

SURROGATE MODEL CONSTRUCTED USING NEURAL NETWORKS FOR THE FORWARD AND INVERSE PROBLEMS

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Surrogate Model

Problem:

The mapping between input and output spaces is not achievable, but can be approximated.

Goal:

Analyzes the convergence of probability densities solving uncertainty quantification problems using surrogate model.

Example

Consider the following ODE:

$$\begin{aligned} y' &= -\lambda xy \\ y(0) &= 1 \end{aligned}$$

where $0.3 \leq x \leq 0.7$, λ is an uncertainty parameter.

Exact Solution:

$$y(x, \lambda) = e^{-\lambda x^2/2}$$

Forward UQ Problem

Assume $\lambda \sim \text{Beta}(2, 2)$,
QoI is $Q(\lambda) = y(0.5; \lambda)$.

For demonstration purpose, assume we do not know the exact solution describing the relation from x , λ to y .

Surrogate Model:

Neural Network: 400 training data for x is from $U[0.3, 0.7]$; 400 training data for λ is from $\text{Beta}(2, 2)$.

The neural network is trained under different numbers of epochs, 1000, 2000, 5000, 10000, to create surrogate models [1].

Convergence of surrogate model

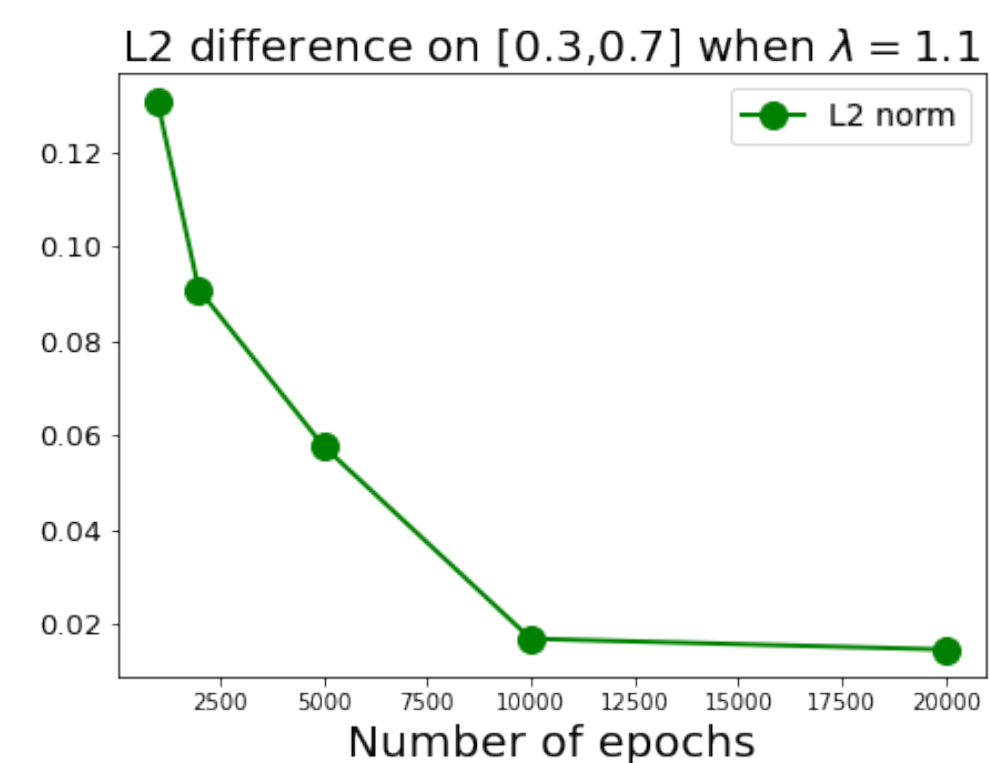


Fig. 1: L2 difference between true solution and surrogate model.

Since our quantity of interest is uncertain due to the randomness from λ , to verify the convergence of push-forward densities using approximate maps, we create the error plot by using the L^2 distance on $[0.3, 0.7]$ for a fixed λ value 1.1.

Conclusion:

Surrogate model converges in the L^2 sense.

