EE2703 : Applied Programming Lab

Assignment 8

Harisankar K J
EE20B043

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Abstract

The aim of the assignment is:

- Analyse signals using the Fast Fourier Transform(FFT) using Numpy fft module.
- FFT is an implementation of the DFT see how python can be used for symbolic Algebra.
- Find Error in the approximation of fft.

Assignment

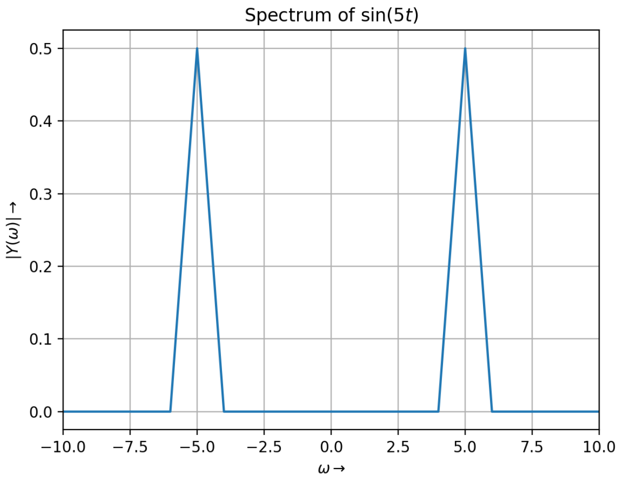
Q1:Finding the spectrum of sin(5t)

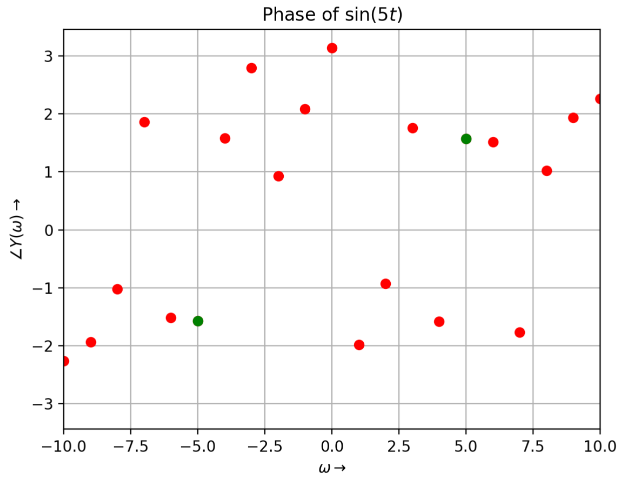
The required code to find the magnitude and phase plot of sin5t is already given. To fix this we sample the input signal at an appropriate frequency. We also shift the phase plot so that it goes from $-\pi$ to π.
We get 2 peaks at +5 and -5 with height 0.5. The phases of the peaks at $\frac{\pi}{2}$ and $-\frac{\pi}{2}$ are also expected based on the expansion of a sine wave.

$$sin(5t) = 0.5(\frac{e^{5t}}{j} - \frac{e^{-5t}}{j}) \tag{1}$$

```
# magnitude and phase plot for the DFT of sin(5t)

N1 = 128
t1 = linspace(-pi, pi, N1+1)
t1 = t1[:-1]
y1 = sin(5*t1)
Y1 = fftshift(fft(y1))/N1
w1 = linspace(-64, 63, N1)

figure(1)
plot(w1, abs(Y1))
xlim([-10, 10])
title(r"Spectrum of $\sin(5t)$")
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()

figure(2)
plot(w1, angle(Y1), 'ro')
```

```
ii = where(abs(Y1) > 1e-3)
plot(w1[ii], angle(Y1[ii]), 'go')
xlim([-10, 10])
title(r"Phase of $\sin(5t)$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()
show()
```





