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INGENIERO:

Mg. BACA LOPEZ, MARCOS GREGORIO

ALUMNOS:

BRIONES VENTURA, ALFONSO ALEXANDER
COTRINA ESCOBAL, ANGEL RICHARD
ESCOBAR AMANCIO, ROYS
SAUCEDO HUAMAN, KEVIN
TASILLA UCEDA, NURI VICTORIA

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Optimal Capacity Expansion

In this section, we further illustrate the dynamic-programming approach by solving a problem of optimal capacity expansion in the electric power industry.

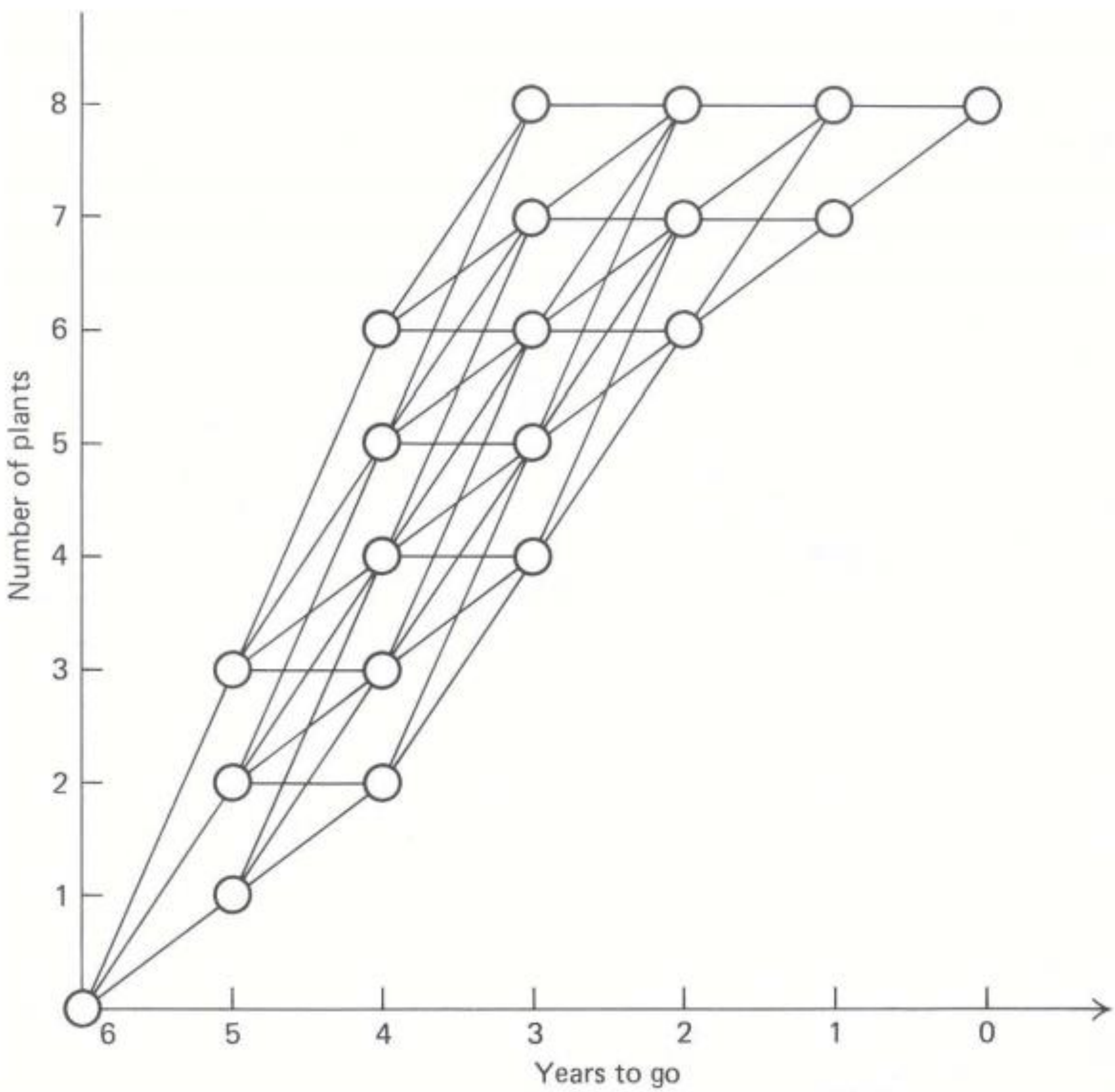
A regional electric power company is planning a large investment in nuclear power plants over the next few years. A total of eight nuclear power plants must be built over the next six years because of both increasing demand in the region and the energy crisis, which has forced the closing of certain of their antiquated fossil fuel plants. Suppose that, for a first approximation, we assume that demand for electric power in the region is known with certainty and that we must satisfy the minimum levels of cumulative demand indicated in Table 11.1. The demand here has been converted into equivalent numbers of nuclear power plants required by the end of each year. Due to the extremely adverse public reaction and subsequent difficulties with the public utilities commission, the power company has decided at least to meet this minimum-demand schedule.

