

Chapter 1

Chapter 1 Maintaining Mathematical Proficiency (p. 1)

- $|8 - 12| = |8 + (-12)| = |-4| = 4$
- $|-6 - 5| = |-6 + (-5)| = |-11| = 11$
- $|4 + (-9)| = |-5| = 5$ 4. $|13 + (-4)| = |9| = 9$
- $|6 - (-2)| = |6 + 2| = |8| = 8$
- $|5 - (-1)| = |5 + 1| = |6| = 6$
- $|-8 - (-7)| = |-8 + 7| = |-1| = 1$
- $|8 - 13| = |8 + (-13)| = |-5| = 5$
- $|-14 - 3| = |-14 + (-3)| = |-17| = 17$

$$\begin{aligned}
 10. A &= \frac{1}{2}bh \\
 &= \frac{1}{2}(14)(22) \\
 &= \frac{1}{2}(308) \\
 &= 154
 \end{aligned}$$

The area of the triangle is 154 square meters.

$$\begin{aligned}
 11. A &= \frac{1}{2}bh \\
 &= \frac{1}{2}(24)(7) \\
 &= \frac{1}{2}(168) \\
 &= 84
 \end{aligned}$$

The area of the triangle is 84 square yards.

$$\begin{aligned}
 12. A &= \frac{1}{2}bh \\
 &= \frac{1}{2}(25)(16) \\
 &= \frac{1}{2}(400) \\
 &= 200
 \end{aligned}$$

The area of the triangle is 200 square inches.

13. $|x - y| > 0$ is true for $x \neq y$. $|x - y| = 0$ is true for $x = y$.
 $|x - y| < 0$ is never true because the absolute value of any number is never negative.

Chapter 1 Mathematical Practices (p. 2)

$$\begin{aligned}
 1. A &= \frac{1}{2}bh \\
 &= \frac{1}{2}(2 \text{ cm})(2 \text{ cm}) \\
 &= \frac{1}{2} \left[(2 \text{ cm}) \left(\frac{1 \text{ in.}}{2.54 \text{ cm}} \right) \right] \left[(2 \text{ cm}) \left(\frac{1 \text{ in.}}{2.54 \text{ cm}} \right) \right] \\
 &\approx \frac{1}{2}(0.79 \text{ cm})(0.79 \text{ cm}) \\
 &\approx 0.31 \text{ cm}^2
 \end{aligned}$$

The area of the triangle is about 0.31 square inch.

$$\begin{aligned}
 2. A &= bh \\
 &= (2.5 \text{ in.})(2 \text{ in.}) \\
 &= \left[(2.5 \text{ in.}) \left(\frac{2.54 \text{ cm}}{1 \text{ in.}} \right) \right] \left[(2 \text{ in.}) \left(\frac{2.54 \text{ cm}}{1 \text{ in.}} \right) \right] \\
 &= (6.35 \text{ cm})(5.08 \text{ cm}) \\
 &\approx 32.26 \text{ cm}^2
 \end{aligned}$$

The area of the parallelogram is about 32.26 square centimeters.

$$\begin{aligned}
 3. \quad 1 \text{ mile} &= 1.609344 \text{ km} \\
 120 \text{ miles} &= 120(1.609344 \text{ km}) \\
 &= 193.12 \text{ km} \\
 &\approx 193 \text{ km}
 \end{aligned}$$

The distance between the cities is about 193 kilometers.

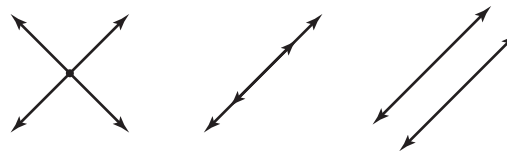
1.1 Explorations (p. 3)

1. A line consists of a set of points that extends in opposite directions infinitely.

A line segment is a subset of a line that has a beginning point and ending point.

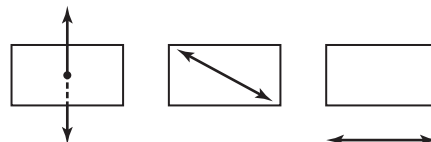
A ray is a subset of a line that has a beginning point and extends infinitely in one direction.

2. a. Two lines can intersect at a point, overlap, or not intersect.



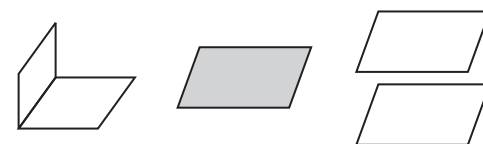
Sample answer: The line formed by the floor and front wall intersects the line formed by the front wall and side wall; The line formed by the bottom of the front wall overlaps the line formed by the front edge of the floor; The line formed by the front wall and the floor does not intersect the line formed by the side wall and back wall.

- b. A line and a plane can intersect at a point, overlap, or not intersect.



Sample answer: A line formed by two walls intersects the floor at a point; A line formed by a wall and the floor overlaps the floor; A line formed by a wall and the floor does not intersect the ceiling.

- c. Two planes can intersect in a line, overlap, or not intersect.



Sample answer: The floor and a wall intersect in a line; The door and the wall overlap; The floor and the ceiling do not intersect.

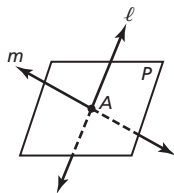
3. *Sample answer:* A segment bisector is a point, ray, line, line segment, or plane that intersects the segment at its midpoint.
4. Dynamic geometry software can be used to illustrate geometric shapes and figures and explain their characteristics.

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1.1 Monitoring Progress (pp. 4–7)

- Two other names for \overleftrightarrow{ST} are \overleftrightarrow{SP} or \overleftrightarrow{PT} . V is a point that is not coplanar with points Q , S , and T .
- Another name for \overleftrightarrow{KL} is \overleftrightarrow{LK} .
- no; \overrightarrow{KP} and \overrightarrow{PK} are not the same rays because they each have a different endpoint and are going in different directions. yes; \overrightarrow{NP} and \overrightarrow{NM} are the same ray because they have the same endpoint and are going in the same direction.

4. Sample answer:



- The intersection of \overleftrightarrow{PQ} and line k is M .
- The intersection of plane A and plane B is line k .
- The intersection of line k and plane A is line k .
- Sample answer: Two planes that contain line s are planes GJ and GIL .
- Sample answer: Three planes that contain point K are planes GK , JKL , and KJI .
- Sample answer: Two planes that contain \overleftrightarrow{HJ} are planes GHJ and JHK .

1.1 Exercises (pp. 8–10)

Vocabulary and Core Concept Check

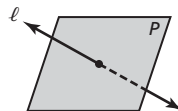
- Collinear points lie on the same line. Coplanar points lie in the same plane.
- The term that does not belong is plane CDE . \overleftrightarrow{AB} , \overleftrightarrow{FG} , and \overleftrightarrow{HI} have one dimension and plane CDE has two dimensions.

Monitoring Progress and Modeling with Mathematics

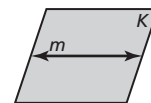
- Sample answer: Four points are A , B , C , and D .
- Two lines are \overleftrightarrow{BC} and \overleftrightarrow{DE} .
- Plane S contains points A , B , and C .
- Plane T contains points A , D , and E .
- Two other names for \overleftrightarrow{WQ} are \overleftrightarrow{QW} and line g .
- Sample answer: Another name for plane V is plane RTS .
- Three collinear points are R , Q , and S . The fourth point not collinear with these points is T or W .
- W is not coplanar with R , S , and T .

- Another name for \overleftrightarrow{BD} is \overleftrightarrow{DB} .
- Another name for \overleftrightarrow{AC} is \overleftrightarrow{CA} .
- Another name for \overleftrightarrow{AE} is \overleftrightarrow{AC} .
- Rays with endpoint E are \overrightarrow{EA} , \overrightarrow{EB} , \overrightarrow{EC} , and \overrightarrow{ED} .
- Two pairs of opposite rays are \overrightarrow{EA} and \overrightarrow{EC} and \overrightarrow{EB} and \overrightarrow{ED} .
- Sample answer: One pair of rays that are not opposite rays are \overrightarrow{EC} and \overrightarrow{ED} .

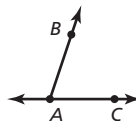
17. Sample answer:



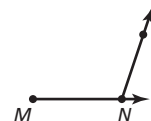
18. Sample answer:



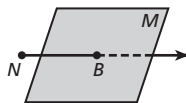
19. Sample answer:



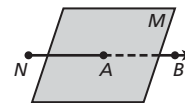
20. Sample answer:



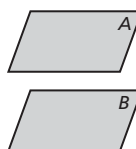
21. Sample answer:



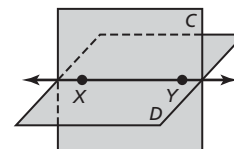
22. Sample answer:



23. Sample answer:



24. Sample answer:

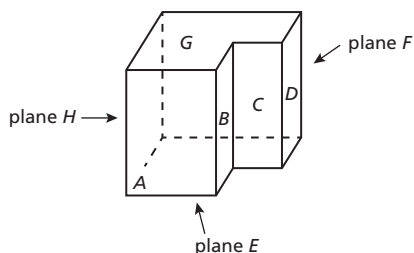


- A , D , and C are noncollinear, so \overleftrightarrow{AD} and \overleftrightarrow{AC} are not opposite rays. \overleftrightarrow{AD} and \overleftrightarrow{AB} are opposite rays because A , B , and D are collinear, and A is between B and D .
- \overrightarrow{YC} and \overrightarrow{YE} are opposite rays. \overline{YC} and \overline{YE} are segments.
- J is collinear with E and H .
- C is collinear with B and I .
- A , B , C , D , F , G , and I are not collinear with E and H .
- A , D , E , F , G , H , and J are not collinear with B and I .
- I and C are coplanar with D , A , and B .
- B and I are coplanar with C , G , and F .
- The intersection of plane AEH and plane FBE is \overleftrightarrow{AE} .
- The intersection of plane BGF and plane HDG is \overleftrightarrow{CG} .
- point 36. plane 37. segment 38. point

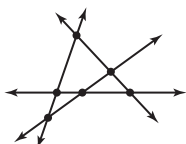
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39. Q , R , S , and P are not coplanar with N , K , and L .
40. R , S , K , and L are not coplanar with P , Q , and N .
41. K , L , M , and N are not coplanar with P , Q , and R .
42. P , M , L , and S are not coplanar with R , K , and N .
43. Q , L , M , and R are not coplanar with P , S , and K .
44. S , R , N , and M are not coplanar with Q , K , and L .
45. yes; Use the point not on the line and two points on the line to draw the plane.
46. yes; Infinitely many planes can intersect any given point.
47. Three of the four legs end in the plane of the floor, but the fourth leg may be above the floor. For a three-legged chair, because three points define a plane, all three legs will be flat on the floor, even if one leg is shorter than the other two.

48. *Sample answer:*



49. The greatest number of intersection points that exist for four coplanar lines is 6.



50. a. *Sample answer:* Both could have traveled east or west on Apple Avenue or north or south on Cherry Court.
- b. Opposite rays could not have been formed because both friends were not traveling along the same line (road).

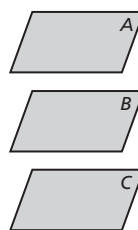
51. $x \leq 3$ ray

52. $-7 \leq x \leq 4$ segment

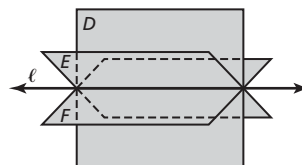
53. $x \geq 5$ or $x \leq -2$ rays

54. $|x| \leq 0$ point

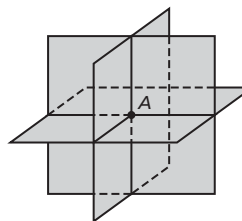
55. a. K and N are collinear with P .
- b. *Sample answer:* Planes JKL and JKN contain J .
- c. All points that are in more than one plane are J , K , L , M , N , P , and Q .
56. never; A line has no endpoints.
57. sometimes; The point may be on the line.
58. sometimes; The point may be in the plane.
59. sometimes; The planes may not intersect.
60. always; Two points determine exactly one line.
61. sometimes; Three noncollinear points determine exactly one plane. Three collinear points determine a line.
62. always; Three noncollinear points determine exactly one plane.
63. sometimes; Lines in parallel planes do not intersect, and may not be parallel.
64. Three planes never intersect if the three planes are parallel.



Three planes can intersect in a line.



Three planes can intersect in a point.



Maintaining Mathematical Proficiency

65. $|6 + 2| = |8| = 8$

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66. $|3 - 9| = |3 + (-9)| = |-6| = 6$
67. $|-8 - 2| = |-8 + (-2)| = |-10| = 10$
68. $|7 - 11| = |7 + (-11)| = |-4| = 4$
69. $18 + x = 43$
 $\begin{array}{r} -18 \\ -18 \\ \hline x = 25 \end{array}$
70. $36 + x = 20$
 $\begin{array}{r} -36 \\ -36 \\ \hline x = -16 \end{array}$
71. $x - 15 = 7$
 $\begin{array}{r} +15 \\ +15 \\ \hline x = 22 \end{array}$
72. $x - 23 = 19$
 $\begin{array}{r} +23 \\ +23 \\ \hline x = 42 \end{array}$

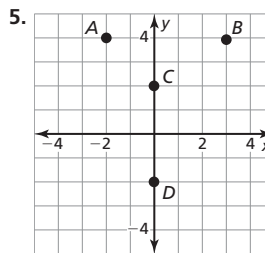
1.2 Explorations (p. 11)

1. a. Check students' work.
 - b. The line segment is $4\frac{4}{5}$ paper clips. Sliding the paper clip end-to-end, the 6-inch segment is just less than 5 lengths of the paper clip.
 - c. 1 paper clip = $1\frac{1}{4}$ inches
 $1 \text{ inch} = \frac{4}{5} \text{ paper clip}$
 - d. Use the pencil and straightedge to draw a segment longer than 6 inches. Starting at one end, measure about $4\frac{4}{5}$ paper clips. Mark the endpoint.
2. a. Check students' work.
 - b. $c^2 = a^2 + b^2$
 $c^2 = 3^2 + 5^2$
 $c^2 = 9 + 25$
 $c^2 = 34$
 $c = \sqrt{34} \approx 5.8 \text{ in.}$
 The length of the diagonal with the ruler is $5\frac{13}{16} \approx 5.8$ inches.
 - c. Width = 4 paper clips
 Height = $2\frac{2}{5}$ paper clips
 - d. $c^2 = a^2 + b^2$
 $c^2 = \left(2\frac{2}{5}\right)^2 + (4)^2$
 $c^2 = \left(\frac{12}{5}\right)^2 + (4)^2$
 $c^2 = \frac{144}{25} + 16$
 $c^2 = \frac{544}{25}$
 $c = \sqrt{\frac{544}{25}} \approx 4.7$
 The length of the diagonal is about 4.7 paper clips. The Pythagorean Theorem works for any unit of measure because it states a relationship between lengths, regardless of how they are measured.
3. Sample answer: 1 diag \approx 5.8 inches; height = 60 inches
 $\left(\frac{\text{height}}{1} \text{ inches}\right) \cdot \left(\frac{1 \text{ diag}}{5.8 \text{ inches}}\right) = \frac{60 \text{ inches}}{1} \left(\frac{1 \text{ diag}}{5.8 \text{ inches}}\right)$
 $= \frac{60}{5.8} \approx 10.3 \text{ diags}$

4. Take an unmarked straightedge and mark the ends of the segment being measured onto the straightedge. Then draw a segment based on the markings.

1.2 Monitoring Progress (pp. 12–15)

1. $MN = 1\frac{5}{8} \text{ in.}$
2. $PQ = 1\frac{3}{8} \text{ in.}$
3. $UV = \frac{7}{8} \text{ in.}$
4. $WX = 1\frac{1}{4} \text{ in.}$



$$AB = |3 - (-2)| = |3 + 2| = |5| = 5$$

$$CD = |2 - (-2)| = |2 + 2| = |4| = 4$$

$$\overline{AB} \neq \overline{CD}$$

\overline{AB} is not congruent to \overline{CD} .

6. $xz = xy + xz$
 $xz = 23 + 50$
 $xz = 73$
7. no; In order to use segment addition, the points have to be collinear. W, Y, and Z are not collinear.
8. $JL = JK + KL$
 $144 = 37 + KL$
 $107 = KL$
9. $PC = PA + AC$
 $680 = PA + 231$
 $449 = PA$
 The distance from Albuquerque, New Mexico, to Provo, Utah, is 449 miles.

1.2 Exercises (pp. 16–18)

Vocabulary and Core Concept Check

1. \overline{XY} represents the segment XY, whereas XY represents the distance between points X and Y (the length of \overline{XY}).
2. "Find $BC - AC$ " is different.
 $BC - AC = 7 - 3 = 4$
 $AC + CB = 3 + 7 = 10$

Monitoring Progress and Modeling with Mathematics

3. 3.5 cm
4. 6 cm
5. 4.5 cm
6. 7 cm

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7. Using a straightedge, draw a ray where the left endpoint will be the beginning of the segment.

With a compass, measure the segment in Exercise 3 by placing the point of the compass on the segment endpoint on the left and the pencil point on the segment endpoint on the right.

Without changing the compass setting, place the point of the compass on the endpoint on the left of the ray you drew and mark the ray with an arc.

Where the arc and ray intersect is the right endpoint of the segment in Exercise 3.

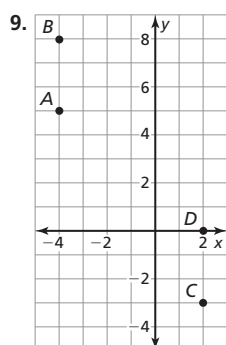


8. Using a straightedge, draw a ray where the left endpoint will be the beginning of the segment.

With a compass, measure the segment in Exercise 4 by placing the point of the compass on the segment endpoint on the left and the pencil point on the segment endpoint on the right.

Without changing the compass setting, place the point of the compass on the endpoint on the left of the ray you drew and mark the ray with an arc.

Where the arc and ray intersect is the right endpoint of the segment in Exercise 4.

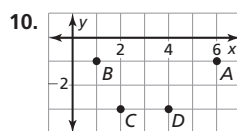


$$AB = |5 - 8| = |-3| = 3$$

$$CD = |-3 - 0| = |-3| = 3$$

$$\overline{AB} \cong \overline{CD}$$

$$\overline{AB} \text{ is congruent to } \overline{CD}.$$

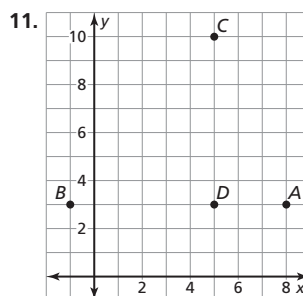


$$AB = |6 - 1| = |5| = 5$$

$$CD = |4 - 2| = |2| = 2$$

$$\overline{AB} \not\cong \overline{CD}$$

$$\overline{AB} \text{ is not congruent to } \overline{CD}.$$

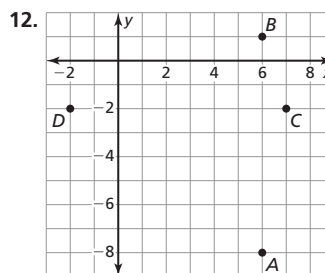


$$AB = |8 - (-1)| = |8 + 1| = 9$$

$$CD = |10 - 3| = |7| = 7$$

$$\overline{AB} \not\cong \overline{CD}$$

$$\overline{AB} \text{ is not congruent to } \overline{CD}.$$

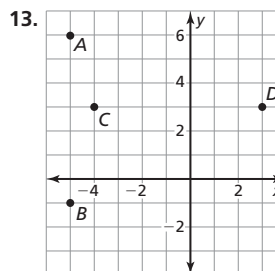


$$AB = |1 - (-8)| = |1 + 8| = |9| = 9$$

$$CD = |7 - (-2)| = |7 + 2| = |9| = 9$$

$$\overline{AB} \cong \overline{CD}$$

$$\overline{AB} \text{ is congruent to } \overline{CD}.$$

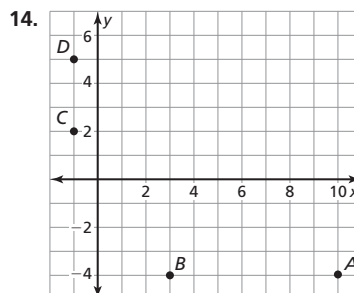


$$AB = |6 - (-1)| = |6 + 1| = 7$$

$$CD = |3 - (-4)| = |3 + 4| = 7$$

$$\overline{AB} \cong \overline{CD}$$

$$\overline{AB} \text{ is congruent to } \overline{CD}.$$



$$AB = |10 - 3| = |7| = 7$$

$$CD = |5 - 2| = |3| = 3$$

$$\overline{AB} \not\cong \overline{CD}$$

$$\overline{AB} \text{ is not congruent to } \overline{CD}.$$

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15. $FH = FG + GH$

$$FH = 8 + 14$$

$$FH = 22$$

17. $FH = FG + GH$

$$FH = 11 + 12$$

$$FH = 23$$

19. $FH + HG = FG$

$$FH + 13 = 37$$

$$FH = 24$$

21. $FH + HG = FG$

$$FH + 22 = 42$$

$$FH = 20$$

16. $FH = FG + GH$

$$FH = 19 + 7$$

$$FH = 26$$

18. $FH = FG + GH$

$$FH = 4 + 15$$

$$FH = 19$$

20. $FH + HG = FG$

$$FH + 15 = 22$$

$$FH = 7$$

22. $FH + HG = FG$

$$FH + 40 = 53$$

$$FH = 13$$

23. The absolute value should have been taken.

$$AB = |1 - 4.5| = |-3.5| = 3.5$$

24. The difference should have been taken.

$$AB = |1 - 4.5| = |-3.5| = 3.5$$

25. The abdomen is about $2\frac{1}{4}$ inches. The thorax is about $|4 - 2\frac{1}{4}| = |1\frac{3}{4}| = 1\frac{3}{4}$ inches.

$$\begin{aligned} 2\frac{1}{4} - 1\frac{3}{4} &= \frac{9}{4} - \frac{7}{4} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

The abdomen is $\frac{1}{2}$ inch longer than the thorax.

$$\begin{aligned} \frac{9}{4} \div \frac{7}{4} &= \frac{9}{4} \cdot \frac{4}{7} \\ &= \frac{9}{7} \\ &= 1\frac{2}{7} \end{aligned}$$

The abdomen is $1\frac{2}{7}$ times longer than the thorax.

26. a. $AC = 1282 + 601 = 1883$

The distance the model plane flew was 1883 miles.

b. $\frac{2000 \text{ mi}}{40 \text{ h}} = \frac{50 \text{ mi}}{\text{h}}$

The estimated average speed is 50 miles per hour.

$$\frac{1883 \text{ mi}}{38 \text{ h}} \approx 49.55 \frac{\text{mi}}{\text{h}}$$

The actual average speed is about 49.55 miles per hour.

27. a. true; A, B, and C are collinear.

b. false; C, B, and E are not collinear.

c. true; A, D, E, and H are all collinear.

d. false; C, E, and F are not collinear.

28. a. $AC = AB + BC$

$$= x + 2 + 7x - 3$$

$$= 8x - 1$$

b. $PR = PQ + QR$

$$13y + 25 = 8y + 5 + QR$$

$$5y + 25 = 5 + QR$$

$$5y + 20 = QR$$

29. 

a. $RT = RS + ST$

$$21 = 2x + 10 + x - 4$$

$$21 = 3x + 6$$

$$15 = 3x$$

$$x = 5$$

$$RS = 2x + 10$$

$$= 2(5) + 10$$

$$= 10 + 10 = 20$$

$$ST = x - 4$$

$$= 5 - 4 = 1$$

$$RT = 20 + 1 = 21$$

b. $RT = RS + ST$

$$60 = 3x - 16 + 4x - 8$$

$$60 = 7x - 24$$

$$84 = 7x$$

$$x = 12$$

$$RS = 3x - 16$$

$$= 3(12) - 16$$

$$= 36 - 16 = 20$$

$$ST = 4x - 8$$

$$= 4(12) - 8$$

$$= 48 - 8 = 40$$

$$RT = 20 + 40 = 60$$

c. $RT = RS + ST$

$$x + 10 = 2x - 8 + 11$$

$$x + 10 = 2x + 3$$

$$10 = x + 3$$

$$x = 7$$

$$RS = 2(7) - 8$$

$$= 14 - 8 = 6$$

$$ST = 11$$

$$RT = x + 10$$

$$= 7 + 10 = 17$$

d. $RT = RS + ST$

$$8x - 14 = 4x - 9 + 19$$

$$8x - 14 = 4x + 10$$

$$4x - 14 = 10$$

$$4x = 24$$

$$x = 6$$

$$RS = 4x - 9$$

$$= 4(6) - 9$$

$$= 24 - 9 = 15$$

$$ST = 19$$

$$RT = 8x - 14$$

$$= 8(6) - 14$$

$$= 48 - 14 = 34$$

30. yes; The legs could all be set at different angles with the table.

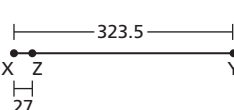


31. a. $86 - 22 = 64 \text{ ft}$

b. $\frac{64 \text{ ft}}{4.4 \text{ ft/sec}} = 14.55 \cancel{\text{sec}} \cdot \frac{1 \text{ min}}{60 \cancel{\text{sec}}} = 0.24 \text{ min}$

c. You might walk slower if other people are in the hall.

32. Your cousin is correct using absolute differences. If you do not line up an object at zero, then take the absolute value of the difference of the measurements at both ends of the object.

33. 

$$\text{Round trip} = 647 \text{ miles}$$

$$1 \text{ way} = 323.5 \text{ miles}$$

$$ZY = 323.5 - 27 = 296.5 \text{ miles}$$

Chapter 1

34. Using the Ruler Postulate, identify the beginning and ending point of the loss bar. Take the absolute value of the ending point of the loss bar minus the beginning point of the loss bar to find the number of losses.

Using the Segment Addition Postulate, subtract the number of wins from the total number of games played to find the number of losses.

35. The length of the segment containing points (a, b) and (c, b) is $|a - c|$, because $b - b = 0$. The length of the segment containing (d, e) and (d, f) is $|e - f|$, because $d - d = 0$.

The equation assuming the segments are congruent is $|a - c| = |e - f|$.

The letters not used in the equation are b and d because $b - b = 0$ and $d - d = 0$.

36. Because $AD = 12$ and $\overline{AC} \cong \overline{CD}$, $AC = CD = 6$. Because $\overline{AB} \cong \overline{BC}$, $AB = BC = 3$. Because $BC = 3$ and $CD = 6$, $BD = 3 + 6 = 9$.

Counting all segment groupings, there are a total of 6 segments \overline{AB} , \overline{AC} , \overline{AD} , \overline{BC} , \overline{BD} , and \overline{CD} . Out of these 6 segments, 4 have a length greater than 3. The probability of choosing a segment with length greater than 3 is $\frac{4}{6}$ or $\frac{2}{3}$.

37. yes; $FB = FC + CB$, the whole segment FB is greater than the two individual parts, FC and CB , that make up the whole segment. Therefore, $FB > CB$.

$AC > DB$ is not necessarily true. $AC = AD + DF + FC$ and $DB = DF + FC + CB$. Both segments share common smaller segments DF and FC , so, in order for $AC > DB$ to be true, $AD > CB$.

Maintaining Mathematical Proficiency

38. $\frac{-4 + 6}{2} = \frac{2}{2} = 1$

39. $\sqrt{20 + 5} = \sqrt{25} = 5$

40. $\sqrt{25 + 9} = \sqrt{34}$

41. $\frac{7 + 6}{2} = \frac{13}{2}$

42. $5x + 7 = 9x - 17$

$7 = 4x - 17$

$24 = 4x$

$x = 6$

43. $\frac{3 + y}{2} = 6$

$3 + y = 12$

$y = 9$

44. $\frac{-5 + x}{2} = -9$

$-5 + x = -18$

$x = -13$

45. $-6x - 13 = -x - 23$

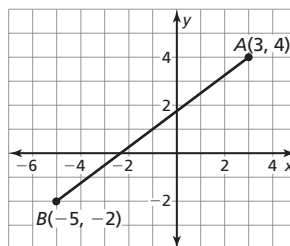
$-5x - 13 = -23$

$-5x = -10$

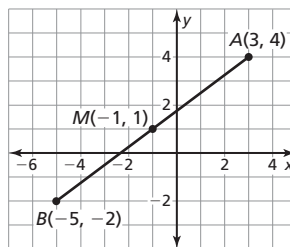
$x = 2$

1.3 Explorations (p. 19)

1. a.



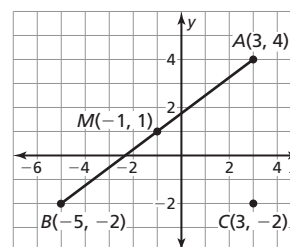
- b. *Sample answer:* Measure the distance between points A and B with a centimeter ruler. Divide the distance by 2. Measure this distance from point A or B and plot point M. The coordinates of M are the midpoint.



c. $M(-1, 1)$

- d. The x-coordinate of M is the average of the x-coordinates of points A and B. The y-coordinate of M is the average of the y-coordinates of points A and B.

2. a.



b. $(AB)^2 = (AC)^2 + (BC)^2$

$(AB)^2 = 6^2 + 8^2$

$(AB)^2 = 36 + 64$

$(AB)^2 = 100$

$AB = \sqrt{100} = 10$

\overline{AB} is 10 centimeters.

- c. Measuring with a centimeter ruler, \overline{AB} is 10 centimeters.

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d. $(BM)^2 = 3^2 + 4^2$

$$(BM)^2 = 9 + 16$$

$$(BM)^2 = 25$$

$$BM = \sqrt{25} = 5$$

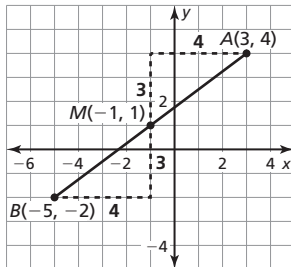
$$(AM)^2 = 3^2 + 4^2$$

$$(AM)^2 = 9 + 16$$

$$(AM)^2 = 25$$

$$AM = \sqrt{25} = 5$$

Sample answer: The Pythagorean Theorem can be used to find the length of any line segment in the coordinate plane.



3. *Sample answer:* To find the x -coordinate of the midpoint, find the average of the original x -coordinates. To find the y -coordinate of the midpoint, find the average of the original y -coordinates. For the distance, find the difference of the two x -coordinates and y -coordinates, square each difference, add the results of the squares, and then calculate the square root of the result.

$$\begin{aligned} 4. \text{ a. } M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{-10 + 14}{2}, \frac{-4 + 6}{2}\right) \\ &= \left(\frac{4}{2}, \frac{2}{2}\right) = (2, 1) \end{aligned}$$

$$\begin{aligned} DE &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[14 - (-10)]^2 + [6 - (-4)]^2} \\ &= \sqrt{(14 + 10)^2 + (6 + 4)^2} \\ &= \sqrt{24^2 + 10^2} \\ &= \sqrt{576 + 100} \\ &= \sqrt{676} \\ &= 26 \end{aligned}$$

$$\begin{aligned} \text{b. } M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{-4 + 9}{2}, \frac{8 + 0}{2}\right) \\ &= \left(\frac{5}{2}, \frac{8}{2}\right) = \left(\frac{5}{2}, 4\right) \end{aligned}$$

$$\begin{aligned} FG &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[9 - (-4)]^2 + (0 - 8)^2} \\ &= \sqrt{(9 + 4)^2 + (-8)^2} \\ &= \sqrt{13^2 + 64} \\ &= \sqrt{169 + 64} \\ &= \sqrt{233} \approx 15.26 \end{aligned}$$

1.3 Monitoring Progress (pp. 20–23)

1. The segment bisector of \overline{PQ} is \overline{MN} .

$$PM = MQ = \frac{1}{2}PQ$$

$$PM = \frac{1}{2}PQ$$

$$1\frac{7}{8} = \frac{1}{2}PQ$$

$$\frac{2}{1} \cdot \frac{15}{8} = PQ$$

$$\frac{15}{4} = PQ$$

$$PQ = 3\frac{3}{4}$$

2. The segment bisector of \overline{PQ} is point M .

$$MQ = \frac{1}{2}PQ$$

$$2\frac{2}{7} = \frac{1}{2}PQ$$

$$\frac{2}{1}\left(\frac{16}{7}\right) = PQ$$

$$\frac{32}{7} = PQ$$

$$PQ = 4\frac{4}{7}$$

3. The segment bisector of \overline{PQ} is line ℓ .

$$PM = MQ$$

$$5x - 3 = 11 - 2x$$

$$7x - 3 = 11$$

$$7x = 14$$

$$x = 2$$

$$MQ = 11 - 2x = 11 - 2(2) = 11 - 4 = 7$$

4. The segment bisector of \overline{RS} is line n .

$$RM = MS$$

$$4x + 3 = 6x - 12$$

$$3 = 2x - 12$$

$$15 = 2x$$

$$x = 7\frac{1}{2} \text{ or } 7.5$$

$$RS = 4x + 3 + 6x - 12$$

$$= 10x - 9 = 10(7.5) - 9 = 66$$

5. $A(1, 2)$, $B(7, 8)$

$$\begin{aligned} M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{1 + 7}{2}, \frac{2 + 8}{2}\right) \\ &= \left(\frac{8}{2}, \frac{10}{2}\right) \\ &= (4, 5) \end{aligned}$$

6. $C(-4, 3)$, $D(-6, 5)$

$$\begin{aligned} M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{-4 + (-6)}{2}, \frac{3 + 5}{2}\right) \\ &= \left(-\frac{10}{2}, \frac{8}{2}\right) \\ &= (-5, 4) \end{aligned}$$

Chapter 1

7. $M(2, 4)$, $T(1, 1)$, $u(x, y)$

$$(2, 4) = \left(\frac{1+x}{2}, \frac{1+y}{2} \right)$$

$$2 = \frac{1+x}{2} \quad 4 = \frac{1+y}{2}$$

$$4 = 1 + x \quad 8 = 1 + y$$

$$x = 3 \quad y = 7$$

$$u(3, 7)$$

8. $M(-1, -2)$, $W(4, 4)$, $V(x, y)$

$$(-1, -2) = \left(\frac{4+x}{2}, \frac{4+y}{2} \right)$$

$$-1 = \frac{4+x}{2} \quad -2 = \frac{4+y}{2}$$

$$-2 = 4 + x \quad -4 = 4 + y$$

$$-6 = x \quad -8 = y$$

$$V(-6, -8)$$

9. Park = $P(3, -4)$, school = $S(4, -1)$

$$PS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4 - 3)^2 + [-1 - (-4)]^2}$$

$$= \sqrt{1^2 + (-1 + 4)^2}$$

$$= \sqrt{1^2 + 3^2}$$

$$= \sqrt{1 + 9}$$

$$= \sqrt{10}$$

$$\approx 3.2$$

The distance between the park and the school is about 3.2 miles.