Forward UQ Problem (Continued)

Convergence of push-forward density

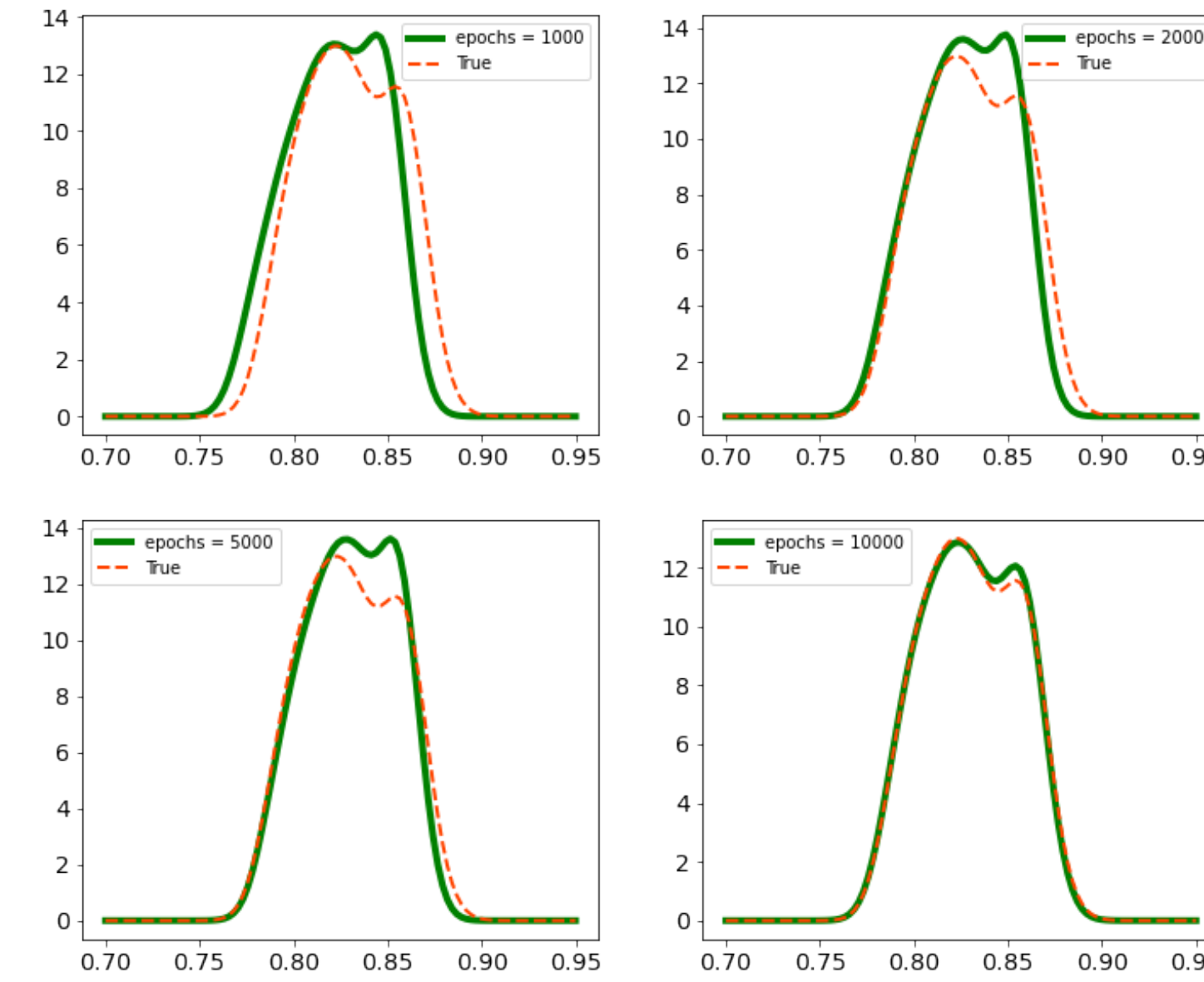


Fig. 2: Push-forward density.

Conclusion:

Surrogate model constructed using neural networks helps approximate true push-forward density.