The building of nuclear power plants takes approximately one year. In addition to a cost directly associated with the construction of a plant, there is a common cost of $1.5 million incurred when any plants are constructed in any year, independent of the number of plants constructed. This common cost results from contract preparation and certification of the impact statement for the Environmental Protection Agency. In any given year, at most three plants can be constructed. The cost of construction per plant is given in Table 11.1 for each year in the planning horizon. These costs are currently increasing due to the elimination of an investment tax credit designed to speed investment in nuclear power. However, new technology should be available by 1984, which will tend to bring the costs down, even given the elimination of the investment tax credit.

We can structure this problem as a dynamic program by defining the state of the system in terms of the cumulative capacity attained by the end of a particular year. Currently, we have no plants under construction, and by the end of each year in the planning horizon we must have completed a number of plants equal to or greater than the cumulative demand. Further, it is assumed that there is no need ever to construct more than eight plants. Figure 11.8 provides a graph depicting the allowable capacity (states) over time. Any node of this graph is completely described by the corresponding year number and level of cumulative capacity, say the node (n, p). Note that we have chosen to measure time in terms of years to go in the planning horizon. The cost of traversing any upward-sloping arc is the common cost of $1.5 million

Table 11.1 Demand and cost per plant ($ ×1000)

Year	*Cumulative demand* (in number of plants)	*Cost per plant* ($ × 1000)
1981	1	5400
1982	2	5600
1983	4	5800
1984	6	5700
1985	7	5500
1986	8	5200



plus the plant costs, which depend upon the year of construction and whether 1, 2, or 3 plants are completed. Measured in thousands of dollars, these costs are

$$1500 + c_n x_n,$$

where cn is the cost per plant in the year n and xn is the number of plants constructed. The cost for traversing any horizontal arc is zero, since these arcs correspond to a situation in which no plant is constructed in the current year.

Rather than simply developing the optimal-value function in equation form, as we have done previously, we will perform the identical calculations in tableau form to highlight the dynamic-programming methodology. To begin, we label the final state zero or, equivalently define the ‘‘stage-zero" optimal-value function to be zero for all possible states at stage zero. We will define a state as the cumulative total number of plants completed. Since the only permissible final state is to construct the entire cumulative demand of eight plants, we have S_0 = 8 and

$$v_0(8) = 0.$$

Now we can proceed recursively to determine the optimal-value function with one stage remaining. Since the demand data requires 7 plants by 1985, with one year to go the only permissible states are to have completed 7 or 8 plants. We can describe the situation by Tableau 1.

The dashes indicate that the particular combination of current state and decision results in a state that is not permissible. In this table there are no choices, since, if we have not already completed eight plants, we will construct one more to meet the demand. The cost of constructing the one additional plant is the $1500 common cost plus the $5200 cost per plant, for a total of $6700. (All costs are measured in thousands of dollars.) The column headed d^*_1 (S_1) gives the optimal decision function, which specifies the optimal number of plants to construct, given the current state of the system.