1.3 Exercises (pp. 24–26)

Vocabulary and Core Concept Check

1. A point, ray, line segment, or plane divides the segment at its midpoint into two equal parts.
2. To find the length of \overline{AB} , with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the Distance Formula.

Monitoring Progress and Modeling with Mathematics

3. Line k bisects \overline{RS} at point M .

$$RS = 2(17) = 34$$

4. \overrightarrow{MA} bisects \overline{RS} at point M . 5. Point M bisects \overline{RS} .

$$RS = 2(9) = 18$$

$$RS = 2(22) = 44$$

6. Line s bisects \overline{RS} at point M .

$$RS = 2(12) = 24$$

7. Point M bisects \overline{JK} .

$$JM = MK \quad JM = 7(5) + 5$$

$$7x + 5 = 8x \quad = 35 + 5$$

$$5 = x \quad = 40$$

8. Line ℓ bisects \overline{JK} at point M .

$$JM = MK \quad JM = 3(-2) + 15$$

$$3x + 15 = 8x + 25 \quad = -6 + 15$$

$$15 = 5x + 25 \quad = 9$$

$$-10 = 5x$$

$$-2 = x$$

9. \overrightarrow{MN} bisects \overline{XY} at point M .

$$XM = MY \quad XM = 3(5) + 1 = 16$$

$$3x + 1 = 8x - 24 \quad MY = 8(5) - 24 = 16$$

$$1 = 5x - 24 \quad XY = 16 + 16 = 32$$

$$25 = 5x$$

$$5 = x$$

10. Line n bisects \overline{XY} at point M .

$$XM = MY \quad XM = 5x + 8$$

$$5x + 8 = 9x + 12 \quad = 5(-1) + 8 = 3$$

$$-4x + 8 = 12 \quad MY = 9x + 12$$

$$-4x = 4 \quad = 9(-1) + 12 = 3$$

$$x = -1 \quad XY = 3 + 3 = 6$$

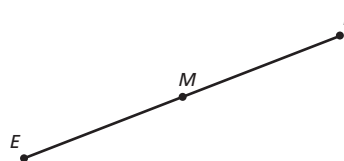
11. Draw the segment \overline{AB} on a piece of paper. Fold the paper so that B is on top of A . Label the midpoint M .



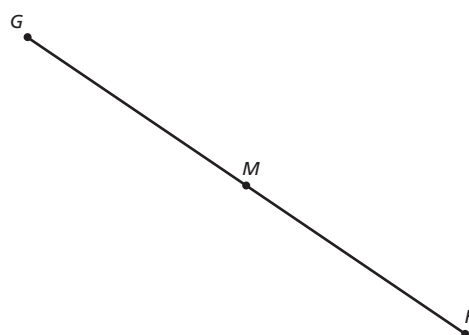
12. Draw the segment \overline{CD} on a piece of paper. Fold the paper so that C is on top of D . Label the midpoint M .



13. Draw the segment \overline{EF} on a piece of paper. Fold the paper so that E is on top of F . Label the midpoint M .



14. Draw the segment \overline{GH} on a piece of paper. Fold the paper so that G is on top of H . Label the midpoint M .



Chapter 1

15. $C(3, -5), D(7, 9)$

$$\begin{aligned} M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{3 + 7}{2}, \frac{-5 + 9}{2}\right) \\ &= \left(\frac{10}{2}, \frac{4}{2}\right) \\ &= (5, 2) \end{aligned}$$

16. $C(-4, 7), D(0, -3)$

$$\begin{aligned} M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{-4 + 0}{2}, \frac{7 + (-3)}{2}\right) \\ &= \left(\frac{-4}{2}, \frac{4}{2}\right) \\ &= (-2, 2) \end{aligned}$$

17. $C(-2, 0), D(4, 9)$

$$\begin{aligned} M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{-2 + 4}{2}, \frac{0 + 9}{2}\right) \\ &= \left(\frac{2}{2}, \frac{9}{2}\right) \\ &= \left(1, \frac{9}{2}\right) \end{aligned}$$

18. $C(-8, -6), D(-4, 10)$

$$\begin{aligned} M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{-8 + (-4)}{2}, \frac{-6 + 10}{2}\right) \\ &= \left(\frac{-12}{2}, \frac{4}{2}\right) \\ &= (-6, 2) \end{aligned}$$

19. $M(4, 3), G(5, -6), H(x, y)$

$$\begin{aligned} (4, 3) &= \left(\frac{5 + x}{2}, \frac{-6 + y}{2}\right) \\ 4 &= \frac{5 + x}{2} & 3 &= \frac{-6 + y}{2} \\ 8 &= 5 + x & 6 &= -6 + y \\ 3 &= x & 12 &= y \\ H(3, 12) \end{aligned}$$

20. $M(-2, 5), H(-3, 7), G(x, y)$

$$\begin{aligned} (-2, 5) &= \left(\frac{-3 + x}{2}, \frac{7 + y}{2}\right) \\ -2 &= \frac{-3 + x}{2} & 5 &= \frac{7 + y}{2} \\ -4 &= -3 + x & 10 &= 7 + y \\ -1 &= x & 3 &= y \\ G(-1, 3) \end{aligned}$$

21. $M(8, 0), H(-2, 9), G(x, y)$

$$\begin{aligned} (8, 0) &= \left(\frac{-2 + x}{2}, \frac{9 + y}{2}\right) \\ 8 &= \frac{-2 + x}{2} & 0 &= \frac{9 + y}{2} \\ 16 &= -2 + x & 0 &= 9 + y \\ 18 &= x & -9 &= y \\ G(18, -9) \end{aligned}$$

22. $M\left(-\frac{13}{2}, -6\right), G(-4, 1), H(x, y)$

$$\begin{aligned} \left(-\frac{13}{2}, -6\right) &= \left(\frac{-4 + x}{2}, \frac{1 + y}{2}\right) \\ -\frac{13}{2} &= \frac{-4 + x}{2} & -6 &= \frac{1 + y}{2} \\ -13 &= -4 + x & -12 &= 1 + y \\ -9 &= x & -13 &= y \\ H(-9, -13) \end{aligned}$$

23. $A(13, 2), B(7, 10)$

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(7 - 13)^2 + (10 - 2)^2} \\ &= \sqrt{(-6)^2 + 8^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

24. $C(-6, 5), D(-3, 1)$

$$\begin{aligned} CD &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-3 - (-6)]^2 + (1 - 5)^2} \\ &= \sqrt{(-3 + 6)^2 + (-4)^2} \\ &= \sqrt{3^2 + 16} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} = 5 \end{aligned}$$

25. $E(3, 7), F(6, 5)$

$$\begin{aligned} EF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 3)^2 + (5 - 7)^2} \\ &= \sqrt{3^2 + (-2)^2} \\ &= \sqrt{9 + 4} \\ &= \sqrt{13} \approx 3.6 \end{aligned}$$

26. $G(-5, 4), H(2, 6)$

$$\begin{aligned} GH &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[2 - (-5)]^2 + (6 - 4)^2} \\ &= \sqrt{(2 + 5)^2 + 2^2} \\ &= \sqrt{7^2 + 2^2} \\ &= \sqrt{49 + 4} \\ &= \sqrt{53} \approx 7.3 \end{aligned}$$

Chapter 1

27. $J(-8, 0), K(1, 4)$

$$\begin{aligned} JK &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[1 - (-8)]^2 + (0 - 4)^2} \\ &= \sqrt{(1 + 8)^2 + (-4)^2} \\ &= \sqrt{9^2 + 16} \\ &= \sqrt{81 + 16} \\ &= \sqrt{97} \approx 9.8 \end{aligned}$$

28. $L(7, -1), M(-2, 4)$

$$\begin{aligned} LM &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 7)^2 + [4 - (-1)]^2} \\ &= \sqrt{(-9)^2 + (4 + 1)^2} \\ &= \sqrt{81 + 25} \\ &= \sqrt{106} \approx 10.3 \end{aligned}$$

29. $R(0, 1), S(6, 3.5)$

$$\begin{aligned} RS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 0)^2 + (3.5 - 1)^2} \\ &= \sqrt{6^2 + 2.5^2} \\ &= \sqrt{36 + 6.25} \\ &= \sqrt{42.25} = 6.5 \end{aligned}$$

30. $T(13, 1.6), V(5.4, 3.7)$

$$\begin{aligned} TV &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5.4 - 13)^2 + (3.7 - 1.6)^2} \\ &= \sqrt{(-7.6)^2 + (2.1)^2} \\ &= \sqrt{57.76 + 4.41} \\ &= \sqrt{62.17} \approx 7.9 \end{aligned}$$

31. The square root should have been taken.

$$\begin{aligned} AB &= \sqrt{(6 - 1)^2 + [2 - (-4)]^2} \\ &= \sqrt{5^2 + 6^2} \\ &= \sqrt{25 + 36} \\ &= \sqrt{61} \approx 7.8 \end{aligned}$$

32. The difference of the x -values and the difference of the y -values should be used.

$$\begin{aligned} AB &= \sqrt{(6 - 1)^2 + [2 - (-4)]^2} \\ &= \sqrt{5^2 + 6^2} \\ &= \sqrt{25 + 36} \\ &= \sqrt{61} \approx 7.8 \end{aligned}$$

33. $A(0, 2), B(-3, 8)$

$$\begin{aligned} AB &= \sqrt{(-3 - 0)^2 + (8 - 2)^2} \\ &= \sqrt{(-3)^2 + 6^2} \\ &= \sqrt{9 + 36} \\ &= \sqrt{45} \approx 6.7 \end{aligned}$$

- $C(-2, 2), D(0, -4)$

$$\begin{aligned} CD &= \sqrt{[0 - (-2)]^2 + (-4 - 2)^2} \\ &= \sqrt{2^2 + (-6)^2} \\ &= \sqrt{4 + 36} \\ &= \sqrt{40} \approx 6.3 \end{aligned}$$

The segments are not congruent and $AB > CD$.

34. $E(1, 4), F(5, 1)$

$$\begin{aligned} EF &= \sqrt{(5 - 1)^2 + (1 - 4)^2} \\ &= \sqrt{4^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

- $G(-3, 1), H(1, 6)$

$$\begin{aligned} GH &= \sqrt{[1 - (-3)]^2 + (6 - 1)^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \approx 6.4 \end{aligned}$$

The segments are not congruent and $GH > EF$.

35. a. To find the x -coordinate of the midpoint, add the x -coordinates of the endpoints, and divide by 2. To find the y -coordinate of the midpoint, add the y -coordinates of the endpoints, and divide by 2.
- b. To find the x -coordinate of the other endpoint, multiply the x -coordinate of the midpoint by 2, and subtract the x -coordinate of the given endpoint. To find the y -coordinate of the other endpoint, multiply the y -coordinate of the midpoint by 2, and subtract the y -coordinate of the given endpoint.

$$36. T = \frac{60 + 42}{2} = \frac{102}{2} = 51$$

The height of T is 51 inches.

Chapter 1

37. a. Player A: $A(8, 4)$, $B(18, 7)$

$$\begin{aligned} AB &= \sqrt{(18 - 8)^2 + (7 - 4)^2} \\ &= \sqrt{10^2 + 3^2} \\ &= \sqrt{100 + 9} \\ &= \sqrt{109} \approx 10.4 \end{aligned}$$

Player B: $B(18, 7)$, $C(24, 14)$

$$\begin{aligned} BC &= \sqrt{(24 - 18)^2 + (14 - 7)^2} \\ &= \sqrt{6^2 + 7^2} \\ &= \sqrt{36 + 49} \\ &= \sqrt{85} \approx 9.2 \end{aligned}$$

Player A threw the ball about 10.4 meters. Player B threw the ball about 9.2 meters.

- b. $A(8, 4)$, $C(24, 14)$

$$\begin{aligned} AC &= \sqrt{(24 - 8)^2 + (14 - 4)^2} \\ &= \sqrt{16^2 + 10^2} \\ &= \sqrt{256 + 100} \\ &= \sqrt{356} \approx 18.9 \end{aligned}$$

Player A would have had to throw the ball about 18.9 meters to player C.

38. School: $S(20, -12)$

Mall: $M(-7, 10)$

$$\begin{aligned} SM &= \sqrt{[20 - (-7)]^2 + (-12 - 10)^2} \\ &= \sqrt{27^2 + (-22)^2} \\ &= \sqrt{729 + 484} \\ &= \sqrt{1213} \approx 34.82 \text{ blocks} \end{aligned}$$

$$\begin{aligned} \text{Distance in miles} &= 34.82 \times 0.1 \\ &= 3.482 \approx 3.5 \end{aligned}$$

The distance between the school and the mall is about 3.5 miles.

39. a. $PQ = |50 - 10| = 40$

$$QR = |80 - 10| = 70$$

$$\begin{aligned} PR &= \sqrt{40^2 + 70^2} \\ &= \sqrt{1600 + 4900} \\ &= \sqrt{6500} \approx 81 \end{aligned}$$

$$\text{Total distance} \approx 40 + 70 + 81 = 191$$

The distance around the park is about 191 yards.

b. Midpoint of $PR = \left(\frac{80 + 10}{2}, \frac{50 + 10}{2} \right)$
 $= \left(\frac{90}{2}, \frac{60}{2} \right) = (45, 30)$

$M(45, 30)$, $Q(10, 10)$

$$\begin{aligned} QM &= \sqrt{(10 - 45)^2 + (10 - 30)^2} \\ &= \sqrt{(-35)^2 + (-20)^2} \\ &= \sqrt{1225 + 400} \\ &= \sqrt{1625} \approx 40 \end{aligned}$$

The length of \overline{QM} is about 40 yards.

- c. $M(45, 30)$, $R(80, 10)$

$$\begin{aligned} MR &= \sqrt{(80 - 45)^2 + (10 - 30)^2} \\ &= \sqrt{35^2 + (-20)^2} \\ &= \sqrt{1225 + 400} \\ &= \sqrt{1625} \approx 40 \end{aligned}$$

So, $PQ = 40$, $QM \approx 40$, $MR \approx 40$, $RQ = 70$, and $QP = 40$. The total distance is

$$40 + 40 + 40 + 70 + 40 = 230 \text{ yards.}$$

$$\begin{aligned} \text{Travel time} &= \frac{\text{Distance traveled}}{\text{Rate of travel}} \\ &= \frac{230 \text{ yd}}{150 \text{ yd}} \\ &= \frac{230 \text{ yd}}{1} \cdot \frac{1 \text{ min}}{150 \text{ yd}} \\ &= \frac{230}{150} \approx 1.5 \end{aligned}$$

It takes the man about 1.5 minutes.

40. no; You have to take the absolute value of the difference of the y-coordinates.

$$41. \text{Midpoint} = \left(\frac{a + b}{2}, \frac{c + d}{2} \right) = \left(\frac{a + b}{2}, \frac{2c}{2} \right) = \left(\frac{a + b}{2}, c \right)$$

$$\text{Distance: } |b - a|$$

42. a. $AM = MB$; The midpoint M divides \overline{AB} into two equal parts.

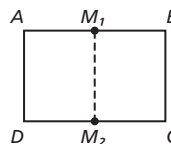
b. $AC < MB$; Because point C is between A and M , and $AC + CM = AM$, $AC < AM$. Because $AM = MB$, $AC < MB$.

c. impossible to tell; The problem does not provide any information about whether C or D is closer to M .

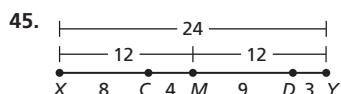
d. $MB > DB$; Because point D is between M and B , and $MD + DB = MB$, $MB > DB$.

43. location D for lunch; The total distance traveled if you return home is $AM + AM + AB + AB$. The total distance traveled if you go to location D for lunch is $AB + DB + DB + AB$. Because $DB < AM$, the second option involves less traveling.

44. *Sample answer:* Connect the midpoints of the lengths, connect the midpoints of the widths, connect opposite corners to create a diagonal; The regions do not have to be triangles.



Chapter 1



$$XC = \frac{2}{3}XM = \frac{2}{3}(12) = 8 \text{ cm}$$

$$MD = \frac{3}{4}MY = \frac{3}{4}(12) = 9 \text{ cm}$$

$$CM = 4 \text{ cm}$$

$$MD = 9 \text{ cm}$$

$$CD = CM + MD = 4 + 9 = 13 \text{ cm}$$

Maintaining Mathematical Proficiency

46. $P = 4 \cdot 5 = 20 \text{ cm}$

$$A = 5 \cdot 5 = 25 \text{ cm}^2$$

47. $P = 2 \cdot 10 + 2 \cdot 3$

$$= 20 + 6 = 26 \text{ ft}$$

$$A = 10 \cdot 3 = 30 \text{ ft}^2$$

48. $P = 3 + 4 + 5 = 12 \text{ m}$

$$A = \frac{1}{2}(3 \cdot 4)$$

$$= \frac{1}{2}(12)$$

$$= 6 \text{ m}^2$$

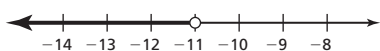
49. $P = 13 + 13 + 10 = 36 \text{ yd}$

$$A = \frac{1}{2}(12)(10)$$

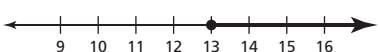
$$= 6 \cdot 10$$

$$= 60 \text{ yd}^2$$

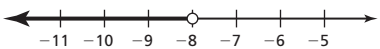
50. $a + 18 < 7$
 $a < -11$



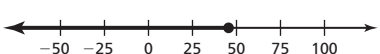
51. $y - 5 \geq 8$
 $y \geq 13$



52. $-3x > 24$
 $x < -8$

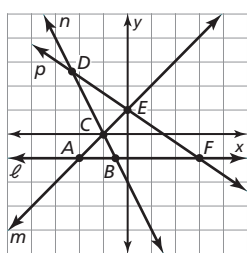


53. $\frac{z}{4} \leq 12$
 $z \leq 48$



1.1–1.3 What Did You Learn? (p. 27)

1. Sample answer:



2. To find the length of a segment when the y -values of two endpoints are the same, find the absolute value of the difference of the x -values. When the x -values are the same, find the absolute value of the difference of the y -values. Because the segments are congruent, you set these two quantities equal to each other.

3. Assume that all of the locations lie on a straight line and that D is between M and B . As such, you can assume that $DB < MB$. Based on the definition of midpoint, $AM = MB$. So, $DB < AM$.

1.1–1.3 Quiz (p. 28)

1. Any four of the points: $A, B, C, D, E, F, G, H, K, L$

2. Three collinear points are H, G, F or A, L, B or C, D, E .

3. Sample answer: Two lines are \overleftrightarrow{HF} and \overleftrightarrow{AB} .

4. Sample answer: Three coplanar points are C, E , and K .

5. Sample answer: The plane shaded green is AGB .

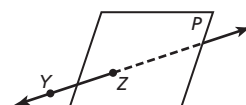
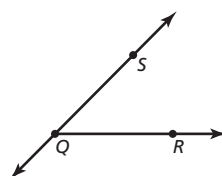
6. Sample answer: Two names for the plane shaded blue are KCE and KDE .

7. Three line segments can be any three of $\overline{HG}, \overline{GF}, \overline{CD}, \overline{DE}, \overline{AL}, \overline{LB}, \overline{AB}, \overline{CE}$, and \overline{HF} .

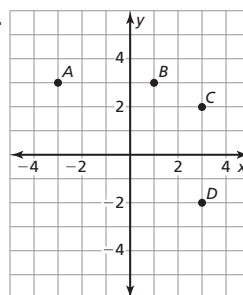
8. Sample answer: Three rays are $\overrightarrow{CE}, \overrightarrow{CD}$, and \overrightarrow{LB} .

9. Sample answer:

10. Sample answer:



11.



$$AB = |1 - (-3)|$$

$$= |1 + 3|$$

$$= |4|$$

$$= 4$$

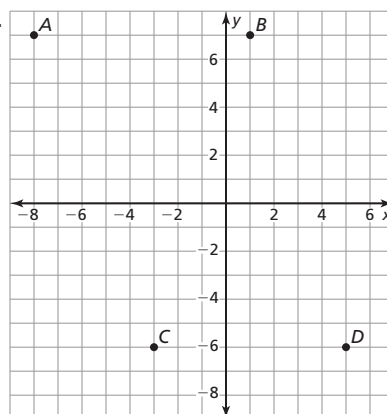
$$CD = |2 - (-2)|$$

$$= |2 + 2|$$

$$= |4| = 4$$

$$\overline{AB} \cong \overline{CD}$$

12.



$$AB = |1 - (-8)|$$

$$= |1 + 8|$$

$$= 9$$

$$CD = |5 - (-3)|$$

$$= |5 + 3|$$

$$= 8$$

$$\overline{AB} \not\cong \overline{CD}$$

13. $AC = AB + BC$

$$= 13 + 26$$

$$= 39$$

14. $AB = AC + CB$

$$62 = AC + 11$$

$$51 = AC$$

Chapter 1

$$15. M(x, y) = \left(\frac{4+2}{2}, \frac{3+(-3)}{2} \right) = \left(\frac{6}{2}, \frac{0}{2} \right) = (3, 0)$$

$$\begin{aligned} JK &= \sqrt{(4-2)^2 + [3-(-3)]^2} \\ &= \sqrt{2^2 + 6^2} \\ &= \sqrt{4 + 36} \\ &= \sqrt{40} \approx 6.3 \end{aligned}$$

$$16. M(x, y) = \left(\frac{-4+5}{2}, \frac{5+(-3)}{2} \right) = \left(\frac{1}{2}, \frac{2}{2} \right) = \left(\frac{1}{2}, 1 \right)$$

$$\begin{aligned} LN &= \sqrt{(-4-5)^2 + [5-(-3)]^2} \\ &= \sqrt{(-9)^2 + 8^2} \\ &= \sqrt{81 + 64} \\ &= \sqrt{145} \approx 12.0 \end{aligned}$$

$$17. M(x, y) = \left(\frac{-6+1}{2}, \frac{-1+2}{2} \right) = \left(-\frac{5}{2}, \frac{1}{2} \right)$$

$$\begin{aligned} PQ &= \sqrt{(-6-1)^2 + (-1-2)^2} \\ &= \sqrt{(-7)^2 + (-3)^2} \\ &= \sqrt{49 + 9} \\ &= \sqrt{58} \approx 7.6 \end{aligned}$$

18. Point M bisects \overline{RS} .

$$RM = MS$$

$$RM = 6(3) - 2$$

$$6x - 2 = 3x + 7$$

$$= 18 - 2 = 16$$

$$6x = 3x + 9$$

$$MS = 3(3) + 7$$

$$3x = 9$$

$$= 9 + 7 = 16$$

$$x = 3$$

$$RS = 16 + 16 = 32$$

$$19. M(0, 1) = \left(\frac{-6+x}{2}, \frac{3+y}{2} \right)$$

$$0 = \frac{-6+x}{2}$$

$$1 = \frac{3+y}{2}$$

$$0 = -6 + x$$

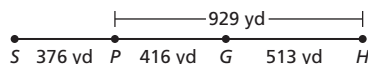
$$2 = 3 + y$$

$$6 = x$$

$$-1 = y$$

$$K(6, -1)$$

20. Let S = school, P = post office, G = grocery store, and H = home.



a. You should stop at the post office first.

b. The distance between the post office and grocery store is 416 yards.

c. The distance between the school and your house is $376 + 929 = 1305$ yards.

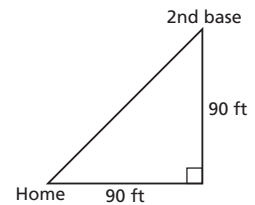
$$d. \frac{1305 \text{ yd}}{1} \cdot \frac{1 \text{ min}}{75 \text{ yd}} = \frac{1305}{75} \text{ min} = 17\frac{2}{5} \text{ min} = 17.4 \text{ min}$$

It will take $17\frac{2}{5}$ or 17.4 minutes to walk from the school to your house.

21. Home plate to second base:

$$\begin{aligned} d &= \sqrt{90^2 + 90^2} \\ &= \sqrt{8100 + 8100} \\ &= 90\sqrt{2} \approx 127 \text{ ft} \end{aligned}$$

The distance from home plate to second base is about 127 feet.



Home plate to the pitching mound is $\frac{1}{2}$ the distance from home plate to second base, $\frac{1}{2}(127) \approx 64$ feet.

The distance between home plate and the pitching mound is about 64 feet.

1.4 Explorations (p. 29)

1. a. Check students' work.

$$\begin{aligned} b. AB &= \sqrt{(-3-1)^2 + (1-4)^2} \\ &= \sqrt{(-4)^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-3-0)^2 + [1-(-3)]^2} \\ &= \sqrt{(-3)^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} = 5 \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(0-4)^2 + (-3-0)^2} \\ &= \sqrt{(-4)^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

$$\begin{aligned} DA &= \sqrt{(4-1)^2 + (0-4)^2} \\ &= \sqrt{3^2 + (-4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} = 5 \end{aligned}$$

So, the perimeter is $5 + 5 + 5 + 5 = 20$ centimeters.

c. Yes, because the slopes of \overline{AB} and \overline{CD} are $\frac{3}{4}$, and the slopes of \overline{AD} and \overline{BC} are $-\frac{4}{3}$. Because $\frac{3}{4}(-\frac{4}{3}) = -1$, the sides are perpendicular.

d. A square is a quadrilateral with four congruent sides and four right angles. Quadrilateral $ABCD$ is a square because $AB = BC = CD = DA = 5$, and all four angles are right angles. The area of quadrilateral $ABCD$ is $5 \cdot 5 = 25$ square centimeters.

2. a. $PQRS$: $P(1, 1)$, $Q(1, 0)$, $R(0, 0)$, $S(0, 1)$

$$CDR: C(0, -3), D(4, 0), R(0, 0)$$

$$CSB: C(0, -3), S(0, 1), B(-3, 1)$$

$$BPA: B(-3, 1), P(1, 1), A(1, 4)$$

$$AQD: A(1, 4), Q(1, 0), D(4, 0)$$

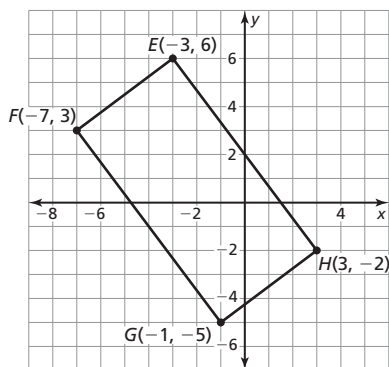
Chapter 1

- b. Area of $\triangle BPA = \frac{1}{2}(3)(4) = \frac{1}{2}(12) = 6 \text{ cm}^2$
 Area of $\triangle AQD = \frac{1}{2}(3)(4) = \frac{1}{2}(12) = 6 \text{ cm}^2$
 Area of $\triangle DRC = \frac{1}{2}(3)(4) = \frac{1}{2}(12) = 6 \text{ cm}^2$
 Area of $\triangle CSB = \frac{1}{2}(3)(4) = \frac{1}{2}(12) = 6 \text{ cm}^2$
 Area of square $PQRS = 1 \text{ cm}^2$

- c. yes; The area of the square $ABCD$ is the sum of the five smaller regions $4(6) + 1 = 25$ square centimeters and using the formula for the area of a square, $(\text{side})^2 = 5^2 = 25$ square centimeters.

3. To find the perimeter, use the Distance Formula to find the lengths of the sides and add the lengths together. To find the area, use the appropriate area formula and the dimensions of the figure, or partition the figure into shapes that have easily determined areas and add the areas together.

4. a.



$$\begin{aligned} \text{b. } EF &= \sqrt{[-3 - (-7)]^2 + (6 - 3)^2} \\ &= \sqrt{(-3 + 7)^2 + 3^2} \\ &= \sqrt{4^2 + 9} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

$$\begin{aligned} FG &= \sqrt{(-7 + 1)^2 + (3 + 5)^2} \\ &= \sqrt{(-6)^2 + 8^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} = 10 \end{aligned}$$

$$\begin{aligned} HE &= \sqrt{(-3 - 3)^2 + [6 - (-2)]^2} \\ &= \sqrt{(-6)^2 + 8^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} = 10 \end{aligned}$$

$$\begin{aligned} GH &= \sqrt{(-1 - 3)^2 + [-5 - (-2)]^2} \\ &= \sqrt{(-4)^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

So, the perimeter is $5 + 10 + 5 + 10 = 30$ centimeters.

- c. Yes, because the slopes of adjacent sides are negative reciprocals, $\left(\frac{3}{4}\right)\left(-\frac{4}{3}\right) = -1$, the sides are perpendicular.

$$\text{Slope of } EF = \frac{6 - 3}{-3 - (-7)} = \frac{3}{4}$$

$$\text{Slope of } EH = \frac{-2 - 6}{3 - (-3)} = \frac{-8}{6} = -\frac{4}{3}$$

$$\text{Slope of } HG = \frac{-2 - (-5)}{3 - (-1)} = \frac{3}{4}$$

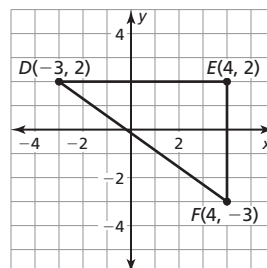
$$\text{Slope of } GF = \frac{-5 - 3}{-1 - (-7)} = \frac{-8}{6} = -\frac{4}{3}$$

- d. A square is a quadrilateral with four right angles and four congruent sides. Quadrilateral $EFHG$ is not a square. Two opposite sides have length 5 and the other opposite sides have length 10. The area is $5 \cdot 10 = 50$ square centimeters.

1.4 Monitoring Progress (pp. 30–33)

- The polygon has five sides; it is a pentagon. The polygon is concave.
- The polygon has seven sides; it is a heptagon. The polygon is convex.

3.



$$DE = |-3 - 4| = |-7| = 7$$

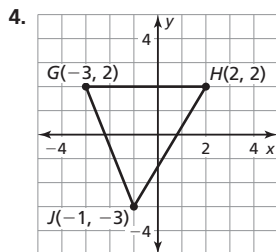
$$EF = |2 - (-3)| = |2 + 3| = |5| = 5$$

$$\begin{aligned} DF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[4 - (-3)]^2 + (-3 - 2)^2} \\ &= \sqrt{(4 + 3)^2 + (-5)^2} \\ &= \sqrt{7^2 + 25} \\ &= \sqrt{49 + 25} \\ &= \sqrt{74} \approx 8.60 \end{aligned}$$

$$DE + EF + FD \approx 7 + 5 + 8.60 = 20.60$$

The perimeter of $\triangle DEF$ is about 20.60 units.

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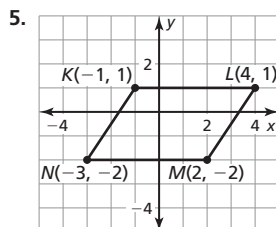
$$GH = |-3 - 2| = |-5| = 5$$

$$\begin{aligned} GJ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-3 - (-1)]^2 + [2 - (-3)]^2} \\ &= \sqrt{(-3 + 1)^2 + (2 + 3)^2} \\ &= \sqrt{(-2)^2 + 5^2} \\ &= \sqrt{4 + 25} \\ &= \sqrt{29} \approx 5.39 \end{aligned}$$

$$\begin{aligned} HJ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1 - 2)^2 + (-3 - 2)^2} \\ &= \sqrt{(-3)^2 + (-5)^2} \\ &= \sqrt{9 + 25} \\ &= \sqrt{34} \approx 5.83 \end{aligned}$$

$$GH + HJ + JG \approx 5 + 5.83 + 5.39 = 16.22$$

The perimeter of $\triangle GHJ$ is about 16.22 units.



$$KL = |-1 - 4| = |-5| = 5$$

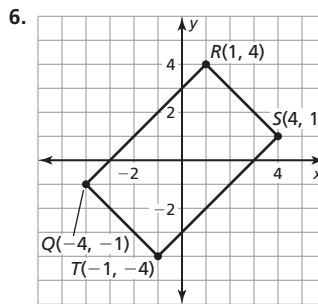
$$\begin{aligned} KN &= \sqrt{[-3 - (-1)]^2 + (-2 - 1)^2} \\ &= \sqrt{(-3 + 1)^2 + (-3)^2} \\ &= \sqrt{(-2)^2 + 9} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13} \approx 3.61 \end{aligned}$$

$$NM = |2 - (-3)| = |2 + 3| = |5| = 5$$

$$\begin{aligned} LM &= \sqrt{(4 - 2)^2 + [1 - (-2)]^2} \\ &= \sqrt{2^2 + 3^2} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13} \approx 3.61 \end{aligned}$$

$$KL + LM + MN + NK \approx 5 + 3.61 + 5 + 3.61 = 17.22$$

The perimeter of quadrilateral $KLMN$ is about 17.22 units.



$$\begin{aligned} QR &= \sqrt{(-4 - 1)^2 + (-1 - 4)^2} \\ &= \sqrt{(-5)^2 + (-5)^2} \\ &= \sqrt{25 + 25} \\ &= \sqrt{50} \approx 7.07 \end{aligned}$$

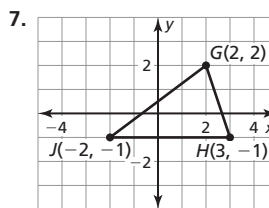
$$\begin{aligned} RS &= \sqrt{(1 - 4)^2 + (4 - 1)^2} \\ &= \sqrt{(-3)^2 + 3^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \approx 4.24 \end{aligned}$$

$$\begin{aligned} ST &= \sqrt{[4 - (-1)]^2 + [1 - (-4)]^2} \\ &= \sqrt{5^2 + 5^2} \\ &= \sqrt{25 + 25} \\ &= \sqrt{50} \approx 7.07 \end{aligned}$$

$$\begin{aligned} TQ &= \sqrt{[-4 - (-1)]^2 + [-1 - (-4)]^2} \\ &= \sqrt{(-3)^2 + (-3)^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \approx 4.24 \end{aligned}$$

$$QR + RS + ST + TQ \approx 7.07 + 4.24 + 7.07 + 4.24 = 22.62$$

The perimeter of quadrilateral $QRST$ is about 22.62 units.



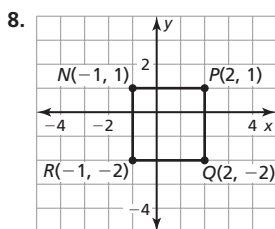
$$\text{Base } JH = |3 - (-2)| = |3 + 2| = |5| = 5$$

Height is the distance from point G to JH , which is 3 units.

$$\begin{aligned} A &= \frac{1}{2} \cdot b \cdot h \\ &= \frac{1}{2}(5)(3) \\ &= \frac{1}{2}(15) = 7.5 \end{aligned}$$

The area of $\triangle GHJ$ is 7.5 square units.