Intro to Data Consistent Inversion

Data Consistent Inversion is a novel framework that uses push-forward and pullback measures to ensure solutions are consistent with the observed distribution of data.

Data Consistent Inversion Approach

Using the exact model [2]:

$$\pi_{\Lambda}^u(\lambda) = \pi_{\Lambda}^i(\lambda) \frac{\pi_{\mathcal{D}}(Q(\lambda))}{\pi_{\mathcal{D}}^Q(Q(\lambda))}$$

Using the surrogate model [3]:

$$\pi_{\Lambda}^{u,n}(\lambda) = \pi_{\Lambda}^i(\lambda) \frac{\pi_{\mathcal{D}}(Q_n(\lambda))}{\pi_{\mathcal{D}}^{Q_n}(Q_n(\lambda))}$$

Demonstration

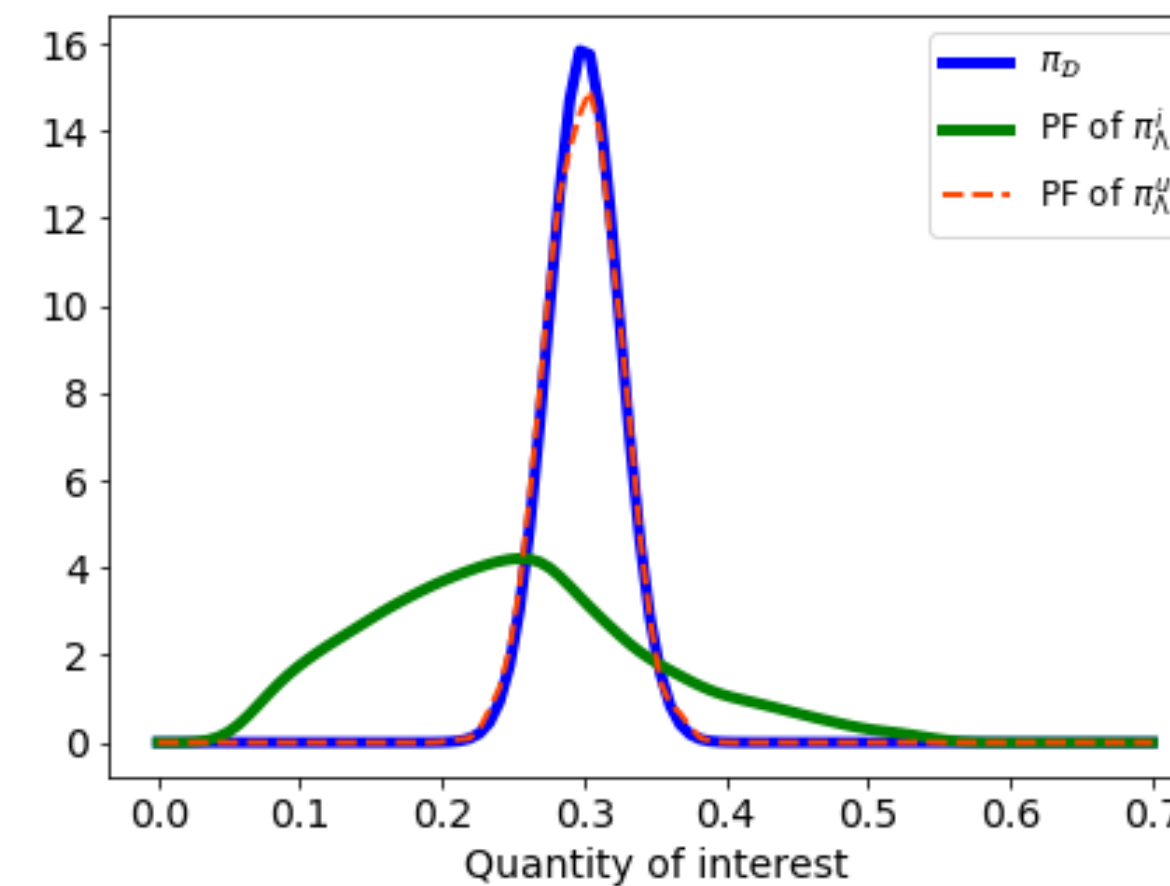


Fig. 3: Data Consistent Inversion Method.