Now let us consider what action we should take with two years (stages) to go. Tableau 2 indicates the possible costs of each state:

		1981	1982	1983	1984	1985	1986
	year	1981	1982	1983	1984	1985	1986
Dn	Demand x year	1	2	3	4	5	6
CXPn	const x plant	5400	5600	5800	5700	5500	5200
CCXP	common costo x plant		1500				

$F(S_n, d_n) = d_n * CXP_n + CCXP + F_{n+1}(S_n + d_n)$

n	6
Sn	the cumulative total number of plants completed in level n
dn	Number de plants to construct in level n
Fn	Minimixe expenses in the construction of nuclear power plants

n=6	$F(S_6,d_6)=d_6*CXP_6 + CCXP + 0$			
S_6	$d_6=0$	$d_6=1$	$F^*_6(S_6)$	d_6^*
8	0		0	0
7		6700	6700	1

n=5	$F(S_5,d_5)=d_5*CXP_5 + CCXP + F_6(S_5 + d_5)$				
S_5	$d_5=0$	$d_5=1$	$d_5=2$	$F^*_5(S_5)$	d_5^*
8	0			0	0
7	6700	7000		6700	0
6		13700	12500	12500	2

If we have already completed eight plants with two years to go, then clearly we will not construct any more. If we have already completed seven plants with two years to go, then we can either construct the one plant we need this year or postpone its construction. Constructing the plant now costs \$1500 in common costs plus \$5500 in variable costs, and results in state 8 with one year to go. Since the cost of state 8 with one year to go is zero, the total cost over the last two years is \$7000. On the other hand, delaying construction costs zero this year and results in state 7 with one year to go. Since the cost of state 7 with one year to go is \$6700, the total cost over the last two years is \$6700. If we arrive at the point where we have two years to go and have completed seven plants, it pays to delay the production of the last plant needed. In a similar way, we can determine that the optimal decision when in state 6 with two years to go is to construct two plants during the next year.

To make sure that these ideas are firmly understood, we will determine the optimal-value function and optimal decision with three years to go. Consider Tableau 3 for three years to go:

n=4	$F(S_4,d_4)=d_4*CXP_4 + CCXP + F_5(S_4 + d_4)$					
S_4	$d_4=0$	$d_4=1$	$d_4=2$	$d_4=3$	$F^*_4(S_4)$	d_4^*
8	0				0	0
7	6700	7200			6700	0
6	12500	13900	12900		12500	0
5		19700	19600	18600	18600	3
4			25400	25300	25300	3

Now suppose that, with three years to go, we have completed five plants. We need to construct at least one plant this year in order to meet demand. In fact, we can construct either 1, 2, or 3 plants. If we construct one plant, it costs $1500 in common costs plus $5700 in plant costs, and results in state 6 with two years to go. Since the minimum cost following the optimal policy for the remaining two years is then $12,500, our total cost for three years would be $19,700. If we construct two plants, it costs the $1500 in common costs plus $11,400 in plant costs and results in state 7 with two years to go. Since the minimum cost following the optimal policy for the remaining two years is then $6700, our total cost for three years would be $19,600. Finally, if we construct three plants, it costs the $1500 in common costs plus $17,100 in plant costs and results in state 8 with two years to go. Since the minimum cost following the optimal policy for the remaining

n=3	$F(S_3,d_3)=d_3*CXP_3 + CCXP + F_4(S_3 + d_3)$					
S_3	$d_3=0$	$d_3=1$	$d_3=2$	$d_3=3$	$F*_3(S_3)$	d_3*
6	12500	14000	13100		12500	0
5	18600	19800	19800	18900	18600	0
4	25300	25900	25600	25600	25300	0
3		32600	31700	31400	31400	3
2			38400	37500	37500	3

n=2	$F(S_2,d_2)=d_2*CXP_2 + CCXP + F_3(S_2 + d_2)$					
S_2	$d_2=0$	$d_2=1$	$d_2=2$	$d_2=3$	$F*_3(S_2)$	d_2*
3	31400	32400	31300	30800	30800	3
2	37500	38500	38000	36900	36900	3
1		44600	44100	43600	43600	3