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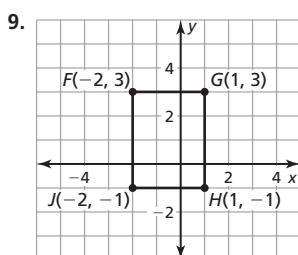


Length: $RQ = |2 - (-1)| = |2 + 1| = |3| = 3$

Width: $PQ = |1 - (-2)| = |1 + 2| = |3| = 3$

Area = ℓw
 $= 3 \cdot 3 = 9$

The area of quadrilateral $NPQR$ is 9 square units.

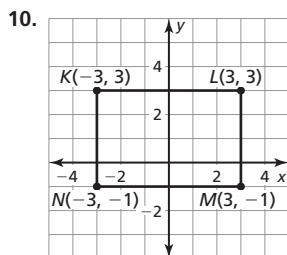


Length: $JH = |-2 - 1| = |-3| = 3$

Width: $GH = |3 - (-1)| = |3 + 1| = 4$

Area = ℓw
 $= 3 \cdot 4 = 12$

The area of quadrilateral $FGHI$ is 12 square units.



Length: $NM = |3 - (-3)| = |3 + 3| = |6| = 6$

Width: $LM = |3 - (-1)| = |3 + 1| = |4| = 4$

Area = ℓw
 $= 6 \cdot 4 = 24$

The area of quadrilateral $KLMN$ is 24 square units.

11. Length: $|2 - 6| = |-4| = 4$
 Width: $|2 - (-3)| = |2 + 3| = 5$
 Area = ℓw
 $= 4 \cdot 5 = 20$

The area of the patio is 20 square feet.

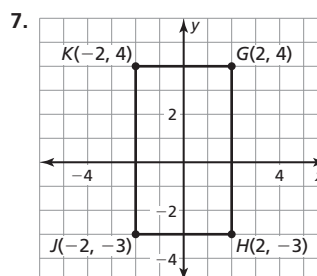
1.4 Exercises (pp. 34–36)

Vocabulary and Core Concept Check

- The perimeter of a square with side length s is $P = 4s$.
- The formulas to use to find the area of a triangle in a coordinate plane are the Distance Formula to find either the height and/or base. Then use the area formula, $A = \frac{1}{2}bh$.

Monitoring Progress and Modeling with Mathematics

- The polygon has four sides; it is a quadrilateral. The polygon is concave.
- The polygon has three sides; it is a triangle. The polygon is convex.
- The polygon has five sides; it is a pentagon. The polygon is convex.
- The polygon has six sides; it is a hexagon. The polygon is concave.



$KG = |-2 - 2| = |-4| = 4$

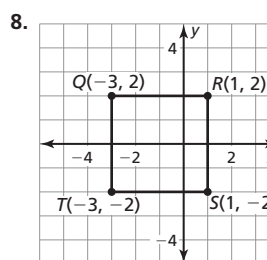
$GH = |4 - (-3)| = |4 + 3| = 7$

$HJ = |2 - (-2)| = |2 + 2| = 4$

$JK = |-3 - 4| = |-7| = 7$

$KG + GH + HJ + JK = 4 + 7 + 4 + 7 = 22$

The perimeter of quadrilateral $KGHJ$ is 22 units.



$QR = |1 - (-3)| = |1 + 3| = |4| = 4$

$RS = |-2 - 2| = |-4| = 4$

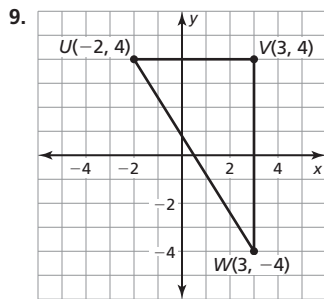
$ST = |1 - (-3)| = |1 + 3| = 4$

$TQ = |-2 - 2| = |-4| = 4$

$QR + RS + ST + TQ = 4 + 4 + 4 + 4 = 16$

The perimeter of quadrilateral $QRST$ is 16 units.

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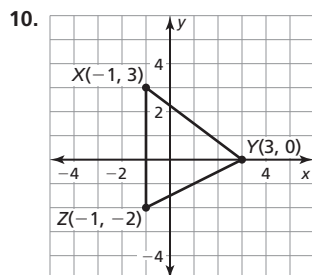
$$UV = |-2 - 3| = |-5| = 5$$

$$VW = |4 - (-4)| = |4 + 4| = |8| = 8$$

$$\begin{aligned} UW &= \sqrt{(-2 - 3)^2 + [4 - (-4)]^2} \\ &= \sqrt{(-5)^2 + 8^2} \\ &= \sqrt{25 + 64} \\ &= \sqrt{89} \\ &\approx 9.43 \end{aligned}$$

$$\begin{aligned} UV + VW + UW &\approx 5 + 8 + 9.43 \\ &= 22.43 \end{aligned}$$

The perimeter of $\triangle UVW$ is about 22.43 units.



$$XZ = |3 - (-2)| = |3 + 2| = 5$$

$$\begin{aligned} XY &= \sqrt{(-1 - 3)^2 + (3 - 0)^2} \\ &= \sqrt{(-4)^2 + 3^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

$$\begin{aligned} YZ &= \sqrt{[3 - (-1)]^2 + [0 - (-2)]^2} \\ &= \sqrt{4^2 + 2^2} \\ &= \sqrt{16 + 4} \\ &= \sqrt{20} \\ &\approx 4.47 \end{aligned}$$

$$XZ + XY + YZ \approx 5 + 5 + 4.47 = 14.47$$

The perimeter of $\triangle XYZ$ is about 14.47 units.

11.

$$\begin{aligned} PL &= \sqrt{(-1 - 1)^2 + (-2 - 4)^2} \\ &= \sqrt{(-2)^2 + (-6)^2} \\ &= \sqrt{4 + 36} \\ &= \sqrt{40} \approx 6.32 \end{aligned}$$

$$\begin{aligned} LM &= \sqrt{(1 - 4)^2 + (4 - 0)^2} \\ &= \sqrt{(-3)^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} = 5 \end{aligned}$$

$$MN = |4 - 2| = 2$$

$$\begin{aligned} PN &= \sqrt{[2 - (-1)]^2 + [0 - (-2)]^2} \\ &= \sqrt{(2 + 1)^2 + 2^2} \\ &= \sqrt{9 + 4} \\ &= \sqrt{13} \approx 3.61 \end{aligned}$$

$$PL + LM + MN + PN \approx 6.32 + 5 + 2 + 3.61 = 16.93$$

The perimeter of quadrilateral $PLMN$ is about 16.93 units.

12.

$$\begin{aligned} AB &= \sqrt{(0 - 2)^2 + (4 - 0)^2} \\ &= \sqrt{(-2)^2 + 4^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \approx 4.47 \end{aligned}$$

$$BC = |0 - (-2)| = |0 + 2| = 2$$

$$CD = |2 - 0| = |2| = 2$$

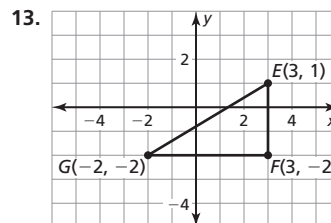
$$\begin{aligned} DE &= \sqrt{[0 - (-2)]^2 + (-2 - 2)^2} \\ &= \sqrt{2^2 + (-4)^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \approx 4.47 \end{aligned}$$

$$EF = |4 - 2| = |2| = 2$$

$$FA = |0 - (-2)| = |2| = 2$$

$$\begin{aligned} AB + BC + CD + DE + EF + FA \\ \approx 4.47 + 2 + 2 + 4.47 + 2 + 2 \\ = 16.94 \end{aligned}$$

The perimeter of polygon $ABCDEF$ is about 16.94 units.



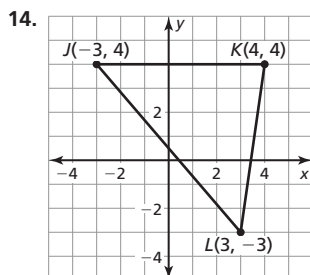
$$\text{Height: } EF = |1 - (-2)| = |1 + 2| = |3| = 3$$

$$\text{Base: } GF = |3 - (-2)| = |3 + 2| = |5| = 5$$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(3)(5) \\ &= \frac{1}{2}(15) = 7.5 \end{aligned}$$

The area of $\triangle EFG$ is 7.5 square units.

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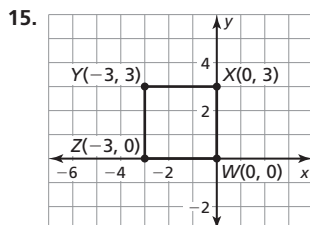
The height is the distance from point L to \overline{JK} :

$$|4 - (-3)| = |4 + 3| = 7$$

$$\text{Base: } JK = |-3 - 4| = |-7| = 7$$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(7 \cdot 7) \\ &= \frac{1}{2}(49) = 24.5 \end{aligned}$$

The area of $\triangle JKL$ is 24.5 square units.

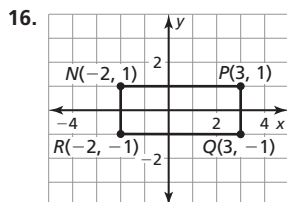


$$\text{Length: } ZW = |-3 - 0| = |-3| = 3$$

$$\text{Width: } WX = |3| = 3$$

$$\begin{aligned} \text{Area} &= \ell w \\ &= 3 \cdot 3 = 9 \end{aligned}$$

The area of quadrilateral $XWZY$ is 9 square units.



$$\text{Length: } RQ = |3 - (-2)| = |3 + 2| = 5$$

$$\text{Width: } PQ = |1 - (-1)| = |1 + 1| = |2| = 2$$

$$\begin{aligned} \text{Area} &= \ell w \\ &= 5 \cdot 2 = 10 \end{aligned}$$

The area of quadrilateral $NPRQ$ is 10 square units.

17. $CD = |-1 - (-5)| = |-1 + 5| = |4| = 4$

$$\begin{aligned} DE &= \sqrt{(4 - 2)^2 + [-3 - (-5)]^2} \\ &= \sqrt{2^2 + (-3 + 5)^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \approx 2.83 \end{aligned}$$

$$\begin{aligned} CE &= \sqrt{(4 - 2)^2 + [-1 - (-3)]^2} \\ &= \sqrt{2^2 + (-1 + 3)^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \approx 2.83 \end{aligned}$$

$$CD + DE + CE \approx 4 + 2.83 + 2.83 = 9.66$$

The perimeter of $\triangle CDE$ is about 9.66 units.

18. $BC = \sqrt{(0 - 4)^2 + [3 - (-1)]^2}$

$$\begin{aligned} &= \sqrt{(-4)^2 + (3 + 1)^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \approx 5.66 \end{aligned}$$

$$\begin{aligned} CE &= \sqrt{(4 - 2)^2 + [-1 - (-3)]^2} \\ &= \sqrt{2^2 + (-1 + 3)^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \approx 2.83 \end{aligned}$$

$$\begin{aligned} EF &= \sqrt{[2 - (-2)]^2 + (-3 - 1)^2} \\ &= \sqrt{(2 + 2)^2 + (-4)^2} \\ &= \sqrt{4^2 + 16} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \approx 5.66 \end{aligned}$$

$$\begin{aligned} BF &= \sqrt{[0 - (-2)]^2 + (3 - 1)^2} \\ &= \sqrt{2^2 + 2^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \approx 2.83 \end{aligned}$$

$$\begin{aligned} BC + CE + EF + BF &\approx 5.66 + 2.83 + 5.66 + 2.83 \\ &= 16.98 \end{aligned}$$

The perimeter of rectangle $BCEF$ is about 16.98 units.

19. $AB = \sqrt{[0 - (-5)]^2 + (3 - 4)^2}$

$$\begin{aligned} &= \sqrt{(0 + 5)^2 + (-1)^2} \\ &= \sqrt{5^2 + 1} \\ &= \sqrt{25 + 1} \\ &= \sqrt{26} \approx 5.10 \end{aligned}$$

$$\begin{aligned} BF &= \sqrt{[0 - (-2)]^2 + (3 - 1)^2} \\ &= \sqrt{2^2 + 2^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \approx 2.83 \end{aligned}$$

$$\begin{aligned} FA &= \sqrt{[-2 - (-5)]^2 + (1 - 4)^2} \\ &= \sqrt{(-2 + 5)^2 + (-3)^2} \\ &= \sqrt{3^2 + (-3)^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \approx 4.24 \end{aligned}$$

$$AB + BF + FA \approx 5.10 + 2.83 + 4.24 = 12.17$$

The perimeter of $\triangle ABF$ is about 12.17 units.

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$$\begin{aligned} 20. AB &= \sqrt{(-5 - 0)^2 + (4 - 3)^2} \\ &= \sqrt{(-5)^2 + 1^2} \\ &= \sqrt{25 + 1} \\ &= \sqrt{26} \approx 5.10 \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(0 - 4)^2 + [3 - (-1)]^2} \\ &= \sqrt{(-4)^2 + (3 + 1)^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \approx 5.66 \end{aligned}$$

$$CD = |-5 - (-1)| = |-5 + 1| = |-4| = 4$$

$$\begin{aligned} DA &= \sqrt{[4 - (-5)]^2 + (-5 - 4)^2} \\ &= \sqrt{(4 + 5)^2 + (-9)^2} \\ &= \sqrt{9^2 + (-9)^2} \\ &= \sqrt{81 + 81} \\ &= \sqrt{162} \approx 12.73 \end{aligned}$$

$$AB + BC + CD + DA \approx 5.10 + 5.66 + 4 + 12.73 = 27.49$$

The perimeter of quadrilateral $ABCD$ is about 27.49 units.

$$21. \text{Base: } CD = |-5 - (-1)| = |-5 + 1| = |-4| = 4$$

The height is the distance from point E to \overline{CD} :

$$|2 - 4| = |-2| = 2$$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \cdot 4 \cdot 2 \\ &= \frac{1}{2}(8) \\ &= 4 \end{aligned}$$

The area of $\triangle CDE$ is 4 square units.

$$\begin{aligned} 22. \text{Length: } EF &= \sqrt{[2 - (-2)]^2 + (-3 - 1)^2} \\ &= \sqrt{4^2 + (-4)^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \end{aligned}$$

$$\begin{aligned} \text{Width: } CE &= \sqrt{(4 - 2)^2 + [-1 - (-3)]^2} \\ &= \sqrt{2^2 + (-1 + 3)^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \end{aligned}$$

$$\begin{aligned} A &= \ell w \\ &= \sqrt{32} \cdot \sqrt{8} \\ &= \sqrt{256} \\ &= 16 \end{aligned}$$

The area of rectangle $BCEF$ is 16 square units.

$$\begin{aligned} 23. \text{Base: } AF &= \sqrt{[-2 - (-5)]^2 + (1 - 4)^2} \\ &= \sqrt{(-2 + 5)^2 + (-3)^2} \\ &= \sqrt{3^2 + 9} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \end{aligned}$$

$$\begin{aligned} \text{Height: } FB &= \sqrt{(-2 - 0)^2 + (1 - 3)^2} \\ &= \sqrt{(-2)^2 + (-2)^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(\sqrt{18} \cdot \sqrt{8}) \\ &= \frac{1}{2}\sqrt{144} \\ &= \frac{1}{2}(12) \\ &= 6 \end{aligned}$$

The area of $\triangle ABF$ is 6 square units.

$$24. \text{Area of } \triangle ABF = 6$$

$$\text{Area of rectangle } BCEF = 16$$

$$\text{Area of } \triangle CED = 4$$

$$\begin{aligned} \text{Area of } ABCD &= \text{Area of } \triangle ABF + \text{Area of rectangle } BCEF \\ &\quad + \text{Area of } \triangle CED \\ &= 6 + 16 + 4 \\ &= 26 \end{aligned}$$

The area of quadrilateral $ABCD$ is 26 square units.

$$25. \text{The length is 5 not 4.}$$

$$\text{Length} = |4 - (-1)| = |4 + 1| = 5$$

$$\begin{aligned} P &= 2(5) + 2(3) \\ &= 10 + 6 \\ &= 16 \end{aligned}$$

So, the perimeter is 16 units.

$$26. \text{The height is the distance from point } A \text{ to the base } BC.$$

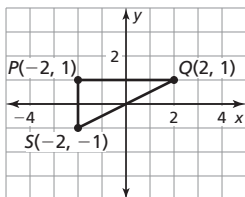
$$\text{Height} = |3 - 1| = 2$$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(4)(2) \\ &= 2(2) \\ &= 4 \end{aligned}$$

So, the area is 4 square units.

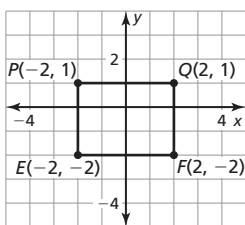
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27. B; The third point S has the coordinates $(-2, -1)$. That would give a height of 2 and the base (PQ) is 4.



$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(4)(2) \\ &= \frac{1}{2}(8) \\ &= 4 \text{ square units} \end{aligned}$$

28. C; The points $E(-2, 2)$ and $F(2, -2)$ are the remaining vertices of a rectangle with a perimeter of 14 units.



$$\begin{aligned} PQ &= |-2 - 2| = |-4| = 4 \\ QF &= |1 - (-2)| \\ &= |1 + 2| = |3| = 3 \\ FE &= |-2 - 2| = |-4| = 4 \\ EP &= |-2 - 1| = |-3| = 3 \end{aligned}$$

$$\begin{aligned} PQ + QF + FE + EP &= 4 + 3 + 4 + 3 \\ &= 14 \end{aligned}$$

29. a. Square $EFGH$ has a side length 2. So, the area of square $EFGH$ is $2^2 = 4$ square units. Square $EJKL$ has a side length 4. So, the area of square $EJKL$ is $4^2 = 16$ square units. The perimeter of $EFGH$ is $4 \cdot 2 = 8$. If the perimeter doubles, then each side is 4 units and the area is 16 square units. So, the area of the original square is multiplied by 4.

- b. yes; Doubling the perimeter of the square, which is the same as doubling the side length, quadruples the area in all cases.

$$P = 4s \text{ and } A = s^2$$

If the side is doubled ($2s$), then $P = 4(2s) = 8s$ and $A = (2s)^2 = 4s^2$.

30. a. $TS = |7 - 1| = 6$

$$SR = |13 - 1| = 12$$

$$\text{Area} = \ell w = 6 \cdot 12 = 72$$

The area of the garden is 72 square feet.

- b. $72 \div 9 = 8$

So, 8 zucchini plants can be planted.

- c. $TW = |4 - 1| = 3$

$$WV = |4 - 1| = 3$$

$$\text{Area} = \ell w = 3 \cdot 3 = 9$$

The area to grow lettuce is 9 square feet. There are 4 heads of lettuce per square foot.

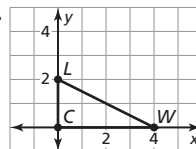
$$4(9) = 36 \text{ heads of lettuce}$$

The area to grow zucchini is $72 - 9 = 63$ square feet.

$$63 \div 9 = 7 \text{ zucchini plants}$$

So, 36 heads of lettuce and 7 zucchini plants can be planted.

31. a.



- b. The distance between the car and the waterfall is 4 miles. The distance from the car to the lookout point is 2 miles. The distance from the lookout point to the waterfall is

$$(LW)^2 = 4^2 + 2^2$$

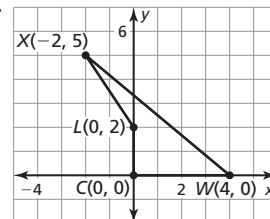
$$(LW)^2 = 16 + 4$$

$$LW = \sqrt{20} \approx 4.47 \text{ miles.}$$

$$CW + LC + LW \approx 4 + 2 + 4.47 = 10.47$$

So, the total distance is about 10.47 miles.

- c.



$$CW = |0 - 4| = 4$$

$$WX = \sqrt{[4 - (-2)]^2 + (0 - 5)^2}$$

$$= \sqrt{6^2 + (-5)^2}$$

$$= \sqrt{36 + 25}$$

$$= \sqrt{61} \approx 7.81$$

$$XL = \sqrt{(-2 - 0)^2 + (5 - 2)^2}$$

$$= \sqrt{(-2)^2 + 3^2}$$

$$= \sqrt{4 + 9}$$

$$= \sqrt{13} \approx 3.61$$

$$LC = |2 - 0| = 2$$

$$CW + WX + XL + LC \approx 4 + 7.81 + 3.61 + 2 = 17.42$$

The total distance traveled is about 17.42 miles.

32. The rectangle has a greater area than the triangle. The base and height are the same length for each. So, the triangle's area will be half of the rectangle's area. The rectangle has a greater perimeter than the triangle. Two of the dimensions are the same, and the triangle's third side is shorter than the total of the rectangle's remaining two sides.

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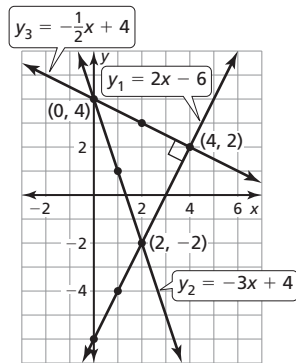
33. a. $y_1 = 2x - 6$ has a slope of 2.

$$y_2 = -3x + 4 \text{ has a slope of } -3.$$

$$y_3 = -\frac{1}{2}x + 4 \text{ has a slope of } -\frac{1}{2}.$$

To determine if two lines are perpendicular, the product of their slopes is equal to -1 . Because $2\left(-\frac{1}{2}\right) = -1$, y_1 and y_3 are perpendicular.

- b. The vertices are $(0, 4)$, $(4, 2)$, and $(2, -2)$.



- c. Distance between $(0, 4)$ and $(4, 2)$:

$$\begin{aligned} d &= \sqrt{(0 - 4)^2 + (4 - 2)^2} \\ &= \sqrt{(-4)^2 + 2^2} \\ &= \sqrt{16 + 4} \\ &= \sqrt{20} \approx 4.47 \end{aligned}$$

Distance between $(4, 2)$ and $(2, -2)$:

$$\begin{aligned} d &= \sqrt{(4 - 2)^2 + [2 - (-2)]^2} \\ &= \sqrt{2^2 + 4^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \approx 4.47 \end{aligned}$$

Distance between $(0, 4)$ and $(2, -2)$:

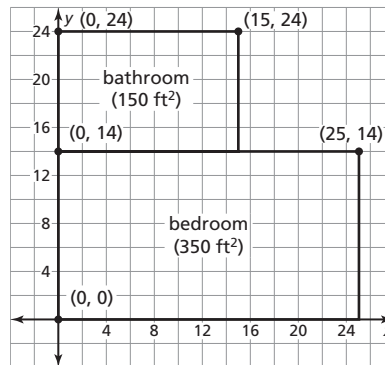
$$\begin{aligned} d &= \sqrt{(0 - 2)^2 + [4 - (-2)]^2} \\ &= \sqrt{(-2)^2 + (4 + 2)^2} \\ &= \sqrt{4 + 36} \\ &= \sqrt{40} \approx 6.32 \end{aligned}$$

$$P = \sqrt{20} + \sqrt{20} + \sqrt{40} \approx 15.27$$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(\sqrt{20})(\sqrt{20}) \\ &= \frac{1}{2}(\sqrt{400}) \\ &= \frac{1}{2}(20) \\ &= 10 \end{aligned}$$

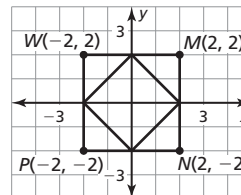
The perimeter of the triangle is about 15.27 units. The area of the triangle is 10 square units.

34. Sample answer:



35. a. The side length of the square is 4 units. So, the perimeter is $4 \cdot 4 = 16$ units and the area is $4^2 = 16$ square units.

- b.



yes; The sides are all the same length because each one is the hypotenuse of a right triangle with legs that are each 2 units long. Because the slopes of the lines of each side are either 1 or -1 , they are perpendicular.

- c. $c^2 = 2^2 + 2^2$

$$c^2 = 4 + 4$$

$$c^2 = 8$$

$$c = \sqrt{8}$$

$$\text{Perimeter} = 4\sqrt{8} \approx 11.31 \text{ units}$$

$$\text{Area} = (\sqrt{8})^2 = 8 \text{ square units}$$

The area of the inner square is half the area of the larger square.

36. $QR = |1 - (-2)| = |1 + 2| = |3| = 3$

$$RS = |-2 - 2| = |-4| = |4| = 4$$

$$QS = \sqrt{(QR)^2 + (RS)^2}$$

$$QS = \sqrt{3^2 + 4^2}$$

$$QS = \sqrt{9 + 16}$$

$$QS = \sqrt{25} = 5$$

$$\text{Perimeter of } \triangle QRS = 3 + 4 + 5 = 12 \text{ units}$$

$$\text{Area of } \triangle QRS = \frac{1}{2}(3)(4) = \frac{12}{2} = 6 \text{ square units}$$

A rectangle that is 1×5 will have the same perimeter as the triangle (12 units), but not the same area (5 square units). So, your friend is not correct.

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37. Find the lengths of the sides in terms of x .

$$\begin{aligned} BC &= \sqrt{(-1-2)^2 + [2-(-2)]^2} \\ &= \sqrt{(-3)^2 + 4^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{[x-(-1)]^2 + (2-2)^2} \\ &= \sqrt{(x+1)^2} \\ &= x+1 \end{aligned}$$

$$\begin{aligned} AB &= \sqrt{(x-2)^2 + [2-(-2)]^2} \\ &= \sqrt{(x-2)^2 + 4^2} \\ &= \sqrt{(x-2)^2 + 16} \end{aligned}$$

Triangle ABC has a perimeter of 12 units.

$$\begin{aligned} BC + AC + AB &= 12 \\ 5 + x + 1 + \sqrt{(x-2)^2 + 16} &= 12 \\ \sqrt{(x-2)^2 + 16} &= 6 - x \\ (x-2)^2 + 16 &= (6-x)^2 \\ x^2 - 4x + 20 &= x^2 - 12x + 36 \\ -4x + 20 &= -12x + 36 \\ 8x &= 16 \\ x &= 2 \end{aligned}$$

Maintaining Mathematical Proficiency

38. $3x - 7 = 2$

$$3x = 9$$

$$x = 3$$

39. $5x + 9 = 4$

$$5x = -5$$

$$x = -1$$

40. $x + 4 = x - 12$

$$4 \neq -12$$

The equation has no solution.

41. $4x - 9 = 3x + 5$

$$x - 9 = 5$$

$$x = 14$$

42. $11 - 2x = 5x - 3$

$$11 = 7x - 3$$

$$14 = 7x$$

$$2 = x$$

43. $\frac{x+1}{2} = 4x - 3$

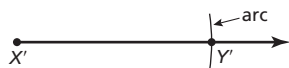
$$x + 1 = 8x - 6$$

$$1 = 7x - 6$$

$$7 = 7x$$

$$1 = x$$

44. Draw a ray using a straightedge and label the left endpoint of the ray x' . Open the compass and place the point of the compass on x and the pencil point on y . Without changing the setting of the compass, place the point on x' and mark the ray with an arc.



Label the intersection of the ray and the arc as y' .

1.5 Explorations (p. 37)

- $m\angle AOB = 35^\circ$ and it is acute.
 - $m\angle AOC = 65^\circ$ and it is acute.
 - $m\angle BOC = |65 - 35| = 30^\circ$ and it is acute.
 - $m\angle BOE = |145 - 35| = 110^\circ$ and it is obtuse.
 - $m\angle COE = |145 - 65| = 80^\circ$ and it is acute.
 - $m\angle COD = |110 - 65| = 45^\circ$ and it is acute.
 - $m\angle BOD = |110 - 35| = 75^\circ$ and it is acute.
 - $m\angle AOE = 145^\circ$ and it is obtuse.
- Check students' work.
 - Check students' work.
 - yes; for a hexagon, $n = 6$ and $180(6 - 2) = 180(4) = 720^\circ$. Each interior vertex angle of the hexagon is 120° , so, $6(120) = 720^\circ$.
 - First hexagon: The sum of the angle measures for each trapezoid is 360° . So, the total interior angle measure is $2 \cdot 360 = 720^\circ$.
Second hexagon: The sum of the angle measures for each triangle is 180° . The sum of the angle measures for the rectangle is 360° . So, the total sum of the interior angles is $180 + 360 + 180 = 720^\circ$.
Third hexagon: The sum of the angle measures for each triangle is 180° . So, the total interior angle measure is $6 \cdot 180^\circ = 1080^\circ$.
no; The first two hexagons have an interior polygon angle sum of 720° ; however, the third hexagon includes an interior point which is the vertex meeting point of the six triangles. This common vertex intersection adds an additional 360° to the combined sum of interior angles.
- Angles can be measured using a protractor. When the measure is greater than 0° and less than 90° , the angle is acute. When the measure is equal to 90° , the angle is right. When the measure is greater than 90° and less than 180° , the angle is obtuse. When the measure is 180° , the angle is straight.

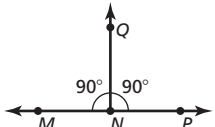
1.5 Monitoring Progress (pp. 38–42)

- Three names for the angle are $\angle PQR$, $\angle RQP$, and $\angle Q$.
- Three names for the angle are $\angle 1$, $\angle Y$, and $\angle XYZ$ (or $\angle ZYX$).
- Three names for the angle are $\angle 2$, $\angle E$, and $\angle DEF$ (or $\angle FED$).
- \overrightarrow{HJ} lines up with 0° on the inner scale of the protractor. \overrightarrow{HM} lines up with 145° on the inner scale. So, $m\angle JHM = 145^\circ$. It is an obtuse angle.
- \overrightarrow{HM} lines up with 145° on the inner scale of the protractor. \overrightarrow{HK} lines up with 55° on the inner scale. So, $m\angle JHM = |145 - 55| = 90^\circ$. It is a right angle.

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6. \overrightarrow{HM} lines up with 145° on the inner scale of the protractor. \overrightarrow{HL} lines up with 90° on the inner scale. So, $m\angle MHL = |145 - 90| = 55^\circ$. It is an acute angle.
7. $\angle DAB$ is not congruent to $\angle FEH$. $\angle DAB$ appears to be a right angle and $\angle FEH$ is obtuse.
8. $m\angle KLN + m\angle NLM = m\angle KLM$
 $(10x - 5)^\circ + (4x + 3)^\circ = 180^\circ$
 $14x - 2 = 180$
 $14x = 182$
 $x = 13$
 $m\angle KLN = (10x - 5)^\circ$
 $= (10 \cdot 13 - 5)^\circ = (130 - 5)^\circ = 125^\circ$
 $m\angle NLM = (4x + 3)^\circ$
 $= (4 \cdot 13 + 3)^\circ = (52 + 3)^\circ = 55^\circ$

9. $m\angle EFH + m\angle HFG = m\angle EFG$
 $(2x + 2)^\circ + (x + 1)^\circ = 90^\circ$
 $3x + 3 = 90$
 $3x = 87$
 $x = 29$
 $m\angle EFH = (2x + 2)^\circ = (2 \cdot 29 + 2)^\circ = (58 + 2)^\circ = 60^\circ$
 $m\angle HFG = (x + 1)^\circ = (29 + 1)^\circ = 30^\circ$

10.  $m\angle MNQ = 90^\circ$
 $m\angle QNP = 90^\circ$

1.5 Exercises (pp. 43–46)

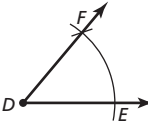
Vocabulary and Core Concept Check

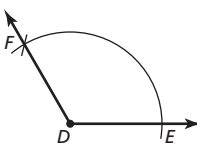
- Two angles are congruent angles when they have the same measure.
- $\angle BCA$ does not belong because $\angle 1$, $\angle BAC$, and $\angle CAB$ all name the same angle.

Monitoring Progress and Modeling with Mathematics

- The names for the angle are $\angle B$, $\angle ABC$, and $\angle CBA$.
- The names for the angle are $\angle G$, $\angle FGH$, and $\angle HGF$.
- The names for the angle are $\angle K$, $\angle 1$, and $\angle JKL$ (or $\angle LKJ$).
- The names for the angle are $\angle R$, $\angle 8$, and $\angle QRS$ (or $\angle SRQ$).
- Three different angles are $\angle HMN$, $\angle HMK$, and $\angle KMN$.
- Three different angles are $\angle GJF$, $\angle GJC$, and $\angle CJF$.
- Line up \overrightarrow{OA} on 0° using the outer scale of the protractor and \overrightarrow{OC} lines up with 30° on the outer scale. $m\angle AOC$ is 30° and it is an acute angle.

- Line up \overrightarrow{OB} on 0° using the inner scale of the protractor and \overrightarrow{OD} lines up with 65° on the inner scale. $m\angle BOD = 65^\circ$ and it is an acute angle.
- Line up \overrightarrow{OC} on 150° using the inner scale of the protractor and \overrightarrow{OD} lines up with 65° on the inner scale. $m\angle COD = |150 - 65| = 85^\circ$ and it is an acute angle.
- Line up \overrightarrow{OD} on 60° using the inner scale of the protractor and \overrightarrow{OE} lines up with 40° on the inner scale. $m\angle EOD = |65 - 40| = 25^\circ$ and it is an acute angle.
- The error was the outer scale was used. The inner scale should have been used because \overrightarrow{OB} passes through 0° on the inner scale; $m\angle BOG = 150^\circ$.
- Because neither side passes through 0° , the measure of the angle should be calculated as the absolute value of the difference between the numbers on the protractor that are matched with the sides; $|65 - 40| = 25^\circ$.

15.  Label the vertex of the original angle as A. Draw a segment and label a point D on the segment. Draw an arc with the center at A. Label the two intersecting points as B and C. Using the same radius, draw an arc with center D. Label the intersecting point as E. Draw an arc with radius BC and center E. Label the intersection F. Draw \overrightarrow{DF} .

16.  Label the vertex of the original angle as A. Draw a segment and label a point D on the segment. Draw an arc with the center at A. Label the two intersecting points as B and C. Using the same radius, draw an arc with center D. Label the intersecting point as E. Draw an arc with radius BC and center E. Label the intersection F. Draw \overrightarrow{DF} .

- Label the vertex of the original angle as A. Draw a segment and label a point D on the segment. Draw an arc with the center at A. Label the two intersecting points as B and C. Using the same radius, draw an arc with center D. Label the intersecting point as E. Draw an arc with radius BC and center E. Label the intersection F. Draw \overrightarrow{DF} .
- The angles congruent to $\angle AED$ are $\angle EDA$, $\angle BDC$, and $\angle BCD$.
 - The angles congruent to $\angle EAD$ are $\angle ADB$ and $\angle DBC$.

