("Target is NOT reachable from source " +
        "within max depth")
```

2.1.1 Console Output

```
Target is reachable from source within max depth
```

2.2 Iterative deepening search

```
// C++ program to search if a target node is reachable from
// a source with given max depth.
#include<bits/stdc++.h>
using namespace std;

// Graph class represents a directed graph using adjacency
// list representation.
class Graph
{
    int V;    // No. of vertices

    // Pointer to an array containing
    // adjacency lists
    list<int> *adj;

    // A function used by IDDFS
    bool DLS(int v, int target, int limit);

public:
    Graph(int V);   // Constructor
    void addEdge(int v, int w);

    // IDDFS traversal of the vertices reachable from v
    bool IDDFS(int v, int target, int max_depth);
};

Graph::Graph(int V)
{
    this->V = V;
    adj = new list<int>[V];
}

void Graph::addEdge(int v, int w)
{
    adj[v].push_back(w); // Add w to v’s list.
}

// A function to perform a Depth-Limited search
// from given source 'src'
bool Graph::DLS(int src, int target, int limit)
{
    if (src == target)
        return true;

    // If reached the maximum depth, stop recursing.
    if (limit <= 0)
        return false;

    // Recur for all the vertices adjacent to source vertex
    for (auto i = adj[src].begin(); i != adj[src].end(); ++i)
       if (DLS(*i, target, limit-1) == true)
          return true;

     return false;
```

```cpp
}

// IDDFS to search if target is reachable from v.
// It uses recursive DFSUtil().
bool Graph::IDDFS(int src, int target, int max_depth)
{
    // Repeatedly depth-limit search till the
    // maximum depth.
    for (int i = 0; i <= max_depth; i++)
       if (DLS(src, target, i) == true)
          return true;

    return false;
}

// Driver code
int main()
{
    // Let us create a Directed graph with 7 nodes
    Graph g(7);
    g.addEdge(0, 1);
    g.addEdge(0, 2);
    g.addEdge(1, 3);
    g.addEdge(1, 4);
    g.addEdge(2, 5);
    g.addEdge(2, 6);

    int target = 6, maxDepth = 3, src = 0;
    if (g.IDDFS(src, target, maxDepth) == true)
        cout << "Target is reachable from source "
                "within max depth";
    else
        cout << "Target is NOT reachable from source "
                "within max depth";
    return 0;
}
```

2.2.1 Console Output

Target is reachable from source within max depth

2.3 A-Star Search Algorithm

```python
# This class represent a graph
class Graph:
    # Initialize the class
    def __init__(self, graph_dict=None, directed=True):
        self.graph_dict = graph_dict or {}
        self.directed = directed
        if not directed:
            self.make_undirected()
    # Create an undirected graph by adding symmetric edges
    def make_undirected(self):
        for a in list(self.graph_dict.keys()):
            for (b, dist) in self.graph_dict[a].items():
                self.graph_dict.setdefault(b, {})[a] = dist
    # Add a link from A and B of given distance, and also add the inverse link
 if the graph is undirected
    def connect(self, A, B, distance=1):
        self.graph_dict.setdefault(A, {})[B] = distance
        if not self.directed:
            self.graph_dict.setdefault(B, {})[A] = distance
    # Get neighbors or a neighbor
    def get(self, a, b=None):
        links = self.graph_dict.setdefault(a, {})
        if b is None:
            return links
        else:
            return links.get(b)
    # Return a list of nodes in the graph
    def nodes(self):
        s1 = set([k for k in self.graph_dict.keys()])
        s2 = set([k2 for v in self.graph_dict.values() for k2, v2 in v.items()
])
        nodes = s1.union(s2)
        return list(nodes)
# This class represent a node
class Node:
    # Initialize the class
    def __init__(self, name:str, parent:str):
        self.name = name
        self.parent = parent
        self.g = 0 # Distance to start node
        self.h = 0 # Distance to goal node
        self.f = 0 # Total cost
    # Compare nodes
    def __eq__(self, other):
        return self.name == other.name
    # Sort nodes
    def __lt__(self, other):
         return self.f < other.f
    # Print node
    def __repr__(self):
        return ('({0},{1})'.format(self.name, self.f))
# A* search
def astar_search(graph, heuristics, start, end):

    # Create lists for open nodes and closed nodes
    open = []
    closed = []
    # Create a start node and an goal node
    start_node = Node(start, None)
    goal_node = Node(end, None)
    # Add the start node
    open.append(start_node)