n=1	$F(S_1,d_1)=d_1*CXP_1 + CCXP + F_1(S_1 + d_1)$					
S_1	$d_1=0$	$d_1=1$	$d_1=2$	$d_1=3$	$F*_2(S_1)$	d_1*
0		50500	49200	48500	48500	3

two years is then zero, our total cost for three years would be $18,600. Hence, the optimal decision, having completed five plants (being in state 5) with three years (stages) to go, is to construct three plants this year. The remaining tableaus for the entire dynamic-programming solution are determined in a similar manner.

Since we start the construction process with no plants (i.e., in state 0) with six years (stages) to go, we can proceed to determine the optimal sequence of decisions by considering the tableaus in the reverse order. With six years to go it is optimal to construct three plants, resulting in state 3 with five years to go. It is then optimal to construct three plants, resulting in state 6 with four years to go, and so forth. The optimal policy is then shown in the tabulation below:

Year	Nº Construct
6	3
5	3
4	0
3	0
2	2
1	0

Hence, from Tableau 6, the total cost of the policy is $48.8 million.

Distributing Medical Teams to Countries

The WORLD HEALTH COUNCIL is devoted to improving health care in the underdeveloped countries of the world. It now has five medical teams available to allocate among three such countries to improve their medical care, health education, and training programs. Therefore, the council needs to determine how many teams (if any) to allocate to each of these countries to maximize the total effectiveness of the five teams. The teams must be kept intact, so the number allocated to each country must be an integer.

The measure of performance being used is additional person-years of life. (For a particular country, this measure equals the increased life expectancy in years times the country's population.) Table 11.1 gives the estimated additional person-years of life (in multiples of 1,000) for each country for each possible allocation of medical teams.

Which allocation maximizes the measure of performance?

TABLE 11.1 Data for the World Health Council problem

Medical Teams	Thousands of Additional Person-Years of Life		
	Country		
	1	2	3
0	0	0	0
1	45	20	50
2	70	45	70
3	90	75	80
4	105	110	100
5	120	150	130

Formulation

This problem requires making three interrelated decisions, namely, how many medical teams to allocate to each of the three countries. Therefore, even though there is no fixed sequence, these three countries can be considered as the three stages in a dynamic programming formulation. The decision variables X_n (n 1, 2, 3) are the number of teams to allocate to stage (country) n.

The identification of the states may not be readily apparent. To determine the states, we ask questions such as the following. What is it that changes from one stage to the next? Given that the decisions have been made at the previous stages, how can the status of the situation at the current stage be described? What information about the current state of affairs is necessary to determine the optimal policy hereafter? On these bases, an appropriate choice for the "state of the system" is

S_n number of medical teams still available for allocation to remaining countries (n, . . ., 3).

Thus, at stage 1 (country 1), where all three countries remain under consideration for allocations, s1 5. However, at stage 2 or 3 (country 2 or 3), S_n is just 5 minus the number of teams allocated at preceding stages, so that the sequence of states is

S_1=5 S_2=5-X_1 S_3=S2-X_2

With the dynamic programming procedure of solving backward stage by stage, when we are solving at stage 2 or 3, we shall not yet have solved for the allocations at the preceding stages. Therefore, we shall consider every possible state we could be in at stage 2 or 3, namely, S_n 0, 1, 2, 3, 4, or 5.

Figure 11.4 shows the states to be considered at each stage. The links (line segments) show the possible transitions in states from one stage to the next from making a feasible allocation of medical teams to the country involved. The numbers shown next to the links are the corresponding contributions to the measure of performance, where these numbers come from Table 11.1. From the perspective of this figure, the overall problem is to find the path from the initial state 5 (beginning stage 1) to the final state 0 (after stage 3) that maximizes the sum of the numbers along the path.