- $m\angle AED = 34^\circ$ $20. m\angle EAD = 112^\circ$
 $m\angle AED = m\angle BDC$ $m\angle EAD = m\angle ADB$
 $m\angle BDC = 34^\circ$ $m\angle ADB = 112^\circ$
- $m\angle ABC = m\angle ABD + m\angle DBC$
 $= 37^\circ + 21^\circ$
 $= 58^\circ$
- $m\angle LMN = m\angle LMP + m\angle PMN$
 $= 85^\circ + 23^\circ$
 $= 108^\circ$

Chapter 1

23. $m\angle RST = m\angle RSV + m\angle VST$

$$114^\circ = m\angle RSV + 72^\circ$$

$$42^\circ = m\angle RSV$$

24. $m\angle GHK = m\angle GHL + m\angle LNK$

$$180^\circ = 79^\circ + m\angle LNK$$

$$101^\circ = m\angle LNK$$

25. $m\angle ABC = m\angle ABD + m\angle DBC$

$$95^\circ = (2x + 23)^\circ + (9x - 5)^\circ$$

$$95 = 11x + 18$$

$$77 = 11x$$

$$7 = x$$

$$m\angle ABD = (2 \cdot 7 + 23)^\circ = (14 + 23)^\circ = 37^\circ$$

$$m\angle DBC = (9 \cdot 7 - 5)^\circ = (63 - 5)^\circ = 58^\circ$$

26. $m\angle XYZ = m\angle XYW + m\angle WYZ$

$$117^\circ = (6x + 44)^\circ + (-10x + 65)^\circ$$

$$117 = -4x + 109$$

$$8 = -4x$$

$$-2 = x$$

$$m\angle XYW = [6(-2) + 44]^\circ = (-12 + 44)^\circ = 32^\circ$$

$$m\angle WYZ = [-10(-2) + 65]^\circ = (20 + 65)^\circ = 85^\circ$$

27. $m\angle LMN = m\angle LMP + m\angle PMN$

$$180^\circ = (-16x + 13)^\circ + (-20x + 23)^\circ$$

$$180 = -36x + 36$$

$$144 = -36x$$

$$-4 = x$$

$$m\angle LMP = [-16(-4) + 13]^\circ = (64 + 13)^\circ = 77^\circ$$

$$m\angle PMN = [-20(-4) + 23]^\circ = (80 + 23)^\circ = 103^\circ$$

28. $m\angle ABC = m\angle ABX + m\angle XBC$

$$180^\circ = (14x + 70)^\circ + (20x + 8)^\circ$$

$$180 = 34x + 78$$

$$102 = 34x$$

$$3 = x$$

$$m\angle ABX = (14 \cdot 3 + 70)^\circ = (42 + 70)^\circ = 112^\circ$$

$$m\angle XBC = (20 \cdot 3 + 8)^\circ = (60 + 8)^\circ = 68^\circ$$

29. $m\angle RST = m\angle RSQ + m\angle QST$

$$90^\circ = (15x - 43)^\circ + (8x + 18)^\circ$$

$$90 = 23x - 25$$

$$115 = 23x$$

$$5 = x$$

$$m\angle RSQ = (15 \cdot 5 - 43)^\circ = (75 - 43)^\circ = 32^\circ$$

$$m\angle QST = (8 \cdot 5 + 18)^\circ = (40 + 18)^\circ = 58^\circ$$

30. $m\angle FED = m\angle FEH + m\angle HED$

$$90^\circ = (10x + 21)^\circ + 13x^\circ$$

$$90 = 23x + 21$$

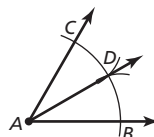
$$69 = 23x$$

$$3 = x$$

$$m\angle FEH = (10 \cdot 3 + 21)^\circ = (30 + 21)^\circ = 51^\circ$$

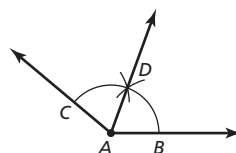
$$m\angle HED = (13 \cdot 3)^\circ = 39^\circ$$

31.



Label the vertex of the angle as A. Place the compass at A. Draw an arc that intersects both sides of the angle. Label the intersections B and C. Place the compass at C and draw an arc, then place the compass point at B. Using the same radius, draw another arc. Label the intersection D. Use a straightedge to draw a ray through A and D. \overrightarrow{AD} bisects $\angle A$.

32.



Label the vertex of the angle as A. Place the compass at A. Draw an arc that intersects both sides of the angle. Label the intersections B and C. Place the compass at C and draw an arc. Then place the compass point at B. Using the same radius, draw another arc. Label the intersection D. Use a straightedge to draw a ray through A and D. \overrightarrow{AD} bisects $\angle A$.

33. $m\angle RQS = m\angle PQS = 63^\circ$

$$m\angle PQR = m\angle PQS + m\angle RQS = 63^\circ + 63^\circ = 126^\circ$$

34. $m\angle PQS = m\angle RQS = 71^\circ$

$$m\angle PQR = m\angle PQS + m\angle RQS = 71^\circ + 71^\circ = 142^\circ$$

35. $m\angle PQS = \frac{1}{2}m\angle PQR = \frac{1}{2}(124^\circ) = 62^\circ$

$$m\angle RQS = \frac{1}{2}m\angle PQR = \frac{1}{2}(124^\circ) = 62^\circ$$

36. $m\angle PQS = \frac{1}{2}m\angle PQR = \frac{1}{2}(119^\circ) = 59.5^\circ$

$$m\angle SQR = \frac{1}{2}m\angle PQR = \frac{1}{2}(119^\circ) = 59.5^\circ$$

37. $m\angle ABD = m\angle DBC$

$$(6x + 14)^\circ = (3x + 29)^\circ$$

$$3x + 14 = 29$$

$$3x = 15$$

$$x = 5$$

$$m\angle ABD = (6 \cdot 5 + 14)^\circ = (30 + 14)^\circ = 44^\circ$$

$$m\angle DBC = (3 \cdot 5 + 29)^\circ = (15 + 29)^\circ = 44^\circ$$

$$m\angle ABC = (44 + 44)^\circ = 88^\circ$$

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38. $m\angle ABD = m\angle CBD$

$$(3x + 6)^\circ = (7x - 18)^\circ$$

$$6 = 4x - 18$$

$$24 = 4x$$

$$6 = x$$

$$m\angle ABD = (3 \cdot 6 + 6)^\circ = (18 + 6)^\circ = 24^\circ$$

$$m\angle CBD = (7 \cdot 6 - 18)^\circ = (42 - 18)^\circ = 24^\circ$$

$$m\angle ABC = (24 + 24)^\circ = 48^\circ$$

39. $m\angle ABD = m\angle CBD$

$$(-4x + 33)^\circ = (2x + 81)^\circ$$

$$33 = 6x + 81$$

$$-48 = 6x$$

$$-8 = x$$

$$m\angle ABD = [-4(-8) + 33]^\circ = (32 + 33)^\circ = 65^\circ$$

$$m\angle CBD = [2(-8) + 81]^\circ = (-16 + 81)^\circ = 65^\circ$$

$$m\angle ABC = (65 + 65)^\circ = 130^\circ$$

40. $m\angle ABD = m\angle DBC$

$$(8x + 35)^\circ = (11x + 23)^\circ$$

$$35 = 3x + 23$$

$$12 = 3x$$

$$4 = x$$

$$m\angle ABD = (8 \cdot 4 + 35)^\circ = (32 + 35)^\circ = 67^\circ$$

$$m\angle DBC = (11 \cdot 4 + 23)^\circ = (44 + 23)^\circ = 67^\circ$$

$$m\angle ABC = (67 + 67)^\circ = 134^\circ$$

41. Use the Angle Addition Postulate to find $m\angle ABD$.

$$m\angle ABC = m\angle ABD + m\angle CBD, \text{ then subtract } m\angle CBD \text{ from } m\angle ABC.$$

42. Let x be the angle at which Malcom Way intersects Park Road.

$$162 = 87 + x$$

$$75 = x$$

The angle at which Malcom Way intersects Park Road is 75° .

43. $m\angle LMN = m\angle LMP + m\angle PMN$

$$76^\circ = m\angle LMP + 36^\circ$$

$$40^\circ = m\angle LMP$$

44. a. $m\angle ABC = m\angle DEF = 112^\circ$

b. \overrightarrow{BG} bisects $\angle ABC$.

$$m\angle ABG = \frac{1}{2}m\angle ABC = \frac{1}{2}(112) = 56^\circ$$

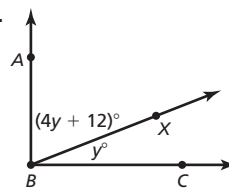
c. $m\angle CBG = \frac{1}{2}m\angle ABC = \frac{1}{2}(112) = 56^\circ$

d. $m\angle DEG = \frac{1}{2}m\angle DEF = \frac{1}{2}(112) = 56^\circ$

45. $\angle DGF$ is a straight angle. \overrightarrow{GB} bisects $\angle DGF$, so $\angle DGE \cong \angle DGF$. Because both angles are congruent and supplementary, each angle measures 90° .

46. Sample answer: $\angle DEG$ is an acute angle, $\angle ABC$ is an obtuse angle, $\angle DGE$ is a right angle, and $\angle DGF$ is a straight angle.

47. a.



b. $m\angle ABC = m\angle ABX + m\angle XBC$

$$92^\circ = (4y + 12)^\circ + y^\circ$$

$$92 = 5y + 12$$

$$80 = 5y$$

$$16 = y$$

$$m\angle CBX = 16^\circ$$

$$m\angle ABX = (4 \cdot 16 + 12)^\circ = (64 + 12)^\circ = 76^\circ$$

48. Sample answer: 9:00 or 3:00; The hour hand will be pointing at the 3 or 9 exactly, and the minute hand will be on the 12 exactly.

49. a. acute; When bisecting an acute angle, each angle is acute because the original angle is less than 90° .

b. acute; When bisecting a right angle, each angle is acute because the angles are complementary.

c. acute; When bisecting an obtuse angle, each angle is acute because the original angle is less than 180° and greater than 90° .

d. right; When bisecting a straight angle, each angle is a right angle because the angles are supplementary.

50. a. If a ray is drawn in the interior of an acute angle, then the two angles formed are each acute. An acute angle is less than 90° , so each of the smaller angles will be less than 90° .

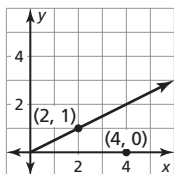
b. If a ray is drawn in the interior of a right angle, each angle will have a measure less than 90° because the angles are complementary.

c. If a ray is drawn in the interior of an obtuse angle, then the two angles formed could be both acute, one right and one acute, or one obtuse and one acute. An obtuse angle has a measure greater than 90° but less than 180° .

d. If a ray is drawn in the interior of a straight angle, the two angles formed could be both right angles or one acute and one obtuse. The sum of the angle measures is 180° .

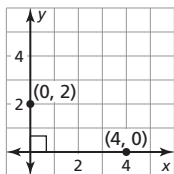
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51. a. *Sample answer:*



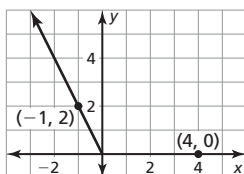
The point (2, 1) will create an acute angle.

b. *Sample answer:*



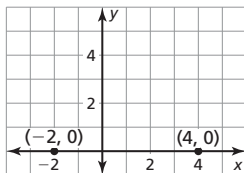
The point (0, 2) will create a right angle.

c. *Sample answer:*



The point (-1, 2) will create an obtuse angle.

d. *Sample answer:*



The point (-2, 0) will create a straight angle.

52. no; Obtuse angles have a measure greater than 90° and less than 180° . So, adding two obtuse angle measures gives a sum greater than 180° .

53. The sum of two acute angles could yield another acute angle ($30^\circ + 50^\circ = 80^\circ$), a right angle ($60^\circ + 30^\circ = 90^\circ$), or an obtuse angle ($55^\circ + 45^\circ = 100^\circ$).

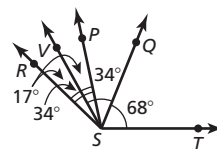
54. a. $m\angle XYW + m\angle WYV + m\angle VYZ = 90^\circ + 46^\circ + 46^\circ$
 $= 182^\circ$

The three angles appear to form a straight angle; however, the sum of the three angles is 182° .

b. *Sample answer:* Change $m\angle VYZ$ to 45° .

55. *Sample answer:* You draw a segment, ray, or line in the interior of an angle so that the two angles created are congruent to each other. Angle bisectors and segment bisectors can be segments, rays, or lines, but only a segment bisector can be a point. The two angles/segments created are congruent to each other, and their measures are each half the measure of the angle/segment.

56.



$$m\angle RSQ = m\angle TSQ$$

$$m\angle RSP = m\angle PSQ = \frac{1}{2}m\angle RSQ$$

$$m\angle RSV = m\angle VSP = \frac{1}{2}m\angle RSP$$

$$m\angle VSP = 17^\circ$$

$$m\angle RSP = 2(17^\circ) = 34^\circ$$

$$m\angle RSQ = 2 \cdot m\angle PSQ = 2(34^\circ) = 68^\circ$$

$$m\angle TSQ = m\angle RSQ = 68^\circ$$

57. acute; It is likely that the angle with the horizontal is very small because levels are typically used when something appears to be horizontal but still needs to be checked.

Maintaining Mathematical Proficiency

58. $x + 67 = 180$
 $x = 113$

59. $x + 58 = 90$
 $x = 32$

60. $16 + x = 90$
 $x = 74$

61. $109 + x = 180$
 $x = 71$

62. $6x + 7 + 13x + 21 = 180$
 $19x + 28 = 180$
 $19x = 152$
 $x = 8$

63. $3x + 15 + 4x - 9 = 90$
 $7x + 6 = 90$
 $7x = 84$
 $x = 12$

64. $11x - 25 + 24x + 10 = 90$
 $35x - 15 = 90$
 $35x = 105$
 $x = 3$

65. $14x - 18 + 5x + 8 = 180$
 $19x - 10 = 180$
 $19x = 190$
 $x = 10$

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1.6 Explorations (p. 47)

1. a. The angles measuring x° and y° make a straight angle;
The angles measuring y° and z° make a straight angle; The
angles measuring x° and z° appear to be congruent.
b. $x = 180 - 108 = 72$
 $y = 180 - 72 = 108$
 $z = 180 - 108 = 72$
 $w = 180 - 108 = 72$
 $v + z + w = 180$
 $v + 72 + 72 = 180$
 $v + 144 = 180$
 $v = 36$
2. a. The angles measuring a° and b° make a right angle; The
angles measuring c° and d° make a straight angle; The
angles measuring c° and e° appear to be congruent.
b. $c = 180 - 90 = 90$
 $d = 180 - c = 180 - 90 = 90$
 $e = 180 - 90 = 90$
3. When two lines intersect, four angles and two pairs of
opposite rays are formed. The angles that are next to each
other have measures that add up to 180° . The angles that are
across from each other are congruent and have the
same measure.
4. The sum of adjacent complementary angles is 90° . The sum
of adjacent supplementary angles is 180° . Vertical angles
have the same measure.

1.6 Monitoring Progress (pp. 49–51)

1. $\angle FGK$ and $\angle GKL$ are complementary because
 $41^\circ + 49^\circ = 90^\circ$.
 $\angle GKL$ and $\angle HGK$ are supplementary because
 $49^\circ + 131^\circ = 180^\circ$.
 $\angle FGK$ and $\angle KGH$ are adjacent because they share a
common ray \overrightarrow{GK} .
2. $\angle KGH$ and $\angle LKG$ are not adjacent angles because they do
not share a common side.
 $\angle FGK$ and $\angle FGH$ are not adjacent angles because they
have interior points in common.
3. $m\angle 1 + m\angle 2 = 90^\circ$
 $m\angle 1 + 5^\circ = 90^\circ$
 $m\angle 1 = 85^\circ$
4. $m\angle 3 + m\angle 4 = 180^\circ$
 $148^\circ + m\angle 4 = 180^\circ$
 $m\angle 4 = 32^\circ$
5. $m\angle LMN + m\angle PQR = 90^\circ$
 $(4x - 2)^\circ + (9x + 1)^\circ = 90^\circ$
 $13x - 1 = 90$
 $13x = 91$
 $x = 7$
 $m\angle LMN = (4 \cdot 7 - 2)^\circ = (28 - 2)^\circ = 26^\circ$
 $m\angle PQR = (9 \cdot 7 + 1)^\circ = (63 + 1)^\circ = 64^\circ$

6. no; There are no numbered adjacent angles whose noncommon
sides are opposite rays. There are three pairs of vertical
angles, $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 6$. Their sides
form two pairs of opposite rays.
7. Let x° be the angle, and its complement be $(90 - x)^\circ$.
 $x^\circ = 2(90 - x)^\circ$
 $x = 180 - 2x$
 $3x = 180$
 $x = 60$
 $90 - x = 90 - 60 = 30$
The angle has a measure of 60° and its complement measures 30° .
8. The first angle is $\frac{3}{2}x^\circ$ and the second angle is x° .
 $\frac{3}{2}x^\circ + x^\circ = 180^\circ$
 $\frac{5}{2}x = 180$
 $x = 72$
 $\frac{3}{2}x = \frac{3}{2}(72) = 108$
The angle measurements are 108° and 72° .

1.6 Exercises (pp. 52–54)

Vocabulary and Core Concept Check

1. Adjacent angles share a common side and are next to each
other. Vertical angles form two pairs of opposite rays and
are across from each other.
2. The linear pair with $\angle 3$ and $\angle 4$ does not belong. This pair
of angles is supplementary, and in the other three pairs, the
angles are complementary.

Monitoring Progress and Modeling with Mathematics

3. $\angle LJM$ and $\angle MJN$ are adjacent complementary angles
because $56^\circ + 34^\circ = 90^\circ$.
4. $\angle NJL$ and $\angle LJK$ (or $\angle KJM$ and $\angle MJN$) are adjacent
supplementary angles.
 $m\angle NJL = 56^\circ + 34^\circ = 90^\circ$
 $m\angle LJK = 90^\circ$
 $m\angle NJK + m\angle LJK = 90^\circ + 90^\circ = 180^\circ$
5. $\angle FGE$ and $\angle NJP$ are nonadjacent complementary angles.
 $m\angle FGE + m\angle NJP = 41^\circ + 49^\circ = 90^\circ$
6. $\angle FGH$ and $\angle LJM$ are nonadjacent supplementary angles.
 $m\angle FGH + m\angle LJM = 124^\circ + 56^\circ = 180^\circ$
7. $m\angle 1 + m\angle 2 = 90^\circ$
 $23^\circ + m\angle 2 = 90^\circ$
 $m\angle 2 = 67^\circ$
8. $m\angle 3 + m\angle 4 = 90^\circ$
 $46^\circ + m\angle 4 = 90^\circ$
 $m\angle 4 = 44^\circ$
9. $m\angle 5 + m\angle 6 = 180^\circ$
 $78^\circ + m\angle 6 = 180^\circ$
 $m\angle 6 = 102^\circ$
10. $m\angle 7 + m\angle 8 = 180^\circ$
 $109^\circ + m\angle 8 = 180^\circ$
 $m\angle 8 = 71^\circ$

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11. $m\angle QRT + m\angle TRS = 180^\circ$

$$(3x + 5)^\circ + (10x - 7)^\circ = 180^\circ$$

$$13x - 2 = 180$$

$$13x = 182$$

$$x = 14$$

$$m\angle QRT = (3 \cdot 14 + 5)^\circ = (42 + 5)^\circ = 47^\circ$$

$$m\angle TRS = (10 \cdot 14 - 7)^\circ = (140 - 7)^\circ = 133^\circ$$

12. $m\angle BAC + m\angle CAD = 90^\circ$

$$(15x - 2)^\circ + (7x + 4)^\circ = 90^\circ$$

$$22x + 2 = 90$$

$$22x = 88$$

$$x = 4$$

$$m\angle BAC = (15 \cdot 4 - 2)^\circ = (60 - 2)^\circ = 58^\circ$$

$$m\angle CAD = (7 \cdot 4 + 4)^\circ = (28 + 4)^\circ = 32^\circ$$

13. $m\angle UVW + m\angle XYZ = 90^\circ$

$$(x - 10)^\circ + (4x - 10)^\circ = 90^\circ$$

$$5x - 20 = 90$$

$$5x = 110$$

$$x = 22$$

$$m\angle UVW = (22 - 10)^\circ = 12^\circ$$

$$m\angle XYZ = (4 \cdot 22 - 10)^\circ = (88 - 10)^\circ = 78^\circ$$

14. $m\angle EFG + m\angle LMN = 180^\circ$

$$(3x + 17)^\circ + \left(\frac{1}{2}x - 5\right)^\circ = 180^\circ$$

$$\frac{7}{2}x + 12 = 180$$

$$\frac{7}{2}x = 168$$

$$x = 48$$

$$m\angle EFG = (3 \cdot 48 + 17)^\circ = (144 + 17)^\circ = 161^\circ$$

$$m\angle LMN = \left(\frac{1}{2} \cdot 48 - 5\right)^\circ = (24 - 5)^\circ = 19^\circ$$

15. $\angle 1$ and $\angle 5$ form a linear pair.

16. $\angle 7$ and $\angle 6$ and $\angle 7$ and $\angle 8$ are linear pairs.

17. yes; $\angle 6$ and $\angle 8$ are vertical angles because their sides form two pairs of opposite rays.

18. no; Only one pair of opposite rays is formed by these two angles.

19. Let the first angle be x° and the second angle be $2x^\circ$.

$$x^\circ + 2x^\circ = 180^\circ$$

$$3x = 180$$

$$x = 60$$

$$2x = 2(60) = 120$$

The measures of the two angles are 60° and 120° .

20. Let the first angle be x° and the second angle be $\left(\frac{1}{3}x\right)^\circ$.

$$x^\circ + \left(\frac{1}{3}x\right)^\circ = 180^\circ$$

$$\frac{4}{3}x = 180$$

$$x = 135$$

$$\frac{1}{3}x = \frac{1}{3}(135) = 45$$

The measures of the two angles are 135° and 45° .

21. Let the first angle be x° and the complement be $(90 - x)^\circ$.

$$x^\circ = 9(90 - x)^\circ$$

$$x = 810 - 9x$$

$$10x = 810$$

$$x = 81$$

$$90 - x = 90 - 81 = 9$$

The measures of the two angles are 81° and 9° .

22. Let the first angle be x° and the complement be $(90 - x)^\circ$.

$$x^\circ = \frac{1}{4}(90 - x)^\circ$$

$$4x = 90 - x$$

$$5x = 90$$

$$x = 18$$

$$90 - x = 90 - 18 = 72$$

The measures of the two angles are 18° and 72° .

23. $\angle 2$ and $\angle 4$ are nonadjacent angles because they do not share a common side. $\angle 1$ and $\angle 2$ are adjacent angles.

24. $\angle 1$ and $\angle 3$ do not form a linear pair because they are not adjacent. They are vertical angles.

25. $m\angle 1 + x^\circ = 180^\circ$

$$58 + x = 180$$

$$x = 122$$

The supplement of $m\angle 1$ is 122° .

26. $m\angle 2 + x^\circ = 180^\circ$

$$24 + x = 180$$

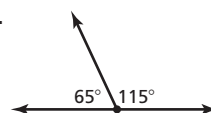
$$x = 156$$

The supplement of $m\angle 2$ is 156° .

27. C; Because $90^\circ - 42^\circ = 48^\circ$, the arm of the crossing gate must move an additional 48° for it to be horizontal.

28. Because $90^\circ - 27^\circ = 63^\circ$, the angle measure between the first base foul line and the path of the baseball is 63° .

29.



30.



Chapter 1

31. B; $m\angle U + m\angle V = 90^\circ$

$$(2x)^\circ + 4(2x)^\circ = 90^\circ$$

$$2x + 8x = 90$$

$$10x = 90$$

$$x = 9$$

$$m\angle U = (2 \cdot 9)^\circ = 18^\circ$$

$$m\angle V = (8 \cdot 9)^\circ = 72^\circ$$

The value $x = 9$ makes $\angle U$ and $\angle V$ complements of each other because $18^\circ + 72^\circ = 90^\circ$.

32. Angle: x°

Complement: $(90 - x)^\circ$

$$x^\circ = (90 - x)^\circ - 6^\circ$$

$$x = 84 - x$$

$$2x = 84$$

$$x = 42$$

$$90 - x = 90 - 42 = 48$$

The measures of the two angles are 42° and 48° .

33. Angle: x°

Complement: $(90 - x)^\circ$

$$x^\circ = 2(90 - x)^\circ + 12^\circ$$

$$x = 180 - 2x + 12$$

$$x = 192 - 2x$$

$$3x = 192$$

$$x = 64$$

$$90 - x = 90 - 64 = 26$$

The measures of the two angles are 64° and 26° .

34. Angle: x°

Supplement: $(180 - x)^\circ$

$$x^\circ = \frac{1}{2}(180 - x)^\circ + 3^\circ$$

$$2x = 180 - x + 6$$

$$3x = 186$$

$$x = 62$$

$$180 - x = 180 - 62 = 118$$

The measures of the two angles are 62° and 118° .

35. First angle = x°

Second angle = $\left(\frac{2}{3}x - 15\right)^\circ$

$$x^\circ + \left(\frac{2}{3}x - 15\right)^\circ = 180^\circ$$

$$\frac{5}{3}x - 15 = 180$$

$$\frac{5}{3}x = 195$$

$$x = \left(\frac{3}{5}\right) \cdot 195$$

$$x = 117$$

$$\frac{2}{3}(x) - 15 = \frac{2}{3}(117) - 15 = 78 - 15 = 63$$

The measures of the two angles are 117° and 63° .

36. sometimes; They could share a common side and form a right angle.

37. always; Angles that form a linear pair are supplementary.

38. never; Vertical angles are formed by two pairs of opposite rays.

39. sometimes; This is possible if the lines are perpendicular.

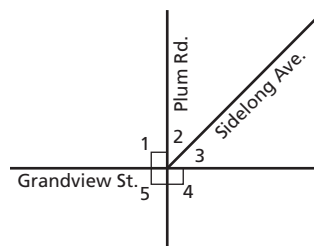
40. never; an acute angle is less than 90° , so its complement will also be acute, and its supplement will be obtuse.

41. always; Complementary angles have a combined measure of 90° , so the two angles that are complementary and congruent each will have a measure of 45° ($2x = 90$, $x = 45$).

42. An acute angle is always less than 90° . The supplement of an acute angle is greater than 90° and less than 180° , which makes it obtuse.

43. Obtuse angles have a measure greater than 90° and less than 180° . The sum of two complementary angles equals 90° .

44. Sample answer:



Supplementary angles: $\angle 4$ and $\angle 5$, $\angle 4$ and $\angle 1$, $\angle 5$ and $\angle 1$

Complementary angles: $\angle 2$ and $\angle 3$

Vertical angles: $\angle 1$ and $\angle 4$

45. a. $m\angle UWV = 180^\circ - 130^\circ = 50^\circ$

$$m\angle TWU = 90^\circ - 50^\circ = 40^\circ$$

$$m\angle TWX = 180^\circ - 40^\circ = 140^\circ$$

- b. There are two sets of angles that are supplementary, $\angle UWV$ and $\angle VWX$ and $\angle UWT$ and $\angle TWX$.

There are a total of four angles, $\angle TWU$, $\angle TWX$, $\angle UWV$, and $\angle VWX$.

Total possible outcomes:

$\angle UWV$ and $\angle VWX$; $\angle VWX$ and $\angle UWT$

$\angle UWV$ and $\angle UWT$; $\angle VWX$ and $\angle TWX$

$\angle UWV$ and $\angle TWX$; $\angle UWT$ and $\angle TWX$

There are only two favorable outcomes, $\angle UWV$ and $\angle VWX$ and $\angle UWT$ and $\angle TWX$.

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total possible outcomes}} = \frac{2}{6} = \frac{1}{3}$$

The probability that you choose supplementary angles is $\frac{1}{3}$.

Chapter 1

46. a. yes; \overline{CA} and \overline{AF} are marked as congruent.
 b. yes; Point A lies on \overline{CF} .
 c. no; $\angle CAB$ and $\angle EAF$ are congruent because they are vertical angles. $\angle CAD$ and $\angle EAF$ are not vertical angles and their measures are unknown.
 d. no; They are not marked as congruent.
 e. yes; They appear to intersect at point A in the diagram.
 f. yes; They are adjacent angles and $\angle BAD$ is marked as a right angle.
 g. yes; $\angle BAD$ and $\angle DAE$ are a linear pair, and $\angle BAD$ is marked as a right angle.

47. yes; Because $m\angle KJL + m\angle LJM = 90^\circ$ and $m\angle MJN + m\angle LJM = 90^\circ$, then by substitution,

$$m\angle KJL + m\angle LJM = m\angle MJN + m\angle LJM$$

$$m\angle KJL = m\angle MJN$$

By the definition of congruence, $\angle KJL \cong \angle MJN$.

48. yes; The angle of incidence is always congruent to the angle of reflection, so their complements will always be congruent.

49. a. $m\angle BAE = y^\circ$
 $m\angle DAE = (180 - y)^\circ$
 $m\angle CAB = (180 - y)^\circ$
 b. $\angle DAE$ and y are supplementary, $m\angle DAE + y^\circ = 180^\circ$ and $\angle CAB$ and y are supplementary, $m\angle CAB + y^\circ = 180^\circ$. By substitution, $m\angle DAE + y^\circ = m\angle CAB + y^\circ$, subtracting y from each side of the equation will result in $m\angle DAE = m\angle CAB$. Vertical angles have equal angle measures.

50. a. $m\angle 1 = x$, $m\angle 2 = y_1$, $m\angle 3 = y_2$

$$m\angle 2 + m\angle 1 = 90^\circ$$

$$y_1 + x = 90$$

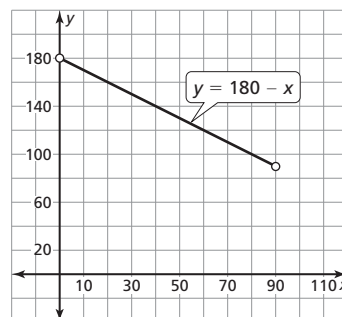
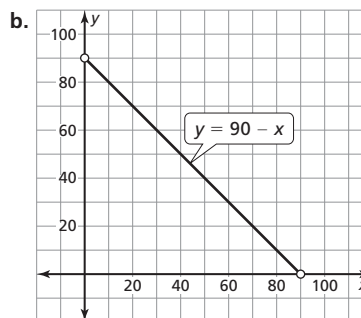
$$y_1 = 90 - x$$

$$m\angle 3 + m\angle 1 = 180^\circ$$

$$y_2 + x = 180$$

$$y_2 = 180 - x$$

The domain of y_1 is $0 < x < 90$ and the domain of y_2 is $0 < x < 90$.



The range for y_1 is $0 < y < 90$. The range for y_2 is $90 < y < 180$.

51. If two angles are complementary, then their sum is 90° . If x is one of the angles, then $(90 - x)$ is the complement. Write and solve the equation $90 = [x - (90 - x)] + 74$.

$$90 = [x - (90 - x)] + 74$$

$$16 = x - (90 - x)$$

$$16 = x - 90 + x$$

$$16 = 2x - 90$$

$$106 = 2x$$

$$53 = x$$

The measures of the angles are 53° and $90^\circ - 53^\circ = 37^\circ$.

Maintaining Mathematical Proficiency

52. sometimes; Positive integers are whole numbers. Negative integers are not.
53. never; Integers are positive or negative whole numbers. Irrational numbers are decimals that never terminate and never repeat.
54. always; The real number set is made up of all rational and irrational numbers.
55. never; The whole numbers are positive or zero.
56. sometimes; The set of rational numbers includes integers, as well as fractions and decimals that terminate or repeat.
57. always; The set of integers includes all natural numbers and their opposites (and zero).

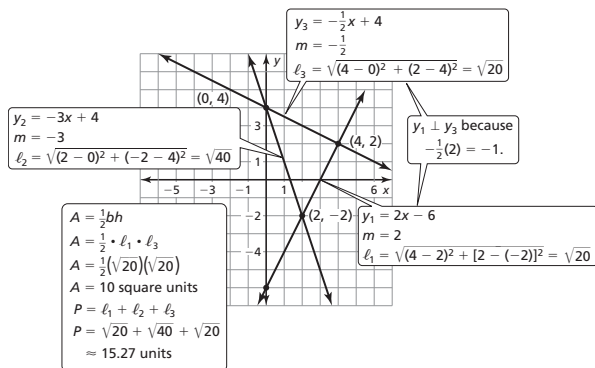
Chapter 1

58. always; The set of rational numbers includes whole numbers.

59. sometimes; Irrational numbers can be positive or negative.

1.4–1.6 What Did You Learn? (p. 55)

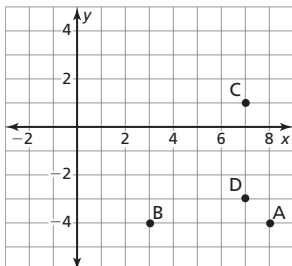
1. Show your friend a graph of the lines and the written work for determining which sides are perpendicular and for finding the lengths of the sides and the perimeter and area of the triangle. Use arrows and labels, as shown.



2. Use a protractor to verify your answers.
3. disagree; The angles are complementary because they form a right angle, and the sum of their measures is 90° .

Chapter 1 Review (pp. 56–58)

1. Sample answer: Another name for plane M is plane XZN .
2. Line g or \overleftrightarrow{XZ} lie in the plane.
3. Line h or \overleftrightarrow{PY} intersect plane M .
4. Two rays are \overrightarrow{YX} and \overrightarrow{YZ} (or \overrightarrow{YP}).
5. \overrightarrow{YX} and \overrightarrow{YZ} are opposite rays.
6. Point P is not contained in plane M .
7. $XY + YZ = XZ$
 $17 + 24 = XZ$
 $41 = XZ$
8. $AZ = AX + XZ$
 $38 = 27 + XZ$
 $11 = XZ$
9. $AB = |8 - 3| = 5$
 $CD = |-3 - 1| = |-4| = 4$
 \overline{AB} is not congruent to \overline{CD} .



10. $S(-2, 4), T(3, 9)$

$$M(x, y) = \left(\frac{-2 + 3}{2}, \frac{4 + 9}{2} \right) = \left(\frac{1}{2}, \frac{13}{2} \right)$$

$$ST = \sqrt{[3 - (-2)]^2 + (9 - 4)^2}$$

$$= \sqrt{(3 + 2)^2 + 5^2}$$

$$= \sqrt{5^2 + 5^2}$$

$$= \sqrt{25 + 25}$$

$$= \sqrt{50} \approx 7.1$$

11. $S(6, -3), T(7, -2)$

$$M(x, y) = \left(\frac{6 + 7}{2}, \frac{-3 + (-2)}{2} \right) = \left(\frac{13}{2}, -\frac{5}{2} \right)$$

$$ST = \sqrt{(6 - 7)^2 + [-3 - (-2)]^2}$$

$$= \sqrt{(-1)^2 + (-3 + 2)^2}$$

$$= \sqrt{1 + (-1)^2}$$

$$= \sqrt{2} \approx 1.4$$

12. $M(6, 3) = \left(\frac{14 + x}{2}, \frac{y + 9}{2} \right)$

$$6 = \frac{14 + x}{2} \quad 3 = \frac{y + 9}{2}$$

$$12 = 14 + x \quad 6 = y + 9$$

$$-2 = x \quad -3 = y$$

$$K(-2, -3)$$

13. $AM = MB$

$$3x + 8 = 6x - 4$$

$$8 = 3x - 4$$

$$12 = 3x$$

$$4 = x$$

$$AM = 3(4) + 8 = 12 + 8 = 20$$

$$MB = 6(4) - 4 = 24 - 4 = 20$$

$$AB = 20 + 20 = 40$$

14. $XW = |6 - (-1)| = |6 + 1| = 7$

$$WY = |5 - 2| = |3| = 3$$

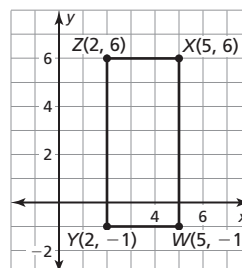
$$YZ = |6 - (-1)| = |6 + 1| = 7$$

$$ZX = |5 - 2| = |3| = 3$$

$$XW + WY + YZ + ZX = 7 + 3 + 7 + 3 = 20$$

$$A = bh = 7 \cdot 3 = 21$$

The perimeter of quadrilateral $WXZY$ is 20 units and the area is 21 square units.