    # Loop until the open list is empty
    while len(open) > 0:
        # Sort the open list to get the node with the lowest cost first
        open.sort()
        # Get the node with the lowest cost
        current_node = open.pop(0)
        # Add the current node to the closed list
        closed.append(current_node)

        # Check if we have reached the goal, return the path
        if current_node == goal_node:
            path = []
            while current_node != start_node:
                path.append(current_node.name + ': ' + str(current_node.g))
                current_node = current_node.parent
            path.append(start_node.name + ': ' + str(start_node.g))
            # Return reversed path
            return path[::-1]
        # Get neighbours
        neighbors = graph.get(current_node.name)
        # Loop neighbors
        for key, value in neighbors.items():
            # Create a neighbor node
            neighbor = Node(key, current_node)
            # Check if the neighbor is in the closed list
            if(neighbor in closed):
                continue
            # Calculate full path cost
            neighbor.g = current_node.g + graph.get(current_node.name, neighbo
r.name)
            neighbor.h = heuristics.get(neighbor.name)
            neighbor.f = neighbor.g + neighbor.h
            # Check if neighbor is in open list and if it has a lower f value
            if(add_to_open(open, neighbor) == True):
                # Everything is green, add neighbor to open list
                open.append(neighbor)
    # Return None, no path is found
    return None
# Check if a neighbor should be added to open list
def add_to_open(open, neighbor):
    for node in open:
        if (neighbor == node and neighbor.f > node.f):
            return False
    return True
# The main entry point for this module
def main():
    # Create a graph
    graph = Graph()
    # Create graph connections (Actual distance)
    graph.connect('Frankfurt', 'Wurzburg', 111)
    graph.connect('Frankfurt', 'Mannheim', 85)
    graph.connect('Wurzburg', 'Nurnberg', 104)
    graph.connect('Wurzburg', 'Stuttgart', 140)
    graph.connect('Wurzburg', 'Ulm', 183)
    graph.connect('Mannheim', 'Nurnberg', 230)
    graph.connect('Mannheim', 'Karlsruhe', 67)
    graph.connect('Karlsruhe', 'Basel', 191)
    graph.connect('Karlsruhe', 'Stuttgart', 64)
    graph.connect('Nurnberg', 'Ulm', 171)
    graph.connect('Nurnberg', 'Munchen', 170)
    graph.connect('Nurnberg', 'Passau', 220)
    graph.connect('Stuttgart', 'Ulm', 107)
    graph.connect('Basel', 'Bern', 91)
    graph.connect('Basel', 'Zurich', 85)
    graph.connect('Bern', 'Zurich', 120)
    graph.connect('Zurich', 'Memmingen', 184)
    graph.connect('Memmingen', 'Ulm', 55)
    graph.connect('Memmingen', 'Munchen', 115)
    graph.connect('Munchen', 'Ulm', 123)
    graph.connect('Munchen', 'Passau', 189)
    graph.connect('Munchen', 'Rosenheim', 59)
    graph.connect('Rosenheim', 'Salzburg', 81)
    graph.connect('Passau', 'Linz', 102)
    graph.connect('Salzburg', 'Linz', 126)
    # Make graph undirected, create symmetric connections
    graph.make_undirected()
    # Create heuristics (straight-line distance, air-travel distance)
    heuristics = {}
    heuristics['Basel'] = 204
    heuristics['Bern'] = 247
    heuristics['Frankfurt'] = 215
    heuristics['Karlsruhe'] = 137
    heuristics['Linz'] = 318
    heuristics['Mannheim'] = 164
    heuristics['Munchen'] = 120
    heuristics['Memmingen'] = 47
    heuristics['Nurnberg'] = 132
    heuristics['Passau'] = 257
    heuristics['Rosenheim'] = 168
    heuristics['Stuttgart'] = 75
    heuristics['Salzburg'] = 236
    heuristics['Wurzburg'] = 153
    heuristics['Zurich'] = 157
    heuristics['Ulm'] = 0
    # Run the search algorithm
    path = astar_search(graph, heuristics, 'Frankfurt', 'Ulm')
    print(path)
    print()
# Tell python to run main method
if __name__ == "__main__": main()
```

2.3.1 Console Output

```
['Frankfurt: 0', 'Wurzburg: 111', 'Ulm: 294']
```

2.4 Genetic Algorithm

```
# Python3 program to create target string, starting from
# random string using Genetic Algorithm

import random

# Number of individuals in each generation
POPULATION_SIZE = 100

# Valid genes
GENES = '''abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOP
QRSTUVWXYZ 1234567890, .-;:_!"#%&/()=?@${[]}'''