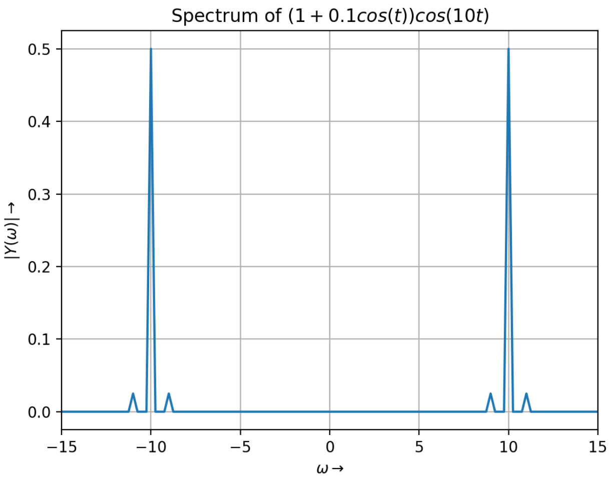
Q1: Finding the spectrum of Amplitude Modulated Wave

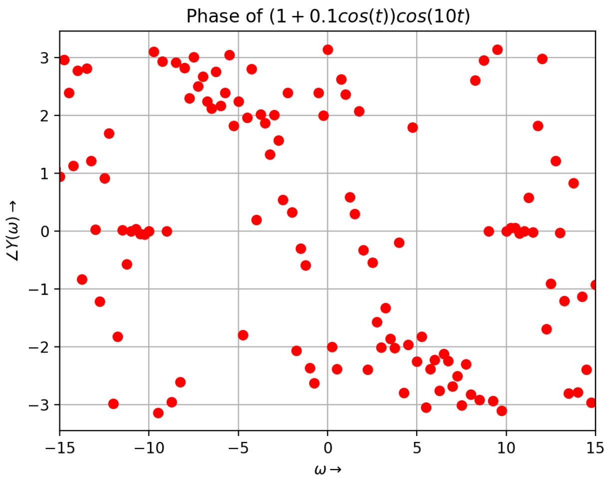
Consider the signal:

$$f(t) = (1 + 0.1\cos(t))\cos(10t) \tag{2}$$

```
# magnitude and phase plot for the DFT of (1 + 0.1cos(t))cos(10t
    )

N2 = 512
t2 = linspace(-4*pi, 4*pi, N2+1)
t2 = t2[:-1]
y2 = (1 + 0.1*cos(t2))*cos(10*t2)
Y2 = fftshift(fft(y2))/N2
w2 = linspace(-64, 64, N2+1)
w2 = w2[:-1]

figure(3)
plot(w2, abs(Y2))
xlim([-15, 15])
```

```
title(r"Spectrum of $(1 + 0.1cos(t))cos(10t)$")
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()

figure(4)
plot(w2, angle(Y2), 'ro')
xlim([-15, 15])
title(r"Phase of $(1 + 0.1cos(t))cos(10t)$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()
show()
```