Chapter 1

$$15. GF = |6 - (-1)| = |6 + 1| = 7$$

$$FE = |5 - (-2)| = |5 + 2| = 7$$

$$GE = \sqrt{[6 - (-1)]^2 + (-2 - 5)^2}$$

$$= \sqrt{(6 + 1)^2 + (-7)^2}$$

$$= \sqrt{49 + 49}$$

$$= \sqrt{98}$$

$$\approx 9.9$$

$$GF + FE + GE = 7 + 7 + 9.9 = 23.9$$

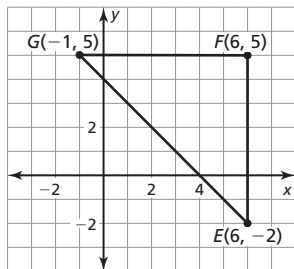
$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(7)(7)$$

$$= \frac{1}{2}(49)$$

$$= 24.5$$

The perimeter of $\triangle GFE$ is about 23.9 units and the area is 24.5 square units.



$$16. m\angle ABC = m\angle ABD + m\angle DBC$$

$$77^\circ = (3x + 22)^\circ + (5x - 17)^\circ$$

$$77 = 8x + 5$$

$$72 = 8x$$

$$9 = x$$

$$m\angle ABD = (3 \cdot 9 + 22)^\circ = (27 + 22)^\circ = 49^\circ$$

$$m\angle DBC = (5 \cdot 9 - 17)^\circ = (45 - 17)^\circ = 28^\circ$$

$$17. m\angle ABC = m\angle ABD + m\angle DBC$$

$$111^\circ = (-10x + 58)^\circ + (6x + 41)^\circ$$

$$111 = -4x + 99$$

$$12 = -4x$$

$$-3 = x$$

$$m\angle ABD = [-10(-3) + 58]^\circ = (30 + 58)^\circ = 88^\circ$$

$$m\angle CBD = [6(-3) + 41]^\circ = (-18 + 41)^\circ = 23^\circ$$

18. The measure of the angle is 127° .

$$19. m\angle 1 + m\angle 2 = 90^\circ$$

$$12^\circ + m\angle 2 = 90^\circ$$

$$m\angle 2 = 78^\circ$$

$$20. m\angle 1 + m\angle 2 = 90^\circ$$

$$83^\circ + m\angle 2 = 90^\circ$$

$$m\angle 2 = 7^\circ$$

$$21. m\angle 3 + m\angle 4 = 180^\circ$$

$$116^\circ + m\angle 4 = 180^\circ$$

$$m\angle 4 = 64^\circ$$

$$22. m\angle 3 + m\angle 4 = 180^\circ$$

$$56^\circ + m\angle 4 = 180^\circ$$

$$m\angle 4 = 124^\circ$$

Chapter 1 Test (p. 59)

1. Use the Segment Addition Postulate (Post. 1.2).

$$QS = QR + RS$$

$$= 12 + 19$$

$$= 31$$

2. Use the Segment Addition Postulate (Post. 1.2).

$$QR = QS + SR$$

$$59 = QS + 47$$

$$12 = QS$$

3. $A(-4, -8), B(-1, 4)$

$$M(x, y) = \left(\frac{-4 + (-1)}{2}, \frac{-8 + 4}{2} \right) = \left(\frac{-5}{2}, \frac{-4}{2} \right) = \left(-\frac{5}{2}, -2 \right)$$

$$AB = \sqrt{[-1 - (-4)]^2 + [4 - (-8)]^2}$$

$$= \sqrt{(-1 + 4)^2 + (4 + 8)^2}$$

$$= \sqrt{3^2 + 12^2}$$

$$= \sqrt{9 + 144}$$

$$= \sqrt{153} \approx 12.4$$

4. $C(-1, 7), D(-8, -3)$

$$M(x, y) = \left(\frac{-8 + (-1)}{2}, \frac{-3 + 7}{2} \right) = \left(\frac{-9}{2}, \frac{4}{2} \right) = \left(-\frac{9}{2}, 2 \right)$$

$$CD = \sqrt{[-1 - (-8)]^2 + [7 - (-3)]^2}$$

$$= \sqrt{(-1 + 8)^2 + (7 + 3)^2}$$

$$= \sqrt{7^2 + 10^2}$$

$$= \sqrt{49 + 100}$$

$$= \sqrt{149} \approx 12.2$$

$$5. M(1, -1) = \left(\frac{-3 + x}{2}, \frac{2 + y}{2} \right)$$

$$1 = \frac{-3 + x}{2} \quad -1 = \frac{2 + y}{2}$$

$$2 = -3 + x \quad -2 = 2 + y$$

$$5 = x \quad -4 = y$$

$$F(5, -4)$$

6. The statement is false.

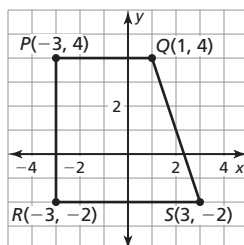
7. The statement is true.

8. The statement is false.

9. The statement is false.

Chapter 1

10.



$$PQ = |-3 - 1| = |-4| = 4$$

$$PR = |4 - (-2)| = |4 + 2| = 6$$

$$RS = |-3 - 3| = |-6| = 6$$

$$QS = \sqrt{(3 - 1)^2 + (-2 - 4)^2}$$

$$= \sqrt{4 + 36}$$

$$= \sqrt{40}$$

$$\approx 6.3$$

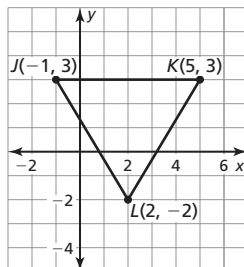
$$PQ + PR + RS + QS \approx 4 + 6 + 6 + 6.3 = 22.3$$

To find the area of the trapezoid, the height is $PR = 6$, the top base is $PQ = 4$, and the bottom base is $RS = 6$.

$$A = \frac{1}{2}h(b + B) = \frac{1}{2}(6)(4 + 6) = 3(10) = 30$$

The perimeter of quadrilateral $PQSR$ is about 22.3 units and the area is 30 square units.

11.



$$JK = |5 - (-1)| = |6| = 6$$

$$KL = \sqrt{(5 - 2)^2 + [3 - (-2)]^2}$$

$$= \sqrt{3^2 + 5^2}$$

$$= \sqrt{9 + 25}$$

$$= \sqrt{34} \approx 5.83$$

$$LJ = \sqrt{[2 - (-1)]^2 + (-2 - 3)^2}$$

$$= \sqrt{(2 + 1)^2 + (-5)^2}$$

$$= \sqrt{9 + 25}$$

$$= \sqrt{34} \approx 5.83$$

$$JK + KL + LJ \approx 6 + 5.83 + 5.83 = 17.66$$

To find the area, the base is $JK = 6$ and the height is the distance from point L to \overline{JK} , which is 5.

$$A = \frac{1}{2}bh = \frac{1}{2}(6)(5) = 3 \cdot 5 = 15$$

The perimeter of $\triangle JKL$ is about 17.66 units and the area is 15 square units.

12. Supplementary angles:

$$\angle AFB, \angle BFE$$

$$\angle AFC, \angle CFE$$

$$\angle AFD, \angle DFE$$

Complementary angles:

$$\angle AFB, \angle BFC$$

$$\angle CFD, \angle DFE$$

$$m\angle DFE = 90^\circ - 27^\circ = 63^\circ$$

$$m\angle BFC = 90^\circ - 39^\circ = 51^\circ$$

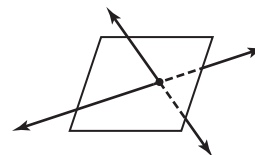
$$m\angle BFE = m\angle BFC + m\angle CFE = 51^\circ + 90^\circ = 141^\circ$$

13. a. The measure of the acute angle created when the clock is at 10:00 is 60° .

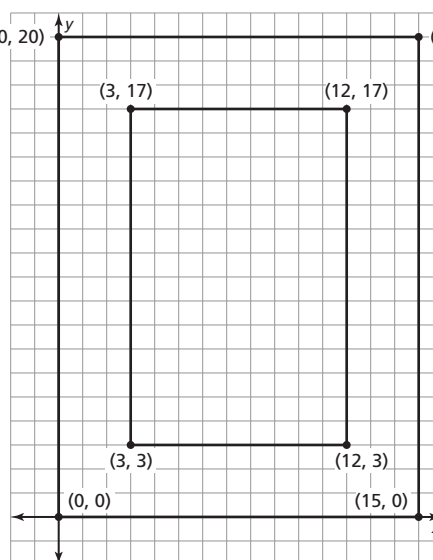
b. When the time is 5:00, the obtuse angle is 150° .

c. *Sample answer:* When the time is 6:00, the hour and minute hands create a straight angle.

14. *Sample answer:*



15. (0, 20) (15, 20)



The width of the pool is $15 - 6 = 9$ and the length of the pool is $20 - 6 = 14$.

$$P = 2w + 2\ell = 2(9) + 2(14) = 18 + 28 = 46$$

$$A = \ell w = 9(14) = 126$$

The perimeter of the pool is 46 feet and the area is 126 square feet.

16. The yellow ball is closer to the pallino. Because the green ball is the midpoint between the red ball and the pallino, it is equidistant from them. So, it is 10 inches from the pallino. Because the yellow ball is only 8 inches from the pallino, it is closer.

Chapter 1

Chapter 1 Standards Assessment (pp. 60–61)

1. $CD = MN = 12$; $EF = PQ = KL = 3$
So, $\overline{CD} \cong \overline{MN}$ and $\overline{EF} \cong \overline{PQ} \cong \overline{KL}$.

2. point, segment, ray, line, plane

3. B; $(-6, 13)$, $(11, 5)$

$$M(x, y) = \left(\frac{-6 + 11}{2}, \frac{13 + 5}{2} \right) = \left(\frac{5}{2}, \frac{18}{2} \right) = \left(\frac{5}{2}, 9 \right)$$

$$\begin{aligned} d &= \sqrt{(-6 - 11)^2 + (13 - 5)^2} \\ &= \sqrt{(-17)^2 + 8^2} \\ &= \sqrt{289 + 64} \\ &= \sqrt{353} \approx 18.8 \end{aligned}$$

4. $QR = |-4 - 2| = |-6| = 6$

$$\begin{aligned} RS &= \sqrt{(4 - 2)^2 + (-3 - 3)^2} \\ &= \sqrt{(2)^2 + (-6)^2} \\ &= \sqrt{4 + 36} \\ &= \sqrt{40} \approx 6.32 \end{aligned}$$

$$TS = |-2 - 4| = |-6| = 6$$

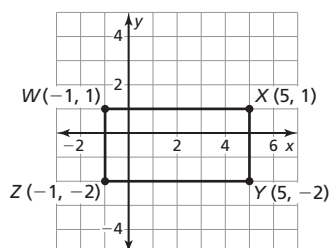
$$\begin{aligned} QT &= \sqrt{(-2 + 4)^2 + (-3 - 3)^2} \\ &= \sqrt{(2)^2 + (-6)^2} \\ &= \sqrt{4 + 36} \\ &= \sqrt{40} \approx 6.32 \end{aligned}$$

$$QR + RS + TS + QT = 6 + 6.32 + 6 + 6.32 = 24.64$$

$$A = bh = 6 \cdot 6 = 36$$

The perimeter is about 24.6 units and the area is 36 square units.

- 5.



The quadrilateral WXYZ is a rectangle; yes; If each unit on the coordinate plane represents 15 feet on the basketball court, then each square unit represents $15^2 = 225$ square feet. The rectangle on the coordinate plane that is 3 units by 6 units makes a good model of a basketball court that is 45 feet by 90 feet with the given perimeter and area.

6. In step 1, a compass is used to draw an arc. The two points (C and B) where this arc intersects the sides of the angle are the same distance from vertex A. In step 2, the compass is used to draw an arc of points equidistant from B and a separate arc of points equidistant from C. The point where these two arcs intersect is a point that is equidistant from each side of the angle. In step 3, a straightedge is used to draw the angle bisector, a ray that has an endpoint at the vertex and that passes through the point that is equidistant from each side of the angle.

7. $\angle NMK \cong \angle AEC$, $\angle KMO \cong \angle CED$, $\angle MOP \cong \angle QOR$, $\angle POQ \cong \angle MOR$; right angles: $\angle MKJ \cong \angle MKL \cong \angle LKH \cong \angle HKJ \cong \angle KHG \cong \angle GHF \cong \angle FHI \cong \angle KHI$; straight angles: $\angle NMO \cong \angle MOQ \cong \angle POR \cong \angle MKH \cong \angle KHF \cong \angle JKL \cong \angle GHI \cong \angle AED$

8. a. Vertical angles: $\angle KJL$ and $\angle PJN$, $\angle LJN$ and $\angle KJP$

- b. Linear pairs: $\angle KJM$ and $\angle MJN$, $\angle KJP$ and $\angle PJN$, $\angle KJL$ and $\angle LJN$, $\angle LJM$ and $\angle MJP$, $\angle LJN$ and $\angle NJP$, $\angle KJL$ and $\angle KJP$

- c. You turn at $\angle KJL$.

Chapter 2

Chapter 2 Maintaining Mathematical Proficiency (p. 63)

1. The first term is 3 and the common difference is 6.

$$a_n = a_1 + (n - 1)d$$

$$a_n = 3 + (n - 1)6$$

$$a_n = 3 + 6n - 6$$

$$a_n = 6n - 3$$

Use the equation to find a_{50} .

$$a_{50} = 6 \cdot 50 - 3 = 300 - 3 = 297$$

2. The first term is -29 and the common difference is 17.

$$a_n = a_1 + (n - 1)d$$

$$a_n = -29 + (n - 1)17$$

$$a_n = -29 + 17n - 17$$

$$a_n = 17n - 46$$

Use the equation to find a_{50} .

$$a_{50} = 17 \cdot 50 - 46 = 850 - 46 = 804$$

3. The first term is 2.8 and the common difference is 0.6.

$$a_n = a_1 + (n - 1)d$$

$$a_n = 2.8 + (n - 1)0.6$$

$$a_n = 2.8 + 0.6n - 0.6$$

$$a_n = 0.6n + 2.2$$

Use the equation to find a_{50} .

$$a_{50} = 0.6 \cdot 50 + 2.2 = 30 + 2.2 = 32.2$$

4. The first term is $\frac{1}{2}$ and the common difference is $\frac{1}{6}$.

$$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$

$$a_n = a_1 + (n - 1)d$$

$$a_n = \frac{1}{2} + (n - 1)\frac{1}{6}$$

$$a_n = \frac{2}{6} + (n - 1)\frac{1}{6}$$

$$a_n = \frac{2}{6} + \frac{1}{6}n - \frac{1}{6}$$

$$a_n = \frac{1}{6}n + \frac{1}{6}$$

Use the equation to find a_{50} .

$$a_{50} = \frac{1}{6}(50) + \frac{1}{6} = \frac{51}{6} = 8\frac{1}{2}$$

5. The first term is 26 and the common difference is -4 .

$$a_n = a_1 + (n - 1)d$$

$$a_n = 26 + (n - 1)(-4)$$

$$a_n = 26 - 4n + 4$$

$$a_n = -4n + 30$$

Use the equation to find a_{50} .

$$a_{50} = -4 \cdot 50 + 30 = -200 + 30 = -170$$

6. The first term is 8 and the common difference is -6 .

$$a_n = a_1 + (n - 1)d$$

$$a_n = 8 + (n - 1)(-6)$$

$$a_n = 8 - 6n + 6$$

$$a_n = -6n + 14$$

Use the equation to find a_{50} .

$$a_{50} = -6 \cdot 50 + 14 = -300 + 14 = -286$$

7. $2y - 2x = 10$

$$-2x = -2y + 10$$

$$\frac{-2}{-2}x = \frac{-2}{-2}y + \frac{10}{-2}$$

$$x = y - 5$$

8. $20y + 5x = 15$

$$5x = -20y + 15$$

$$\frac{5}{5}x = \frac{-20}{5}y + \frac{15}{5}$$

$$x = -4y + 3$$

9. $4y - 5 = 4x + 7$

$$4y - 12 = 4x$$

$$\frac{4}{4}y - \frac{12}{4} = \frac{4}{4}x$$

$$y - 3 = x$$

10. $y = 8x - x$

$$y = 7x$$

$$\frac{y}{7} = \frac{7x}{7}$$

$$\frac{1}{7}y = x$$

11. $y = 4x + zx + 6$

$$y = x(4 + z) + 6$$

$$y - 6 = x(4 + z)$$

$$\frac{y - 6}{4 + z} = \frac{x(4 + z)}{4 + z}$$

$$\frac{y - 6}{4 + z} = x$$

12. $z = 2x + 6xy$

$$z = x(2 + 6y)$$

$$\frac{z}{2 + 6y} = \frac{x(2 + 6y)}{2 + 6y}$$

$$\frac{z}{2 + 6y} = x$$

13. no; The sequence does not have a common difference.

Chapter 2 Mathematical Practices (p. 64)

1. true

2. flawed; There are no squares that are trapezoids. Trapezoids have only one pair of parallel sides, whereas squares have right angles, congruent sides, and parallel opposite sides.

3. flawed; Only some rectangles are squares.

4. flawed; $ABCD$ may be a non-square rectangle.

2.1 Explorations (p. 65)

1. a. true; Thursday always follows Wednesday.
b. false; An acute angle can have any measure greater than 0° and less than 90° .
c. false; The month could be September, April, June, or November.
d. true; All even numbers are divisible by 2, and 9 is not a perfect cube. Because both the hypothesis and conclusion are false, the conditional statement is true.

Chapter 2

2. a. true

$$\begin{aligned} AB &= \sqrt{[-4 - (-4)]^2 + (5 - 0)^2} \\ &= \sqrt{(-4 + 4)^2 + 5^2} \\ &= \sqrt{0 + 25} \\ &= \sqrt{25} = 5 \end{aligned}$$

$$BC = |4 - (-4)| = |4 + 4| = |8| = 8$$

$$\begin{aligned} AC &= \sqrt{(-4 - 4)^2 + (5 - 0)^2} \\ &= \sqrt{(-8)^2 + 5^2} \\ &= \sqrt{64 + 25} \\ &= \sqrt{89} \end{aligned}$$

Because $5^2 + 8^2 = \sqrt{89}$, then by the Pythagorean Theorem, $\triangle ABC$ is a right triangle.

b. false

$$\begin{aligned} BD &= \sqrt{[0 - (-4)]^2 + (5 - 0)^2} \\ &= \sqrt{(0 + 4)^2 + 5^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \end{aligned}$$

$$\begin{aligned} DC &= \sqrt{[0 - (-4)]^2 + (5 - 0)^2} \\ &= \sqrt{(0 + 4)^2 + 5^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \end{aligned}$$

$$BC = |4 - (-4)| = 8$$

Because all three sides are not congruent, $\triangle BDC$ is not an equilateral triangle.

c. true

$$\begin{aligned} BD &= \sqrt{[0 - (-4)]^2 + (5 - 0)^2} \\ &= \sqrt{(0 + 4)^2 + 5^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \end{aligned}$$

$$\begin{aligned} DC &= \sqrt{[0 - (-4)]^2 + (5 - 0)^2} \\ &= \sqrt{(0 + 4)^2 + 5^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \end{aligned}$$

$$BC = |4 - (-4)| = 8$$

Because at least two sides are congruent, $\triangle BDC$ is an isosceles triangle.

d. true

$$\text{Slope of } \overline{AD}: \frac{5 - 5}{0 - (-4)} = \frac{0}{4} = 0$$

$$\text{Slope of } \overline{BC}: \frac{0 - 0}{4 - (-4)} = \frac{0}{8} = 0$$

Because the slope of \overline{AD} is equal to the slope of \overline{BC} , $\overline{AD} \parallel \overline{BC}$ and $ABCD$ is a trapezoid.

e. false

Slope of \overline{AB} is undefined.

$$\text{Slope of } \overline{DC}: \frac{5 - 0}{0 - 4} = \frac{5}{-4}$$

\overline{AB} is not parallel to \overline{DC} , because the slopes are not equal.

3. a. true; The Pythagorean Theorem is valid for all right triangles.

b. false; Two angles are complementary when the sum of their measures is 90° .

c. false; The sum of the angle measures of a quadrilateral is always 360° .

d. true; Collinear points are points on the same line.

e. true; Every pair of intersecting lines forms two pairs of opposite rays and therefore two pairs of vertical angles.

4. A conditional statement is true if both the hypothesis and the conclusion are true or if both are false or if a false hypothesis yields a true conclusion. A conditional statement is false when a true hypothesis yields a conclusion that is false.

5. Sample answer:

True: If two angles are supplementary, then the sum of the angles is 180° .

False: If two angles have a sum of 180° , then the angles form a linear pair. (The angles may have a sum of 180° but not be adjacent angles.)

2.1 Monitoring Progress (pp. 66–70)

1. Hypothesis: All 30° angles

Conclusion: Acute angles

If-then form: If an angle measures 30° , then it is an acute angle.

2. Hypothesis: $x = -3$

Conclusion: $2x + 7 = 1$

If-then form: If $x = -3$, then $2x + 7 = 1$.

3. The shirt is not green.

4. The shoes are red.

5. a. Conditional: If the stars are visible then it is night; true.

b. Converse: If it is night, then the stars are visible; false (could be cloudy).

c. Inverse: If the stars are not visible, then it is not night; false (could be cloudy).

d. Contrapositive: If it is not night, then the stars are not visible; true.

6. true; The diagram shows that $\angle JMF$ and $\angle FMG$ are a linear pair. By definition, angles that form a linear pair are supplementary.

7. false; The midpoint cannot be assumed from a diagram without markings that indicate $FM = MH$.

Chapter 2

8. true; Because M lies on \overrightarrow{FH} and \overrightarrow{JG} , two pairs of opposite rays are formed.
9. false; Right angles and perpendicular lines cannot be assumed from a diagram without being marked as such.
10. An angle is a right angle if and only if its measure is 90° .
11. Two line segments have the same length if and only if they are congruent segments.
12. Mary is in the fall play if and only if she is taking theater class.
13. You can run for President if and only if you are at least 35 years old.

14.

p	q	$\sim q$	$p \rightarrow \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

15.

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

2.1 Exercises (pp. 71–74)

Vocabulary and Core Concept Check

1. A conditional statement and its contrapositive, as well as the converse and inverse of a conditional statement are both true or both false.
2. The statement that does not belong is “If you are an athlete, then you play soccer.” This statement is false and the other three are true.

Monitoring Progress and Modeling with Mathematics

3. Hypothesis: A polygon is a pentagon.
Conclusion: It has five sides.
4. Hypothesis: Two lines form vertical angles.
Conclusion: They intersect.
5. Hypothesis: You run.
Conclusion: You are fast.
6. Hypothesis: You like math.
Conclusion: You like science.
7. If $x = 2$, then $9x + 5 = 23$.

8. If today is Friday, then tomorrow is the weekend.
9. If you are in a band, then you play the drums.
10. If two angles are right angles, then they are supplementary.
11. If you are registered, then you are allowed to vote.
12. If two angles are complementary, then their measures sum to 90° .
13. The sky is not blue.
14. The lake is not cold.
15. The ball is pink.
16. The dog is a lab.
17. Conditional statement: If two angles are supplementary, then the measures of the angles sum to 180° ; true.
Converse: If the measures of two angles sum to 180° , then the two angles are supplementary; true.
Inverse: If two angles are not supplementary, then their measures do not sum to 180° ; true.
Contrapositive: If the measures of two angles do not sum to 180° , then they are not supplementary; true.
18. Conditional statement: If you are in a math class, then you are in Geometry; false.
Converse: If you are in Geometry, then you are in math class; true.
Inverse: If you are not in math class, then you are not in Geometry; true.
Contrapositive: If you are not in Geometry, then you are not in math class; false.
19. Conditional statement: If you do your math homework, then you will do well on your test; false.
Converse: If you do well on your test, then you did your math homework; false.
Inverse: If you do not do your math homework, then you will not do well on your test; false.
Contrapositive: If you do not do well on your test, then you did not do your math homework; false.
20. Conditional statement: If you are not an only child, then you have a sibling; true.
Converse: If you have a sibling, then you are not an only child; true.
Inverse: If you are an only child, then you do not have a sibling; true.
Contrapositive: If you do not have a sibling, then you are an only child; true.
21. Conditional statement: If it does not snow, then I will run outside; false.
Converse: If I run outside, then it is not snowing; true.
Inverse: If it snows, then I will not run outside; true.
Contrapositive: If I do not run outside, then it is snowing; false.

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22. Conditional statement: If the Sun is out, then it is daytime; true.

Converse: If it is daytime, then the Sun is out; false.

Inverse: If the Sun is not out, then it is not daytime; false.

Contrapositive: If it is not daytime, then the Sun is not out; true.

23. Conditional statement: If $3x - 7 = 20$, then $x = 9$; true.

Converse: If $x = 9$, then $3x - 7 = 20$; true.

Inverse: If $3x - 7 \neq 20$, then $x \neq 9$; true.

Contrapositive: If $x \neq 9$, then $3x - 7 \neq 20$; true.

24. Conditional statement: If it is Valentine's Day, then it is February; true.

Converse: If it is February, then it is Valentine's Day; false.

Inverse: If it is not Valentine's Day, then it is not February; false.

Contrapositive: If it is not February, then it is not Valentine's Day; true.

25. true; By definition of right angle, the measure of the right angle shown is 90° .

26. true; If two lines form a right angle, then the lines are perpendicular.

27. true; If two adjacent angles form a linear pair, then the sum of the measures of the two angles is 180° .

28. false; The midpoint cannot be assumed unless \overline{AM} and \overline{MB} are marked as congruent.

29. A point is the midpoint of a segment if and only if the point divides the segment into two congruent segments.

30. Two angles are vertical angles if and only if their sides form two pairs of opposite rays.

31. Two angles are adjacent angles if and only if they share a common vertex and side, but have no common interior points.

32. Two angles are supplementary angles if and only if the sum of the two angle measures is 180° .

33. A polygon has three sides if and only if it is a triangle.

34. A polygon is a quadrilateral if and only if it has four sides.

35. An angle is a right angle if and only if the angle measures 90° .

36. An angle has a measure between 90° and 180° if and only if it is obtuse.

37. Taking four English courses is a requirement regardless of the total amount of courses the student takes, and the courses do not have to be taken simultaneously. The correct if-then form is: If students are in high school, then they will take four English courses before they graduate.

38. The inverse was used instead of the converse. The correct converse is: If I bring an umbrella, then it is raining.

39.

p	$\sim p$	q	$\sim p \rightarrow q$
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F

40.

p	q	$\sim q$	$\sim q \rightarrow p$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

41.

p	$\sim p$	q	$\sim q$	$\sim p \rightarrow \sim q$	$\sim(\sim p \rightarrow \sim q)$
T	F	T	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	T	T	F

42.

p	q	$\sim q$	$p \rightarrow \sim q$	$\sim(p \rightarrow \sim q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	T	F
F	F	T	T	F

43.

p	q	$\sim p$	$q \rightarrow \sim p$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	T

44.

p	q	$q \rightarrow p$	$\sim(q \rightarrow p)$
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	F

45. a. If a rock is igneous, then it is formed from the cooling of molten rock.

If a rock is sedimentary, then it is formed from pieces of other rocks.



If a rock is metamorphic, then it is formed by changing temperature, pressure, or chemistry.

- b. If a rock is formed from the cooling of molten rock, then it is igneous; true; All rocks formed from cooling molten rock are called igneous.

If a rock is formed from pieces of other rocks, then it is sedimentary; true; All rocks formed from pieces of other rocks are called sedimentary.

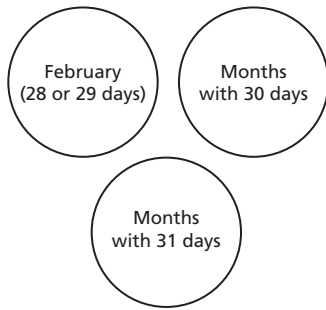
If a rock is formed by changing temperature, pressure, or chemistry, then it is metamorphic; true; All rocks formed by changing temperature, pressure, or chemistry are called metamorphic.

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- c. *Sample answer:* If a rock is not sedimentary, then it was not formed from pieces of other rocks; This is the inverse of one of the conditional statements in part (a). So, the converse of this statement will be the contrapositive of the conditional statement. Because the contrapositive is equivalent to the conditional statement and the conditional statement was true, the contrapositive will also be true.
46. A biconditional statement is true only if the conditional and converse statements are both true. The shirt could have been purchased at another location other than the mall, so the sister is correct.
47. no; The contrapositive is equivalent to the original conditional statement. In order to write a conditional statement as a true biconditional statement, you must know that the converse (or inverse) is true.
48. The if-then statement is the inverse of the conditional statement:
Conditional statement: $p \rightarrow q$; inverse: $\sim p \rightarrow \sim q$
49. If you tell the truth, then you don't have to remember anything.
Hypothesis: You tell the truth.
Conclusion: You don't have to remember anything.
50. If you expect things of yourself, then you can do them.
Hypothesis: You expect things of yourself.
Conclusion: You can do them.
51. If one is lucky, then a solitary fantasy can totally transform one million realities.
Hypothesis: One is lucky.
Conclusion: A solitary fantasy can totally transform one million realities.
52. If you are happy, then you will make others happy too.
Hypothesis: You are happy.
Conclusion: You will make others happy too.
53. no; "If $x^2 - 10 = x + 2$, then $x = 4$ " is a false statement because $x = -3$ is also possible. The converse, however, of the original conditional statement is true. In order for a biconditional statement to be true, both the conditional statement and its converse must be true.
54. a. *Sample answer:* If a natural arch is the largest in the United States, then it is the Landscape Arch. If a natural arch is the Landscape Arch, then it spans 290 feet.
b. Contrapositive: If a natural arch is not the Landscape Arch, then it is not the largest in the United States. If a natural arch does not span 290 feet, then it is not the Landscape Arch.
- c. Converse: If a natural arch is the Landscape Arch, then it is the largest in the United States.
Inverse: If a natural arch is not the largest in the United States, then it is not the Landscape Arch.
Both of these statements are true because there is only one arch that fits both criteria.
Converse: If a natural arch spans 290 feet, then it is the Landscape Arch.
Inverse: If a natural arch is not the Landscape Arch, then it does not span 290 feet.
Both of these statements are false because it is possible for a natural arch in another country to span 290 feet.
55. A; You can rewrite the given statement in if-then form as: If you do your homework, then you can watch a movie afterward.
56. *Sample answer:*
If $4x = 28$, then $x = 7$. (true)
If $5y = 25$, then $y = 4$. (false)
If 6 times your age is subtracted from 5 times my age, then the result is 0. (Whether the statement is true or false depends on the ages of the people. If your age is 15 and my age is 18, then this statement is true, however if your age is 18 and my age is 15, then this statement is false.)
57. If yesterday was February 28, then today is March 1.
58. *Sample answer:* If a person is in chorus, then the person is a musician.
If a person is in jazz band, then the person is in band.
If a person is in band, then the person is a musician.
59. a. 
If you see a cat, then you went to the zoo to see a lion; The original statement is true, because a lion is a type of cat, but the converse is false, because you could see a cat without going to the zoo.
- b. 
If you wear a helmet, then you play a sport; Both the original statement and the converse are false, because not all sports require helmets and sometimes helmets are worn for activities that are not considered a sport, such as construction work.

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c.



If this month is not February, then it has 31 days; The original statement is true, because February never has 31 days, but the converse is false, because a month that is not February could have 30 days.

60. a. true (as long as $x \neq y$)

b. If the mean of the data is between x and y , then x and y are the least and greatest values in your data set. This converse is false, because x and y could be any two values in the set as long as one is higher and one is lower than the mean.

c. If a data set has a mean, median, and a mode, then the mode of the data set will always be a data value. The mean is always a calculated value that is not necessarily equal to any of the data values, and the median is a calculated value when there are an even number of data values. The mode is the data value with the greatest frequency, so it is always a data value.

61. *Sample answer:*

Conditional statement: If the course is Biology, then the class is a science class.

Converse: If the class is a science class, then the course is Biology.

62. By definition of linear pairs, $\angle 1$ and $\angle 2$ are supplementary. So, if $m\angle 1 = 90^\circ$, then $m\angle 2 = 90^\circ$. Also, by definition of linear pairs, $\angle 2$ and $\angle 3$ are supplementary. So, if $m\angle 2 = 90^\circ$, then $m\angle 3 = 90^\circ$. Finally, by definition of linear pairs, $\angle 3$ and $\angle 4$ are supplementary. So, if $m\angle 3 = 90^\circ$, then $m\angle 4 = 90^\circ$.

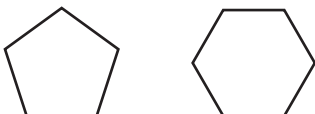
63. *Sample answer:*

Slogan: "This treadmill is a fat-burning machine!"

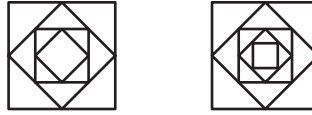
Conditional statement: If you use this treadmill, then you will burn fat quickly.

Maintaining Mathematical Proficiency

64. The pattern is to add a side to the previous polygon.



65. The pattern is to add a square that connects the midpoints of the previously added square.



66. The pattern is to add 2 to the previous number.

1

$$1 + 2 = 3$$

$$3 + 2 = 5$$

$$5 + 2 = 7$$

$$7 + 2 = 9$$

$$9 + 2 = 11$$

The next two numbers in the pattern are 9 and 11.

67. The pattern is to add 11 to the previous number:

12

$$12 + 11 = 23$$

$$23 + 11 = 34$$

$$34 + 11 = 45$$

$$45 + 11 = 56$$

$$56 + 11 = 67$$

The next two numbers in the pattern are 56 and 67.