# Target string to be generated
TARGET = "I love GeeksforGeeks"

class Individual(object):
    '''
    Class representing individual in population
    '''
    def __init__(self, chromosome):
        self.chromosome = chromosome
        self.fitness = self.cal_fitness()

    @classmethod
    def mutated_genes(self):
        '''
        create random genes for mutation
        '''
        global GENES
        gene = random.choice(GENES)
        return gene
```

```
@classmethod
def create_gnome(self):
    '''
    create chromosome or string of genes
    '''
    global TARGET
    gnome_len = len(TARGET)
    return [self.mutated_genes() for _ in range(gnome_len)]

def mate(self, par2):
    '''
    Perform mating and produce new offspring
    '''

    # chromosome for offspring
    child_chromosome = []
    for gp1, gp2 in zip(self.chromosome, par2.chromosome):

        # random probability
        prob = random.random()

        # if prob is less than 0.45, insert gene
        # from parent 1
        if prob < 0.45:
            child_chromosome.append(gp1)

        # if prob is between 0.45 and 0.90, insert
        # gene from parent 2
        elif prob < 0.90:
            child_chromosome.append(gp2)

        # otherwise insert random gene(mutate),
        # for maintaining diversity
        else:
            child_chromosome.append(self.mutated_genes())

    # create new Individual(offspring) using
    # generated chromosome for offspring
    return Individual(child_chromosome)

def cal_fitness(self):
    '''
    Calculate fittness score, it is the number of
    characters in string which differ from target
    string.
    '''
    global TARGET
```

```
        fitness = 0
        for gs, gt in zip(self.chromosome, TARGET):
            if gs != gt: fitness+= 1
        return fitness

# Driver code
def main():
    global POPULATION_SIZE

    #current generation
    generation = 1

    found = False
    population = []

    # create initial population
    for _ in range(POPULATION_SIZE):
                gnome = Individual.create_gnome()
                population.append(Individual(gnome))

    while not found:

        # sort the population in increasing order of fitness score
        population = sorted(population, key = lambda x:x.fitness)

        # if the individual having lowest fitness score ie.
        # 0 then we know that we have reached to the target
        # and break the loop
        if population[0].fitness <= 0:
            found = True
            break

        # Otherwise generate new offsprings for new generation
        new_generation = []

        # Perform Elitism, that mean 10% of fittest population
        # goes to the next generation
        s = int((10*POPULATION_SIZE)/100)
        new_generation.extend(population[:s])

        # From 50% of fittest population, Individuals
        # will mate to produce offspring
        s = int((90*POPULATION_SIZE)/100)
        for _ in range(s):
            parent1 = random.choice(population[:50])
            parent2 = random.choice(population[:50])
            child = parent1.mate(parent2)
            new_generation.append(child)
```

```python
        population = new_generation

        print("Generation: {}\tString: {}\tFitness: {}".\
              format(generation,
              "".join(population[0].chromosome),
              population[0].fitness))

        generation += 1


    print("Generation: {}\tString: {}\tFitness: {}".\
          format(generation,
          "".join(population[0].chromosome),
          population[0].fitness))

if __name__ == '__main__':
    main()
```

2.4.1 Console Output

```
Generation: 1    String: tO{"-?=jH[k8=B4]Oe@}    Fitness: 18

Generation: 2    String: tO{"-?=jH[k8=B4]Oe@}    Fitness: 18

Generation: 3    String: .#lRWf9k_Ifslw #O$k_    Fitness: 17

Generation: 4    String: .-1Rq?9mHqk3Wo]3rek_    Fitness: 16

Generation: 5    String: .-1Rq?9mHqk3Wo]3rek_    Fitness: 16

Generation: 6    String: A#ldW) #lIkslw cVek)    Fitness: 14

Generation: 7    String: A#ldW) #lIkslw cVek)    Fitness: 14

Generation: 8    String: (, o x _x%Rs=, 6Peek3   Fitness: 13

Generation: 29    String: I lope Geeks#o, Geeks   Fitness: 3

Generation: 30    String: I loMe GeeksfoBGeeks    Fitness: 2

Generation: 31    String: I love Geeksfo0Geeks    Fitness: 1

Generation: 32    String: I love Geeksfo0Geeks    Fitness: 1

Generation: 33    String: I love Geeksfo0Geeks    Fitness: 1

Generation: 34    String: I love GeeksforGeeks    Fitness: 0
```

2.5 Min-Max and Alpha-Beta pruning