To state the overall problem mathematically, let pi(xi) be the measure of performance from allocating xi medical teams to country i, as given in Table 11.1. Thus, the objective is to choose x1, x2, x3 so as to

$$Maximize\ \sum_{i=0}^{3} P_i(X_i),\ \text{ subject to } \sum_{i=1}^{3} X_i = 5$$

And

Xi are nonnegative integers.

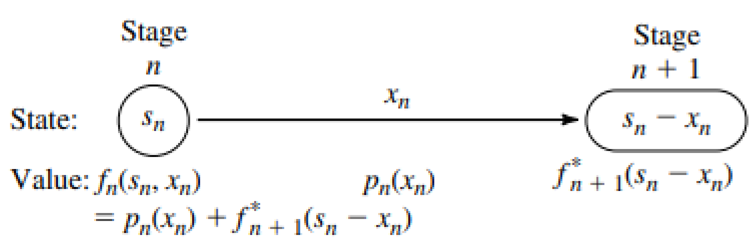
Using another notation, we have:

$$\boldsymbol{Fn(Sn, Xn) = Pn(Xn) + MAX \sum_{i=n+1}^{3} Pi(Xi)}$$

Where the maximum is taken over $\mathbf{X_{n+1}, \ldots, X_3}$ such that $\sum_{i=n}^{3} \boldsymbol{Xi = Sn}$ and the Xi are nonnegative integers, for n = 1,2,3. In addition $\boldsymbol{F_n^*(Sn) = MAX\ F_n\ (Sn, Xn)\ ;}$ *Xn = 0,1,…….,Sn.*

Therefore,

$$\boldsymbol{F_n^*(Sn, Xn) = P_n\ (Xn) + F_{n+1}^*(Sn - Xn)}$$



Consequently, the recursive relationship relating functions $F_1^*, F_2^*\ and\ F_3^*$ for this problem is

$$F_n^*(Sn) = \max\{P_n\ (Xn) + F_{n+1}^*(Sn - Xn)\},\ \text{for X} = 1;\quad \text{Xn} = 0, 1, \ldots, \text{Sn}$$

For the last stage (n=3)