68. The pattern is to multiply the previous number by $\frac{2}{3}$:

2

$$2 \cdot \frac{2}{3} = \frac{4}{3}$$

$$\frac{4}{3} \cdot \frac{2}{3} = \frac{8}{9}$$

$$\frac{8}{9} \cdot \frac{2}{3} = \frac{16}{27}$$

$$\frac{16}{27} \cdot \frac{2}{3} = \frac{32}{81}$$

$$\frac{32}{81} \cdot \frac{2}{3} = \frac{64}{243}$$

The next two numbers in the pattern are $\frac{32}{81}$ and $\frac{64}{243}$.

69. The pattern is n^2 , where $n \geq 1$.

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

The next two numbers in the pattern are 25 and 36.

Chapter 2

2.2 Explorations (p. 75)

1. a. The circle is rotating from one vertex in the triangle to the next in a clockwise direction.



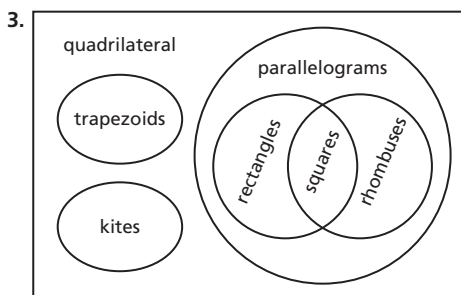
- b. The pattern alternates between a curve in an odd quadrant and a line segment with a negative slope in an even quadrant. The quadrants with a curve or a line segment follow the pattern I, IV, III, II, and the curves follow the pattern of two concave down and two concave up.



- c. The pattern alternates between the first three arrangements, then their respective mirror images.



2. a. true; Because all of Property B is inside Property A, all items with Property B must also have Property A.
 b. false; There is a region for items that have Property A but not B.
 c. false; There is a region for items that have Property A but not C.
 d. true; There is a region for items that have Property A but not B.
 e. true; There is no intersection of the regions for Properties C and B.
 f. true; There is a region that is the intersection of Properties A and C.
 g. false; There is no intersection of the regions for Properties B and C.



Sample answer:

If a quadrilateral is a rectangle, then it is a parallelogram.

If a quadrilateral is a kite, then it is not a parallelogram.

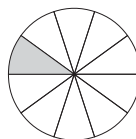
If a quadrilateral is a square, then it is a rhombus, a rectangle, and a parallelogram.

4. You can look for a pattern and then use a “rule” based on that pattern to predict what will happen if the pattern continues.

5. *Sample answer:* You noticed that you did much better on your math tests when you were able to study for at least one hour the night before as opposed to when you were only able to study for less than an hour. So now you make sure that you study for at least one hour the night before a test.

2.2 Monitoring Progress (pp. 76–79)

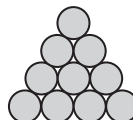
1. Divide the circle into 10 equal parts. Shade the section just above the horizontal segment on the left.



2.



3. Add 4 circles at the bottom.



4. The product of any three negative integers will yield a negative integer.

$$\text{Tests: } (-2) \cdot (-6) \cdot (-4) = -48$$

$$(-5) \cdot (-2) \cdot (-1) = -10$$

5. The sum of any five consecutive integers is 5 times the middle (third) number.

$$\text{Tests: } 2 + 3 + 4 + 5 + 6 = 20 = 5(4)$$

$$-2 + (-1) + 0 + 1 + 2 = 0 = 5(0)$$

6. *Sample answer:* If $x = \frac{1}{2}$, then $x^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$. $\frac{1}{4}$ is less than $\frac{1}{2}$, not greater.

7. *Sample answer:* The sum of -1 and -3 is $-1 + (-3) = -4$. The difference of -1 and -3 is $-1 - (-3) = -1 + 3 = 2$. Because $-4 < 2$, the sum is not greater than the difference.

8. $\angle R$ is obtuse.

9. If you get an A on your math test, then you can watch your favorite actor.

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10. Conjecture: The sum of a number and itself is 2 times the number. If n is the number, then $n + n = 2n$.

Inductively: $4 + 4 = 8$, $10 + 10 = 20$,
 $45 + 45 = 90$, $n + n = 2n$

Deductively: Let n be any number. By the Reflexive Property, $n = n$. If n is added to each side by the Addition Property, then $n + n = n + n$. Combining like terms yields $2n = 2n$. Therefore, $n + n = 2n$, which means the sum of any number and itself is 2 times the number.

11. Deductive reasoning is used because the Law of Detachment is used to reach the conclusion.

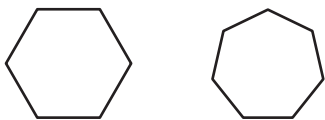
2.2 Exercises (pp. 80–82)

Vocabulary and Core Concept Check

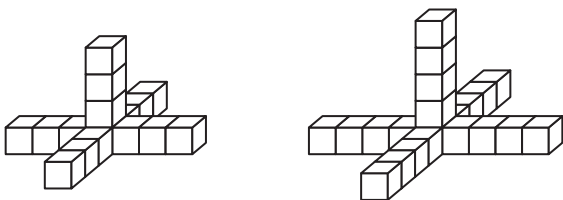
- Because the prefix *counter-* means “opposing,” a counterexample opposes the truth of the statement.
- Inductive reasoning uses patterns to write a conjecture. Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.

Monitoring Progress and Modeling with Mathematics

- The absolute value of each number in the list is 1 greater than the absolute value of the previous number in the list, and the signs alternate from positive to negative. The next two numbers are: $-6, 7$.
- The numbers are increasing by successive multiples of 2. The sequence is: $0 + 2 = 2$, $2 + 4 = 6$, $6 + 6 = 12$, $12 + 8 = 20$, $20 + 10 = 30$, $30 + 12 = 42$, etc. So, the next two numbers are: $30, 42$.
- The pattern is the alphabet written backward. The next two letters are: U, T.
- The letters represent the first letter of each month of the year, and they are in the order of the months. The next two letters are: J, J.
- The pattern is regular polygons having one more side than the previous polygon.



- The pattern is the addition of 5 blocks to the previous figure. One block is added to each of the four ends of the base and one block is added on top. So, the next two figures will have 16 blocks and then 21 blocks.



9. The product of any two even integers is an even integer.

Tests: $2 \cdot 8 = 16$, $22 \cdot 20 = 440$

10. The sum of an even integer and an odd integer is an odd integer.

Tests: $3 + 4 = 7$, $6 + 13 = 19$

11. The quotient of a number and its reciprocal is the square of that number.

Tests: $\frac{10}{\left(\frac{1}{10}\right)} = \frac{10}{1} \cdot \frac{10}{1} = 100 = 10^2$
 $\frac{\left(\frac{2}{3}\right)}{\left(\frac{3}{2}\right)} = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9} = \left(\frac{2}{3}\right)^2$

12. The quotient of two negative numbers is a positive rational number.

Tests: $\frac{-24}{-12} = 2$, $\frac{-33}{-3} = 11$

13. *Sample answer:* Let the two positive numbers be $\frac{1}{2}$ and $\frac{1}{6}$.

The product is $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$. Because $\frac{1}{12} < \frac{1}{2}$ and $\frac{1}{12} < \frac{1}{6}$, the product of two positive numbers is not always greater than either number.

14. *Sample answer:* Let $n = -1$.

$\frac{-1 + 1}{-1} = 0$
 $0 \nless 1$

15. Each angle could be 90° . Then neither are acute.

16. If line s intersects \overline{MN} at any point other than the midpoint, it is not a segment bisector.

17. You passed the class.

18. not possible; You may get to the movies by other means.

19. not possible; $QRST$ could be a rectangle.

20. P is the midpoint of \overline{LH} .

21. not possible

22. If $\frac{1}{2}a = 1\frac{1}{2}$, then $5a = 15$.

23. If a figure is a rhombus, then the figure has two pairs of opposite sides that are parallel.

24. not possible

25. The law of logic used was the Law of Syllogism.

26. The law of logic used was the Law of Detachment.

27. The law of logic used was the Law of Detachment.

Chapter 2

28. The law of logic used was Law of Syllogism.

29. $1 + 3 = 4$, $3 + 5 = 8$, $7 + 9 = 16$

Conjecture: The sum of two odd integers is an even integer. Let m and n be integers, then $(2m + 1)$ and $(2n + 1)$ are odd integers.

$$\begin{aligned}(2m + 1) + (2n + 1) &= 2m + 2n + 2 \\ &= 2(m + n + 1)\end{aligned}$$

Any number multiplied by 2 is an even number. Therefore, the sum of two odd integers is an even integer.

30. $1 \cdot 3 = 3$, $3 \cdot 5 = 15$, $7 \cdot 9 = 63$

Conjecture: The product of two odd integers is an odd integer. Let m and n be integers. Then $(2m + 1)$ and $(2n + 1)$ are odd integers.

$$\begin{aligned}(2m + 1)(2n + 1) &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1\end{aligned}$$

Any number multiplied by 2 is an even number, and adding 1 will yield an odd number. Therefore, the product of two odd integers is an odd integer.

31. inductive reasoning; The conjecture is based on the assumption that a pattern, observed in specific cases, will continue.

32. deductive reasoning; The conclusion is based on mathematical definitions and properties.

33. deductive reasoning; Laws of nature and the Law of Syllogism were used to draw the conclusion.

34. inductive reasoning; The conjecture is based on the assumption that a pattern, observed in specific cases, will continue.

35. The Law of Detachment cannot be used because the hypothesis is not true; *Sample answer:* Using the Law of Detachment, because a square is a rectangle, you can conclude that a square has four sides.

36. The conjecture was based on a pattern in specific cases, not rules or laws about the general case; Using inductive reasoning, you can make a conjecture that you will arrive at school before your friend tomorrow.

37. Using inductive reasoning, you can make a conjecture that male tigers weigh more than female tigers because this was true in all of the specific cases listed in the table.

38. a. yes; Bases on inductive reasoning, the pattern in all of the years shown is that the number of girls participating is more than the year before.

b. no; There is no information in the graph about how the number of girl participants compares with the number of boy participants.

39. 1: $2 = 1(2)$

$$2: 2 + 4 = 6 = 2(3)$$

$$3: 2 + 4 + 6 = 12 = 3(4)$$

$$4: 2 + 4 + 6 + 8 = 20 = 4(5)$$

$$5: 2 + 4 + 6 + 8 + 10 = 30 = 5(6)$$

$$\vdots \quad \vdots$$

$$n: n(n + 1)$$

So, the sum of the first n positive even integers is $n(n + 1)$.

40. a. $1 + 1 = 2$, $2 + 1 = 3$, $3 + 2 = 5$, $5 + 3 = 8$,
 $8 + 5 = 13$, $13 + 8 = 21$, $21 + 13 = 34$

Each number in the sequence is the sum of the previous two numbers in the sequence.

b. $21 + 34 = 55$

$$34 + 55 = 89$$

$$55 + 89 = 144$$

c. *Sample answer:* A spiral can be drawn by connecting the opposite corners of squares with side lengths that follow the Fibonacci sequence. This spiral is similar to the spiral seen on nautilus shells. It is also similar to the golden spiral, which is sometimes found in spiraling galaxies.

41. Argument 2: This argument uses the Law of Detachment to say that when the hypothesis is met, the conclusion is true.

42. Pattern 1: Multiply each term by 2.

$$\frac{1}{4} \cdot 2 = \frac{1}{2}, \frac{1}{2} \cdot 2 = \frac{2}{2} = 1, 1 \cdot 2 = 2, 2 \cdot 2 = 4, 2 \cdot 4 = 8$$

Pattern 2: Add $\frac{1}{4}$ to the previous term.

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

$$\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$$

$$\frac{5}{4} + \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$$

Pattern 3: Multiply each term by half the reciprocal of the previous term.

$$\frac{1}{4} \cdot \left(\frac{1}{2} \cdot 4\right) = \frac{1}{4} \cdot 2 = \frac{1}{2}$$

$$\frac{1}{2} \cdot \left(\frac{1}{2} \cdot 2\right) = \frac{1}{2} \cdot 2 = \frac{1}{2}$$

$$\frac{1}{2} \cdot \left(\frac{1}{2} \cdot 2\right) = \frac{1}{2} \cdot 2 = \frac{1}{2}$$

$$\frac{1}{2} \cdot \left(\frac{1}{2} \cdot 2\right) = \frac{1}{2} \cdot 2 = \frac{1}{2}$$

43. The value of y is 2 more than three times the value of x ;

$$y = 3x + 2;$$

Sample answer: If $x = 10$, then $y = 3(10) + 2 = 32$;

If $x = 72$, then $y = 3(72) + 2 = 218$.

Chapter 2

44. a. Figure 1 has a perimeter of 4.
Figure 2 has a perimeter of 8.
Figure 3 has a perimeter of 12.
Figure 4 has a perimeter of 16.
Figure 5 has a perimeter of 20.
Figure 6 has a perimeter of 24.
Figure 7 has a perimeter of 28.
The perimeter is equal to the product of 4 and the figure number.
- b. The 20th figure has a perimeter of $4(20) = 80$.
45. a. true; Based on the Law of Syllogism, if you went camping at Yellowstone, and Yellowstone is in Wyoming, then you went camping in Wyoming.
- b. false; When you go camping, you go canoeing, but even though your friend always goes camping when you do, he or she may not choose to go canoeing with you.
- c. true; We know that if you go on a hike, your friend goes with you, and we know that you went on a hike. So, based on the Law of Detachment, your friend went on a hike.
- d. false; We know that you and your friend went on a hike, but we do not know where. We just know that there is a 3-mile long trail near where you went camping.
46. a. Mineral *C* must be Talc. Because it was scratched by all three of the other minerals, it must have the lowest hardness rating. Because Mineral *B* has a higher hardness rating than Mineral *A*, Mineral *A* could be either Gypsum or Calcite, and Mineral *B* could be either Calcite or Fluorite.
- b. Check Mineral *B* and Mineral *D*. If Mineral *D* scratches Mineral *B*, then Mineral *D* is Fluorite, Mineral *B* is Calcite, and Mineral *A* is Gypsum. If Mineral *B* scratches Mineral *D*, then Mineral *B* is Fluorite, and you have to check Mineral *D* and Mineral *A*. The one that scratches the other has the higher hardness rating and is therefore Calcite. The one that gets scratched is Gypsum.

Maintaining Mathematical Proficiency

47. Segment Addition Postulate (Post. 1.2)
48. Angle Addition Postulate (Post. 1.4)
49. Ruler Postulate (Post. 1.1)
50. Protractor Postulate (Post. 1.3)

2.3 Explorations (p. 83)

1. The diagram can be turned at any angle to the right or to the left and the lines will appear to be perpendicular.
2. a. true; For every set of two intersecting lines, there is exactly one plane that is defined, so it can be assumed that all of the points shown are coplanar.

- b. false; For every two points there is exactly one line, the third point does not necessarily have to be on the same line as the other two.
- c. true; All three points lie on the same line, \overleftrightarrow{AH} .
- d. true; $\angle GFH$ is marked as a right angle.
- e. true; By definition of a linear pair, the sides of $\angle BCA$ and $\angle ACD$ form a straight line (straight angle).
- f. false; \overleftrightarrow{AF} and \overleftrightarrow{BD} are not necessarily perpendicular because the angle is not marked.
- g. false; \overleftrightarrow{EG} and \overleftrightarrow{BD} are not necessarily parallel, there is not enough information about the related angles.
- h. true; \overleftrightarrow{AF} and \overleftrightarrow{BD} are coplanar.
- i. false; \overleftrightarrow{EG} and \overleftrightarrow{BD} could possibly intersect.
- j. true; \overleftrightarrow{AF} and \overleftrightarrow{BD} intersect at point *C*.
- k. false; \overleftrightarrow{EG} and \overleftrightarrow{AH} are perpendicular. So, \overleftrightarrow{EG} cannot be perpendicular to two different lines that intersect.
- l. true; $\angle ACD$ and $\angle BCF$ form two pairs of opposite rays.
- m. true; \overleftrightarrow{AC} and \overleftrightarrow{FH} are the same line because the points *A*, *C*, *F*, and *H* are all collinear.
3. You can assume intersecting lines, opposite rays, vertical angles, linear pairs, adjacent angles, coplanar (points, lines, rays, etc.), collinear points, which point is between two other points, and which points are in the interior of an angle. You have to have a label for identifying angle measures, segment lengths, perpendicular lines, parallel lines, and congruent segments or angles.
4. *Sample answer:* $\angle ACD$ and $\angle DCF$ form a linear pair, because these angles share a vertex and a side but no common interior points and $\angle ACF$ is a straight angle. $\angle CFE$ and $\angle GFH$ are vertical angles, because \overleftrightarrow{FG} and \overleftrightarrow{FE} are opposite rays as well as \overleftrightarrow{FC} and \overleftrightarrow{FH} ; $\angle DCF$ is a right angle, which cannot be assumed because angle measurements have to be marked. $\overline{BC} \cong \overline{CD}$, which cannot be assumed because lengths of segments have to be labeled.

2.3 Monitoring Progress (pp. 84–86)

1. Plane Intersection Postulate (Post. 2.7)
2. a. Line *n* passes through points *A* and *B*.
b. Line *n* contains points *A* and *B*.
c. Line *m* and line *n* intersect at point *A*.
3. Mark each segment with double tick marks to show that $\overline{PW} \cong \overline{WQ}$.
4. *Sample answer:* $\angle TWP$ and $\angle PWV$ are supplementary.
5. Yes, by the Plane Intersection Postulate (Post. 2.7), plane *T* intersects plane *S* at \overline{BC} .
6. Because of the right angle symbol you know that plane *T* is perpendicular to plane *S*. If \overline{AB} is perpendicular to plane *S* and \overline{AB} intersects \overline{BC} in plane *S* at point *B*, then $\overline{AB} \perp \overline{BC}$.

Chapter 2

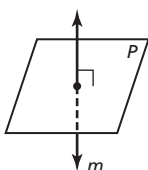
2.3 Exercises (pp. 87–88)

Vocabulary and Core Concept Check

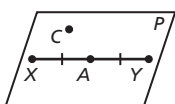
- Through any three noncollinear points, there exists exactly one plane.
- Two points determine a line, which could be on infinitely many planes, but only one plane will go through those two points and a third noncollinear point.

Monitoring Progress and Modeling With Mathematics

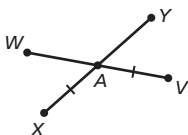
- Two Point Postulate (Post. 2.1): Through any two points there exists exactly one line.
- Plane-Point Postulate (Post. 2.5): A plane contains at least three noncollinear points.
- Sample answer:* Line p contains points H and G .
- Sample answer:* Lines p and q intersect at point H .
- Sample answer:* Through points J , G , and L there is exactly one plane, which is plane M .
- Sample answer:* Points J and K lie in plane M , so line q lies in plane M .
- Plane P and line m intersect at a 90° angle.



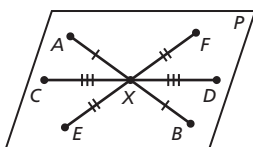
- Plane P contains \overline{XY} , point A bisects \overline{XY} , and point C is not on \overline{XY} .



- \overline{XY} intersects \overline{WV} at point A , so that $XA = VA$.



- \overline{AB} , \overline{CD} , and \overline{EF} are all in plane P and point X is the midpoint of each segment.



- Yes, planes W and X intersect at \overleftrightarrow{KL} .

- Yes, N , K , and M are collinear with L not on the line, so these points are coplanar.
- No, Q is a point contained in plane W , M is a point contained in plane X , and J is a point on the intersection of the planes, so they are three noncollinear points.
- No, \overleftrightarrow{RP} and \overleftrightarrow{MN} both intersect \overleftrightarrow{JL} (which is contained in both planes) at two different points.
- Yes, the line of intersection is contained in both planes.
- No, there is not enough information given.
- Yes, $\angle NKL$ and $\angle JKM$ are vertical angles.
- Yes, the nonadjacent sides form a straight angle.
- In order to determine that M is the midpoint of \overline{AC} or \overline{BD} , the segments that would have to be marked as congruent are \overline{AM} and \overline{MC} or \overline{DM} and \overline{MB} , respectively; Based on the diagram and markings, you can assume \overline{AC} and \overline{DB} intersect at point M , such that $\overline{AM} \cong \overline{MB}$ and $\overline{DM} \cong \overline{MC}$.
- In order to assume that an angle measures 90° , the angle must be marked as such; Based on the diagram, you can assume two pairs of vertical angles, $\angle DMC$ and $\angle AMB$ or $\angle DMA$ and $\angle CMB$, and you can assume linear pairs, such as $\angle DMC$ and $\angle CMB$.
- The statements that cannot be concluded are: C , D , F , and H .
- one; Based on the Line-Point Postulate (Post. 2.2), line m contains at least two points. Because these two points are noncollinear with point C , based on the Three Point Postulate (Post. 2.4), there is exactly one plane that goes through line m and point C .
- Two Point Postulate (Post. 2.1)
- Line Intersection Postulate (Post. 2.3)
- Two Point Postulate: Through any two points, there exists exactly one line.
 - Conditional statement: If there are two points, then there exists exactly one line that passes through them.
 - Converse: If there exists exactly one line that passes through a given point or points, then there are two points. (False)
Inverse: If there are not two points, then there is not exactly one line that passes through them. (False)
Contrapositive: If there is not exactly one line that passes through a given point or points, then there are not two points. (True)
- Plane-Point Postulate: A plane contains at least three noncollinear points.
 - Conditional statement: If a plane exists, then it contains at least three noncollinear points.

Chapter 2

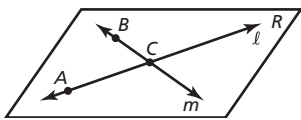
b. Converse: If a plane contains at least three noncollinear points, then the plane exists. (True)

Inverse: If no plane exists, then there are not three noncollinear points. (True)

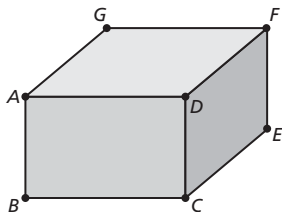
Contrapositive: If there are not three noncollinear points, then a plane has not been defined. (True)

29. Number of points to determine a line < number of points to determine a plane

30. yes; Let two lines ℓ and m intersect at point C . There must be a second point on each line, A in ℓ and B in m . Through the three noncollinear points A , B , and C , there exists exactly one plane R . Because A and C are in R , ℓ is in R . Because B and C are in R , m is in R .

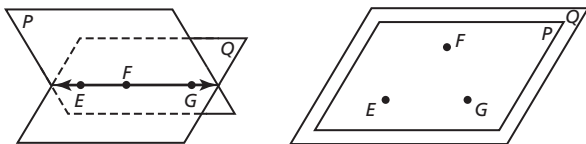


31. Yes, for example, three planes, $ABCD$, $DCEF$, and $DFGA$, have point D in common.



32. no; The postulate states that if two planes intersect, they will intersect in a line. But planes can be parallel and never intersect. For example, the ceiling and floor of a room are parallel.

33. Points E , F , and G must be collinear. They must be on the line that intersects plane P and plane Q ; Points E , F , and G can be either collinear or not collinear.



34. *Sample answer:* The Line Intersection Postulate (Post. 2.3) would have to be altered. In spherical geometry, if two lines intersect, then their intersection is exactly two points. The two points of intersection would be the endpoints of a diameter.

Maintaining Mathematical Proficiency

35. Addition Property of Equality

$$t - 6 = -4$$

$$t - 6 + 6 = -4 + 6$$

$$t = 2$$

36. Division Property of Equality

$$3x = 21$$

$$\frac{3x}{3} = \frac{21}{3}$$

$$x = 7$$

37. Subtraction Property of Equality

$$9 + x = 13$$

$$9 - 9 + x = 13 - 9$$

$$x = 4$$

38. Multiplication Property of Equality

$$\frac{x}{7} = 5$$

$$7 \cdot \frac{x}{7} = 5 \cdot 7$$

$$x = 35$$

2.1–2.3 What Did You Learn? (p. 89)

1. “If you are in math class, then you are in geometry,” is false. You could be in another math class, for example, you could be in Algebra I or Calculus.

“If you do your math homework, then you will do well on the test,” is false. Some students can do all their homework, however, they may have test anxiety, in which case they may not do well on the test.

“If it does not snow, then I will run outside” is false. On a day that it is not snowing you may feel too sick to run outside.

2. a. p : You go to the zoo to see a lion.

q : You will see a cat.

p	q	$q \rightarrow p$
T	T	T
T	F	F
F	T	T
F	F	T

b. p : You play a sport.

q : You wear a helmet.

p	q	$q \rightarrow p$
T	T	T
T	F	F
F	T	T
F	F	T

c. p : This month has 31 days.

q : It is not February.

p	q	$q \rightarrow p$
T	T	T
T	F	F
F	T	T
F	F	T

3. *Sample answer:* What about parallel lines? Do they intersect?

Chapter 2

2.1–2.3 Quiz (p. 90)

1. If-then form: If an angle measures 167° , then the angle is an obtuse angle. (True)

Converse: If an angle is obtuse, then the angle measures 167° . (False)

Inverse: If an angle does not measure 167° , then the angle is not an obtuse angle. (False)

Contrapositive: If an angle is not obtuse, then the measure of the angle is not 167° . (True)

2. If-then form: If you are in physics class, then you always have homework. (True)

Converse: If you always have homework, then you are in physics class. (False)

Inverse: If you are not in physics class, then you do not always have homework. (False)

Contrapositive: If you do not always have homework, then you are not in physics class. (True)

3. If-then form: If I take my driving test, then I will get my driver's license. (False)

Converse: If I get my driver's license, then I took my driving test. (True)

Inverse: If I do not take my driving test, then I will not get my driver's license. (True)

Contrapositive: If I do not get my driver's license, then I did not take my driving test. (False)

4. *Sample answer:* $5 + (-14) = -9$

5. *Sample answer:* A figure with four sides that is not a rectangle is a trapezoid.

6. The sum of two negative integers is a negative integer.

Inductive reasoning: $-2 + (-4) = -6$, $-23 + (-14) = -37$

Deductive reasoning: When you add two integers with the same sign, the rule is that you first add the absolute values, and then give the sum the same sign as the addends. So, the sum will be negative.

7. The difference of two even integers is an even integer.

Inductive reasoning: $4 - 2 = 2$, $84 - 62 = 22$

Deductive reasoning: Let m and n be integers. Then $2n$ and $2m$ are even integers because they are the product of 2 and an integer. $2n - 2m$ represents the difference of the two even integers. By the Distributive Property, $2n - 2m = 2(n - m)$, and $2(n - m)$ is an even integer because it is the product of 2 and an integer $(n - m)$.

8. Yes, points D , B , and C are coplanar, because three noncollinear points determine a plane.

9. No, in order for the planes to be parallel, it would have to be shown that the line that contains the points G , B , and A is perpendicular to each plane.

10. Yes, two lines intersect in one point.

11. Yes, if two points lie in the plane, then the line containing them lies in the plane.

12. No, there is no indication that \overleftrightarrow{BG} is perpendicular to \overleftrightarrow{BD} .

13. no; The converse of that would be, "If I used the green ball, then I got a strike," and only one counterexample of using the green ball and not getting a strike or getting a strike with another color ball would be all you need to disprove the biconditional statement for the given conditional statement.

14. a. *Sample answer:* The males' running times are faster than the females' running times.

- b. Inductive reasoning was used, because the conjecture was based on the specific cases represented in the table.

15. Two Point Postulate (Post. 2.1): Points C and D contain one line, \overleftrightarrow{CD} .

Line-Point Postulate (Post. 2.2): \overleftrightarrow{CD} contains at least two points, C and D .

Line Intersection Postulate (Post. 2.3): Line m and line n intersect at exactly one point, G .

Three Point Postulate (Post. 2.4): Through points A , B , and C , there exists exactly one plane, X .

Plane-Point Postulate (Post. 2.5): Plane X contains the noncollinear points A , B , and D .

Plane-Line Postulate (Post. 2.6): Points A and B lie in plane X , therefore the line containing them, \overleftrightarrow{AB} , also lies in plane X .

Plane Intersection Postulate (Post. 2.7): Plane Y and plane X intersect in \overleftrightarrow{CD} .

2.4 Explorations (p. 91)

1. Distribution Property

Simplify.

Subtraction Property of Equality

Combine like terms.

Subtraction Property of Equality

Combine like terms.

Division Property of Equality

Simplify.

Symmetric Property of Equality

Chapter 2

2. The diamond represents multiplication because $0 \times 5 = 0$.
The circle represents addition because $0 + 5 = 5$.

Commutative Property of Multiplication

Commutative Property of Addition

Associative Property of Multiplication

Associative Property of Addition

Zero Property of Multiplication

Identity Property of Addition

Identity Property of Multiplication

Distributive Property

3. Algebraic properties help you solve an equation by isolating the variable on one side of the equation.

4. Equation	Reason
$3(x + 1) - 1 = -13$	Write the equation.
$3x + 3 - 1 = -13$	Distributive Property
$3x + 2 = -13$	Combine like terms.
$3x + 2 - 2 = -13 - 2$	Subtraction Property of Equality
$3x = -15$	Combine like terms.
$\frac{3x}{3} = \frac{-15}{3}$	Division Property of Equality
$x = -5$	Simplify.

2.4 Monitoring Progress (pp. 92–95)

1. Equation	Explanation and Reason
$6x - 11 = -35$	Write the equation; Given
$6x - 11 + 11 = -35 + 11$	Add 11 to each side; Addition Property of Equality
$6x = -24$	Combine like terms; Simplify.
$x = -4$	Divide each side by 6; Division Property of Equality

2. Equation	Explanation and Reason
$-2p - 9 = 10p - 17$	Write the equation; Given
$-2p - 10p - 9 = 10p - 10p - 17$	Subtract $10p$ from each side; Subtraction Property of Equality
$-12p - 9 = -17$	Combine like terms; Simplify.
$-12p - 9 + 9 = -17 + 9$	Add 9 to each side; Addition Property of Equality
$-12p = -8$	Combine like terms; Simplify.
$p = \frac{2}{3}$	Divide each side by -12 ; Division Property of Equality

3. Equation	Explanation and Reason
$39 - 5z = -1 + 5z$	Write the equation; Given
$39 - 5z - 5z = -1 + 5z - 5z$	Subtract $5z$ from each side; Subtraction Property of Equality
$39 - 10z = -1$	Combine like terms; Simplify.
$39 - 39 - 10z = -1 - 39$	Subtract 39 from each side; Subtraction Property of Equality
$-10z = -40$	Combine like terms; Simplify.
$z = 4$	Divide each side by -10 ; Division Property of Equality

4. Equation	Explanation and Reason
$3(3x + 14) = -3$	Write the equation; Given
$9x + 42 = -3$	Multiply; Distributive Property
$9x = -45$	Subtract 42 from each side; Subtraction Property of Equality
$x = -5$	Divide each side by 9; Division Property of Equality

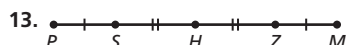
5. Equation	Explanation and Reason
$4 = -10b + 6(2 - b)$	Write the equation; Given
$4 = -10b + 12 - 6b$	Multiply; Distributive Property
$4 = -16b + 12$	Combine like terms; Simplify.
$-8 = -16b$	Subtract 12 from each side; Subtraction Property of Equality
$\frac{1}{2} = b$	Divide each side by -16 ; Division Property of Equality
$b = \frac{1}{2}$	Rewrite the equation; Symmetric Property of Equality

6. Equation	Explanation and Reason
$A = \frac{1}{2}bh$	Write the equation; Given
$2A = bh$	Multiply each side by 2; Multiplication Property of Equality
$\frac{2A}{h} = b$	Divide each side by h ; Division Property of Equality
$b = \frac{2A}{h}$	Rewrite the equation; Symmetric Property of Equality
$b = \frac{2 \cdot 952}{56} = \frac{1904}{56} = 34$	
The base is 34 feet when the area is 952 square feet and the height is 56 feet.	

7. The property illustrated is the Symmetric Property of Equality.
8. The property illustrated is the Reflexive Property of Equality.
9. The property illustrated is the Transitive Property of Equality.

Chapter 2

10. The property illustrated is the Transitive Property of Equality.
11. The property illustrated is the Symmetric Property of Equality.
12. The property illustrated is the Reflexive Property of Equality.



Equation	Explanation and Reason
$SH = HZ$	Marked in diagram; Given
$PS = ZM$	Marked in diagram; Given
$PH = PS + SH$	Add lengths of adjacent segments; Segment Addition Postulate (Post. 1.2)
$HM = ZM + HZ$	Add lengths of adjacent segments; Segment Addition Postulate (Post. 1.2)
$PH = ZM + HZ$	Substitute ZM for PS and HZ for SH ; Substitution Property of Equality
$PH = HM$	Substitute HM for $ZM + HZ$; Substitution Property of Equality

2.4 Exercises (pp. 96–98)

Vocabulary and Core Concept Check

- Reflexive Property of Equality
- “If $e = f$ and $f = g$, then $e = g$ ” is different. It represents the Transitive Property of Equality. The other three statements represent the Symmetric Property of Equality.