```python
# Python3 program to demonstrate
# working of Alpha-Beta Pruning

# Initial values of Aplha and Beta
MAX, MIN = 1000, -1000

# Returns optimal value for current player
#(Initially called for root and maximizer)
def minimax(depth, nodeIndex, maximizingPlayer,
            values, alpha, beta):

    # Terminating condition. i.e
    # leaf node is reached
    if depth == 3:
        return values[nodeIndex]

    if maximizingPlayer:

        best = MIN

        # Recur for left and right children
        for i in range(0, 2):

            val = minimax(depth + 1, nodeIndex * 2 + i,
                          False, values, alpha, beta)
            best = max(best, val)
            alpha = max(alpha, best)

            # Alpha Beta Pruning
            if beta <= alpha:
                break

        return best

    else:
        best = MAX

        # Recur for left and
        # right children
        for i in range(0, 2):

            val = minimax(depth + 1, nodeIndex * 2 + i,
                            True, values, alpha, beta)
            best = min(best, val)
            beta = min(beta, best)

            # Alpha Beta Pruning
            if beta <= alpha:
                break

        return best

# Driver Code
if __name__ == "__main__":

    values = [3, 5, 6, 9, 1, 2, 0, -1]
    print("The optimal value is :", minimax(0, 0, True, values, MIN, MAX))
```

2.5.1 Console Output

The optimal value is: 5

2.6 Hill climbing algorithm

```
# hill climbing search of a one-dimensional objective function
from numpy import asarray
from numpy import arange
from numpy.random import randn
from numpy.random import rand
from numpy.random import seed
from matplotlib import pyplot

# objective function
def objective(x):
    return x[0]**2.0

# hill climbing local search algorithm
def hillclimbing(objective, bounds, n_iterations, step_size):
    # generate an initial point
    solution = bounds[:, 0] + rand(len(bounds)) * (bounds[:, 1] - bounds[:, 0]
)
    # evaluate the initial point
    solution_eval = objective(solution)
    # run the hill climb
    solutions = list()
    solutions.append(solution)
    for i in range(n_iterations):
        # take a step
        candidate = solution + randn(len(bounds)) * step_size
        # evaluate candidate point
        candidte_eval = objective(candidate)
        # check if we should keep the new point
        if candidte_eval <= solution_eval:
            # store the new point
            solution, solution_eval = candidate, candidte_eval
            # keep track of solutions
            solutions.append(solution)
            # report progress
            print('>%d f(%s) = %.5f' % (i, solution, solution_eval))
    return [solution, solution_eval, solutions]

# seed the pseudorandom number generator
seed(5)
# define range for input
bounds = asarray([[-5.0, 5.0]])
# define the total iterations
n_iterations = 1000
# define the maximum step size
step_size = 0.1
# perform the hill climbing search
best, score, solutions = hillclimbing(objective, bounds, n_iterations, step_si
ze)
print('Done!')
print('f(%s) = %f' % (best, score))
# sample input range uniformly at 0.1 increments
inputs = arange(bounds[0,0], bounds[0,1], 0.1)
# create a line plot of input vs result
pyplot.plot(inputs, [objective([x]) for x in inputs], '--')
# draw a vertical line at the optimal input
pyplot.axvline(x=[0.0], ls='--', color='red')
# plot the sample as black circles
pyplot.plot(solutions, [objective(x) for x in solutions], 'o', color='black')
pyplot.show()
```

2.6.1 Console Output