Inverse UQ Problem

Convergence of updated density

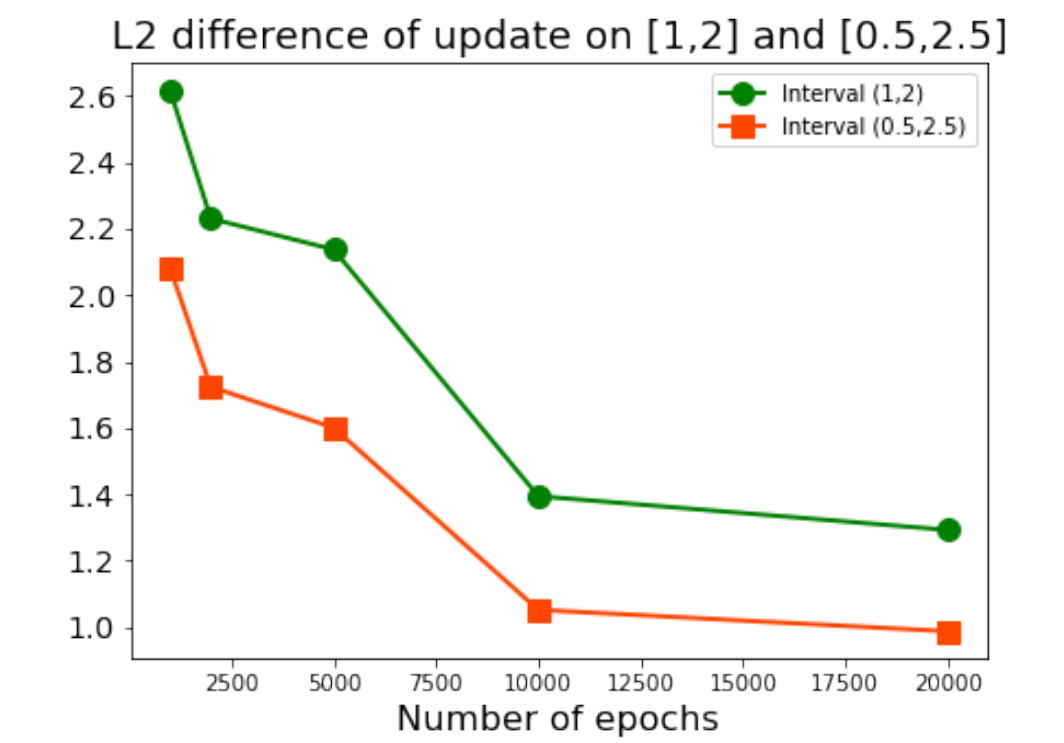


Fig. 4: L2 difference between true update and approximate update.

Conclusion:

Surrogate model constructed using neural networks helps approximate true updated density.

Notation

Notation	Description
$\lambda \in \Lambda$	Parameter Space
\mathcal{D}	Observable Space
Q	Exact model
Q_n	n -th surrogate model
π_{Λ}^i	Initial density guess of λ
π_{Λ}^u	Update pullback density
$\pi_{\Lambda}^{u,n}$	Approximate update pullback density using Q_n
π_{Λ}	Observed density
$\pi_{\mathcal{D}}^Q$	Push-forward density
$\pi_{\mathcal{D}}^{Q_n}$	Approximate push-forward density

References

References

- [1] Lagaris, I., Likas, A., and Fotiadis, D., *Artificial Neural Networks for Solving Ordinary and Partial Differential Equations*, *IEEE Transactions on Neural Networks*, 9(5):987-1000, 1998.
- [2] Butler, T., Jakeman, J., and Wildey, T., *Combining push-forward measures and bayes' rule to construct consistent solutions to stochastic inverse problems*, *SIAM Journal on Scientific Computing*, 40(2):A984-A1011, 2018.
- [3] Butler, T., Jakeman, J., and Wildey, T., *Convergence of probability densities using approximate models for forward and inverse problems in uncertainty quantification*, *SIAM Journal on Scientific Computing*, 40(5):A3523-A3548, 2018.