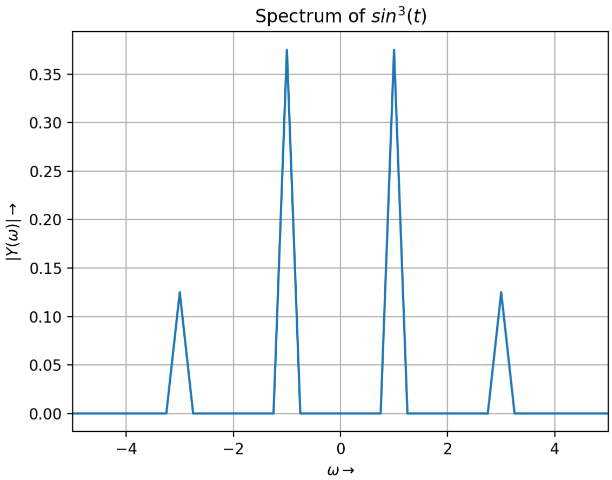


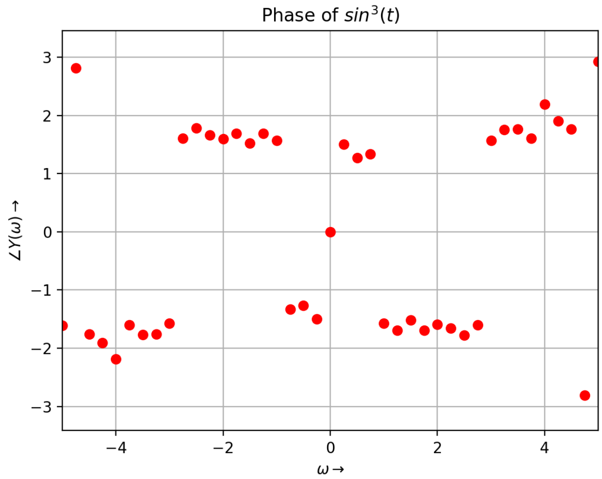
Q2: Spectrum of $sin^3(t)$ and $cos^3(t)$

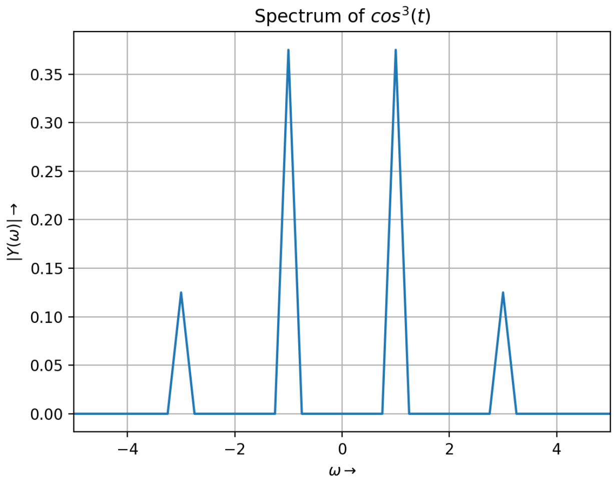
For $sin^3(t)$ can be expressed as a sum of sine waves using this identity: $\sin^3(t) = \frac{3}{4}\sin(t) - \frac{1}{4}\sin(3t)$ 2 peaks can be found at frequencies 1 and 3, and phases similar to that expected from a sum of sinusoids.
For signal $cos^3(t)$ can be expressed as a sum of cosine waves using this identity:
$\sin^3(t) = \frac{3}{4}\cos(t) + \frac{1}{4}\cos(3t)$
2 peaks can be found at frequencies 1 and 3, and phase=0 at the peaks.

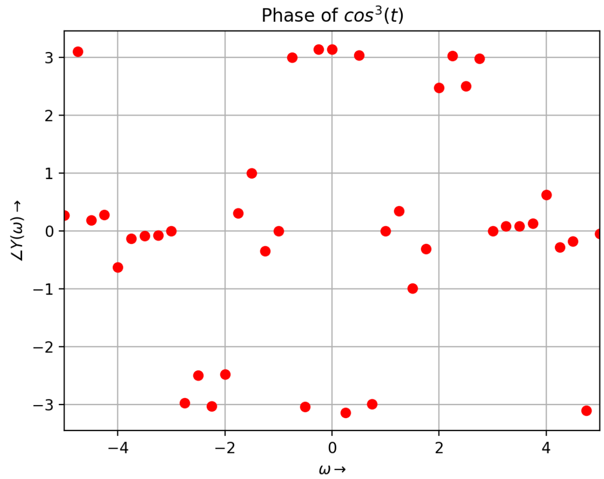
```
# magnitude and phase plot for the DFT of sin^3t
N3 = 512
t3 = linspace(-4*pi, 4*pi, N3+1)
t3 = t3[:-1]
y3 = (3*sin(t3) - sin(3*t3))/4
Y3 = fftshift(fft(y3))/N3
```

```
w3 = linspace(-64, 64, N3+1)
w3 = w3[:-1]

figure(5)
plot(w3, abs(Y3))
xlim([-5, 5])
title(r"Spectrum of $sin^3(t)$")
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()

figure(6)
plot(w3, angle(Y3), 'ro')
xlim([-5, 5])
title(r"Phase of $sin^3(t)$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()
show()

# magnitude and phase plot for the DFT of cos^3t
N4 = 512
t4 = linspace(-4*pi, 4*pi, N4+1)
t4 = t4[:-1]
y4 = (3*cos(t3) + cos(3*t3))/4
Y4 = fftshift(fft(y4))/N4
w4 = linspace(-64, 64, N4+1)
w4 = w4[:-1]

figure(8)
plot(w4, abs(Y4))
xlim([-5, 5])
title(r"Spectrum of $cos^3(t)$")
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()

figure(9)
plot(w4, angle(Y4), 'ro')
xlim([-5, 5])
title(r"Phase of $cos^3(t)$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()
show()
```