$$F_3^*(S_3) = \max P_3 \ (X_3). \ X3 = 0,1,\ldots,S_3$$

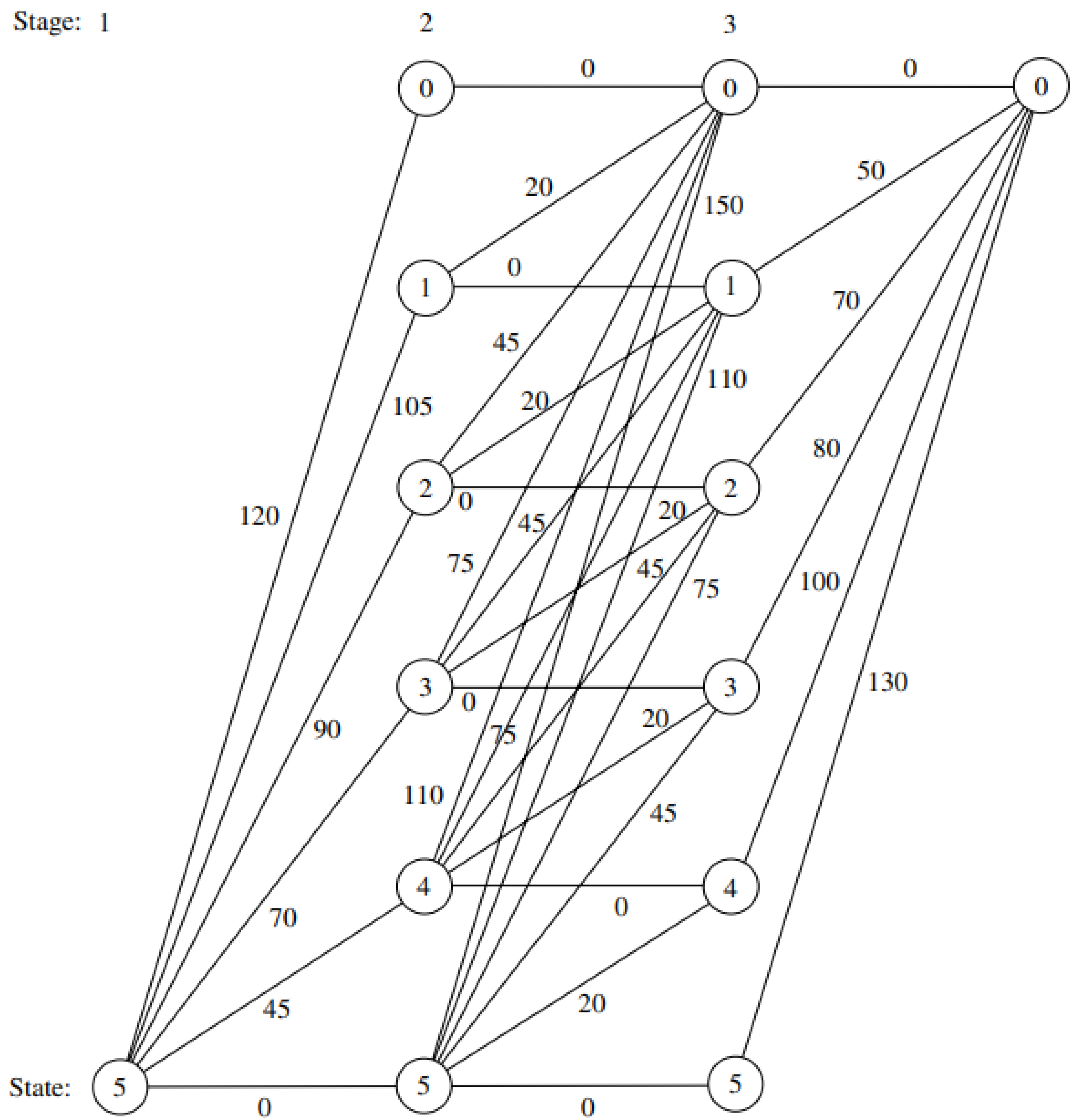


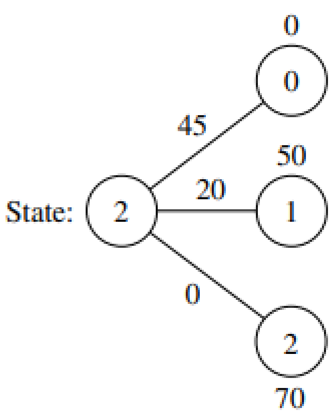
FIGURE 1: Graphical display of the World Health Council problem, showing the possible state at each stage, the passible transition in states, and the corresponding contributions to the measure of performance.

Solution Procedure.

Beginning with the last stage (n = 3), we note that the values of $P_3(X_3)$ are given in the last column of Table 11.1 and these values keep increasing as we move down the column. Therefore, with S_3 medical teams still available for allocation to country 3, the maximum of $P_3(X_3)$ is automatically achieved by allocating all S_3 teams; so $X_3^* = S_3$ and $X_3^*(S_3) = P_3(S_3)$, as shown in the following table.

n=3	F2(S2, X2)=P2(X2)		
S4	X_3=0	$F^*_3(S_3)$	X_3*
0	0	0	0
1	50	50	1
2	70	70	2
3	80	80	3
4	100	100	4
5	130	130	5

We now move backward to start from the next-to-last stage (n = 2). Here, finding X_2* requires calculating and comparing f2(s2, x2) for the alternative values of x2, namely, x2 = 0, 1, . . ., s2. To illustrate, we depict this situation when s2=2 graphically:



This diagram corresponds to Fig. 11.5 except that all three possible states at stage 3 are shown. Thus, if x2 = 0, the resulting state at stage 3 will be s2 - x2 = 2 – 0 = 2, whereas x2 = 1 leads to state 1 and x2 = 2 leads to state 0. The corresponding values of p2(x2) from the country 2 column of Table 11.1 are shown along the links, and the values of f 3 *(s2 - x2) from the n 3 table are given next to the stage 3 nodes. The required calculations for this case of s2 = 2 are summarized below.