```
>1 f([-2.74290923]) = 7.52355

>3 f([-2.65873147]) = 7.06885

>4 f([-2.52197291]) = 6.36035

>5 f([-2.46450214]) = 6.07377

>7 f([-2.44740961]) = 5.98981

>9 f([-2.28364676]) = 5.21504

>12 f([-2.19245939]) = 4.80688

>14 f([-2.01001538]) = 4.04016

>15 f([-1.86425287]) = 3.47544

>22 f([-1.79913002]) = 3.23687

>24 f([-1.57525573]) = 2.48143
```

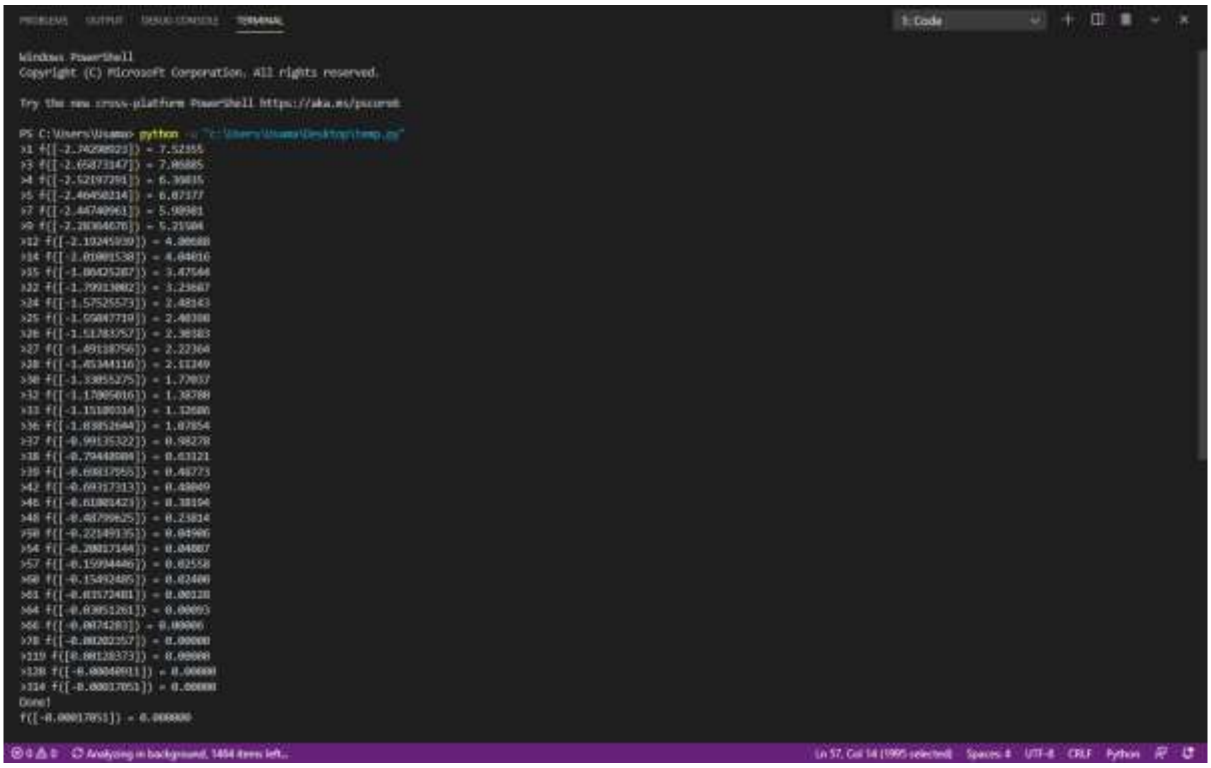
```
>25 f([-1.55047719]) = 2.40398

>26 f([-1.51783757]) = 2.30383

>27 f([-1.49118756]) = 2.22364

>28 f([-1.45344116]) = 2.11249

>30 f([-1.33055275]) = 1.77037

>32 f([-1.17805016]) = 1.38780

>33 f([-1.15189314]) = 1.32686

>36 f([-1.03852644]) = 1.07854

>37 f([-0.99135322]) = 0.98278

>38 f([-0.79448984]) = 0.63121

>39 f([-0.69837955]) = 0.48773

>42 f([-0.69317313]) = 0.48049

>46 f([-0.61801423]) = 0.38194

>48 f([-0.48799625]) = 0.23814

>50 f([-0.22149135]) = 0.04906

>54 f([-0.20017144]) = 0.04007

>57 f([-0.15994446]) = 0.02558

>60 f([-0.15492485]) = 0.02400

>61 f([-0.03572481]) = 0.00128

>64 f([-0.03051261]) = 0.00093

>66 f([-0.0074283]) = 0.00006

>78 f([-0.00202357]) = 0.00000

>119 f([0.00128373]) = 0.00000

>120 f([-0.00040911]) = 0.00000
```

>314 f([-0.00017051]) = 0.00000

Done!

f([-0.00017051]) = 0.000000

2.6.2 Screenshoots



```
Windows PowerShell
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Try the new cross-platform PowerShell https://aka.ms/pscore6