Q3: Spectrum of $\cos(20t + 5\cos(t)$

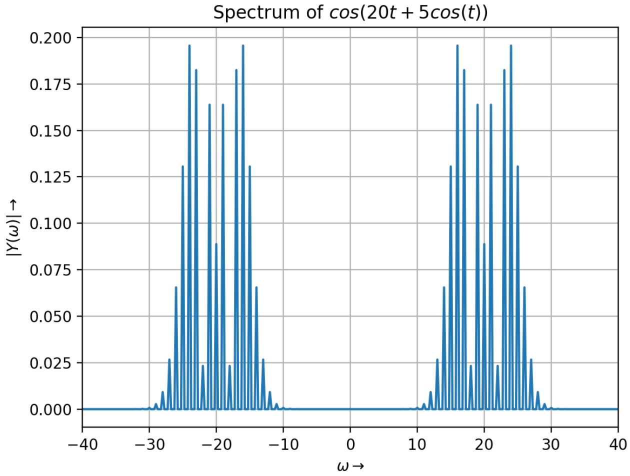
Considering the frequency modulated signal:

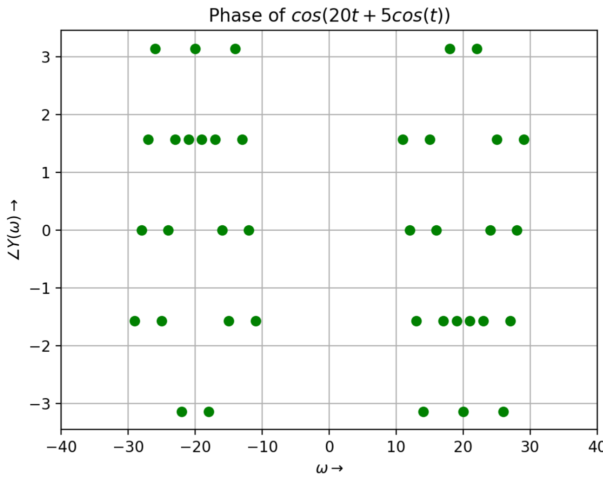
$$f(t) = \cos(20t + 5\cos(t)) \tag{3}$$

Phase points are plotted only when the magnitude is significant.

```
# magnitude and phase plot for the DFT of sin^3t
N3 = 512
t3 = linspace(-4*pi, 4*pi, N3+1)
t3 = t3[:-1]
y3 = (3*sin(t3) - sin(3*t3))/4
Y3 = fftshift(fft(y3))/N3
w3 = linspace(-64, 64, N3+1)
w3 = w3[:-1]
```

```
figure(5)
plot(w3, abs(Y3))
xlim([-5, 5])
title(r"Spectrum of $sin^3(t)$")
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()

figure(6)
plot(w3, angle(Y3), 'ro')
xlim([-5, 5])
title(r"Phase of $sin^3(t)$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()
show()

# magnitude and phase plot for the DFT of cos^3t
N4 = 512
t4 = linspace(-4*pi, 4*pi, N4+1)
t4 = t4[:-1]
y4 = (3*cos(t3) + cos(3*t3))/4
Y4 = fftshift(fft(y4))/N4
w4 = linspace(-64, 64, N4+1)
w4 = w4[:-1]

figure(8)
plot(w4, abs(Y4))
xlim([-5, 5])
title(r"Spectrum of $cos^3(t)$")
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()

figure(9)
plot(w4, angle(Y4), 'ro')
xlim([-5, 5])
title(r"Phase of $cos^3(t)$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid()
show()
```