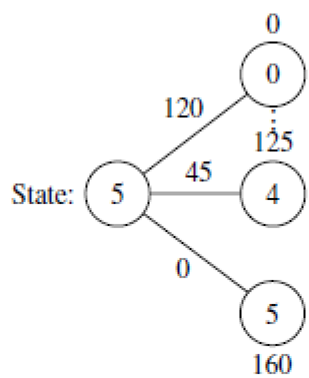
Formula: $f_2(2, x_2) = p_2(x_2) + f_3^*(2 - x_2)$.
$p_2(x_2)$ is given in the country 2 column of Table 11.1.
$f_3^*(2 - x_2)$ is given in the $n = 3$ table (bottom of preceding page).

$x_2 = 0$: $f_2(2, 0) = p_2(0) + f_3^*(2) = 0 + 70 = 70.$
$x_2 = 1$: $f_2(2, 1) = p_2(1) + f_3^*(1) = 20 + 50 = 70.$
$x_2 = 2$: $f_2(2, 2) = p_2(2) + f_3^*(0) = 45 + 0 = 45.$

Because the objective is maximization, x2 * = 0 or 1 with f $_2$*(2) = 70. Proceeding in a similar way with the other possible values of s2 (try it) yields the following table.

n=2	F2(S2, X2)=P2(X2) + F*$_3$(S2 - X2)							
S4	X_2=0	X_2=1	X_2=2	X_2=3	X_2=4	X_2=5	F*$_2$(S_2)	X_2*
0	0						0	0
1	50	20					50	1
2	70	70	45				70	2
3	80	90	95	75			95	3
4	100	100	115	125	110		125	4
5	130	120	125	145	160	150	160	5

We now are ready to move backward to solve the original problem where we are starting from stage 1 ($n = 1$). In this case, the only state to be considered is the starting state of S_1 = 5, as depicted below.



Since allocating x1 medical teams to country 1 leads to a state of 5 - x1 at stage 2, a choice of x1 = 0 leads to the bottom node on the right, x = 1 leads to the next node up, and so forth up to the top node with x1 = 5. The corresponding p1(x1) values from Table 11.1 are shown next to the links. The numbers next to the nodes are obtained from the f 2*(s2) column of the n = 2 table. As with n = 2, the calculation needed for each alternative value of the decision variable involves adding the corresponding link value and node value, as summarized below.

Formula: $f_1(5, x_1) = p_1(x_1) + f_2^*(5 - x_1)$.
$p_1(x_1)$ is given in the country 1 column of Table 11.1.
$f_2^*(5 - x_1)$ is given in the $n = 2$ table.

$x_1 = 0$: $f_1(5, 0) = p_1(0) + f_2^*(5) = 0 + 160 = 160.$
$x_1 = 1$: $f_1(5, 1) = p_1(1) + f_2^*(4) = 45 + 125 = 170.$
$\vdots$
$x_1 = 5$: $f_1(5, 5) = p_1(5) + f_2^*(0) = 120 + 0 = 120.$

The similar calculations for $x_1 = 2, 3, 4$ (try it) verify that $x_1^* = 1$ with $f_1^*(5) = 170$, as shown in the following table.

n=1	$F1(s1,x1) = P1(X1) + F^*_2(S1 - X1)$							
S4	$X_2=0$	$X_2=1$	$X_2=2$	$X_2=3$	$X_2=4$	$X_2=5$	$F^*_1(S_1)$	X_1^*
5	160	170	165	160	155	120	170	1

Thus, the optimal solution has $x_1^* = 1$, which makes $s_2 = 5 - 1 = 4$, so $x_2^* = 3$, which makes $s_3 = 4 - 3 = 1$, so $x_3^* = 1$. Since $f_1^*(5) = 170$, this (1, 3, 1) allocation of medical teams to the three countries will yield an estimated total of 170, 000 additional person years of life, which is at least 5,000 more than for any other allocation. These results of the dynamic programming analysis also are summarized in Fig. 11.6.