PS C:\Users\Usama> python -u "c:\Users\Usama\Desktop\temp.py"
>1 f([-2.74298923]) = 7.52355
>3 f([-2.65873147]) = 7.06885
>4 f([-2.52197291]) = 6.36035
>5 f([-2.46450214]) = 6.07377
>7 f([-2.44740961]) = 5.98981
>9 f([-2.28304676]) = 5.21184
>12 f([-2.19245939]) = 4.80688
>14 f([-2.01001538]) = 4.04016
>15 f([-1.86425287]) = 3.47544
>22 f([-1.79913002]) = 3.23687
>24 f([-1.57525573]) = 2.48143
>25 f([-1.55047719]) = 2.40388
>26 f([-1.51783757]) = 2.30383
>27 f([-1.49118756]) = 2.22364
>28 f([-1.45344116]) = 2.11249
>30 f([-1.33055275]) = 1.77037
>32 f([-1.17805016]) = 1.38788
>33 f([-1.15180314]) = 1.32686
>36 f([-1.03852644]) = 1.07854
>37 f([-0.99135322]) = 0.98278
>38 f([-0.79448080]) = 0.63121
>39 f([-0.69837955]) = 0.48773
>42 f([-0.69317313]) = 0.48049
>46 f([-0.61801423]) = 0.38194
>48 f([-0.48799625]) = 0.23814
>50 f([-0.22149135]) = 0.04906
>54 f([-0.20017144]) = 0.04007
>57 f([-0.15994446]) = 0.02558
>60 f([-0.15492485]) = 0.02400
>61 f([-0.03572481]) = 0.00128
>64 f([-0.03051261]) = 0.00093
>66 f([-0.0074283]) = 0.00006
>78 f([-0.00202357]) = 0.00000
>119 f([0.00120373]) = 0.00000
>128 f([-0.00040911]) = 0.00000
>314 f([-0.00017051]) = 0.00000
Done!
f([-0.00017051]) = 0.000000
```

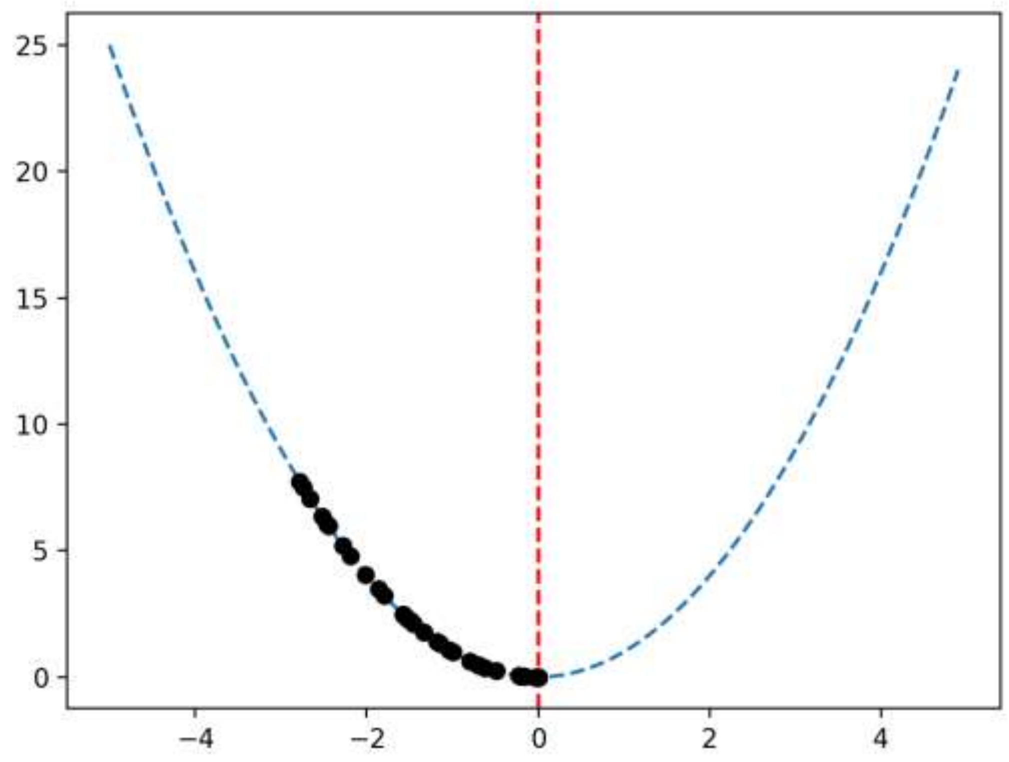
25
20
15
10
5
0
−4
−2
0
2
4