Monitoring Progress and Modeling with Mathematics

- Subtraction Property of Equality
Addition Property of Equality
Division Property of Equality

- Distributive Property
Subtraction Property of Equality
Addition Property of Equality

5. Equation	Explanation and Reason
$5x - 10 = -40$	Write the equation; Given
$5x = -30$	Add 10 to each side; Addition Property of Equality
$x = -6$	Divide each side by 5; Division Property of Equality
6. Equation	Explanation and Reason
$6x + 17 = -7$	Write the equation; Given
$6x = -24$	Subtract 17 from each side; Subtraction Property of Equality
$x = -4$	Divide each side by 6; Division Property of Equality

7. Equation	Explanation and Reason
$2x - 8 = 6x - 20$	Write the equation; Given
$-4x - 8 = -20$	Subtract $6x$ from each side; Subtraction Property of Equality
$-4x = -12$	Add 8 to each side; Addition Property of Equality
$x = 3$	Divide each side by -4 ; Division Property of Equality

8. Equation	Explanation and Reason
$4x + 9 = 16 - 3x$	Write the equation; Given
$7x + 9 = 16$	Add $3x$ to each side; Addition Property of Equality
$7x = 7$	Subtract 9 from each side; Subtraction Property of Equality
$x = 1$	Divide each side by 7; Division Property of Equality

9. Equation	Explanation and Reason
$5(3x - 20) = -10$	Write the equation; Given
$15x - 100 = -10$	Multiply; Distributive Property
$15x = 90$	Add 100 to each side; Addition Property of Equality
$x = 6$	Divide each side by 15; Division Property of Equality

10. Equation	Explanation and Reason
$3(2x + 11) = 9$	Write the equation; Given
$6x + 33 = 9$	Multiply; Distributive Property
$6x = -24$	Subtract 33 from each side; Subtraction Property of Equality
$x = -4$	Divide each side by 6; Division Property of Equality

11. Equation	Explanation and Reason
$2(-x - 5) = 12$	Write the equation; Given
$-2x - 10 = 12$	Multiply; Distributive Property
$-2x = 22$	Add 10 to each side; Addition Property of Equality
$x = -11$	Divide each side by -2 ; Division Property of Equality

12. Equation	Explanation and Reason
$44 - 2(3x + 4) = -18x$	Write the equation; Given
$44 - 6x - 8 = -18x$	Multiply; Distributive Property
$-6x + 36 = -18x$	Combine like terms; Simplify.
$36 = -12x$	Add $6x$ to each side; Addition Property of Equality
$-3 = x$	Divide each side by -12 ; Division Property of Equality
$x = -3$	Rewrite the solution; Symmetric Property of Equality

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13. Equation	Explanation and Reason
$4(5x - 9) = -2(x + 7)$	Write the equation; Given
$20x - 36 = -2x - 14$	Multiply on each side; Distributive Property
$22x - 36 = -14$	Add $2x$ to each side; Addition Property of Equality
$22x = 22$	Add 36 to each side; Addition Property of Equality
$x = 1$	Divide each side by 22; Division Property of Equality
14. Equation	Explanation and Reason
$3(4x + 7) = 5(3x + 3)$	Write the equation; Given
$12x + 21 = 15x + 15$	Multiply on each side; Distributive Property
$-3x + 21 = 15$	Subtract $15x$ from each side; Subtraction Property of Equality
$-3x = -6$	Subtract 21 from each side; Subtraction Property of Equality
$x = 2$	Divide each side by -3 ; Division Property of Equality
15. Equation	Explanation and Reason
$5x + y = 18$	Write the equation; Given
$y = -5x + 18$	Subtract $5x$ from each side; Subtraction Property of Equality
16. Equation	Explanation and Reason
$-4x + 2y = 8$	Write the equation; Given
$2y = 4x + 8$	Add $4x$ to each side; Addition Property of Equality
$y = 2x + 4$	Divide each side by 2; Division Property of Equality
17. Equation	Explanation and Reason
$2y + 0.5x = 16$	Write the equation; Given
$2y = -0.5x + 16$	Subtract $0.5x$ from each side; Subtraction Property of Equality
$y = -0.25x + 8$	Divide each side by 2; Division Property of Equality
18. Equation	Explanation and Reason
$\frac{1}{2}x - \frac{3}{4}y = -2$	Write the equation; Given
$-\frac{3}{4}y = -\frac{1}{2}x - 2$	Subtract $\frac{1}{2}x$ from each side; Subtraction Property of Equality
$y = \frac{2}{3}x + \frac{8}{3}$	Multiply each side by $-\frac{4}{3}$; Multiplication Property of Equality

19. Equation	Explanation and Reason
$12 - 3y = 30x + 6$	Write the equation; Given
$-3y = 30x - 6$	Subtract 12 from each side; Subtraction Property of Equality
$y = -10x + 2$	Divide each side by -3 ; Division Property of Equality
20. Equation	Explanation and Reason
$3x + 7 = -7 + 9y$	Write the equation; Given
$3x + 14 = 9y$	Add 7 to each side; Addition Property of Equality
$\frac{1}{3}x + \frac{14}{9} = y$	Divide each side by 9; Division Property of Equality
$y = \frac{1}{3}x + \frac{14}{9}$	Rewrite the equation; Symmetric Property of Equality
21. Equation	Explanation and Reason
$C = 2\pi r$	Write the equation; Given
$\frac{C}{2\pi} = r$	Divide each side by 2π ; Division Property of Equality
$r = \frac{C}{2\pi}$	Rewrite the equation; Symmetric Property of Equality
22. Equation	Explanation and Reason
$I = Prt$	Write the equation; Given
$\frac{I}{rt} = P$	Divide each side by rt ; Division Property of Equality
$P = \frac{I}{rt}$	Rewrite the equation; Symmetric Property of Equality
23. Equation	Explanation and Reason
$S = 180(n - 2)$	Write the equation; Given
$\frac{S}{180} = n - 2$	Divide each side by 180; Division Property of Equality
$\frac{S}{180} + 2 = n$	Add 2 to each side; Addition Property of Equality
$n = \frac{S}{180} + 2$	Rewrite the equation; Symmetric Property of Equality
24. Equation	Explanation and Reason
$S = 2\pi r^2 + 2\pi rh$	Write the equation; Given
$S - 2\pi r^2 = 2\pi rh$	Subtract $2\pi r^2$ from each side; Subtraction Property of Equality
$\frac{S - 2\pi r^2}{2\pi r} = h$	Divide each side by $2\pi r$; Division Property of Equality
$h = \frac{S - 2\pi r^2}{2\pi r}$	Rewrite the equation; Symmetric Property of Equality
25. The property illustrated is the Multiplication Property of Equality.	

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26. The property illustrated is the Addition Property of Equality.
27. The property illustrated is the Reflexive Property of Equality.
28. The property illustrated is the Symmetric Property of Equality.
29. The property illustrated is the Reflexive Property of Equality.
30. The property illustrated is the Substitution Property of Equality.
31. The property illustrated is the Symmetric Property of Equality.
32. The property illustrated is the Transitive Property of Equality.
33. If $AB = 20$, then $AB + CD = 20 + CD$.
34. If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
35. If $AB = CD$, then $AB + EF = CD + EF$.
36. If $AB = CD$, then $5 \cdot AB = 5 \cdot CD$.
37. If $LM = XY$, then $LM - GH = XY - GH$.
38. If $5(x + 8) = 2$, then $5x + 40 = 2$.
39. $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

40. $m\angle ABC = m\angle ABC$

41. The Subtraction Property of Equality should be used to subtract x from each side of the equation in order to get the second step.

$$7x = x + 24 \quad \text{Given}$$

$$6x = 24 \quad \text{Subtraction Property of Equality}$$

$$x = 4 \quad \text{Division Property of Equality}$$

42. The reasons are wrong.

$$6x + 14 = 32 \quad \text{Given}$$

$$6x = 18 \quad \text{Subtraction Property of Equality}$$

$$x = 3 \quad \text{Division Property of Equality}$$

43. Equation

Explanation and Reason

$$P = 2\ell + 2w \quad \text{Write the equation; Given}$$

$$P - 2w = 2\ell \quad \text{Subtract } 2w \text{ from each side; Subtraction Property of Equality}$$

$$\frac{P - 2w}{2} = \frac{2\ell}{2} \quad \text{Divide each side by 2; Division Property of Equality}$$

$$\frac{P - 2w}{2} = \ell \quad \text{Simplify.}$$

$$\ell = \frac{P - 2w}{2} \quad \text{Rewrite the equation; Symmetric Property of Equality}$$

$$\ell = \frac{32 - 2 \cdot 5}{2} = \frac{32 - 10}{2} = \frac{22}{2} = 11$$

The length is 11 meters.

44. Equation

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$2 \cdot A = 2 \cdot \frac{1}{2}h(b_1 + b_2)$$

$$2A = h(b_1 + b_2)$$

$$2A = hb_1 + hb_2$$

$$2A - hb_2 = hb_1 + hb_2 - hb_2$$

Explanation and Reason

Write the equation; Given

Multiply each side by 2; Multiplication Property of Equality

Simplify.

Multiply; Distributive Property

Subtract hb_2 from each side; Subtraction Property of Equality

Combine like terms; Simplify.

Divide each side by h ; Division Property of Equality

Simplify.

Rewrite the equation; Symmetric Property of Equality

$$b_1 = \frac{2 \cdot 91 - 7 \cdot 20}{7} = \frac{182 - 140}{7} = \frac{42}{7} = 6$$

The other base is 6 meters.

45. Equation

Explanation and Reason

$$m\angle ABD = m\angle CBE$$

Write the equation; Given

$$m\angle ABD = m\angle 1 + m\angle 2$$

Add measures of adjacent angles; Angle Addition Postulate (Post. 1.4)

$$m\angle CBE = m\angle 2 + m\angle 3$$

Add measures of adjacent angles; Angle Addition Postulate (Post. 1.4)

$$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$$

Substitute $m\angle 1 + m\angle 2$ for $m\angle ABD$; Substitution Property of Equality

$$m\angle 1 = m\angle 3$$

Subtract $m\angle 2$ from each side; Subtraction Property of Equality

46. Equation

Explanation and Reason

$$AC = BD$$

Write the equation; Given

$$AC = AB + BC$$

Add measures of adjacent sides; Segment Addition Postulate (Post. 1.2)

$$BD = BC + CD$$

Add measures of adjacent sides; Segment Addition Postulate (Post. 1.2)

$$AC = BC + CD$$

Substitute AC for BD ; Substitution Property of Equality

$$AB + BC = BC + CD$$

Substitute $AB + BC$ for AC ; Substitution Property of Equality

$$AB = CD$$

Subtract BC from each side; Subtraction Property of Equality

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Equation	Reason
$m\angle 1 = m\angle 4$, $m\angle EHF = 90^\circ$, $m\angle GHF = 90^\circ$	Given
$m\angle EHF = m\angle GHF$	Transitive Property of Equality
$m\angle EHF = m\angle 1 + m\angle 2$ $m\angle GHF = m\angle 3 + m\angle 4$	Angle Addition Postulate (Post. 1.4)
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Transitive Property of Equality
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	Substitution Property of Equality
$m\angle 2 = m\angle 3$	Subtraction Property of Equality

48. Both properties state basic ideas about equality. The Reflexive Property of Equality states that something is equal to itself. So, both sides of the equal sign are identical. The Symmetric Property of Equality states that you can switch the two sides of an equation. So, two equations are equivalent if they have the same two expressions set equal to each other, but the expressions are on different sides of the equal sign.

Equation	Explanation and Reason
$DC = BC$, $AD = AB$	Marked in diagram; Given
$AC = AC$	AC is equal to itself; Reflexive Property of Equality
$AC + AB + BC = AC + AB + BC$	Add $AB + BC$ to each side of $AC = AC$; Addition Property of Equality
$AC + AB + BC = AC + AD + DC$	Substitute AD for AB and DC for BC ; Substitution Property of Equality

Equation	Explanation and Reason
$BC = DA$, $CD = AB$	Marked in diagram; Given
$AC = AC$	AC is equal to itself; Reflexive Property of Equality
$AC + AB + BC = AC + AB + BC$	Add $AB + BC$ to each side of $AC = AC$; Addition Property of Equality
$AC + AB + BC = AC + CD + DA$	Substitute CD for AB and DA for BC ; Substitution Property of Equality

51. $YX = 3$, $ZX = 5x + 17$, $YW = 10 - 2x$

$$ZX = ZY + YX$$

$$ZX = ZY + 3$$

$$ZY = ZX - 3$$

$$YW = YX + XW$$

$$YW = 3 + XW$$

$$XW = YW - 3$$

$$ZY = XW$$

$$ZX - 3 = YW - 3$$

$$(5x + 17) - 3 = (10 - 2x) - 3$$

$$5x + 14 = 7 - 2x$$

$$7x + 14 = 7$$

$$7x = -7$$

$$x = -1$$

$$ZY = 5x + 17 - 3 = 5(-1) + 17 - 3 = 9$$

$$XW = 10 - 2x - 3 = 10 - 2(-1) - 3 = 10 + 2 - 3 = 9$$

52. *Sample answer:* Reflexive: Employee 1 worked the same number of hours as Employee 1. Symmetric: If Employee 4 worked the same number of hours as Employee 5, then Employee 5 worked the same number of hours as Employee 4. Transitive: If Employee 2 worked the same number of hours as Employee 4, and Employee 4 worked the same number of hours as Employee 5, then Employee 2 worked the same number of hours as Employee 5.

53. The Symmetric Property of Equality is illustrated by A and B.

54. *Sample answer:* Reflexive: I earned the same number of points as myself on my favorite video game. This is reflexive because a quantity is equal to itself. Symmetric: If John had the same score as Tyeesha on our math quiz, then Tyeesha had the same score as John. This is Symmetric because the same two quantities are equal to each other. Transitive: If Dominic has the same number of pets as Ella, and Ella has the same number of pets as Brady, then Dominic has the same number of pets as Brady. This is transitive because the way we know that two quantities are equal is because they are each equal to a third quantity.

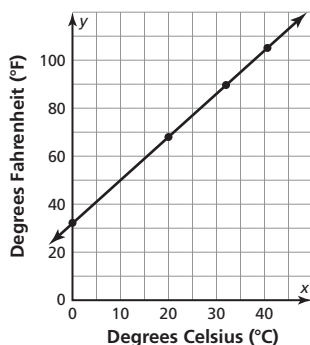
Equation	Explanation and Reason
$C = \frac{5}{9}(F - 32)$	Write the equation; Given
$\frac{9}{5}C = F - 32$	Multiply each side by $\frac{9}{5}$; Multiplication Property of Equality
$\frac{9}{5}C + 32 = F$	Add 32 to each side; Addition Property of Equality
$F = \frac{9}{5}C + 32$	Rewrite the equation; Symmetric Property of Equality

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b.

Degrees Celsius (°C)	$F = \frac{9}{5}C + 32$	Degrees Fahrenheit (°F)
0	$\frac{9}{5} \cdot 0 + 32 = 32$	32
20	$\frac{9}{5} \cdot 20 + 32 = 9 \cdot 4 + 32 = 68$	68
32	$\frac{9}{5} \cdot 32 + 32 = \frac{288}{5} + 32 = 57.6 + 32 = 89.6$	89.6
41	$\frac{9}{5} \cdot 41 + 32 = \frac{369}{5} + 32 = 73.8 + 32 = 105.8$	105.8

c. Yes, this is a linear function.



56. A, B, F; The Addition and Subtraction Properties are true because if you add (or subtract) the same amount to each side of an inequality, the inequality is still true. For the Substitution Property, two equal quantities could be substituted for each other in an inequality, but if one quantity is less than (or greater than) another quantity, you cannot always substitute one for the other into another inequality. The Reflexive Property is not true because quantities are not less than (or greater than) themselves. In order for the Symmetric Property to be true, the sign must be flipped around, as in if $a < b$, then $b > a$. The Transitive Property is true as long as all signs are going in the same direction. For example, if quantity A is less than quantity B , and quantity B is less than quantity C , then quantity A is less than quantity C .

Maintaining Mathematical Proficiency

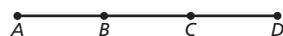
57. Segment Addition Postulate (Post. 1.2)
 58. Angle Bisector
 59. Midpoint
 60. Angle Addition Postulate (Post. 1.4)

2.5 Explorations (p. 99)

2. Segment Addition Postulate (Post. 1.2)
 3. Transitive Property of Equality
 4. Subtraction Property of Equality
1. $m\angle 1 = m\angle 3$
 4. $m\angle 1 + m\angle 2$
 5. $m\angle CBD$
 6. $m\angle EBA = m\angle CBD$
- You can use deductive reasoning to make statements about a given situation and use math definitions, postulates, and theorems as your reason or justification for each statement.

4. **Given** B is the midpoint of \overline{AC} .
 C is the midpoint of \overline{BD} .

Prove $AB = CD$

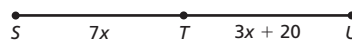


STATEMENTS	REASONS
1. B is the midpoint of \overline{AC} . C is the midpoint of \overline{BD} .	1. Given
2. $\overline{AB} \cong \overline{BC}$, $\overline{BC} \cong \overline{CD}$	2. Definition of midpoint
3. $AB = BC$, $BC = CD$	3. Definition of congruent segments
4. $AB = CD$	4. Transitive Property of Equality

2.5 Monitoring Progress (pp. 100–102)

1. **Given** T is the midpoint of \overline{SU} .

Prove $x = 5$



STATEMENTS	REASONS
1. T is the midpoint of \overline{SU} .	1. Given
2. $\overline{ST} \cong \overline{TU}$	2. Definition of midpoint
3. $ST = TU$	3. Definition of congruent segments
4. $7x = 3x + 20$	4. Substitution Property of Equality
5. $4x = 20$	5. Subtraction Property of Equality
6. $x = 5$	6. Division Property of Equality

Chapter 2

- The property illustrated is the Reflexive Property of Segment Congruence (Thm. 2.1).
- The property illustrated is the Symmetric Property of Angle Congruence (Thm. 2.2).
- Step 5 would be $MB + MB = AB$.
Step 6 would be $2MB = AB$.
Step 7 would be $MB = \frac{1}{2}AB$.

2.5 Exercises (pp. 103–104)

Vocabulary and Core Concept Check

- A postulate is a rule that is accepted to be true without proof and a theorem is a statement that can be proven by using definitions, postulates, and previously proven theorems.
- In a two column proof, each statement is on the left and each reason is on the right.

Monitoring Progress and Modeling with Mathematics

3. Given $PQ = RS$

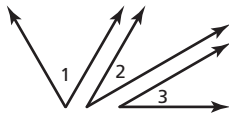
Prove $PR = QS$



STATEMENTS	REASONS
1. $PQ = RS$	1. Given
2. $PQ + QR = RS + QR$	2. Addition Property of Equality
3. $PQ + QR = PR$	3. Segment Addition Postulate (Post. 1.2)
4. $RS + QR = QS$	4. Segment Addition Postulate (Post. 1.2)
5. $PR = QS$	5. Transitive Property of Equality

4. Given $\angle 1$ is a complement of $\angle 2$.
 $\angle 2 \cong \angle 3$

Prove $\angle 1$ is a complement of $\angle 3$.



STATEMENTS	REASONS
1. $\angle 1$ is a complement of $\angle 2$.	1. Given
2. $\angle 2 \cong \angle 3$	2. Given
3. $m\angle 1 + m\angle 2 = 90^\circ$	3. Definition of complementary angles
4. $m\angle 2 = m\angle 3$	4. Definition of congruent angles
5. $m\angle 1 + m\angle 3 = 90^\circ$	5. Substitution Property of Equality
6. $\angle 1$ is a complement of $\angle 3$.	6. Definition of complementary angles

- Transitive Property of Segment Congruence (Thm. 2.1)
- Reflexive Property of Angle Congruence (Thm. 2.2)
- Symmetric Property of Angle Congruence (Thm. 2.2)
- Reflexive Property of Segment Congruence (Thm. 2.1)
- Symmetric Property of Segment Congruence (Thm. 2.1)
- Transitive Property of Angle Congruence (Thm. 2.2)

11. Given Segment AB

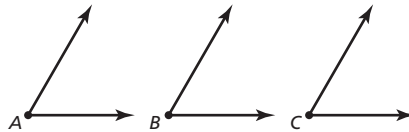


Prove $\overline{AB} \cong \overline{AB}$

STATEMENTS	REASONS
1. A segment exists with endpoints A and B .	1. Given
2. AB equals the length of the segment with endpoints A and B .	2. Ruler Postulate (Post. 1.1)
3. $AB = AB$	3. Reflexive Property of Equality
4. $\overline{AB} \cong \overline{AB}$	4. Definition of congruent segments

12. Given $\angle A \cong \angle B$, $\angle B \cong \angle C$

Prove $\angle A \cong \angle C$

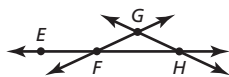


STATEMENTS	REASONS
1. $\angle A \cong \angle B$	1. Given
2. $m\angle A = m\angle B$	2. Definition of congruent angles
3. $\angle B \cong \angle C$	3. Given
4. $m\angle B = m\angle C$	4. Definition of congruent angles
5. $m\angle A = m\angle C$	5. Transitive Property of Equality
6. $\angle A \cong \angle C$	6. Definition of congruent angles

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13. Given $\angle GFH \cong \angle GHF$

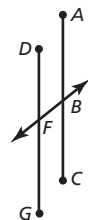
Prove $\angle EFG$ and $\angle GHF$ are supplementary.



STATEMENTS	REASONS
1. $\angle GFH \cong \angle GHF$	1. Given
2. $m\angle GFH = m\angle GHF$	2. Definition of congruent angles
3. $\angle EFG$ and $\angle GFH$ form a linear pair.	3. Given (diagram)
4. $\angle EFG$ and $\angle GFH$ are supplementary.	4. Definition of linear pair
5. $m\angle EFG + m\angle GFH = 180^\circ$	5. Definition of supplementary angles
6. $m\angle EFG + m\angle GHF = 180^\circ$	6. Substitution Property of Equality
7. $\angle EFG$ and $\angle GHF$ are supplementary.	7. Definition of supplementary angles

14. Given $\overline{AB} \cong \overline{FG}$
 \overline{BF} bisects \overline{AC} and \overline{DG} .

Prove $\overline{BC} \cong \overline{DF}$

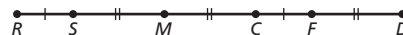


STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{FG}$	1. Given
2. \overline{BF} bisects \overline{AC} and \overline{DG} .	2. Given
3. $\overline{BC} \cong \overline{AB}$, $\overline{FG} \cong \overline{DF}$	3. Definition of segment bisector
4. $\overline{BC} \cong \overline{FG}$	4. Transitive Property of Equality
5. $\overline{BC} \cong \overline{DF}$	5. Transitive Property of Segment Congruence (Thm. 2.1)

15. The Transitive Property of Segment Congruence (Thm. 2.1) should have been used. If $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$, then $\overline{MN} \cong \overline{PN}$ by the Transitive Property of Segment Congruence (Thm. 2.1).

16. a. Given $\overline{RS} \cong \overline{CF}$, $\overline{SM} \cong \overline{MC} \cong \overline{FD}$

Prove $\overline{RM} \cong \overline{CD}$



STATEMENTS	REASONS
1. $\overline{RS} \cong \overline{CF}$	1. Given
2. $RS = CF$	2. Definition of congruent segments
3. $\overline{SM} \cong \overline{FD}$	3. Given
4. $SM = FD$	4. Definition of congruent segments
5. $RM = RS + SM$	5. Segment Addition Postulate (Post. 1.2)
6. $CD = CF + FD$	6. Segment Addition Postulate (Post. 1.2)
7. $RS + SM = CD$	7. Substitution Property of Equality
8. $RM = CD$	8. Substitution Property of Equality
9. $\overline{RM} \cong \overline{CD}$	9. Definition of congruent segments

17. The triangle is an equiangular (or equilateral) triangle. By the Transitive Property of Angle Congruence (Thm. 2.2), because $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, you know that $\angle 1 \cong \angle 3$. Because all three angles are congruent, the triangle is equiangular. (It is also equilateral and acute.)

18. no; The statements have to have one segment in common in order to use the Transitive Property of Segment Congruence (Thm. 2.1), but in this case, the statements are about four different segments. They may or may not all be congruent to each other.

19. The purpose of a proof is to ensure the truth of a statement with such certainty that the theorem or rule proved could be used as a justification in proving another statement or theorem. Because inductive reasoning relies on observations about patterns in specific cases, the pattern may not continue or may change. So, the ideas cannot be used to prove ideas for the general case.

20. a. Given $\triangle JML$ is a right triangle.

Prove The acute angles of a right triangle are complementary.

- b. Given $\triangle JML$ is a right triangle.

N is the midpoint of JM .

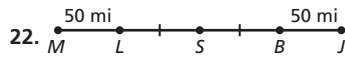
K is the midpoint of JL .

Prove $NK = \frac{1}{2}ML$

Chapter 2

21. a. It is a right angle.

STATEMENTS	REASONS
1. $m\angle 1 + m\angle 1 + m\angle 2 + m\angle 2 = 180^\circ$	1. Angle Addition Postulate (Post. 1.4)
2. $2(m\angle 1 + m\angle 2) = 180$	2. Distributive Property
3. $m\angle 1 + m\angle 2 = 90^\circ$	3. Division Property of Equality



23. Given $\overline{QR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{PQ}$

STATEMENTS	REASONS
1. $\overline{QR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{PQ}$	1. Given
2. $QR = 2x + 5$, $RS = 10 - 3x$	2. Given
3. $QR = PQ$, $RS = PQ$	3. Definition of congruent segments
4. $QR = RS$	4. Transitive Property of Equality
5. $2x + 5 = 10 - 3x$	5. Substitution Property of Equality
6. $5x + 5 = 10$	6. Addition Property of Equality
7. $5x = 5$	7. Subtraction Property of Equality
8. $x = 1$	8. Division Property of Equality

Maintaining Mathematical Proficiency

24. $m\angle 1 + m\angle 4 = 90^\circ$

$$33^\circ + m\angle 4 = 90^\circ$$

$$m\angle 4 = 90^\circ - 33^\circ$$

$$m\angle 4 = 57^\circ$$

25. $m\angle 2 + m\angle 3 = 180^\circ$

$$147^\circ + m\angle 3 = 180^\circ$$

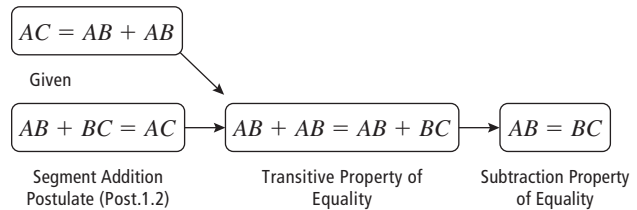
$$m\angle 3 = 180^\circ - 147^\circ$$

$$m\angle 3 = 33^\circ$$

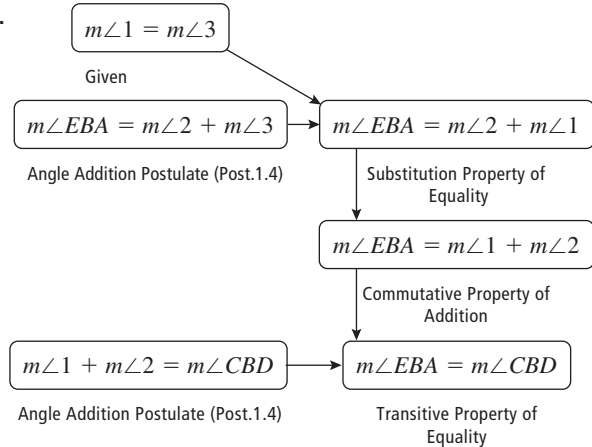
26. A pair of vertical angles are $\angle 1$ and $\angle 3$.

2.6 Explorations (p. 105)

1.



2.



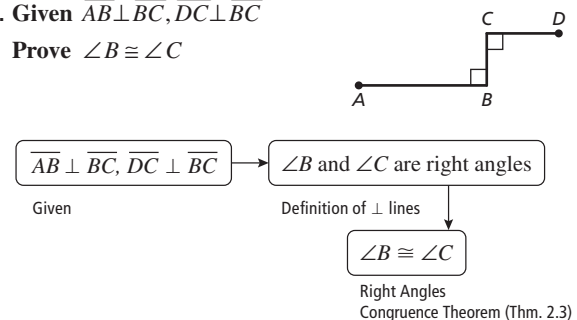
3. A flowchart uses boxes and arrows to show the flow of a logical argument.

4. The flowchart proof, unlike the two-column proof, allows you to show explicitly which statement leads to which, but the two-column proof has a uniform, predictable shape and style and has each statement right below the previous one to allow for easy comparison. Both allow you to provide a logical argument and justification for why something is true.

2.6 Monitoring Progress (pp. 106–110)

1. Given $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$

Prove $\angle B \cong \angle C$



Chapter 2

STATEMENTS	REASONS
1. $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$	1. Given
2. $\angle B$ and $\angle C$ are right angles.	2. Definition of \perp lines
3. $\angle B \cong \angle C$	3. Right Angles Congruence Theorem (Thm. 2.3)

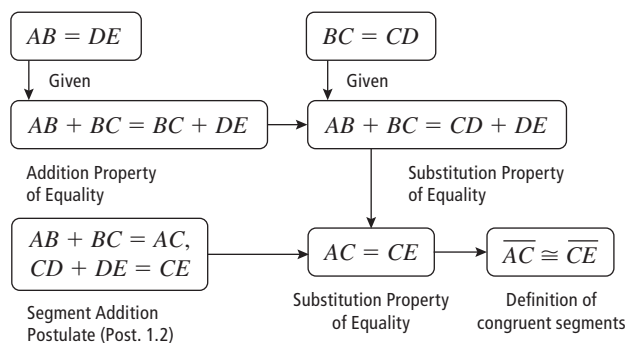
2. Given $AB = DE, BC = CD$

Prove $\overline{AC} \cong \overline{CE}$



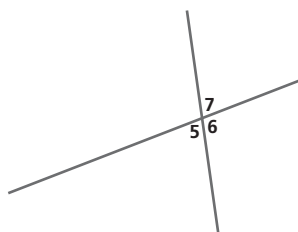
STATEMENTS	REASONS
1. $AB = DE, BC = CD$	1. Given
2. $AB + BC = BC + DE$	2. Addition Property of Equality
3. $AB + BC = CD + DE$	3. Substitution Property of Equality
4. $AB + BC = AC, CD + DE = CE$	4. Segment Addition Postulate (Post. 1.2)
5. $AC = CE$	5. Substitution Property of Equality
6. $\overline{AC} \cong \overline{CE}$	6. Definition of congruent segments

Flowchart proof:



3. Given $\angle 5$ and $\angle 7$ are vertical angles.

Prove $\angle 5 \cong \angle 7$



STATEMENTS	REASONS
1. $\angle 5$ and $\angle 7$ are vertical angles.	1. Given
2. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	2. Definition of linear pair
3. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	3. Linear Pair Postulate (Post. 2.8)
4. $m\angle 5 + m\angle 6 = 180^\circ, m\angle 6 + m\angle 7 = 180^\circ$	4. Definition of supplementary angles
5. $m\angle 6 + m\angle 7 = m\angle 5 + m\angle 6$	5. Transitive Property of Equality
6. $m\angle 5 = m\angle 7$	6. Subtraction Property of Equality
7. $\angle 5 \cong \angle 7$	7. Definition of congruent angles

By using the Congruent Supplement Theorem, you save three steps.

4. By the definition of supplementary angles,
 $m\angle 1 + m\angle 2 = 180^\circ$.

$$117^\circ + m\angle 2 = 180^\circ$$

$$m\angle 2 = 180^\circ - 117^\circ = 63^\circ$$

Vertical angles are congruent, so $\angle 1 \cong \angle 3$.

$$m\angle 3 = 117^\circ$$

By the definition of supplementary angles,
 $m\angle 3 + m\angle 4 = 180^\circ$.

$$117^\circ + m\angle 4 = 180^\circ$$

$$m\angle 4 = 180^\circ - 117^\circ = 63^\circ$$

$$m\angle 2 = 63^\circ, m\angle 3 = 117^\circ, m\angle 4 = 63^\circ$$

5. By the definition of supplementary angles,
 $m\angle 1 + m\angle 2 = 180^\circ$.

$$m\angle 1 + 59^\circ = 180^\circ$$

$$m\angle 1 = 180^\circ - 59^\circ = 121^\circ$$

Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.

$$m\angle 3 = 121^\circ$$

Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.

$$m\angle 4 = 59^\circ$$

$$m\angle 1 = 121^\circ, m\angle 3 = 121^\circ, m\angle 4 = 59^\circ$$

6. By the definition of supplementary angles,
 $m\angle 1 + m\angle 4 = 180^\circ$.

$$m\angle 1 + 88^\circ = 180^\circ$$

$$m\angle 1 = 180^\circ - 88^\circ = 92^\circ$$

Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.

$$m\angle 2 = 88^\circ$$

Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.

$$m\angle 3 = 92^\circ$$

$$m\angle 1 = 92^\circ, m\angle 2 = 88^\circ, m\angle 3 = 92^\circ$$

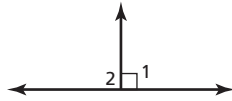
Chapter 2

7. $5w + 3 = 98$ Vertical angles are congruent.
 $5w + 3 - 3 = 98 - 3$ Subtraction Property of Equality
 $5w = 95$ Simplify.
 $\frac{5w}{5} = \frac{95}{5}$ Division Property of Equality
 $w = 19$ Simplify.

8. **Given** $\angle 1$ is a right angle.

Prove $\angle 2$ is a right angle.

$\angle 1$ is a right angle. By the definition of a right angle, $m\angle 1 = 90^\circ$. $\angle 1$ and $\angle 2$ form a linear pair. So, by the Linear Pair Postulate (Post. 2.8), $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180^\circ$. By the Substitution Property of Equality, $90^\circ + m\angle 2 = 180^\circ$. Therefore, by the Subtraction Property of Equality, $m\angle 2 = 90^\circ$. So, by definition, $\angle 2$ is a right angle.



2.6 Exercises (pp. 111–114)

Vocabulary and Core Concept Check

- All right angles have a measure of 90° , and angles with the same measure are congruent.
- Vertical angles and supplementary angles are formed by intersecting lines.

Monitoring Progress and Modeling with Mathematics

- $\angle MSN \cong \angle PSQ$ by definition because they have the same measure; $\angle MSP \cong \angle PSR$ by the Right Angles Congruence Theorem (Thm. 2.3). They form a linear pair, which means they are supplementary by the Linear Pair Postulate (Post. 2.8), and because one is a right angle, so is the other by the Subtraction Property of Equality; $\angle NSP \cong \angle QSR$ by the Congruent Complements Theorem (Thm. 2.5) because they are complementary to congruent angles.
- $\angle FGH \cong \angle WXZ$, because $m\angle WXZ = 90^\circ$ by the Angle Addition Postulate (Post. 1.4), which means that it is a right angle, and so, $\angle FGH$ and $\angle WXZ$ are congruent by the Right Angles Congruence Theorem (Thm. 2.3).
- $\angle GML \cong \angle HMJ$ and $\angle GMH \cong \angle LMJ$ by the Vertical Angles Congruence Theorem (Thm. 2.6); $\angle GMK \cong \angle JMK$ by the Right Angles Congruence Theorem (Thm. 2.3). They form a linear pair, which means they are supplementary by the Linear Pair Postulate (Post. 2.8), and because one is a right angle, so is the other by the Subtraction Property of Equality.
- $\angle ABC \cong \angle DEF$ by the Congruent Supplements Theorem (Thm. 2.4); $\angle CBD \cong \angle FEA$ by the Congruent Supplements Theorem (Thm. 2.4). $\angle DEF$ and $\angle FEA$ are supplementary, because they form a linear pair, and because $\angle CBD$ and $\angle FEA$ are supplementary to congruent angles, they are also congruent to each other.

7. Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.

$$m\angle 3 = 143^\circ$$

By the definition of supplementary angles,
 $m\angle 1 + m\angle 2 = 180^\circ$.

$$143^\circ + m\angle 2 = 180^\circ$$

$$m\angle 2 = 180^\circ - 143^\circ = 37^\circ$$

Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.

$$m\angle 4 = 37^\circ$$

$$m\angle 2 = 37^\circ, m\angle 3 = 143^\circ, m\angle 4 = 37^\circ$$

8. Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.

$$m\angle 1 = 159^\circ$$

By the definition of supplementary angles,
 $m\angle 2 + m\angle 3 = 180^\circ$.

$$m\angle 2 + 159^\circ = 180^\circ$$

$$m\angle 2 = 180^\circ - 159^\circ = 21^\circ$$

Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.

$$m\angle 4 = 21^\circ$$

$$m\angle 1 = 159^\circ, m\angle 2 = 21^\circ, m\angle 4 = 21^\circ$$

9. Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.

$$m\angle 4 = 34^\circ$$

By the definition of supplementary angles,
 $m\angle 2 + m\angle 3 = 180^\circ$.

$$34^\circ + m\angle 3 = 180^\circ$$

$$m\angle 3 = 180^\circ - 34^\circ = 146^\circ$$

Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.

$$m\angle 1 = 146^\circ$$

$$m\angle 1 = 146^\circ, m\angle 3 = 146^\circ, m\angle 4 = 34^\circ$$

10. By the definition of supplementary angles,

$$m\angle 1 + m\angle 4 = 180^\circ$$

$$m\angle 1 + 29^\circ = 180^\circ$$

$$m\angle 1 = 180^\circ - 29^\circ = 151^\circ$$

Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.

$$m\angle 2 = 29^\circ$$

Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.

$$m\angle 3 = 151^\circ$$

$$m\angle 1 = 151^\circ, m\angle 2 = 29^\circ, m\angle 3 = 151^\circ$$

Chapter 2

11. $8x + 7 = 9x - 4$ Given
 $8x + 7 - 8x = 9x - 4 - 8x$ Subtraction Property of Equality
 $7 = x - 4$ Simplify.
 $7 + 4 = x - 4 + 4$ Addition Property of Equality
 $11 = x$ Simplify.
- $5y = 7y - 34$ Given
 $5y - 7y = 7y - 34 - 7y$ Addition Property of Equality
 $-2y = -34$ Simplify.
 $\frac{-2y}{-2} = \frac{-34}{-2}$ Division Property of Equality
 $y = 17$ Simplify.
12. $4x = 6x - 26$ Given
 $4x - 6x = 6x - 26 - 6x$ Subtraction Property of Equality
 $-2x = -26$ Simplify.
 $\frac{-2x}{-2} = \frac{-26}{-2}$ Division Property of Equality
 $x = 13$ Simplify.
- $7y - 12 = 6y + 8$ Given
 $7y - 12 - 6y = 6y + 8 - 6y$ Subtraction Property of Equality
 $y - 12 = 8$ Simplify.
 $y - 12 + 12 = 8 + 12$ Addition Property of Equality
 $y = 20$ Simplify.
13. $10x - 4 = 6(x + 2)$ Given
 $10x - 4 = 6x + 12$ Distributive Property
 $10x - 4 - 6x = 6x + 12 - 6x$ Subtraction Property of Equality
 $4x - 4 = 12$ Simplify.
 $4x - 4 + 4 = 12 + 4$ Addition Property of Equality
 $4x = 16$ Simplify.
 $\frac{4x}{4} = \frac{16}{4}$ Division Property of Equality
 $x = 4$ Simplify.
- $16y = 18y - 18$ Given
 $16y - 18y = 18y - 18 - 18y$ Subtraction Property of Equality
 $-2y = -18$ Simplify.
 $\frac{-2y}{-2} = \frac{-18}{-2}$ Division Property of Equality
 $y = 9$ Simplify.

14. $2(5x - 5) = 6x + 50$ Given
 $10x - 10 = 6x + 50$ Distributive Property
 $10x - 10 - 6x = 6x + 50 - 6x$ Subtraction Property of Equality
 $4x - 10 = 50$ Simplify.
 $4x - 10 + 10 = 50 + 10$ Addition Property of Equality
 $4x = 60$ Simplify.
 $\frac{4x}{4} = \frac{60}{4}$ Division Property of Equality
 $x = 15$ Simplify.
- $5y + 5 = 7y - 9$ Given
 $5y + 5 - 7y = 7y - 9 - 7y$ Subtraction Property of Equality
 $-2y + 5 = -9$ Simplify.
 $-2y + 5 - 5 = -9 - 5$ Subtraction Property of Equality
 $-2y = -14$ Simplify.
 $\frac{-2y}{-2} = \frac{-14}{-2}$ Division Property of Equality
 $y = 7$ Simplify.

15. The expressions should have been set equal to each other because they represent vertical angles.

$$(13x + 45)^\circ = (19x + 3)^\circ$$

$$-6x + 45 = 3$$

$$-6x = -42$$

$$x = 7$$

16. Because the angles form a linear pair, the sum of their measures should be equal to 180° .

$$(13x + 45)^\circ + (12x - 40)^\circ = 180^\circ$$

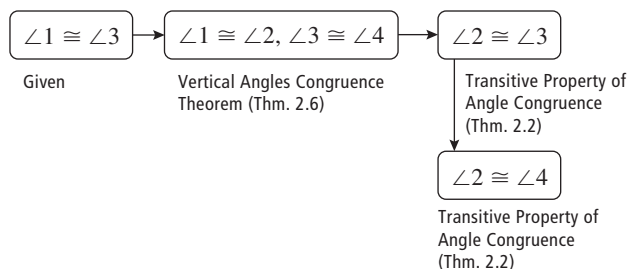
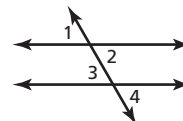
$$25x + 5 = 180$$

$$25x = 175$$

$$\frac{25x}{25} = \frac{175}{25}$$

$$x = 7$$

17. Given $\angle 1 \cong \angle 3$
 Prove $\angle 2 \cong \angle 4$



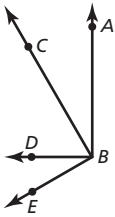
Chapter 2

Two column proof:

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 3$	1. Given
2. $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$	2. Vertical Angles Congruence Theorem (Thm. 2.6)
3. $\angle 2 \cong \angle 3$	3. Transitive Property of Angle Congruence (Thm 2.2)
4. $\angle 2 \cong \angle 4$	4. Transitive Property of Angle Congruence (Thm 2.2)

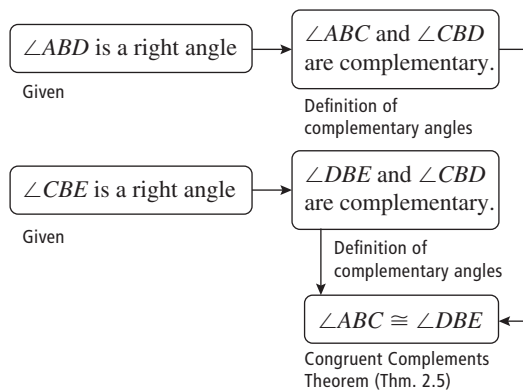
18. **Given** $\angle ABD$ is a right angle.
 $\angle CBE$ is a right angle.

Prove $\angle ABC \cong \angle DBE$



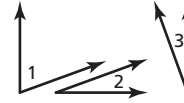
STATEMENTS	REASONS
1. $\angle ABD$ is a right angle. $\angle CBE$ is a right angle.	1. Given
2. $\angle ABC$ and $\angle CBD$ are complementary.	2. Definition of complementary angles
3. $\angle DBE$ and $\angle CBD$ are complementary.	3. Definition of complementary angles
4. $\angle ABC \cong \angle DBE$	4. Congruent Complements Theorem (Thm. 2.5)

Flowchart proof:



19. **Given** $\angle 1$ and $\angle 2$ are complementary.
 $\angle 1$ and $\angle 3$ are complementary.

Prove $\angle 2 \cong \angle 3$



$\angle 1$ and $\angle 2$ are complementary, and $\angle 1$ and $\angle 3$ are complementary. By the definition of complementary angles, $m\angle 1 + m\angle 2 = 90^\circ$ and $m\angle 1 + m\angle 3 = 90^\circ$. By the Transitive Property of Equality, $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$. By the Subtraction Property of Equality, $m\angle 2 = m\angle 3$. So, $\angle 2 \cong \angle 3$ by the definition of congruent angles.

Two-column proof:

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are complementary. $\angle 1$ and $\angle 3$ are complementary.	1. Given
2. $m\angle 1 + m\angle 2 = 90^\circ$ $m\angle 1 + m\angle 3 = 90^\circ$	2. Definition of complementary angles
3. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$	3. Transitive Property of Equality
4. $m\angle 2 = m\angle 3$	4. Subtraction Property of Equality
5. $\angle 2 \cong \angle 3$	5. Definition of congruent angles

20. **Given** $\angle 1$ and $\angle 2$ are supplementary.
 $\angle 3$ and $\angle 4$ are supplementary.
 $\angle 1 \cong \angle 4$

Prove $\angle 2 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary. $\angle 1 \cong \angle 4$	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$ $m\angle 3 + m\angle 4 = 180^\circ$	2. Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Transitive Property of Equality
4. $m\angle 1 = m\angle 4$	4. Definition of congruent angles
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	5. Substitution Property of Equality
6. $m\angle 2 = m\angle 3$	6. Subtraction Property of Equality
7. $\angle 2 \cong \angle 3$	7. Definition of congruent angles

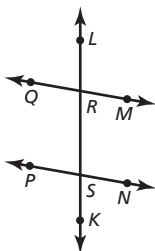
Chapter 2

Paragraph proof:

Because $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 4$ are supplementary, $m\angle 1 + m\angle 2 = 180^\circ$ and $m\angle 3 + m\angle 4 = 180^\circ$ by the definition of supplementary angles. By the Transitive Property of Equality, $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$. Because we are given that $\angle 1 \cong \angle 4$, by definition of congruent angles, $m\angle 1 = m\angle 4$. Therefore, by the Substitution Property of Equality, $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$, and by the Subtraction Property of Equality, $m\angle 2 = m\angle 3$. So, by definition of congruent angles, $\angle 2 \cong \angle 3$.

21. **Given** $\angle QRS$ and $\angle PSR$ are supplementary angles.

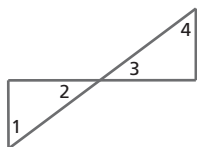
Prove $\angle QRL \cong \angle PSR$



Because $\angle QRS$ and $\angle PSR$ are supplementary, $m\angle QRS + m\angle PSR = 180^\circ$ by the definition of supplementary angles. $\angle QRL$ and $\angle QRS$ form a linear pair and by definition are supplementary, which means that $m\angle QRL + m\angle QRS = 180^\circ$. So, by the Transitive Property of Equality, $m\angle QRS + m\angle PSR = m\angle QRL + m\angle QRS$, and by the Subtraction Property of Equality, $m\angle PSR = m\angle QRL$. So, by definition of congruent angles, $\angle PSR \cong \angle QRL$, and by the Symmetric Property of Angle Congruence (Thm. 2.2), $\angle QRL \cong \angle PSR$.

22. **Given** $\angle 1$ and $\angle 3$ are complementary.
 $\angle 2$ and $\angle 4$ are complementary.

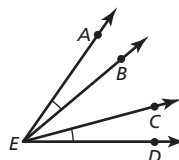
Prove $\angle 1 \cong \angle 4$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 3$ are complementary. $\angle 2$ and $\angle 4$ are complementary.	1. Given
2. $m\angle 1 + m\angle 3 = 90^\circ$ $m\angle 2 + m\angle 4 = 90^\circ$	2. Definition of complementary angles
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	3. Transitive Property of Equality
4. $\angle 2 \cong \angle 3$	4. Vertical Angles Congruence Theorem (Thm. 2.6)
5. $m\angle 2 = m\angle 3$	5. Definition of congruent angles
6. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 4$	6. Substitution Property of Equality
7. $m\angle 1 = m\angle 4$	7. Subtraction Property of Equality
8. $\angle 1 \cong \angle 4$	8. Definition of congruent angles

23. **Given** $\angle AEB \cong \angle DEC$

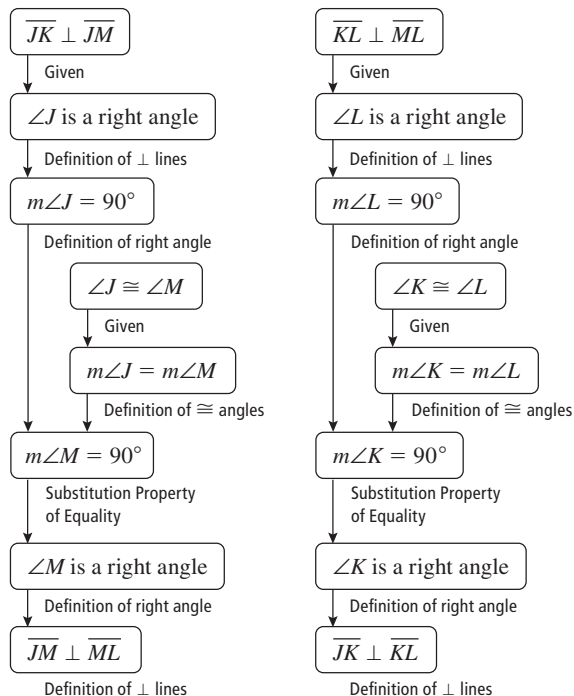
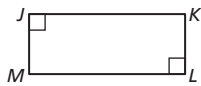
Prove $\angle AEC \cong \angle DEB$



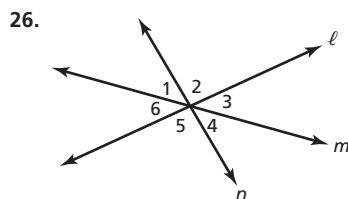
STATEMENTS	REASONS
1. $\angle AEB \cong \angle DEC$	1. Given
2. $m\angle AEB = m\angle DEC$	2. Definition of congruent angles
3. $m\angle DEB = m\angle BEC + m\angle DEC$	3. Angle Addition Postulate (Post. 1.4)
4. $m\angle DEB = m\angle AEB + m\angle BEC$	4. Substitution Property of Equality
5. $m\angle AEC = m\angle BEC + m\angle AEB$	5. Angle Addition Postulate (Post. 1.4)
6. $m\angle AEC = m\angle DEB$	6. Transitive Property of Equality
7. $\angle AEC \cong \angle DEB$	7. Definition of congruent angles

Chapter 2

24. Given $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$, $\angle J \cong \angle M$, $\angle K \cong \angle L$
Prove $\overline{JM} \perp \overline{ML}$ and $\overline{JK} \perp \overline{KL}$



25. Your friend is correct. $\angle 1$ and $\angle 4$ are not vertical angles because they do not form two pairs of opposite rays. So, the Vertical Angles Congruence Theorem (Thm. 2.6) does not apply.



If the measures of any two adjacent angles, such as $\angle 1$ and $\angle 2$ were given, then you could find the other four angle measures. In this case, you could find $m\angle 1 + m\angle 2$ and subtract this sum from 180° in order to find $m\angle 3$ (or $m\angle 6$). You can find the measures of the other three angles because each is a vertical angle with one of the three angles you know. Because vertical angles are congruent, $m\angle 4 = m\angle 3$, $m\angle 5 = m\angle 2$, and $m\angle 6 = m\angle 1$.

27. The converse statement is false: The converse is "If two angles are supplementary, then they are a linear pair." This is false because angles can be supplementary without being adjacent.

28. Time can be saved when writing proofs by using abbreviations and symbols instead of writing out the whole word. Also, when consecutive statements have the same reason, you can put them on the same line so that you only have to write the reason once.

$$\begin{aligned}
 29. \quad (7x + 4)^\circ + (4x - 22)^\circ &= 180^\circ \\
 11x - 18 &= 180 \\
 11x &= 198 \\
 x &= 18
 \end{aligned}$$

$$\begin{aligned}
 (3y + 11)^\circ + 10y^\circ &= 180^\circ \\
 13y + 11 &= 180 \\
 13y &= 169 \\
 y &= 13
 \end{aligned}$$

So, the angle measures are:

$$\begin{aligned}
 10y^\circ &= 10(13) = 130^\circ \\
 (4x - 22)^\circ &= 4(18) - 22 = 50^\circ \\
 (7x + 4)^\circ &= 7(18) + 4 = 130^\circ \\
 (3y + 11)^\circ &= 3(13) + 11 = 50^\circ
 \end{aligned}$$

30. a. The student is trying to prove that $\angle 1$ and $\angle 2$ are right angles by the definition of right angles.
b. No, because the last statement should be what is being proved: $\angle 1$ and $\angle 2$ are right angles. $\angle 1 \cong \angle 2$, was one of the given statements.

Maintaining Mathematical Proficiency

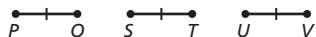
31. Three collinear points are E, J, H or B, I, C .
32. The intersection is \overline{EF} .
33. The two planes that contain \overline{BC} are the planes containing any combination of three of the points A, B, C, D and any combination of three of the points B, C, G, F .
34. The three planes containing point D are the planes containing any combination of three of the points A, B, C, D , any combination of three of the points D, H, E, A and any combination of three of the points D, C, G, H .
35. Three noncollinear points are: J, H, I (or any three points not on the same line).
36. The two planes containing J are the planes containing any combination of three of the points A, D, H, E and any combination of three of the points E, H, G, F .

2.4–2.6 What Did You Learn? (p. 115)

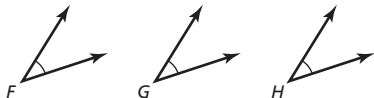
1. Even though the process for solving an equation may be almost automatic, when you have to justify each step, you have to think about the rules you are using and why you do each step in the process. When you think carefully about the rules and steps, you will make fewer mistakes, and this is how you know that your solution is a true statement for the given equation.

Chapter 2

2. If $\overline{PQ} \cong \overline{ST}$ and $\overline{ST} \cong \overline{UV}$, then $\overline{PQ} \cong \overline{UV}$.

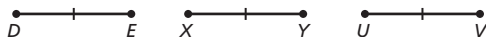


$\angle F \cong \angle F$ If $\angle G \cong \angle H$, then $\angle H \cong \angle G$.

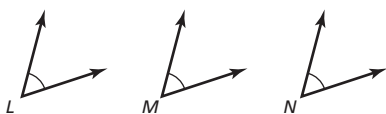


$\overline{DE} \cong \overline{DE}$

If $\overline{XY} \cong \overline{UV}$, then $\overline{UV} \cong \overline{XY}$.



If $\angle L \cong \angle M$ and $\angle M \cong \angle N$, then $\angle L \cong \angle N$.



3. \overleftrightarrow{MQ} and \overleftrightarrow{NP} could be meeting \overleftrightarrow{LK} at different angles. So, unless \overleftrightarrow{MQ} and \overleftrightarrow{NP} are parallel, $\angle QRL$ is not congruent to $\angle PSR$.

Chapter 2 Review (pp. 116–118)

- Conditional statement: If two lines intersect, then their intersection is a point.
Converse: If two lines intersect in a point, then they are intersecting lines.
Inverse: If two lines do not intersect, then they do not intersect in a point.
Contrapositive: If two lines do not intersect in a point, then they are not intersecting lines.
Biconditional: Two lines intersect if and only if they intersect in a point.
- Conditional: If $4x + 9 = 21$, then $x = 3$.
Converse: If $x = 3$, then $4x + 9 = 21$.
Inverse: If $4x + 9 \neq 21$, then $x \neq 3$.
Contrapositive: If $x \neq 3$, then $4x + 9 \neq 21$.
Biconditional: $4x + 9 = 21$ if and only if $x = 3$.
- Conditional: If angles are supplementary, then they sum to 180° .
Converse: If angles sum to 180° , then they are supplementary.
Inverse: If angles are not supplementary, then they do not sum to 180° .
Contrapositive: If angles do not sum to 180° , then they are not supplementary.
Biconditional: Angles are supplementary if and only if they sum to 180° .

4. Conditional: If an angle is a right angle, then its measure is 90° .

Converse: If an angle measures 90° , then it is a right angle.

Inverse: If an angle is not a right angle, then its measure is not 90° .

Contrapositive: If an angle does not measure 90° , then it is not a right angle.

Biconditional: An angle is a right angle if and only if its measure is 90° .

5. Pattern: $5 - 3 = 2$, $17 - 13 = 4$

Conjecture: Odd integer $-$ Odd integer $=$ Even integer

Let m and n be integers. Then $(2m + 1)$ and $(2n + 1)$ are odd integers.

$$\begin{aligned}(2m + 1) - (2n + 1) &= 2m + 1 - 2n - 1 \\ &= 2m - 2n \\ &= 2(m - n)\end{aligned}$$

Any number multiplied by 2 is an even number. So, the difference of any two odd integers is an even integer.

6. Pattern: $2 \cdot 3 = 6$, $4 \cdot 13 = 52$

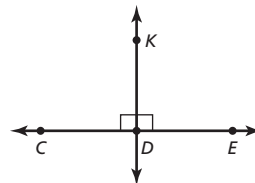
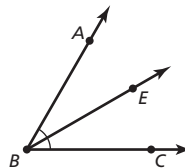
Conjecture: Even integer \times Odd integer $=$ Even integer

Let m and n be integers. Then $2m$ is an even integer and $2n + 1$ is an odd integer.

$$2m \cdot (2n + 1) = 2(2mn + m)$$

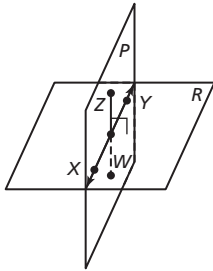
Any number multiplied by 2 is an even number. So, the product of an even integer and an odd integer is an even integer.

- $m\angle B = 90^\circ$
- If $4x = 12$, then $2x = 6$.
- yes; Points A , B , C , and E are coplanar. \overleftrightarrow{AB} and point C , which is not on \overleftrightarrow{AB} , lie in the same plane and point E , which is not on \overleftrightarrow{AB} lie in the same plane.
- yes; The right angle symbol indicates that $\overleftrightarrow{HC} \perp \overleftrightarrow{GE}$.
- no; Points F , B , and G are not collinear.
- no; There is not enough information to conclude that $\overleftrightarrow{AB} \parallel \overleftrightarrow{GE}$.
- $\angle ABC$ is bisected by \overleftrightarrow{BE} .
- $\angle CDE$ is bisected by \overleftrightarrow{DK} .



Chapter 2

15. Plane $P \perp$ plane R and intersect in \overleftrightarrow{XY} , and \overleftrightarrow{ZW} lies in plane P .



- 16. Equation**
 $-9x - 21 = -20x - 87$ Write the equation; Given
 $11x - 21 = -87$ Add $20x$ to each side; Addition Property of Equality
 $11x = -66$ Add 21 to each side; Addition Property of Equality
 $x = -6$ Divide each side by 11 ; Division Property of Equality

- 17. Equation**
 $15x + 22 = 7x + 62$ Write the equation; Given
 $8x + 22 = 62$ Subtract $7x$ from each side; Subtraction Property of Equality
 $8x = 40$ Subtract 22 from each side; Subtraction Property of Equality
 $x = 5$ Divide each side by 8 ; Division Property of Equality

- 18. Equation**
 $3(2x + 9) = 30$ Write the equation; Given
 $6x + 27 = 30$ Multiply; Distributive Property
 $6x = 3$ Subtract 27 from each side; Subtraction Property of Equality
 $x = \frac{3}{6} = \frac{1}{2}$ Divide each side by 6 ; Division Property of Equality

- 19. Equation**
 $5x + 2(2x - 23) = -154$ Write the equation; Given
 $5x + 4x - 46 = -154$ Multiply; Distributive Property
 $9x - 46 = -154$ Combine like terms; Simplify.
 $9x = -108$ Add 46 to each side; Addition Property of Equality
 $x = -12$ Divide each side by 9 ; Division Property of Equality

20. Transitive Property of Equality

21. Reflexive Property of Equality

22. Symmetric Property of Angle Congruence (Thm. 2.2)

23. Reflexive Property of Angle Congruence (Thm 2.2)

24. Transitive Property of Equality

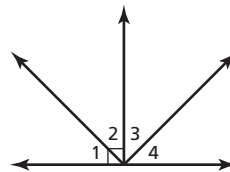
25. Given $\angle A$

Prove $\angle A \cong \angle A$

STATEMENTS	REASONS
1. An angle with vertex A exists.	1. Given
2. $m\angle A$ equals the measure of the angle with vertex A .	2. Protractor Postulate (Post. 1.3)
3. $m\angle A = m\angle A$	3. Reflexive Property of Equality
4. $\angle A \cong \angle A$	4. Definition of congruent angles

26. Given $\angle 3$ and $\angle 2$ are complementary.
 $m\angle 1 + m\angle 2 = 90^\circ$

Prove $\angle 3 \cong \angle 1$



STATEMENTS	REASONS
1. $m\angle 1 + m\angle 2 = 90^\circ$	1. Given
2. $\angle 1$ and $\angle 2$ are complementary.	2. Definition of complementary angles
3. $\angle 3$ and $\angle 2$ are complementary.	3. Given
4. $\angle 3 \cong \angle 1$	4. Congruent Complements Theorem (Thm. 2.5)

Chapter 2 Test (p. 119)

- no; No right angle is marked on \overleftrightarrow{AB} .
- yes; Three noncollinear points determine a plane and all three points lie in plane P .
- yes; Points E , C , and G all are on \overleftrightarrow{GC} .
- yes; The intersection of two planes is a line by Postulate 2.7.
- yes; The two points F and A lie in the same plane, so the line that contains them lies in the same plane.
- no; \overleftrightarrow{FG} is not drawn. So, you cannot be sure about where it intersects \overleftrightarrow{AB} .

Chapter 2

7. Equation

$$9x + 31 = -23 + 3x$$

$$6x + 31 = -23$$

$$6x = -54$$

$$x = -9$$

Explanation and Reason

Write the equation; Given

Subtract $3x$ from each side;
Subtraction Property of Equality

Subtract 31 from each side;
Subtraction Property of Equality

Divide each side by 6; Division
Property of Equality

8. Equation

$$26 + 2(3x + 11) = -18$$

$$26 + 6x + 22 = -18$$

$$6x + 48 = -18$$

$$6x = -66$$

$$x = -11$$

Explanation and Reason

Write the equation; Given

Multiply; Distributive Property

Combine like terms; Simplify.

Subtract 48 from each side;
Subtraction Property of Equality

Divide each side by 6; Division
Property of Equality

9. Equation

$$3(7x - 9) - 19x = -15$$

$$21x - 27 - 19x = -15$$

$$2x - 27 = -15$$

$$2x = 12$$

$$x = 6$$

Explanation and Reason

Write the equation; Given

Multiply; Distributive Property

Combine like terms; Simplify.

Add 27 to each side; Addition
Property of Equality

Divide each side by 2; Division
Property of Equality

10. Conditional: If two planes intersect, then their intersection is a line.

Converse: If two planes intersect in a line, then they are intersecting planes.

Inverse: If two planes do not intersect, then they do not intersect in a line.

Contrapositive: If two planes do not intersect in a line, then they are not intersecting planes.

Biconditional: Two planes intersect if and only if their intersection is a line.

11. Conditional: If a relation pairs each input with exactly one output, then the relation is a function.

Converse: If a relation is a function, then each input is paired with exactly one output.

Inverse: If a relation does not pair each input with exactly one output, then the relation is not a function.

Contrapositive: If a relation is not a function, then each input is not paired with exactly one output.

Biconditional: A relation pairs each input with exactly one output if and only if the relation is a function.

12. Pattern: $3 + 7 + 11 = 21$, $5 + 13 + 15 = 33$

Conjecture: The sum of three odd integers is an odd integer.

Let ℓ , m , and n be integers. Then $(2\ell + 1)$, $(2m + 1)$, and $(2n + 1)$ are odd integers.

$$(2\ell + 1) + (2m + 1) + (2n + 1)$$

$$= 2\ell + 2m + 2n + 3$$

$$= 2(\ell + m + n + 1) + 1$$

The result is 1 more than an even integer (the product of 2 and $(\ell + m + n + 1)$). So, the sum of three odd integers is an odd integer.

13. Pattern: $2 \cdot 4 \cdot 6 = 48$, $2 \cdot 10 \cdot 12 = 240$

Conjecture: The product of three even integers is a multiple of 8.

Let ℓ , m , and n be integers. Then 2ℓ , $2m$, and $2n$ represent even integers.

$$(2\ell)(2m)(2n) = 8\ell mn$$

The result, $8\ell mn$, is the product of 8 and integer ℓmn . So, the product is a multiple of 8.

14. Sample answer: If a figure is a rectangle, then it has four sides $ABCD$ has four sides.

15. Equation

$$A = \frac{1}{2}bh$$

$$2A = bh$$

$$\frac{2A}{b} = h$$

$$h = \frac{2 \cdot 558}{36} = \frac{1116}{36} = 31$$

The height of the sign is 31 inches.

Explanation and Reason

Write the equation; Given

Multiply each side by 2; Multiplication
Property of Equality

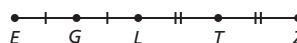
Divide each side by b . Division Property
of Equality

16. Given: G is the midpoint of EL .

L is the midpoint of GT .

T is the midpoint of LZ .

Prove: $EG = TZ$

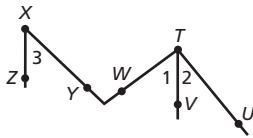


STATEMENTS	REASONS
1. G is the midpoint of EL . L is the midpoint of GT .	1. Given
2. $EG = GL$ $GL = LT$	2. Definition of midpoint
3. $EG = LT$	3. Transitive Property of Equality
4. T is the midpoint of LZ .	4. Given
5. $LT = TZ$	5. Definition of midpoint
6. $EG = TZ$	6. Transitive Property of Equality

Chapter 2

17. Given $\angle 2 \cong \angle 3$
 \overrightarrow{TV} bisects $\angle UTW$.

Prove $\angle 1 \cong \angle 3$



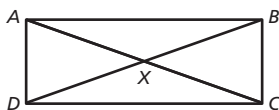
STATEMENTS	REASONS
1. $\angle 2 \cong \angle 3$	1. Given
2. \overrightarrow{TV} bisects $\angle UTW$	2. Given
3. $\angle 1 \cong \angle 2$	3. Definition of angle bisector
4. $\angle 1 \cong \angle 3$	4. Transitive Property of Angle Congruence (Thm. 2.2)

Chapter 2 Standards Assessment (pp. 120–121)

- Through points C and D , there exists exactly one line, \overleftrightarrow{CD} .
- \overleftrightarrow{AF} intersects \overleftrightarrow{CB} at point B .
- Through points E , B , and D , there exists exactly one plane S .
- Points A and F lie in plane T , so \overleftrightarrow{AF} also lies in plane T .
- Planes T and S intersect in \overleftrightarrow{CB} .

2. Given $\overline{AX} \cong \overline{DX}$, $\overline{XB} \cong \overline{XC}$

Prove $\overline{AC} \cong \overline{BD}$



STATEMENTS	REASONS
1. $\overline{AX} \cong \overline{DX}$	1. Given
2. $AX = DX$	2. Definition of congruent segments
3. $\overline{XB} \cong \overline{XC}$	3. Given
4. $XB = XC$	4. Definition of congruent segments
5. $AX + XC = AC$	5. Segment Addition Postulate (Post. 1.2)
6. $DX + XB = DB$	6. Segment Addition Postulate (Post. 1.2)
7. $AC = DX + XB$	7. Substitution Property of Equality
8. $AC = BD$	8. Substitution Property of Equality
9. $\overline{AC} \cong \overline{BD}$	9. Definition of congruent segments

3. a. biconditional statement

b. inverse

c. converse

d. contrapositive

4. $AB = 3 + 1 = 4$

$$BC = 6(4 - 3) = 6(1) = 6$$

$$CD = 4 \cdot 3 - 6 = 12 - 6 = 6$$

$$DE = 2(5 \cdot 3 - 7) - 8$$

$$= 2(15 - 7) - 8 = 2 \cdot 8 - 8 = 16 - 8 = 8$$

$$EF = 3(5 - 3) + 2 = 3(2) + 2 = 6 + 2 = 8$$

$$AB + BC + CD = 4 + 6 + 6 = 16$$

$$DE + EF = 8 + 8 = 16$$

Segment BD is bisected by line ℓ . Segment DF is bisected by line n . Segment AF is bisected by line m .

5. a. $\angle 3 \cong \angle 6$ by the Vertical Angles Congruence Theorem (Thm. 2.6).

b. $m\angle 4 \cong m\angle 7$ by the Vertical Angles Congruence Theorem (Thm. 2.6).

c. $m\angle FHE \neq m\angle AHG$

d. $m\angle AHG + m\angle GHE = 180^\circ$ by the Linear Pair Postulate (Post. 2.8).

6. a. $AB = \sqrt{(-1 - (-6))^2 + (6 - 1)^2}$
 $= \sqrt{(-1 + 6)^2 + (6 - 1)^2}$
 $= \sqrt{(5)^2 + (5)^2}$
 $= \sqrt{25 + 25} = \sqrt{50} \approx 7.07$

- b. $CD = \sqrt{(5 - (-5))^2 + (8 - 8)^2}$
 $= \sqrt{(5 + 5)^2 + 0^2}$
 $= \sqrt{(10)^2} = \sqrt{100} = 10$

- c. $EF = \sqrt{(4 - 2)^2 + (-2 - 7)^2}$
 $= \sqrt{2^2 + (-9)^2}$
 $= \sqrt{4 + 81} = \sqrt{85} \approx 9.22$

- d. $GH = \sqrt{(7 - 7)^2 + (-1 - 3)^2}$
 $= \sqrt{0^2 + (-4)^2}$
 $= \sqrt{16} = 4$

- e. $JK = \sqrt{(1 - (-4))^2 + (-5 - (-2))^2}$
 $= \sqrt{(1 + 4)^2 + (-5 + 2)^2}$
 $= \sqrt{5^2 + (-3)^2}$
 $= \sqrt{25 + 9} = \sqrt{34} \approx 5.83$

- f. $LM = \sqrt{(7 - 3)^2 + (-5 - (-8))^2}$
 $= \sqrt{(4)^2 + (-5 + 8)^2}$
 $= \sqrt{4^2 + (-3)^2}$
 $= \sqrt{16 + 9} = \sqrt{25} = 5$

The line segments in order from longest to shortest are \overline{CD} , \overline{EF} , \overline{AB} , \overline{JK} , \overline{LM} , and \overline{GH} .

Chapter 2

7. $\angle PSK$ and $\angle NSR$ are vertical angles. So, by the Vertical Angles Congruence Theorem (Thm. 2.6), $\angle PSK \cong \angle NSR$. $\angle MRL$ and $\angle QRS$ are vertical angles. So, by the Vertical Angles Congruence Theorem (Thm. 2.6), $\angle MRL \cong \angle QRS$. Because $\angle MRL \cong \angle NSR$, you can conclude by the Transitive Property of Angle Congruence (Thm. 2.2) that $\angle QRS \cong \angle NSR$. So, the other angles that are also congruent to $\angle NSR$ are $\angle PSK$ and $\angle QRS$.
8. no; In order to prove the Vertical Angles Congruence Theorem (Thm. 2.6), you must state that $\angle 1$ and $\angle 3$ each form a linear pair with $\angle 2$, and therefore each is supplementary to $\angle 2$ by the Linear Pair Postulate (Post. 2.8). You can then state that $\angle 1$ and $\angle 3$ are congruent by the Congruent Supplements Theorem (Thm. 2.4).