0.0.1 Q4:Analysing Fourier Transform of a Gaussian

We use the FFT to estimate the CTFT of the Gaussian distribution function

$e^{\frac{-t^2}{2}}$

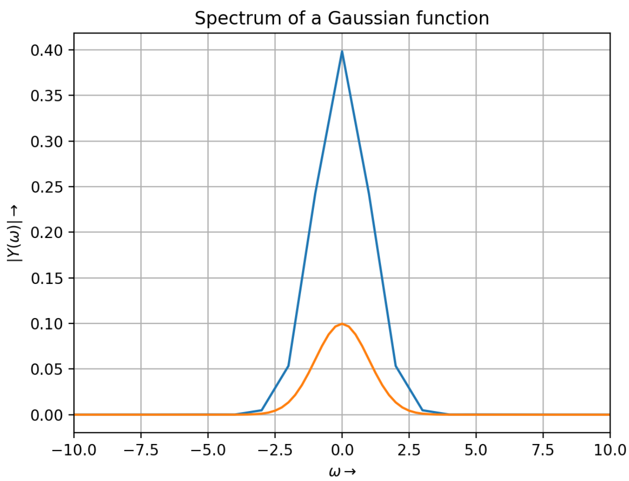
The CTFT of the above nonperiodic function is

$\frac{e^{\frac{-t^2}{2}}}{\sqrt{2\pi}}$,
using the appropriate CTFT Formulation.

We use the FFT to estimate the CTFT of the Gaussian distribution function $e^{\frac{-t^2}{2}}$
The CTFT of the above nonperiodic function is $\frac{e^{\frac{-t^2}{2}}}{\sqrt{2\pi}}$, using the appropriate CTFT Formulation.

```
# Finding the max error in the magnitude of calculated DTF of
    exp(-0.5t^2)
N = 128
T = 2*pi
maxerror = 1e-6
n = 0
error = 0
w = 0
Y = 0
while error < maxerror:

    t = linspace(-T/2, T/2, N+1)[:-1]
    w = N/T * linspace(-pi, pi, N+1)[:-1]
    y = exp(-0.5*t**2)
```

```
    Y_True = (1/sqrt(2*pi))*exp(-0.5*w**2)
    Y = fftshift(fft(y))*T/(2*pi*N)
    error = max(abs(abs(Y)-Y_True))

    T = T*2
    N = N*2

    n = n+1

print(" Max Error: {} \n No. of Interations: {}".format(error, n
    ))
print(" Value for T: {}*pi \n Value for N: {}" .format(T/pi, N))

# magnitude plots for different values of N and Ts
y = exp(-0.5*t1**2)
Y = fftshift(fft(y))/N1
figure(12)
plot(w1, abs(Y))
title(r"Spectrum of a Gaussian function")
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid(True)
xlim([-10, 10])

y = exp(-0.5*t2**2)
Y = fftshift(fft(y))/N2
plot(w2, abs(Y))
title(r"Spectrum of a Gaussian function")
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
grid(True)
xlim([-10, 10])
show()
```



Conclusion:

- We understood how to find the DFTs of sinusoidal signals.
- The magnitude spectrum of the Gaussian almost coincides with fourier transform.
- FFT fourier works well for samples of 2^k as it divides the samples into even and odd and goes on to make DFT.